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Research Article

Comparison of the Performance of Type-1 and Interval Type-2 Fuzzy PI Controllers for Liquid Level Control in a Coupled Tank System

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Abstract

In this study, liquid level control in a coupled tank system is realized with PI controllers whose parameters are adapted using fuzzy logic approximation methods. Type-1 fuzzy PI control, interval type-2 fuzzy PI control and classical PI+Feedforward control methods are applied to the tank system in real time for different reference liquid levels. The performances of the controllers are compared in terms of system response parameters such as rise time, settling time and overshoot percentage. In addition, the performance evaluation of the controllers over a certain period of time is also compared by calculating integral-based concepts that express error performance metrics such as integral square error (ISE), integral time square error (ITSE), integral absolute error (IAE) and integral time absolute error (ITAE). The obtained results show that the type-2 fuzzy PI controller shows the best performance, followed by type-1 fuzzy PI control and classical PI+FF controller, respectively.

Keywords: PI controller, fuzzy PI controller, couple tank system, liquid level control

İkili Tank Sisteminde Sıvı Seviye Kontrolü için Tip 1 ve Aralık Tip 2 Bulanık PI Kontrolörlerin Performanslarının Karşılaştırılması

Öz

Bu çalışmada, ikili tank sisteminde sıvı seviye kontrolü bulanık mantık yaklaşım yöntemleri kullanılarak parametreleri uyarlanan PI kontrolcüler ile gerçekleştirilmiştir. Tip-1 bulanık PI kontrol, aralık tip-2 bulanık PI kontrol ve klasik PI+ileri besleme kontrol yöntemleri farklı referans sıvı sevileri için gerçek zamanlı olarak ikili tank sistemine uygulanmıştır. Kontrolörlerin performansları yükselme zamanı, yerleşme zamanı ve aşım yüzdesi gibi sistem cevabı parametreleri açısından karşılaştırılmıştır. Ayrıca hatanın karelerinin integrali (ISE), zaman ağırlıklı hatanın karelerinin integrali (ITSE), mutlak hatanın integrali (IAE) ve zaman ağırlıklı mutlak hatanın integrali (ITAE) gibi hata performans ölçütlerini ifade eden integral tabanlı kavramların hesaplanmasıyla kontrolcülerin belirli bir süre boyunca gösterdikleri performans değerlendirmesi de sayısal değerlerle ortaya koyularak karşılaştırılmıştır. Elde edilen sonuçlar aralık tip-2 bulanık PI kontrolcünün en iyi performansı gösterdiği ve ardından sırasıyla tip-1 bulanık PI kontrol ve klasik PI+FF kontrolcünün performanslarının iyi olduğu görülmüştür.

Anahtar Kelimeler: PI kontrolör, bulanık PI kontrolör, ikili tank sistemi, sıvı seviye kontrolü

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1. Introduction

Efficient control of liquid level systems is a common problem in industrial applications. In liquid level control, it is expected that the liquid level follows the desired level without error, responds quickly and accurately to changes in system dynamics, is resistant to uncertainties and disturbances in the system, and can adapt to changing conditions in the system. Effective and successful control of liquid level systems is important for the reliability and efficiency of industrial processes. Various control methods have been used to control liquid level systems. Some of these studies are as follows [1,5].

PI and PID control techniques are widely used in liquid level control because of their simple structure and ease of parameter setting [6-8]. These controllers have fixed values of the parameters and are not able to adapt themselves to changing conditions. Therefore, they cannot provide the desired results when used to control nonlinear systems. In nonlinear systems, nonlinear control methods can respond quickly to changes in system dynamics and are more effective against parameter uncertainties [9]. In liquid level systems, liquid level control has been carried out using various nonlinear control methods such as adaptive, sliding mode and fractional order controllers with high accuracy against uncertainties [10-12].

Fuzzy logic control method, which is a nonlinear control method and used in liquid level control, has a structure that expresses uncertain situations better than classical logic systems. Fuzzy logic evaluates many situations at the same time and performs system control by making an appropriate inference [13-16]. In order to extend the limit of the fuzzy logic control method to express uncertainties, the boundaries of the membership functions are extended by a kind of spreading operation and the resulting interval type-2 fuzzy logic control method is used in the control of liquid level systems [17,18]. Fuzzy PI control method, which is a combination of classical PI and fuzzy logic control methods, is used as a more robust and adaptive control method in nonlinear systems and in the presence of uncertainty [19-23]. In this control method, the error and error change rate are converted into fuzzy values with the help of membership functions and K_p and K_i values are calculated from these values according to the defined rule tables.

In this study, PI+FF controller and PI controller whose parameters are adapted with type-1 and interval type-2 fuzzy logic control methods are tested in the liquid level control of tank-1 and tank-2 for different reference signals and the controller performances are compared. The results obtained from the real-time studies show that the PI controller whose parameters are adapted using interval type-2 fuzzy logic is better than the PI controller adapted using type-1 fuzzy logic, and the type-1 fuzzy PI is better than the PI+FF controller.

2. Material and Methods

2.1. Material

The coupled tank liquid level system consists of a pump, a water basin, two water tanks and two pressure sensors to measure the liquid level. There are two configurations of this model of tank system. These are the tank 1 and tank 2 configurations. In this system, the pump draws water from the water basin and pumps the water to tank 1, which is in the upper position. The water from tank 1 flows to tank 2 through the drain hole at the bottom of tank 1 and the water in tank 2 flows to the main water basin through the drain hole at the bottom of tank 2.

2.1.1. Tank 1 Model

In the tank 1 model shown in Figure 1, the level of the liquid in tank 1 is controlled. The system input is the pump voltage and the system output is the water level in tank 1. The volumetric inflow rate (f_{i1}) of water entering the tank and the outflow rate (f_{o1}) of water leaving through the hole in the bottom of the tank are used to determine the mathematical model of the tank 1. Equations (1) and (2) give the volumetric flow rates of water entering and leaving the tank respectively [24].

$$f_{i1} = K_p V_p \tag{1}$$

$$f_{o1} = A_{o1}\, V_{o1} \tag{2}$$

In the equation, K_p is the pump flow constant ($\frac{cm^2}{s}\frac{1}{V}$), V_p is the actual voltage applied to the pump (volt), A_{o1} is the cross-sectional area of the liquid flow path (cm^2) and V_{o1} is the velocity of the water leaving tank 1 to tank 2 (cm/s).

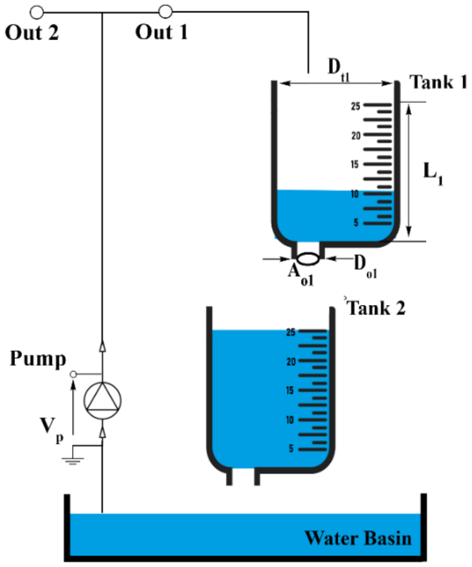

Figure 1. Tank 1 model

The outflow rate is obtained as follows using the Bernoulli equation.

$$V_{o1} = \sqrt{2gL_1} \tag{3}$$

The g in the equation is the gravitational constant. The cross-sectional area of the liquid outlet path for tank 1 can be calculated by using the following equation.

