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AN INNOVATIVE HYBRID APPROACH TO FORECASTING INTERVAL TIME SERIES DATA WITH ELMAN ARTIFICIAL NEURAL NETWORKS AND A MODIFIED ADAPTIVE NETWORK-BASED FUZZY INFERENCE SYSTEM

ELMAN YAPAY SİNİR AĞLARI VE MODİFİYE ADAPTİF AĞ TABANLI BULANIK ÇIKARIM SİSTEMİ İLE ARALIKLI ZAMAN SERİSİ VERİLERİNİN TAHMİNİ İÇİN YENİLİKÇİ BİR HİBRİT YAKLAŞIM

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ABSTRACT

Interval Time Series (ITS) techniques are used in data analysis for modeling and forecasting. This paper introduces a hybrid model that combines two effective forecasting methods: the Modified Adaptive Network-Based Fuzzy Inference System (MANFIS) and the Elman Recurrent Neural Network (ERNN). Unlike non-interval time series data, ITS considers both the highest and lowest values within an interval to represent dynamic data, thereby capturing potential relationships between these bounds. The proposed algorithm incorporates ANFIS and ERNN structures with the following advantages: it utilizes particle swarm optimization to train the model and addresses both linear and nonlinear forecasting aspects, providing model-based and data-based approaches, respectively.The fuzzification process employs the fuzzy c-means clustering technique to systematically derive membership values from input data, thereby enhancing forecasting accuracy. The proposed method has been rigorously validated using seven diverse real-world datasets, and comparative analyses with existing algorithms in the literature confirm its superior performance.

ÖZ

Aralık Değerli Zaman Serisi (ITS) teknikleri, veri analizinde modelleme ve tahmin yapmak için kullanılır. Bu makale, aralık değerli zaman serileri için tahmin üretmeye yönelik iki yöntemi birleştiren hibrit bir model önermektedir: Modifiye Edilmiş Adaptif Ağ Tabanlı Bulanık Çıkarım Sistemi (MANFIS) ve Elman Yapay Sinir Ağı (ERNN). Aralık değerli zaman serileri, geleneksel zaman serilerinden farklı olarak aralığın en yüksek ve en düşük değerlerini dikkate alarak, bu sınırlar arasındaki olası ilişkileri değerlendirir. Önerilen hibrit model iki kısımdan oluşmaktadır: ANFIS algoritması ve ERNN model yapısı. Modelin eğitimi için parçacık sürü optimizasyonunun kullanılması, önerilen yöntemin avantajlarından biridir. Ayrıca, hem doğrusal hem de doğrusal olmayan tahmin yönlerini ele alarak model tabanlı ve veri tabanlı yaklaşımlar sunmaktadır. Girdi değerlerinin bulanıklaştırılması, üyelik değerlerinin bulanık c-ortalamalar kümeleme tekniği kullanılarak sistematik olarak elde edilmesiyle gerçekleştirilir; bu da tahmin performansını artırmaktadır. Yedi farklı veri seti kullanılarak yöntemin etkinliği doğrulanmış ve sonuçlar literatürdeki modellerle karşılaştırılmıştır. Önerilen modelin üstün performans gösterdiği görülmektedir.

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Introduction

Making predictions for planning and strategy formulation is essential and can be achieved by collecting data and constructing a model that best fits the available data. In the literature, various stochastic and non-stochastic methods have been proposed. The type of data available dictates the methods to be used if no transformation is applied. One type of data encountered is interval-valued data, which is observed in several applications. Like, the lower and higher values of stock prices, the systolic and diastolic measurements of patients' blood pressure, and the minimum and maximum measurement heights of a river for flood management. Direct methods that deal with this type of data are necessary to avoid losing information. If the data is recorded over time, Interval Time Series (ITS) is one of the data types used to handle them. ITS is also employed when exact data is unavailable. For example, a person's blood pressure over time can be represented as an interval, or a sensor can measure and record a device's minimum and maximum values each hour. Although such data can be modeled using classical time series methods, converting interval data to crisp values can result in information loss. When a group of people is observed to measure the same variable over time, ITS can be obtained (Arroyo & Mate, 2006). ITS is prevalent in various fields, such as finance (the daily price range of a company's stock), engineering (the daily voltage range in electric currents), medicine (the daily systolic and diastolic blood pressure of patients), meteorology (the monthly minimum and maximum precipitation in a location), and relative humidity (the monthly measured range).

