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Research Article

APPLICATION OF HOMOTOPY PERTURBATION METHOD TO HEAT TRANSFER IN NANOFLUIDS

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ABSTRACT
This paper investigates heat transfer in a nanofluid using the Homotopy Perturbation method. Similarity transformation variables and a stream function are used to transform the partial differential equations governing the fluid flow into ordinary differential equations. He's Homotopy perturbation method is then used to solve the resulting dimensionless equations. It was discovered that an increase in the fraction number, magnetic parameter or Grashof number led to a corresponding increase in the rate of heat transfer regardless of the nanoparticles in the fluid. These results are in agreement with those found in existing literature.

Keywords: *Nanofluid, Similarity Transformation, Homotopy Perturbation Method, Heat Transfer*

1. INTRODUCTION

Nanofluids are formed when nanoparticles such as oxide ceramics, nitrides, graphites etc are mixed with base fluids like water, polymer solutions and lubricants. Nanofluids possess heat transfer properties that can help address the energy demand and emission issues of the present world. They can be used for industrial cooling purposes and this could result in great energy savings and significantly reduce emission.

These properties and their potential benefits have made nanofluids an important area of research. Wang *et al*. (1999) in their research into the thermal conductivity of nanoparticle-fluid mixture provided suggestions to improve the conductivity of nanofluids. Do and Jang (2010) analyzed the effects of thermophysical properties Aluminum Oxide on the heat transfer of a flat micro heat pipe. Uddin *et al.* (2012) discovered that an increase in Newtonian heating enhanced the heat and mass transfer rate of a nanofluid. The dimensionless governing equations were solved using the Runge-Kutta-Fehlberg method coupled with shooting technique.

Hamad (2011) studied free convective flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field. Oahimire *et al.* (2016) extended the work of Hamad by incorporating a thermal radiation parameter into the flow equations and solved them using the Runge-kutta Fehlberg method together with shooting technique. To the best of our knowledge, HPM has not been applied to solve the flow equations of Oahimire *et al.* (2016).

In this present study, HPM is applied to study the effects of volume fraction, magnetic field and buoyancy force on the rate of heat transfer of natural convection flow of a nanofluid over linearly stretching sheet in the presence of magnetic field. The Homotopy Perturbation method (HPM) is a technique based on the concept of the Homotopy from topology that was introduced by Dr. Ji-Huan He in 1998. It is a simple but effective method for solving non-linear partial differential equations. The basic idea is illustrated below. Consider a non-linear differential equation

$$[\boldsymbol{A(u)-f(r)}]=\boldsymbol{0} \tag{1}$$

Where f(r) is a known analytic function and A(u) is a nonlinear differential operator which can be separated into 2 parts, one linear part, L and a non-linear part, N. i.e.

$$\boldsymbol{A(u)=L(u)+N(u)} \tag{2}$$

We construct a homotopy as follows

$$\boldsymbol{H(u,p)=(1-p)[L(u_0)-L(v_0)]+[A(u)-f(r)]=0} \tag{3}$$

where $\boldsymbol{p}$ is an embedding parameter that lies in the unit interval $[\boldsymbol{0},\boldsymbol{1}]$ and $\boldsymbol{v_0}$ is an initial guess of the solution to the equation. Setting the value of our small parameter to 0, we have the initial guess while setting its value to 1 gives us the original equation. This process of changing p from 1 to 0 is called a deformation.

$$\boldsymbol{H(u,0)=L(u)-L(v_0)=0} \tag{4}$$

$$\boldsymbol{H(u,1)=A(u)-f(r)=0} \tag{5}$$

According to the HPM, we assume our solution is in form of a series

$$\boldsymbol{u=u_0+pu_1+p^2u_2+\cdots}$$

We solve for $\boldsymbol{u_n}$ iteratively and setting $\boldsymbol{p=1}$, we have

$$\boldsymbol{u=u_0+u_1+u_2}$$

This is the approximate solution to Eq. (1). We have the freedom of choice for the operator L. However great care must be taken to choose an operator which simplifies the solution process as the solution depends entirely on the choice of the L and the initial guess $\boldsymbol{v_0}$. Ayati and Biazar (2015) showed that in most cases, the HPM solution is convergent.