$$A_{o1} = \frac{1}{4}\,\pi\, {D_{o1}}^2 \tag{4}$$

D_{o1} is the diameter of the liquid outlet path of tank 1. Substituting these expressions into equation (2) gives the liquid flow rate as follows.

$$f_{o1} = A_{o1}\sqrt{2gL_1} \tag{5}$$

The first-order differential equation for L_1 is obtained using the mass balance principle for tank 1 as follows.

$$A_{t1}\frac{dL_1}{dt} = f_{i1} - f_{o1} \tag{6}$$

A_{t1} is the cross-sectional area of the tank and by replacing equations (1) and (5) in equation (6) and rearranging equation (6), the tank 1 system model can be expressed as follows.

$$\frac{dL_1}{dt} = \frac{K_p V_p - A_{o1}\sqrt{2gL_1}}{A_{t1}} \tag{7}$$

2.1.2. Tank 2 Model

In the tank 2 model shown in Figure 2, the level control of tank 2 is realized. The system input is the pump voltage and the system output is the water level of tank 2. The water level in tank 2 depends on the water level in tank 1.

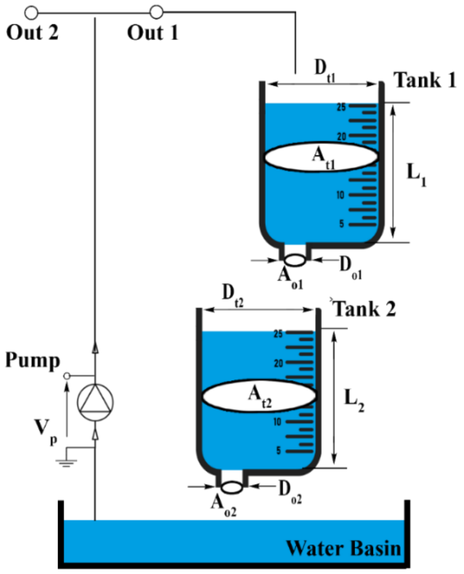

Figure 2. Tank 2 model

The liquid outflow rate for tank 2 is given below.

$$f_{o2} = A_{o2} V_{o2} \tag{8}$$

The liquid outflow rate for tank 2 is obtained using the Bernoulli equation as follows.

$$V_{o2} = \sqrt{2gL_2} \tag{9}$$

The cross-sectional area of the liquid outlet path of tank 2 can be calculated as given in equation (10).

$$A_{o2} = \frac{1}{4}\,\pi\, {D_{o2}}^2 \tag{10}$$

If the cross-sectional area of the liquid outlet path is rewritten in equation (8), the liquid outflow rate of tank 2 is obtained as follows.

$$f_{o2} = A_{o2}\sqrt{2gL_2} \tag{11}$$

The liquid flow rate entering tank 2 is obtained as follows.

$$f_{i2} = A_{o1}\sqrt{2gL_1} \tag{12}$$

If the mass balance principle is written for tank 2, the first order differential equation for L_2 is obtained as follows.

$$A_{t2}\frac{dL_2}{dt} = f_{i2} - f_{o2} \tag{13}$$

If the equations (11) and (12) are replaced in equation (13) and equation (13) is rearranged, the system model for tank 2 is obtained as follows.

$$\frac{dL_2}{dt} = \frac{A_{o1}\sqrt{2gL_1} - A_{o2}\sqrt{2gL_2}}{A_{t2}} \tag{14}$$

2.2. Methods

Liquid level control in the coupled tank system is performed for two different cases. Firstly, the liquid level control of tank 1, whose control block diagram is given in Figure 3, is realized. As can be seen from the figure, the controller determines the pump voltage required for the liquid in tank 1 to follow the reference liquid level.

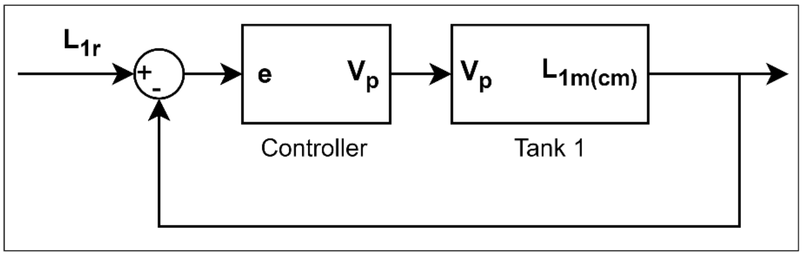

Figure 3. Tank 1 control block diagram

In the second case, the liquid level in tank 2 is controlled. In the control block diagram given in Figure 4, the first controller determines the reference liquid level for tank 1 by using the liquid level error of tank 2. The second controller generates the pump voltage that should be applied to the pump by using the liquid level error of tank 1.

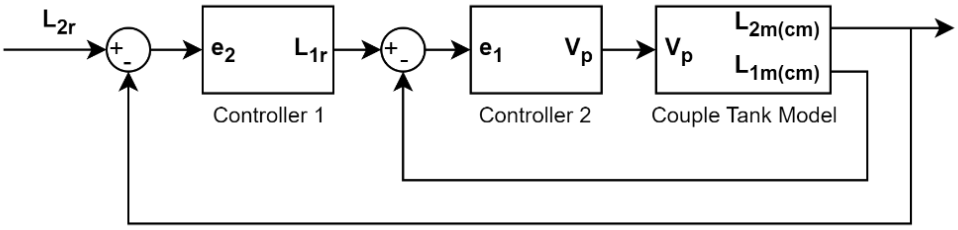

Figure 4. Tank 2 control block diagram

2.2.1. Fuzzy Controller Architecture

The fuzzy controller consists of fuzzification, inference engine and defuzzification, as shown in Figure 5. The crisp values at the fuzzy logic controller input are sent to the inference engine by calculating the fuzzy input sets with the relevant membership functions. The fuzzy input sets received by the inference engine are inferred using the defined rule bases and the fuzzy output sets obtained are sent to the defuzzification interface. The fuzzy output sets received at the fuzzification interface are converted into crisp output values by output membership functions and the selected defuzzification method [25].

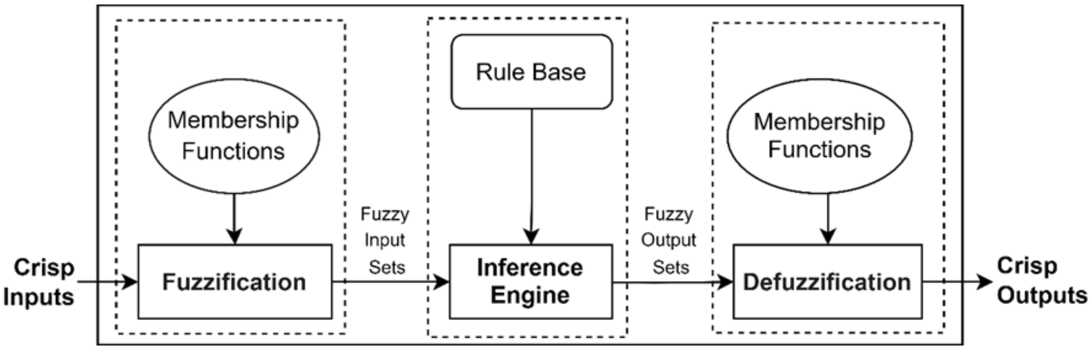

Figure 5. Fuzzy logic model

Different membership functions are used in the fuzzification and defuzzification interfaces. Commonly used membership functions are triangular, trapezoidal, gaussian, generalized bell, π-shaped and s-shaped [26]. In this study, a triangular membership function is used. In the triangular membership function, a defines the start point of the curve, b defines the peak point of the curve and c defines the end point of the curve. The mathematical expression of the triangular membership function is given in equation (15).