Interval data, which plays a crucial role in symbolic data analysis (SDA), has found extensive applications across diverse fields. In 1996, Ichino et al. introduced a symbolic classifier using a region-oriented methodology tailored for symbolic interval data. Lissoir and Rasson (2000) developed a symbolic kernel classifier that employed appropriate dissimilarity functions. Lechevallier and Périnel (2000) proposed a tree-growing algorithm designed specifically for classifying symbolic interval data. Subsequently, Bock (2002) presented a range of clustering algorithms suited for interval variables and introduced a Self-Organizing Map (SOM) for their visual representation. Lechevallier and Chavent (2002) introduced a dynamic clustering algorithm for interval data, utilizing a modified Hausdorff distance to represent class representatives. Measures of dispersion and central tendency were expanded upon by Diday and Billard (2003) and Saracco and Chavent (2008). Irpino (2006) delved into factorial methods, while Groenen et al. (2006) focused on multidimensional scaling. Hierarchical clustering has been explored by several researchers, including Diday and Gowda (1991), Yaguchi and Ichino (1994), Ravi and Gowda (1995, 1999), Guru et al. (2004), and Chavent (2000). Recognizing the correspondence between interval sets and fuzzy sets, their fuzzy aspect was studied by Ismail and El Sonbaty (1998), Yang et al. (2004), and De Carvalho (2007). Hard partition clustering has been researched by De Carvalho et al. (2009), Lechevallier and De Carvalho (2009a, 2009b), De Carvalho et al. (2006), De Carvalho and De Souza (2004), and Verde and Irpino (2008). Regression modeling for interval data has been examined by De Carvalho and Lima Neto (2010) and Arroyo and Mate (2009). De Carvalho and Souza (2004) proposed segregative clustering methods that utilize both adaptive and non-adaptive city block distances. De Carvalho et al. (2006) developed an algorithm specifically for this purpose. De Carvalho (1995) recommended the use of histograms for interval-valued data. Principal component analysis for interval-valued data was suggested by Palumbo and Lauro (2000) and Cazes et al. (1997). Diday (1987) highlighted that statistical units defined by range data can be considered special cases of Symbolic Objects (SO).

In the realm of univariate statistics, Diday and Billard (2003) and Goupil and Bertrand (2000) introduced measures of dispersion and central tendency for symbolic interval data. Diday and Billard (2000) proposed an approach for fitting linear regression models to interval-valued data sets. De Carvalho and Neto (2008) enhanced this method by introducing a novel approach based on two distinct linear regression models. Factorial Discrimination Analysis (FDA) for Interval Time Series (ITS) was adapted by Lauro et al. (2000) and Palumbo and Verde (1999). In the context of ITS, Maia et al. (2008) introduced methodologies including an artificial neural network (ANN), an autoregressive integrated moving average (ARIMA), and a hybrid model combining both ANN and ARIMA. De Carvalho and Maia (2011) proposed three different approaches to forecasting ITS, with the first two relying on multilayer perceptron neural networks (MLP) and Holt's exponential smoothing methods. The third approach combines both the MLP and Holt methods into a hybrid methodology. Chang, Chuang, and Jeng (2023) proposed the Interval Improved Fuzzy Partitions Fuzzy C-Means (IIFPFCM) clustering algorithm, which uses Euclidean and city block distance measures and achieves faster convergence than traditional Interval Fuzzy C-Means (IFCM). The algorithm also addresses group division issues in symbolic interval data and is robust against outliers. Experimental analysis with nine datasets shows that the IIFPFCM algorithm, particularly with the city block distance, outperforms IFCM in convergence speed, efficiency, and handling outliers, offering superior overall performance.

Silva (2011) employed the copula approach for regression models. Initially based on conventional mathematical tools such as differential equations, the Adaptive Neuro-Fuzzy Inference System (ANFIS), first introduced by Jang in 1993, has recently been the subject of numerous studies aiming to hybridize linear and nonlinear methods to construct models. Carvalho and Maia (2011) introduced three approaches to predicting ITSs. Xiong et al. (2017) recommended a hybrid model framework that combines Holt's method with the MSVR.