NOMENCLATURE
a = Constant
g = Acceleration due to gravity
k = Thermal Conductivity
P_r = Prandtl Number
T = Fluid Temperature
T_w = Surface Temperature
T_∞ = Free Stream Temperature
u,v = Velocity Components
x,y = Cartesian Coordinates
f(x) = Dimensionless Stream Function
G_r = Grashof Number
q_r = Heat Flux Radiation
B_o = Magnetic Field of Constant Strength
R = Radiation Parameter
K_s = Rosseland Mean Absorption Coefficient
K = Thermal Conductivity Coefficient
GREEK SYMBOLS
β = Thermal Expansion Coefficient
μ = Dynamic Coefficient of Viscosity
θ (η) = Dimensionless Temperature
η = Similarity Variable
ρ = Fluid Density
ψ = Stream Function
σ^* = Stefan-Bottzman Constant

2. MATHEMATICAL FORMULATION

Consider a steady, two-dimensional flow of an incompressible viscous nanofluid past a linearly semi-infinite stretching sheet. Magnetic field of strength B_0 is applied perpendicularly to the sheet. The nanofluid under consideration is water-based and contains Copper, Silver, Aluminum oxide and Titanium Dioxide. The nanofluid is assumed to be in thermal equilibrium. Following Oahimire *et al.* (2016), the governing equations are:

$$\boldsymbol{\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0} \tag{6}$$

$$\boldsymbol{\rho_{nf}\left[u'\frac{\partial u'}{\partial x'}+v'\frac{\partial v'}{\partial y'}\right]=\mu_{nf}\frac{\partial^2 u'}{\partial y'^2}-\sigma B_0 u'+g\beta_t(T'-T'_\infty)} \tag{7}$$

$$\boldsymbol{(\rho c_p)_{nf}\left[u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}\right]=K_{nf}\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q_r}{\partial y'}} \tag{8}$$

The boundary conditions of the equations are

$$u' = u'_w(x') = ax', v' = 0, T' = T'_w \; at \; y' = 0$$

$$u' \to 0, T' \to T'_\infty, y' \to \infty \qquad (9)$$

Where q_r is the radiative heat flux, T' is the temperature of the fluid, $x' and y'$ are the coordinates along and perpendicular to the sheet while $u' and\; v'$ are the velocity components in the $x' and\; y'$ directions respectively and a is a constant. The effective density (ρ_{nf}), effective dynamic viscosity (μ_{nf}), heat capacitance $(\rho C_p)_{nf}$ and the effective thermal conductivity (k_{nf}) of the nanofluid, in that order, are given as

$$\rho_{nf} = (1-A)\rho_f + A\rho_s$$
$$\mu_{nf} = \frac{\mu_f}{(1-A)^{2.5}}$$
$$(\rho C_p)_{nf} = (1-A)(\rho C_p)_f + A(\rho C_p)_s$$
$$k_{nf} = k_f\left(\frac{k_s + 2k_f - 2A(k_f - k_s)}{k_s + 2k_f + 2A(k_f - k_s)}\right) \qquad (10)$$

Where A is the solid volume fraction ($A \neq 1$), μ_f is the dynamic viscosity of the base fluid, while ρ_f and ρ_s are the densities of the pure fluid and the nanoparticle respectively. The constants k_f and k_s are the thermal conductivities of the base fluid and the nanoparticle respectively. Using Rosseland approximation given by $q_r = \frac{4\sigma'}{3k'}\frac{\partial T'^4}{\partial y'}$ with Taylor's series expansion and differentiation, Eq. (8) becomes

$$(\rho c_p)_{nf}\left[u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'}\right] = K_{nf}\frac{\partial^2 T'}{\partial y'^2} + \frac{16T_\infty^3\sigma'}{3k'}\frac{\partial^2 T'}{\partial y'^2} \qquad (11)$$

The following variables are used for transformation

$$u = \frac{u'}{\sqrt{av_f}} v = \frac{v'}{\sqrt{av_f}}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, x = \frac{x'}{\sqrt{\frac{v_f}{a}}} . y = \frac{y'}{\sqrt{\frac{v_f}{a}}} \qquad (12)$$