$$u(x) = \begin{cases} 0, \ x \le a \\ \frac{x-a}{b-a}, \ a \le x \le b \\ \frac{c-x}{c-b}, \ b \le x \le c \\ 0, \ x \ge c \end{cases} \tag{15}$$

2.2.1.1. Type-1 Fuzzy PI Controller

The parameters of the classical PI controller are fixed values. When there are sudden disturbances or parameter uncertainties in the system, the performance of the PI controller decreases or deteriorates. Therefore, adapting the PI controller parameters depending on the error value will increase the performance of the controller [27]. In this study, the adaptation of the parameters of the PI controller is performed using fuzzy logic. The PI controller can be expressed mathematically as follows.

$$PI = K_p e(t) + K_i \int_0^t e(t)d(t) \tag{16}$$

In the equation, K_p and K_i are the proportional and integral gains, respectively, and e(t) is the error value. Equation (17) gives the PI controller whose parameters are adapted using fuzzy logic [27].

$$Fuzzy\_PI = X_p G_p e(t) + X_i G_i \int_0^t e(t)d(t) \tag{17}$$

In the equation, X_p and X_i are the outputs of the fuzzy logic controller and, G_p and G_i are the learning rate constants for K_p and K_i. Figure 6 shows the block diagram of the type-1 fuzzy PI controller whose parameters are adapted with fuzzy logic for liquid level control of tank 1.

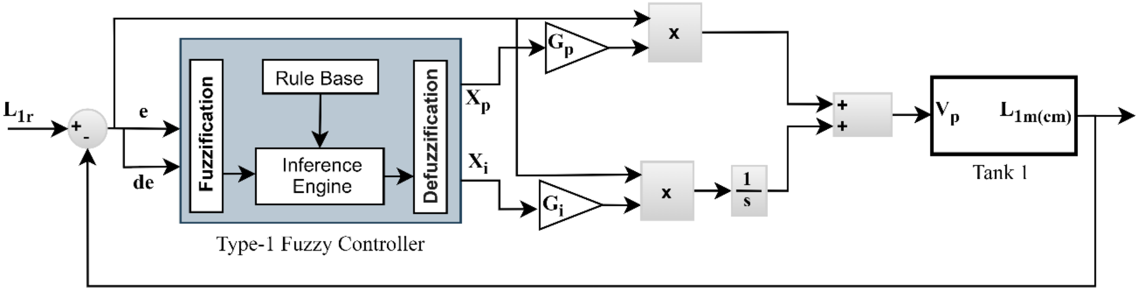

Figure 6. Block diagram of the type-1 fuzzy PI controller for tank 1

The membership functions presented in Table 1 and the fuzzy rules given in Table 2 are used in type-1 fuzzy control for the liquid level control of tank 1. On the other hand, in the liquid level control of the tank 2, there are two different controllers as shown in Figure 4. The structure of each controller is as shown in Figure 6 and the membership functions and rule tables given in Tables 1 and 2 are used in both controllers. However, the learning constants G_p and G_i of the controllers are different values.

Table 1. Input and output membership function.

Input Membership Function		**Output Membership Function**	
e, de		X_p , X_i	
Linguistic Terms	**Range**	**Linguistic Terms**	**Range**
Negative Big	[-1, -0.4]	Very Small	[0, 0.15]
Negative Small	[-0.7, -0.1]	Medium Small	[0.03, 0.3]
Zero	[-0.4, 0.4]	Small	[0.15, 0.5]
Positive Small	[0.1, 0.7]	Medium	[0.3, 0.7]
Positive Big	[0.4, 1]	Big	[0.5, 0.85]
		Medium Big	[0.7, 0.97]
		Positive Big	[0.85, 1]

Table 2*:* IF-THEN rule table

e/de	NB	NS	Z	PS	PB
NB	VS	MS	MS	S	M
NS	MS	MS	S	M	B
Z	MS	S	M	B	MB
PS	S	M	B	MB	MB
PB	M	B	MB	MB	VB

2.2.1.2. Interval Type-2 Fuzzy Controller Architecture

The interval type-2 fuzzy controller, whose block diagram is given in Figure 7, basically consists of fuzzification, inference engine, type reduction and defuzzification interfaces. The fuzzy sets of the crisp values at the controller input are calculated using membership functions and sent to the inference engine. The fuzzy input sets to the inference engine are inferred with the defined rule bases and sent to the type reduction interface. In the type reduction interface, the interval type-2 fuzzy output sets are converted into type-1 fuzzy output sets and sent to the defuzzification interface. The type-1 fuzzy output sets are converted into crisp output values by output membership functions and the selected defuzzification method [16,28].

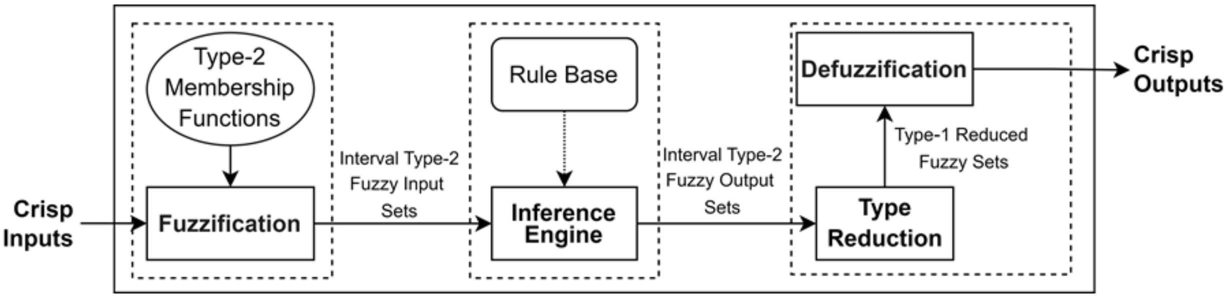

Figure 7. Interval type-2 fuzzy logic model

Interval type-2 fuzzy logic is a generalized form of type-1 fuzzy logic [29]. The set of fuzzy values $\tilde{X}$ in interval type-2, shown in Figure 8, is characterized by the membership function $\mu_{\tilde{X}}(x,u)$ as in equation (19).

$$\tilde{X} = \{((x,u),\ \mu_{\tilde{X}}(x,u))\ |\forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{18}$$

$$\tilde{X} = \int_{x\in D_{\tilde{X}}} \int_{u\in J_x\subseteq[0,1]} \frac{\mu_{\tilde{X}}(x,u)}{(x,u)} \tag{19}$$

In the equation, x is the primary variable, $D_{\tilde{X}}$ is the domain, $u \in [0,1]$ is the secondary variable. J_x denotes the domain and $J_x \subseteq [0,1]$ for each $x \in D_{\tilde{X}}$. J_x is also called the support of the secondary membership function and the amplitude of $\mu_{\tilde{X}}(x,u)$, called the secondary degree of $\tilde{X}$, is equal to 1 for $\forall x \in D_{\tilde{X}}$ and $\forall u \in J_x \subseteq [0,1]$ and is expressed as follows [30,31].

$$\tilde{X} = \int_{x\in D_{\tilde{X}}} \int_{u\in J_x\subseteq[0,1]} \frac{1}{(x,u)} \tag{20}$$

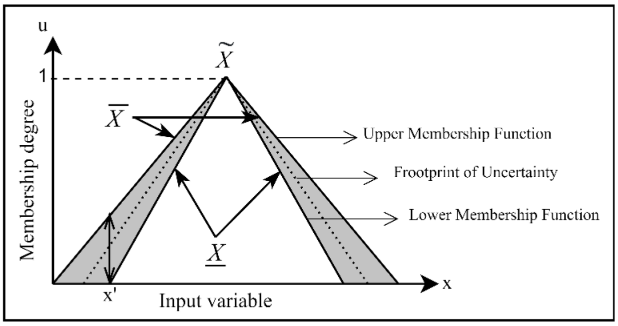

Figure 8. Interval type-2 membership function.