These studies explored various advanced methodologies for handling interval data and forecasting, demonstrating their effectiveness in different domains such as energy, finance, and agriculture. Xiong et al. (2014a) proposed FA-MSVR, a Firefly Algorithm-based approach. Xiong et al. (2014b) introduced a model combining Support Vector Regression (SVR) with BEMD for intermittent electricity demand. Rodrigues and Salish (2015) suggested threshold models for Interval Time Series (ITS) analysis. Sun et al. (2018) employed Interval Data Envelopment (IDE) to estimate crude oil prices, showing superior performance. Xiong T. et al. (2015) combined Vector Error Correction Model (VECM) with MSVR to predict range-valued agricultural commodity prices. Sakaori and Park (2014) applied MSVR with a Memetic Algorithm based on the Firefly Algorithm for interval load estimation. Poczeta and Papageorgiou (2017) introduced a two-stage prediction model using Artificial Neural Networks (ANNs) and evolutionary Fuzzy C-Means (FCM) for multivariate time series prediction. Jiang et al. (2021) proposed a combined forecasting system with four components: optimal sub-model selection, point prediction using a modified multi-objective optimization algorithm, interval forecasting through distribution fitting, and system evaluation. The system leveraged the strengths of sub-models to deliver precise point and interval forecasts. Experimental results showed absolute percentage errors of 2.92%, 3.17%, and 4.84% at Site 1, and 2.27%, 2.59%, and 3.48% at Site 2 for 1-step, 2-step, and 3-step forecasts, respectively. The proposed system outperformed benchmark models, making it highly effective for electric power system scheduling and management. Chinnadurrai et al. (2024) presented a deep-learning ensemble model combining wavelet transformation, Long Short-Term Memory (LSTM), and Elman neural networks for wind speed forecasting using data collected from coastal areas of Western India. The data were pre-processed through wavelet transformation to decompose them into sub-layers, and then trained and tested with the proposed model. The results demonstrated that the proposed method outperformed other models in terms of error metrics, offering an effective solution for integrating wind energy systems and ensuring reliable power system operation. Wu et al. (2023) proposed a novel hybrid approach for short-term power demand prediction, combining the Elman neural network (ELM) and the adaptive network-based fuzzy inference system (ANFIS). By integrating these methods, the hybrid model overcomes their limitations while leveraging their strengths, particularly in handling non-linear data. The approach utilized an enhanced bioinspired algorithm, the improved parasitism-predation algorithm, to optimize the weight coefficients for greater accuracy. Simulation results confirmed that the hybrid method outperformed both ELM and ANFIS individually, as well as other advanced models, delivering superior predictive performance. Wan and Dong (2020) defined the possible degree of comparison between two interval-valued intuitionistic fuzzy numbers (IVIFNs) using a two-dimensional random vector. A new ranking method for IVIFNs was developed based on this concept. The approach employed the Ordered Weighted Average (OWA) and Hybrid Weighted Average (HWA) operators, which are derived using the Karnik-Mendel algorithms, to address multi-attribute group decision-making problems. The method calculates individual overall attribute values of alternatives using the weighted average operator for IVIFNs. The collective values are obtained with the hybrid weighted average operator and used to rank the alternatives.

A numerical example is provided to demonstrate the effectiveness and flexibility of the proposed method. Haiyun et al. (2021) analysed innovation strategies for green supply chain management using Quality Function Deployment (QFD) from a multidimensional perspective. The novelty lies in defining green supply chain criteria for each stage of QFD and proposing a hybrid model combining Interval-Valued Intuitionistic Fuzzy (IVIF) DEMATEL (Decision-Making Trial and Evaluation Laboratory) and IVIF MOORA (Multi-Objective Optimization by Ratio Analysis). The findings reveal that the most crucial innovation strategy for green supply chain management in the energy industry is understanding customer expectations through customer relationship management, followed by benchmarking the competitive market environment. Therefore, energy companies are advised to focus on effective customer relationship management by analyzing customer needs and aligning their products and services accordingly. The study also emphasized the importance of new product and service development, suggesting that companies establish research and development departments and employ qualified personnel while gathering input from customers, employees, and suppliers. Satisfied customers will help energy companies increase their market share.