Eq. (12) transforms Eq. (6), (7) and (11) into the following

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (13)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{(1-A)\rho_f + \rho_s}\left[\frac{1}{(1-A)^{2.5}}\frac{\partial^2 u}{\partial y^2} - Mu + Gr\theta\right] \qquad (14)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{1}{(1-A)(\rho c_p)_f + A(\rho c_p)_s}\left[\frac{k_{nf}}{k_f} + R_d\right]\frac{\partial^2 \theta}{\partial y^2} \qquad (15)$$

Where $M = \frac{\sigma\beta_0}{a}$ is the magnetic field parameter, $p_r = \frac{v_f}{\mu_{nf}}$ is the Prandtl number, $R_d = \frac{16\sigma T_\infty'^3}{3K^*\mu_{nf}}$ is the radiation parameter, $G_r = \frac{g\beta_t(T'_w - T'_\infty)x}{a}$ is the Grashof number and the corresponding boundary conditions are

$$u = x, v = 0, \theta = 1 \; at \; y = 0$$

(16)

$$u \to 0, \theta \to 0 \; as \; y \to \infty$$

To satisfy Eq. (7) we apply the stream function $u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}, \eta = y, \psi = xf(\eta). \theta = \theta(\eta)$ our equations reduce to

$$f''' + (1-A)^{2.5}[ff'' - (f')^2]\left[(1-A)(\rho_f + \rho_s) - (Mf' + Gr\theta)\right] = 0 \qquad (17)$$

$$\frac{1}{Pr}\frac{1}{(1-A)(\rho C_p)_s}\left[\frac{k_{nf}}{k_f} + R_d\right]\theta''(\eta) + f(\eta)\theta'(\eta) = 0 \qquad (18)$$

$$f(0) = 0, f'(0) = 1, \; \theta(0) = 0 \; at \; \eta = 0$$
$$f \to 0, \theta \to 0 \; as \; \eta \to 0 \qquad (19)$$

3. METHOD OF SOLUTION

The transformed non-linear equations can be written as

$$f''' + \alpha\left(\beta\left(ff'' - (f')^2\right)\right) - kf' + Gr\theta = 0$$
$$f\theta' + H\theta'' = 0$$

Where $\alpha = (1-A)^{2.5}, k = M, \beta = (1-A)\rho_f + \rho_s, H = \frac{1}{Pr}\frac{1}{(1-A)(\rho c_p)_f + A(\rho c_p)_s}\left(\frac{k_{nf}}{k_f} + R\right)$

We construct the homotopy of the transformed equations as follows

$$(1-p)(f''' - f'''u_0) + p\left(f''' + \alpha\left(\beta\left(ff'' - (f')^2\right)\right) - kf' + Gr\theta\right) = 0$$

And

$$(1-p)(\theta'' - \theta'' t_0) + p(f\theta' + H\theta'') = 0$$

We assume f and θ in the following form

$$f = f_0 + pf_1 + p^2 f_2$$
$$\theta = \theta_0 + p\theta_1 + p^2\theta_2$$

and group the terms according to the order: For order zero, we have

$$\frac{d^3 f_0}{d\eta^3} - \frac{d^3 v_0}{d\eta^3} = 0$$
$$\frac{d^2\theta_0}{d\eta^2} - \frac{d^2 t_0}{d\eta^2} = 0$$

With boundary conditions

$$f_0(0) = 0, f_0'(0) = 0, f_0'(\infty) = 1, \theta_0(0) = 1, \theta_0(\infty) = 0$$

For order one, we have

$$\frac{d^3 f_1}{d\eta^3} - \frac{d^3 v_0}{d\eta^3} + \alpha\left(Gr\theta_0 - k\frac{df_0}{d\eta} + \beta\left(f_0\frac{d^2 f_0}{d\eta^2} - \left(\frac{df_0}{d\eta}\right)^2\right)\right) = 0$$

$$\frac{d^2\theta_1}{d\eta^2}+f_0\frac{d\theta_0}{d\eta}+(H-1)\frac{d^2\theta_0}{d\eta^2}+\frac{d^2t_0}{d\eta^2}=0$$