In the interval type-2 membership function given in Figure 8, it is seen that there is an interval in the interval type-2 membership function, unlike type-1 membership functions where the membership degree for each $\boldsymbol{x}$ is a number. The interval type-2 membership function is bounded by two type-1 membership functions, the upper membership function ($\overline{\boldsymbol{X}}$) and the lower membership function ($\underline{\boldsymbol{X}}$). The area between $\overline{\boldsymbol{X}}$ and $\underline{\boldsymbol{X}}$ is the footprint of uncertainty [31].

Interval type-2 fuzzy logic IF-THEN rules, inference engine, type reduction, and defuzzification:

The IF-THEN rule for a single-input single-output system is expressed as follows.

R^n: $IF\ x_1 = \widetilde{X}_1^n\ and\ \ldots.and\ x_l = \widetilde{X}_l^n, THEN\ y = Y^n \quad n = 1,2,\ldots,N.$

Where $\widetilde{X}_i^n\ i = 1,\ldots,l$ are interval type-2 fuzzy values, $Y^n = [\underline{y}^n, \overline{y}^n]$ is an interval. For an input vector $x' = (x'_1, x'_2, \ldots, x'_l\ )$ g in Figure 8, the output is calculated as in the following steps [31].

1. For each input vector x'_i, the membership interval X_i^n is calculated in each rule. These membership intervals are expressed as,

$$\left[\mu_{\underline{X}_i^n}(x'_i),\ \mu_{\overline{X}_i^n}(x'_i)\right],\ \ i = 1,2,\ldots l, \text{ and } n = 1,2,\ldots N. \tag{21}$$

   In the equation, $\mu_{\underline{X}_i^n}(x'_i)$ represents the lower membership function and $\mu_{\overline{X}_i^n}(x'_i)$ represents the upper membership function.

2. The firing interval of nth rule F^n is calculated. This is expressed as the product of the membership degrees of all inputs:

$$F^n(x') = \left[\underline{f}^n, \overline{f}^n\right] \equiv \left[\mu_{\underline{X}_1^n}(x'_1) \times \ldots \times \mu_{\underline{X}_l^n}(x'_l),\ \mu_{\overline{X}_1^n}(x'_1) \times \ldots \times\ \mu_{\overline{X}_l^n}(x'_l)\right],$$

$$n = 1,\ldots,N \tag{22}$$

3. Type reduction steps are applied to combine rule trigger intervals $F^n(x')$ and rule results Y^n. There are many types of reduction methods. The most commonly used is the center-of-sets type reduction method [31].

$$Y_{cos} = \frac{\sum_{n=1}^{N} Y^n F^n}{\sum_{n=1}^{N} F^n} = \bigcup_{\substack{y^n \in Y^n \\ f^n \in F^n}} \frac{\sum_{n=1}^{N} y^n f^n}{\sum_{n=1}^{N} f^n} = [y_l, y_r] \tag{23}$$

$$y_l = \min_{k \in [1,N-1]} \frac{\sum_{n=1}^{k} \overline{f}^n \underline{y}^n + \sum_{n=k+1}^{N} \underline{f}^n \underline{y}^n}{\sum_{n=1}^{k} \overline{f}^n + \sum_{n=k+1}^{N} \underline{f}^n} \equiv \frac{\sum_{n=1}^{L} \overline{f}^n \underline{y}^n + \sum_{n=L+1}^{N} \underline{f}^n \underline{y}^n}{\sum_{n=1}^{L} \overline{f}^n + \sum_{n=L+1}^{N} \underline{f}^n} \tag{24}$$

$$y_r = \max_{k \in [1,N-1]} \frac{\sum_{n=1}^{k} \underline{f}^n \overline{y}^n + \sum_{n=k+1}^{N} \overline{f}^n \overline{y}^n}{\sum_{n=1}^{k} \underline{f}^n + \sum_{n=k+1}^{N} \overline{f}^n} \equiv \frac{\sum_{n=1}^{R} \underline{f}^n \overline{y}^n + \sum_{n=R+1}^{N} \overline{f}^n \overline{y}^n}{\sum_{n=1}^{R} \underline{f}^n + \sum_{n=R+1}^{N} \overline{f}^n} \tag{25}$$

In the equation, L and R are the switching points. y_l and y_r can be calculated using Karnik-Mendel algorithms as follows [32].

Table 3. Karnic-Mendel type reduction method [32].

Step	For computing y_l	For computing y_r
1.	Initialize $f^n = \frac{\underline{f}^n + \overline{f}^n}{2}$ and compute $y = \frac{\sum_{n=1}^{N} \underline{y}^n f^n}{\sum_{n=1}^{N} f^n}$	Initialize $f^n = \frac{\underline{f}^n + \overline{f}^n}{2}$ and compute $y = \frac{\sum_{n=1}^{N} \overline{y}^n f^n}{\sum_{n=1}^{N} f^n}$
2.	Find $l \in [1, N-1]$ s.t. $\underline{y}^l \leq y \leq \cdots \underline{y}^{l+1}$	Find $r \in [1, N-1]$ s.t. $\overline{y}^r \leq y \leq \overline{y}^{r+1}$
3.	Set $f^n = \begin{cases} \overline{f}^n, & n \leq l \\ \underline{f}^n, & n > l \end{cases}$ and compute $y' = \frac{\sum_{n=1}^{N} \underline{y}^n f^n}{\sum_{n=1}^{N} f^n}$	Set $f^n = \begin{cases} \underline{f}^n, & n \leq r \\ \overline{f}^n, & n > r \end{cases}$ and compute $y' = \frac{\sum_{n=1}^{N} \overline{y}_n f^n}{\sum_{n=1}^{N} f^n}$
4.	If $y' = y$, stop and set $y_l = y \; and \; L = l$; otherwise, set $y = y'$ and go to Step 2.	If $y' = y$, stop and set $y_r = y \; and \; R = r$; otherwise, set $y = y'$ and go to Step 2.

The main purpose of the Karnic-Mendel type reduction algorithm is to find the y_l and y_r. When calculating y_l, it uses upper membership degrees for $n \leq l$ and lower membership degrees for $n > l$. When calculating y_r, it uses lower membership degrees for $n \leq r$ and upper membership degrees for $n > r$ [32].

4. Finally, a precise output value is calculated using the range of fuzzy results obtained after type reduction. The defuzzied result is calculated as follows.

$$y = \frac{y_l + y_r}{2} \tag{26}$$

2.2.1.3. Interval Type-2 Fuzzy PI Controller

In the interval type-2 fuzzy logic control system with two inputs and two outputs shown in Figure 9, the input values are converted into type-2 fuzzy input sets using the type-2 input membership functions given in Table 4. Interval type-2 output sets are generated using the output membership functions in Table 5 and the rules in Table 2. Type-2 fuzzy values are reduced to type-1 values using the Karnik-Mendel type reduction method given in Table 3. The PI controller parameters K_p and K_i coefficients are adapted using the output values X_p and X_i parameters.