Ali et al. (2021) introduce the principle of Complex Interval-Valued Pythagorean Fuzzy Sets (CIVPFS), a valuable tool for handling inconsistent and uncertain real-world data. CIVPFS combines complex fuzzy sets and interval-valued Pythagorean fuzzy sets, using complex numbers where the real and imaginary parts are sub-intervals of the unit interval. The main advantage of this approach is that the sum of the squares of the real and imaginary parts is constrained within the unit interval. The article explores the algebraic operational laws of CIVPFS and develops Einstein operational laws using t-norm and t-conorm. It also introduces two new operators: Complex Interval-Valued Pythagorean Fuzzy Einstein Weighted Geometric (CIVPFEWG) and Complex Interval-Valued Pythagorean Fuzzy Einstein Ordered Weighted Geometric (CIVPFEOWG). These operators are applied to Multi-Criteria Decision-Making (MCDM) problems. Examples are provided to demonstrate the consistency and reliability of these operators. A comparative analysis and graphical representations are also included to highlight the effectiveness and superiority of the proposed approach.

Garg and Kumar (2020) introduce a novel Multi-Attribute Decision Making (MADM) method in an interval-valued intuitionistic fuzzy (IVIF) set environment by integrating the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. The study applies Set Pair Analysis (SPA), a modern uncertainty theory, which considers "identity," "discrepancy," and "contrary" degrees of connection numbers (CN) to handle data uncertainties. The paper presents exponential-based distance measures using CNs of IVIF sets to enhance the theory of information measurement. A TOPSIS method based on these distance measures is then developed to solve MADM problems in the IVIF context. The approach is validated through a real-life numerical example and compared to existing methods, demonstrating its effectiveness and superiority.

In this research, a new hybrid forecasting method called ERNN–MANFIS, which combines the ERNN and MANFIS models, is recommended for modeling ITS data. The MANFIS model is enhanced and employed in conjunction with ERNN in a hybrid strategy, providing a better alternative for time series forecasting. Membership values are systematically obtained using the fuzzy c-means clustering technique during the fuzzification step of input values, which increases the accuracy of forecasting and makes the approach more systematic. During the training stage, ERNN is utilized, and MANFIS is optimized via particle swarm optimization, thus enhancing the forecasting performance of ITS.

The rest of the manuscript is organized as follows: Section 2 briefly provides the basic concepts of ITS. Section 3 introduces MANFIS, ERNN, and the proposed algorithm to forecast ITS along with auxiliary methods used in the approach. Section 4 presents the novel hybrid ERNN-MANFIS algorithm. Section 5 practices the novel algorithm with different ITS datasets and compares the results with those obtained from existing methods. The conclusion is presented in Section 6.

Preliminary

In this section, a brief introduction to ITS is provided, which is mainly used in Symbolic Data Analysis dealing with multiple analyses, sample recognition, and artificial intelligence. Observations of ITS consist of closed intervals. Considering that the methods using ITS data and fuzzy time series data are different, for example, ITS does not contain membership values. ITS data are often encountered as a new type of data to deal with since they are generally transformed into real numbers. The values of observations are changed quickly in the time domain so they are represented by an interval that contains all values of the observations. The components of ITS data include 4 terms defined as follows:

XU_t, t=1, 2, 3⋯ refers to the upper bound values of the time series,

XL_t, t=1 2, 3⋯ refers to the lower bound values of the time series,

XC_t, t=1, 2, 3⋯ shows center series,

XR_t, t=1, 2, 3⋯ shows range series,

where the range-valued data are ordered sequentially.