With boundary conditions

$$f_1(0)=0, f_1{}'(0)=0, f_1{}'(\infty)=0, \theta_1(0)=1, \theta_1(\infty)=0$$

For order two, we have

$$\frac{d^3f_2}{d\eta^3}+\alpha\left(Gr\theta-k\frac{df_1}{d\eta}+\beta\left(f_1\frac{d^2f_1}{d\eta^2}-\left(\frac{df_1}{d\eta}\right)^2\right)\right)=0$$

$$\frac{d^2\theta_2}{d\eta^2}+f_1\frac{d\theta_1}{d\eta}+(H-1)\frac{d^2\theta_1}{d\eta^2}=0$$

With boundary conditions

$$f_2(0)=0, f_2{}'(0)=0, f_2{}'(\infty)=0, \theta_2(0)=1, \theta_2(\infty)=0$$

Solving the equations with their respective boundary conditions, we have the following solutions

$$f_0=\frac{\eta^2}{12}$$
$$f_1=\frac{c_1\eta^2}{2}-\alpha Gr\frac{\eta^3}{6}+\alpha(Gr+k)\frac{\eta^4}{144}+\alpha\beta\frac{\eta^5}{4320}$$
$$f_2=c_3\frac{\eta^2}{2}+g_8\frac{\eta^4}{24}+g_9\frac{\eta^5}{60}+g_{10}\frac{\eta^6}{120}+g_{11}\frac{\eta^7}{210}+g_{12}\frac{\eta^8}{336}+g_{13}\frac{\eta^9}{504}+g_{14}\frac{\eta^{10}}{720}+g_{15}\frac{\eta^{11}}{990}$$
$$\theta_0=1-\frac{\eta}{6}$$
$$\theta_1=\frac{\eta^4}{864}-\frac{\eta}{4}$$
$$\theta_2=c_2\eta+\frac{g_1}{4}\eta^4-\frac{g_2}{5}\eta^5+\frac{g_3}{6}\eta^6+\frac{g_4}{7}\eta^7+\frac{g_5}{8}\eta^8-\frac{g_6}{9}\eta^9-\frac{g_7}{10}\eta^{10}$$

Where

$$c_1=2\alpha Gr-\alpha k-\frac{\alpha\beta}{4}$$
$$c_2=-\frac{216}{4}g_1+\frac{1296}{5}g_2-1296g_3-\frac{46656}{7}g_4-\frac{279936}{8}g_5+\frac{1679616}{9}g_6+\frac{10077696}{10}g_7$$
$$c_3=-(6g_8+18g_9+\frac{1296}{20}g_{10}+\frac{7776}{30}g_{11}+\frac{46656}{42}g_{12}+\frac{279936}{56}g_{13}+\frac{1679616}{72}g_{14}+\frac{10077696}{90}g_{15})$$
$$g_1=\frac{c_1}{4}-\frac{(H-1)}{72}$$
$$g_2=\frac{\alpha Gr}{24}$$
$$g_3=\frac{\alpha(Gr+k)}{576}$$
$$g_4=\frac{\alpha\beta}{17280}-\frac{c_1}{432}$$
$$g_5=\frac{\alpha Gr}{1296}$$
$$g_6=\frac{\alpha(Gr+k)}{144}$$
$$g_7=\frac{\alpha\beta}{933120}$$
$$g_8=c_1\alpha k+\frac{\alpha Gr}{4}$$
$$g_9=c_1\alpha\beta-\frac{\alpha^2 GrK}{2}-\frac{\alpha\beta\, C_1{}^2}{2}$$
$$g_{10}=\frac{\alpha^2K(Gr+K)}{36}-\frac{\alpha^2C_1\beta\, Gr}{3}$$
$$g_{11}=\frac{\alpha^2K\beta-\alpha Gr}{864}+\alpha\beta\left(\frac{c_1\alpha(Gr+k)+12\alpha^2Gr^2}{144}\right)$$
$$g_{12}=\alpha\beta\left(\frac{4C_1\alpha\beta}{2160}-\frac{\alpha^2Gr(Gr+K)}{144}\right)$$
$$g_{13}=\alpha\beta\left(\frac{\alpha^2(Gr+K)^2}{5184}+\frac{\alpha^2Gr\beta}{270}\right)$$
$$g_{14}=\alpha\beta\left(\frac{\alpha^2\beta(Gr+K)}{77760}\right)$$
$$g_{15}=\frac{\alpha^3\beta^3}{746496}$$

We can calculate the value of the constant coefficients using the boundary conditions. Following standard practice, we replace the boundary condition $\eta=\infty$ with $\eta=6$.