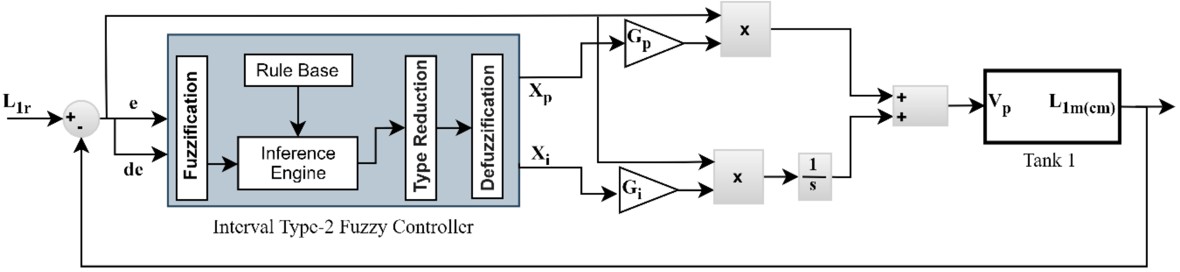

Figure 9. Block diagram of the interval type-2 fuzzy PI controller for tank 1.

Table 4. Input membership function.

Input Membership Function			
e, de			
Linguistic Terms	**Upper Parameters**	**Lower Scale**	**Low Lag**
Negative Big	[-1,-0.4]	0.8	[0.3 0.3]
Negative Small	[-0.7,-0.1]	0.8	[0.3 0.3]
Zero	[-0.4,0.4]	0.8	[0.3 0.3]
Positive Small	[0.1,0.7]	0.8	[0.3 0.3]
Positive Big	[0.4,1]	0.8	[0.3 0.3]

Table 5. Output membership function.

Output Membership Function			
X_p , X_i			
Linguistic Terms	**Upper Parameters**	**Lower Scale**	**Low Lag**
Very Small	[0,0.15]	0.8	[0.3 0.3]
Medium Small	[0.03,0.3]	0.8	[0.3 0.3]
Small	[0.15,0.5]	0.8	[0.3 0.3]
Medium	[0.3,0.7]	0.8	[0.3 0.3]
Big	[0.5,0.85]	0.8	[0.3 0.3]
Medium Big	[0.7,0.97]	0.8	[0.3 0.3]
Very Big	[0.85,1]	0.8	[0.3 0.3]

For tank 2, two different controllers are used as shown in Figure 4. The block structure of each controller is as shown in Figure 9 and the membership functions given in Tables 4 and 5 and the rules given in Table 2 are used in both controllers. However, the learning constants G_p and G_i of the controllers are different values.

3. Results and Discussion

In this study, the liquid level control in a coupled tank system is realised for two different configurations, tank 1 and tank 2 model. The proposed controllers are implemented on both configurations and their performances are tested for different reference liquid levels. The results obtained are compared in terms of system response parameters such as rise time, settling time and maximum overshoot and error performance metrics such as ISE, ITSE, ITAE and IAE. Firstly, the results for the tank 1 model are given in Figures 10-13. Then, the results for the tank 2 model are presented in Figures 14-15.

In order to observe the performance of the controllers in the single tank configuration, a step+sinusoidal reference liquid level was first selected. With the relevant reference, the response of the controllers against constant and slowly changing reference signals was measured. Then, a step+square reference liquid level was applied to observe the performance of the controllers against reference signals that change suddenly with time. The results of PI+FF, type-1 fuzzy PI and interval type-2 fuzzy PI controllers for the related references are given below.

Figure 10 shows that for the step part of the reference signal, the PI+FF controller reaches the reference more quickly than the type-1 fuzzy PI controller, but it overshoots more than the type-1 fuzzy PI controller and settles to the reference signal later. This can also be seen from the values for rise time, overshoot and settling time given in Table 6. It can then be seen that the controller maintains its performance and follows the sinusoidal reference signal with less error. This is also confirmed by the error performance metrics given in Table 7 and Figure 10 (b). On the other hand, Figure 10(c) shows that the type-1 fuzzy PI controller produces a more chattering control signal than PI+FF in order to follow the reference signal. The fact that the amplitude of the chattering component of the generated control signal is not high has prevented a possible negative effect on the system.

Figure 11 shows the results of interval type-2 fuzzy PI controller. Similar to the performance of the type-1 fuzzy PI controller, it is seen that the rise time of PI+FF is lower than the interval type-2 fuzzy PI controller in the tracking of the step part of the reference signal, but in terms of overshoot and settling time, the interval type-2 fuzzy PI controller has lower values. Similarly, it is seen that interval type-2 fuzzy PI controller is better in terms of tracking performance of sinusoidal reference. In addition, compared to the type-1 fuzzy PI controller, the interval type-2 fuzzy PI controller is better in terms of both system response performance and error criterion performance. On the other hand, it is observed that the control signal produced by both fuzzy controllers contains chattering, but the amplitude of the chattering produced by interval type-2 fuzzy PI controller is slightly larger than the type-1 fuzzy PI controller. This small change enables the interval type-2 fuzzy PI controller to follow the reference liquid level better.

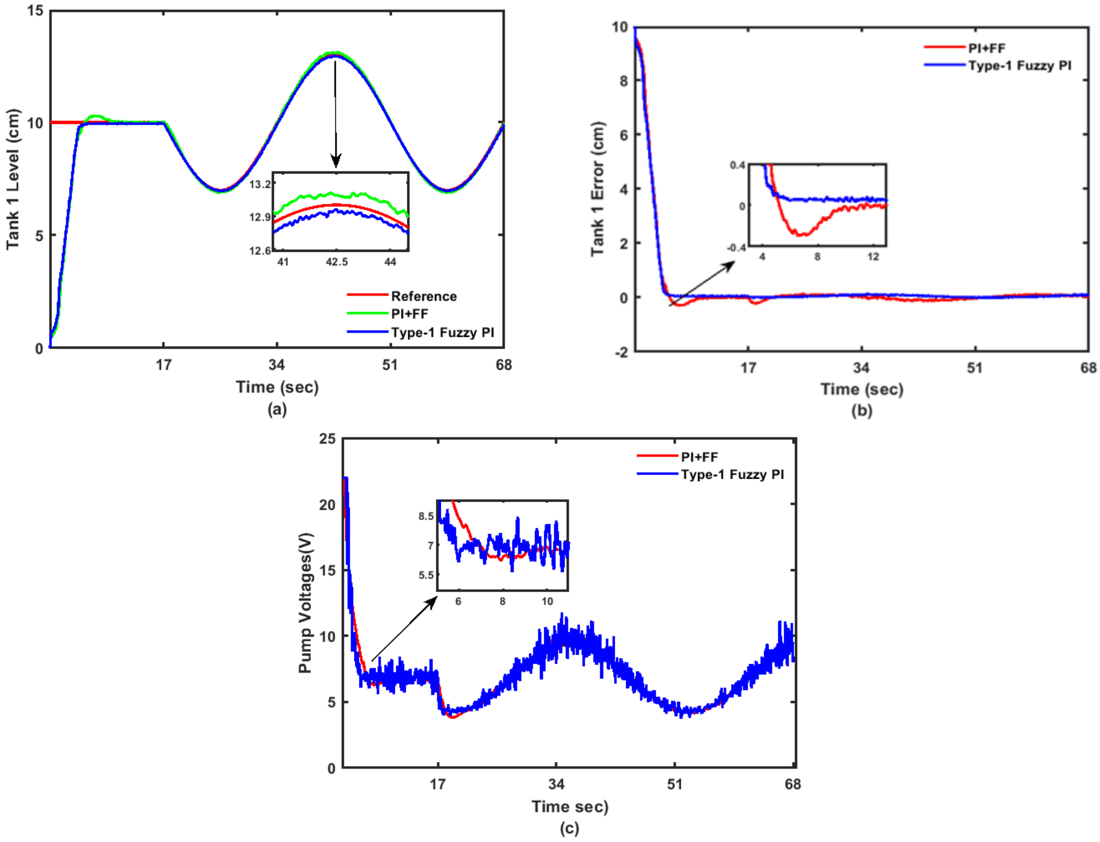

Figure 10. PI+FF and type-1 fuzzy PI controller performances under step + sinusoidal reference for tank 1.

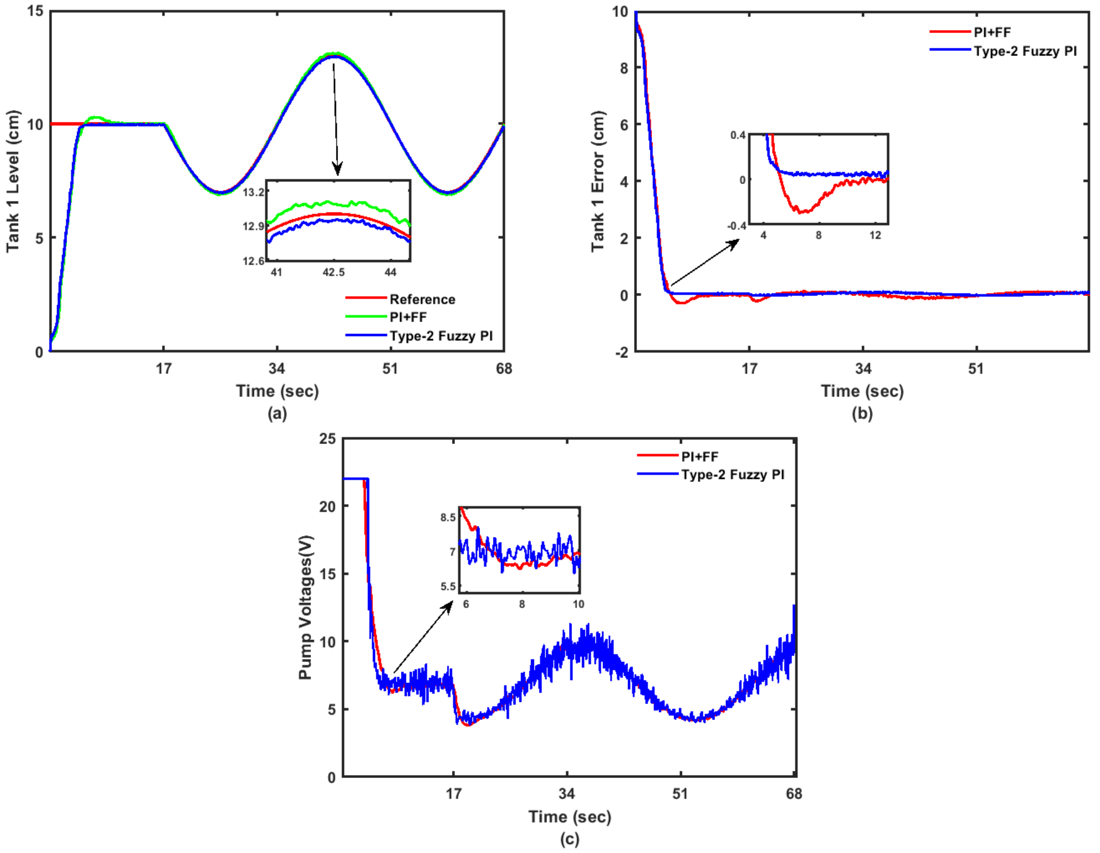

Figure 11. PI+FF and type-2 fuzzy PI controller performances under step + sinusoidal reference for tank 1.

Table 6. System response parameters for step+sinusoidal reference in control of tank 1.

Controllers	Step		
	Rise Time	Overshoots (%)	Settling Time
PI+FF	2.9356	2.7101	7.8981
Tip-1 Fuzzy PI	3.1205	0.22761	4.2772
Interval Tip-2 Fuzzy PI	3.0108	0.14289	4.345

Table 7. Error performance metrics for step+sinusoidal reference in control of tank 1.

Controllers	Step				Sinusoidal			
	ISE	ITSE	ITAE	IAE	ISE	ITSE	ITAE	IAE
PI+FF	174	198.8	43.04	25.02	0.3471	13.47	152.9	3.712
Type-1 Fuzzy PI	163	182.9	40.95	23.79	0.1706	7.248	99.8	2.328
Interval Type-2 Fuzzy PI	162.7	180.6	39.07	23.54	0.1283	5.471	84.1	1.952

Figures 12 and 13 show the results of the controllers for step+square reference liquid level. From figure 12, it is seen that the type-1 fuzzy PI controller follows the reference signal better than the PI+FF controller both in the step part of the reference signal and in the square part of the reference signal. Figure 13 shows that the interval type-2 fuzzy PI controller shows a similar performance with the type-1 fuzzy PI controller. When the system response parameters and error performance metrics given in Tables 8 and 9 are analyzed, it is seen that interval type-2 fuzzy PI controller has the best rise time but type-1 fuzzy PI controller is better in terms of overshoot and settling time. On the other hand, in terms of error performance criteria, PI+FF, type-1 fuzzy PI and type-2 fuzzy PI controllers show the best performance respectively following the step part of the reference signal, but after the addition of the square component in the continuation of the reference signal, the ranking in terms of performance is type-1 fuzzy PI, interval type-2 fuzzy PI and PI+FF. Since the single tank system has a linear model, it is easy to control and therefore the controller performances are close to each other.

Table 8. System response parameters for step+square reference in control of tank 1.

Controllers	Step		
	Rise Time	Overshoots (%)	Settling Time
PI+FF	3.0939	2.982	7.6453
Type-1 Fuzzy PI	3.0171	0.25224	4.2376
Interval Type-2 Fuzzy PI	2.975	0.41267	4.514