4. DISCUSSION AND RESULTS

Numerical evaluation of the solutions was performed with mathematical software "Matlab" and the results are presented in tabular form. This was done to illustrate effect of some governing parameters involved. The rate of heat transfer for different value of volume fraction (A), magnetic parameter(M) and Grashof number(Gr) are obtained as shown in table 2. We notice that an increase in the values of A, M and Gr led to an increase in the values of the heat transfer coefficient $-\theta(0)$.

The therrmophysical properties of nanoparticles used in the evaluation as given by Hamad (2011) are shown below.

Table 1. Thermo physical properties of water and nanoparticles. Hamad (2011)

Compound	ρ (kg/m³)	C_p (J/kgK)	k (W/mK)
Pure water	997.1	4179	0.613
Copper (Cu)	8933	385	401
Alumina (Al_2O_3)	3970	765	40
Silver (Ag)	10500	235	429
Titanium Oxide (TiO_2)	4250	686.2	8.9538

These values where used together with the solutions to obtain the following table showing the effects of varying different flow parameters on the heat transfer of the nanofluid.

Table 2. Effects of variation of A, Gr and M on the rate of heat transfer

			$-\theta(0)$	$-\theta(0)$	$-\theta(0)$	$-\theta(0)$
A	M	Gr	Cu	Al_2o_3	Ag	T_iO_2
0.2	0.5	0.2	-6.3599×10^3	-2.8533×10^3	-7.4671×10^3	-3.0511×10^3
0.3	0.5	0.2	-4.5049×10^3	-1.9933×10^3	-5.2976×10^3	-2.1350×10^3
0.4	0.5	0.2	-3.0301×10^3	-1.3219×10^3	-3.5695×10^3	-1.4183×10^3
0.5	0.5	0.2	-1.8996×10^3	-8.1678×10^3	-2.2415×10^3	-8.7778×10^3
0.6	0.5	0.2	-1.0754×10^3	-4.5554×10^3	-1.2711×10^3	-4.9051×10^3
0.1	0.6	0.2	-8.5324×10^3	-3.8252×10^3	-1.0019×10^4	-4.0908×10^3
0.1	0.7	0.2	-8.4332×10^3	-3.7259×10^3	-9.9194×10^3	-3.9915×10^3
0.1	0.8	0.2	-8.3339×10^3	-3.6266×10^3	-9.8201×10^3	-3.8922×10^3
0.1	0.9	0.2	-8.2346×10^3	-3.5274×10^3	-9.7209×10^3	-3.7929×10^3
0.1	1.0	0.2	-8.1353×10^3	-3.4281×10^3	-9.6216×10^3	-3.6937×10^3
0.1	0.5	0.3	-8.5332×10^3	-3.8260×10^3	-1.0019×10^4	-4.0916×10^3
0.1	0.5	0.4	-8.4348×10^3	-3.7275×10^3	-9.9210×10^3	-3.9931×10^3
0.1	0.5	0.5	-8.3363×10^3	-3.6290×10^3	-9.8225×10^3	-3.8946×10^3
0.1	0.5	0.6	-8.2378×10^3	-3.5306×10^3	-9.7241×10^3	-3.7961×10^3
0.1	0.5	0.7	-8.1393×10^3	-3.4321×10^3	-9.6256×10^3	-3.6977×10^3

5. CONCLUSION

In this work, the dimensionless equations of the governing equations were solved with HPM and the effects of M, Gr and A on heat transfer are presented in Table 2. And we notice that increasing the values of M, Gr and A leads to a corresponding increase in the rate of heat transfer in all of the nanoparticles considered. This is in agreement with the solutions gotten using the Runge-Kutta-Fehlberg method by Oahimire *et al.* (2016). This shows that He's Homotopy Pertubation method is an effective method for solving similar flow problems.

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