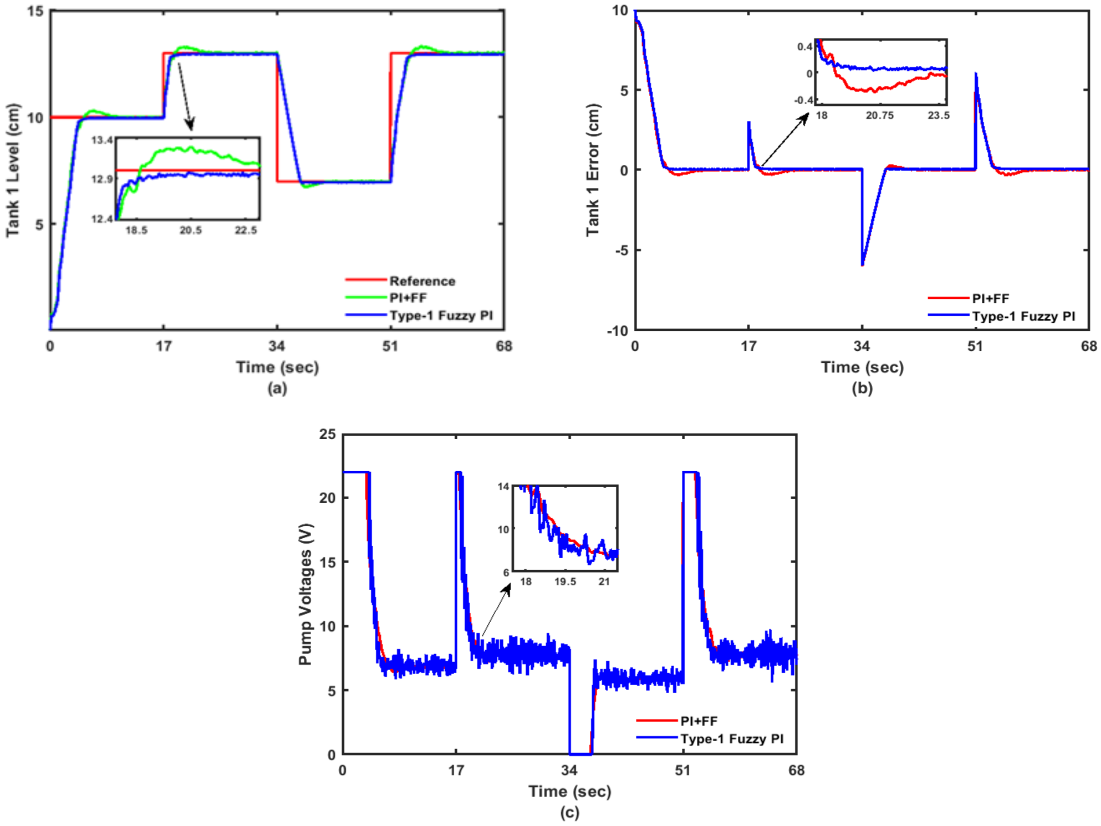

Figure 12. PI+FF and type-1 fuzzy PI controller performances under step + square reference for tank 1.

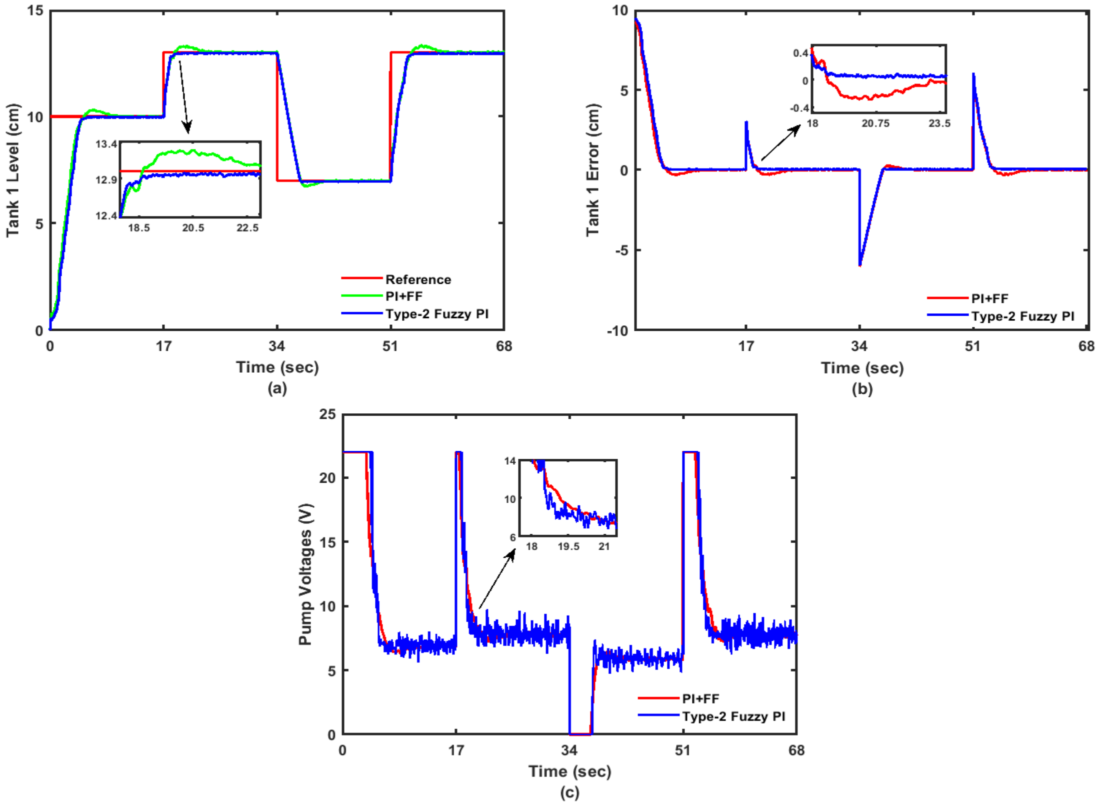

Figure 13. PI+FF and type-2 fuzzy PI controller performances under step + square reference for tank 1.

Table 9. Error performance metrics for step+square reference in control of tank 1.

Controllers	**Step**				**Square**			
	ISE	**ITSE**	**ITAE**	**IAE**	**ISE**	**ITSE**	**ITAE**	**IAE**
PI+FF	157.5	172.1	39.59	23.49	68.35	26.55	832.4	20.97
Type-1 Fuzzy PI	163.4	181.2	38.92	23.57	66.1	2654	828.5	20.59
Interval Type-2 Fuzzy PI	179.1	211	42.4	25.2	66.33	2675	821.5	20.36

In order to observe the performance of the controllers in the tank 2 configuration, a step+sinusoidal reference liquid level is selected and the results of PI+FF, type-1 fuzzy PI and interval type-2 fuzzy PI controllers are given in Figures 14 and 15. When Figure 14 is analyzed, it is seen that the type-1 fuzzy PI controller reaches the reference liquid level faster than the PI+FF controller, but overshoots more and has similar settling times. However, it is seen that the type-1 fuzzy PI controller has a much better performance in tracking the sinusoidal component of the reference signal.

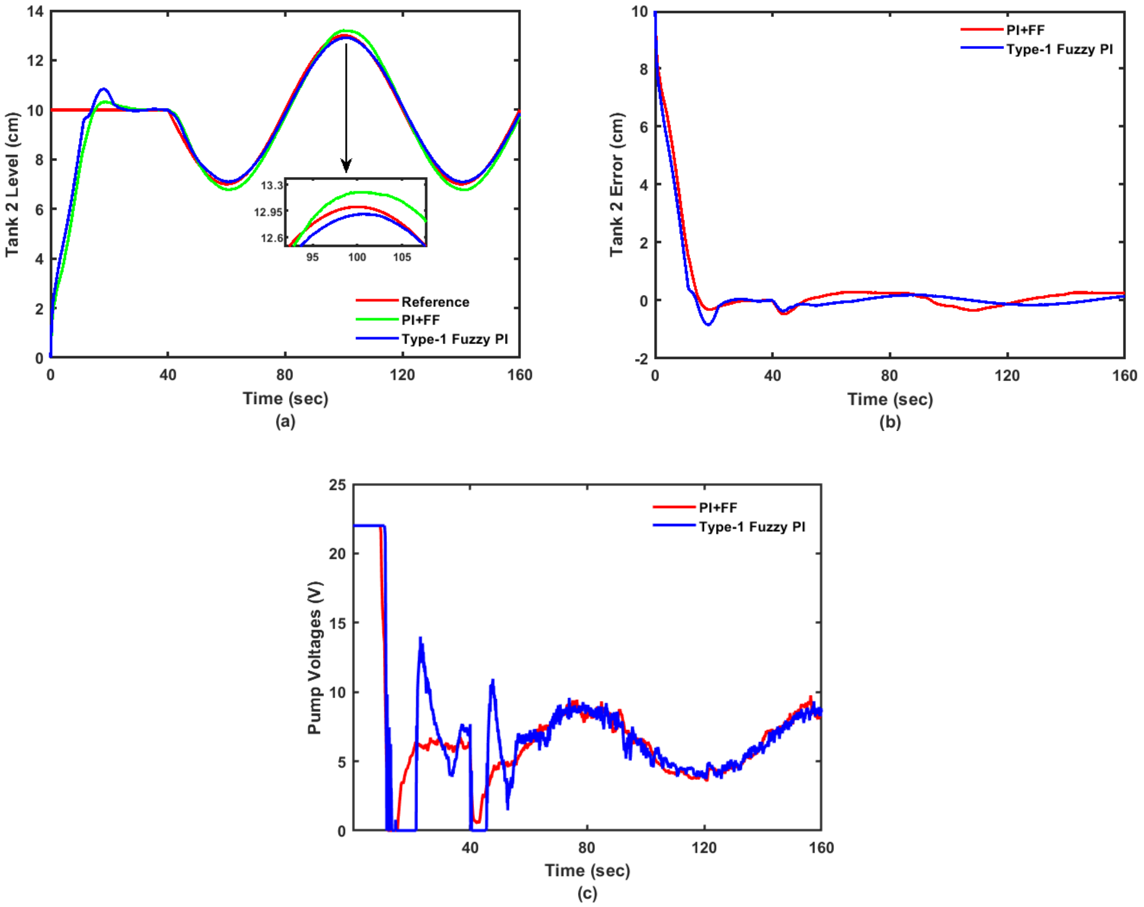

Figure 14. PI+FF and type-1 fuzzy PI controller performances under step + sinusoidal reference for tank 2.

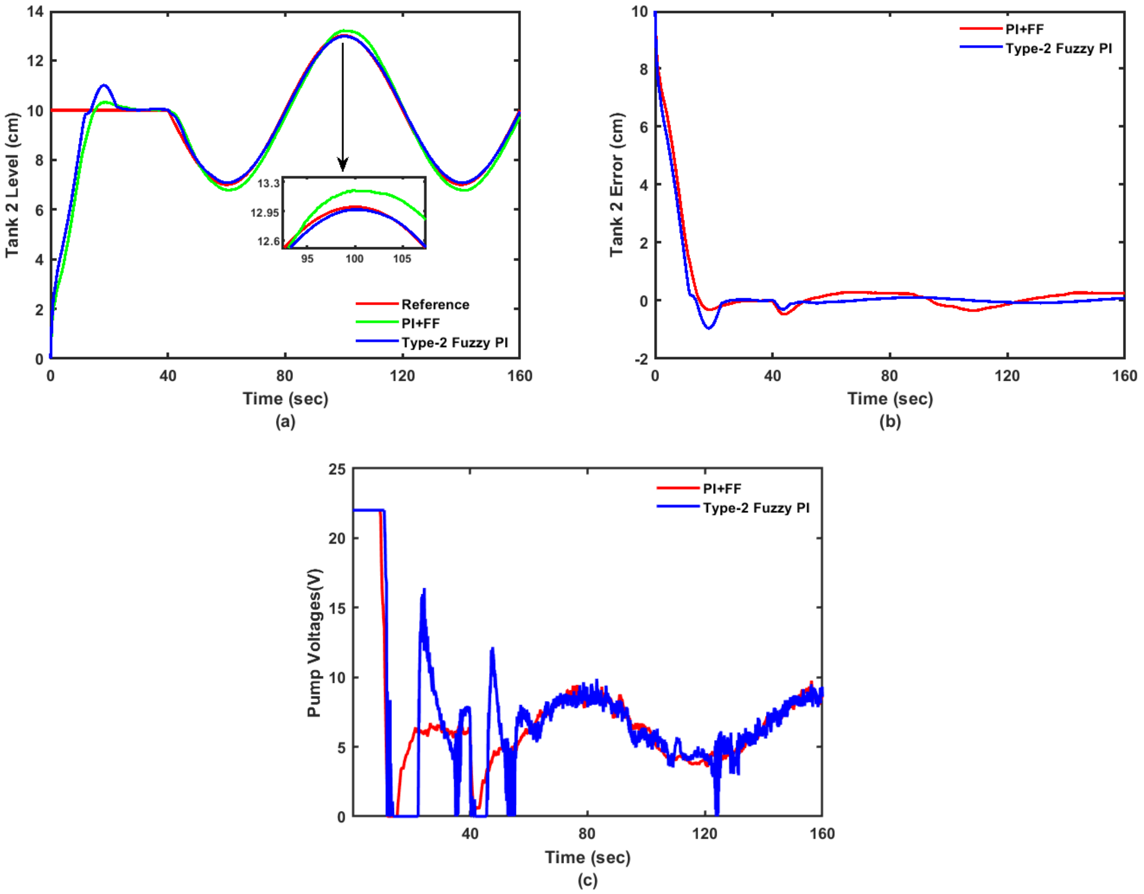

Figure 15. PI+FF and type-2 fuzzy PI controller performances under step + sinusoidal reference for tank 2.

Figure 15 shows that the interval type-2 fuzzy PI controller performs similarly to the type-1 fuzzy PI controller in tracking the step component of the reference signal, but performs much better in tracking the sinusoidal component. This is also seen from the system response parameters given in Table 10 and the error performance metrics given in Table 11. As a result, the interval type-2 fuzzy PI controller performed similar to the type-1 fuzzy PI controller in the control of the single tank configuration, but performed much better than both type-1 fuzzy PI and PI+FF controllers in the tank 2 configuration.

Table 10. System response parameters for step+sinusoidal reference in control of tank 2.

Controllers	Step		
	Rise Time	Overshoots (%)	Settling Time
PI+FF	11.672	5.8034	79.168
Type-1 Fuzzy PI	10.17	10.065	79.124
Interval Type-2 Fuzzy PI	10.228	10.643	79.137

Table 11. Error performance metrics for step+sinusoidal reference in control of tank 2.

Controllers	Step				Sinusoidal			
	ISE	ITSE	ITAE	IAE	ISE	ITSE	ITAE	IAE
PI+FF	365.3	1282	344.1	64.49	6.281	602.1	2469	25.01
Type-1 Fuzzy PI	292.4	928.5	307.8	56,65	2.093	177	1273	13.68
Interval Type-2 Fuzzy PI	292.6	946.2	319.6	57.2	0.7746	57.99	690.6	7.627

4. Conclusion

In this study, PI+FF, type-1 fuzzy PI and interval type-2 fuzzy PI controllers are designed for different reference liquid levels for tank 1 and tank 2 configuration models in a coupled tank system and their performances are compared. The comparison is made in terms of system response parameters such as rise time, settling time and maximum overshoot and error performance metrics such as ISE, ITSE, ITAE and IAE. The obtained results show that the interval type-2 fuzzy PI controller performs the liquid level control in single tank configuration with a similar performance as the type-1 fuzzy PI controller and the performance of both controllers is much better than the PI+FF controller. On the other hand, it is observed that the interval type-2 fuzzy PI controller performs much better than both type-1 fuzzy PI and PI+FF controllers in the tank 2 configuration. As a result, it is observed that the adaptation of the parameters of the PI controller with fuzzy logic improves the control performance of the system.

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