The ITS data for t = 1, 2, 3, …, n is denoted by $(X_{l1}; X_{u_1})$, $(X_{l2}; X_{u_2})$, …, $(X_{l_n}; X_{u_n})$. The concept of ITS was first introduced by Bock and Diday (2000) to be employed in the field of Symbolic Data Analysis. This type of data naturally arises in many situations. Due to its unique structure, most research efforts addressing it are related to artificial intelligence, pattern recognition, and multivariate analysis. ITS data can be used in all types of exploratory data analysis and statistical methods (De Carvalho & Maia, 2011; Liu et al., 2013). ITS histograms were studied by De Carvalho (1995). Basic components analysis methods were recommended by Lauro and Palumbo (2000) and Cazes et al. (1997). Furthermore, appropriate central tendency and dispersion measures were introduced by Bertrand and Goupil (2000) and Billard and Diday (2003). Four novel approaches to model and estimate ITS were recommended by Maia et al. (2008), including AR, ARIMA, ANNs, and Hybrid Models. Monte Carlo simulation was utilized for the evaluation of these four new approaches. By employing ANNs to analyze ITS data in the stock market, the process and applications of the exponential smoothing model were examined (Maia et al., 2008). In the study, two approaches were used. The first approach was based on ANNs, while the second approach involved the exponential smoothing method. Parameters were obtained using nonlinear optimization techniques. These approaches were then applied to daily ITS data. It was observed that the approach employing the exponential smoothing model was superior to the ANNs approach. The center and range methods were applied to the data with symbolic intervals. A new approach was introduced based on the linear regression model (Neto & De Carvalho, 2008).

A linear regression model was applied using midpoints and ranges. The midpoint and the range of the dependent variable were obtained from the estimation of the upper and lower limits. Also, Monte Carlo simulation was used for the evaluation of the recommended estimation methods. A regression model based on the Copula approach for range-valued variables was introduced by Silva et al. (2011). Regression models were used with optimization in interval-valued data. Also, Neto (2011) recommended bivariate symbolic regression models based on Generalized Linear Models. Residual analysis and diagnostic measurements were used to prepare the ground for this regression approach for interval-valued variables. Besides, a regression model based on copula theory was recommended for symbolic interval-valued data. ANNs and Holt methods were applied to forecast ITS by Maia and De Carvalho (2011). Three approaches were used to forecast the ITS. MLP and Holt methods were used. Smoothing parameters in the Holt method with limit constraints were estimated using nonlinear techniques. The third approach was based on a hybrid method that combined the Holt method and MLP to estimate stock market time-series data.

Methodology

Adaptive Neuro-Fuzzy Inference System (ANFIS) excels at managing uncertainty by combining neural networks with fuzzy logic. It creates fuzzy inference rules that can handle imprecise or vague input data, making it robust for problems with noisy or incomplete datasets. The fuzzy rules generated by ANFIS are human-readable, providing insights into the underlying patterns and decision-making process. This interpretability is valuable in applications where understanding the "why" behind predictions or decisions is crucial. ANFIS adapts its parameters through learning algorithms, typically backpropagation for the fuzzy rule parameters and least squares for the consequent parameters, ensuring optimal performance for specific datasets. This adaptability allows it to model nonlinear relationships effectively. Elman Recurrent Neural Network (ERNN) is designed to model dynamic and sequential data by maintaining a context layer that stores information from previous time steps. This makes ERNN particularly effective for time-series forecasting, where historical data influences future predictions. ERNN can learn complex, nonlinear relationships in data through its hidden and recurrent structure.

The recurrent connections in ERNN enable it to retain a "memory" of past inputs, making it better equipped to handle problems where historical trends and patterns are essential. ANFIS handles uncertainty and interpretability, providing a robust framework for feature extraction and rule-based reasoning. ERNN handles temporal dependencies and nonlinear dynamics, making it ideal for modeling sequential relationships. To combine these strengths, a hybrid model can leverage ANFIS given its ability to simplify and clarify complex input relationships while using ERNN to capture and predict dynamic patterns over time, offering a comprehensive solution for challenging forecasting problems.

In this section, we briefly portray the tools used in the recommended method called ERNN-MANFIS. If readers would like to pay more attention to the details of the tools, the books have provided detailed accounts of the tools (Clerck, 2005; Kröse & van der Smagt, 1996; Siddique & Adeli, 2013).

Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a heuristic optimization technique proposed by Eberhart & Kennedy (1995). In PSO, each individual is represented as a particle within a swarm. The method improves upon the original concept introduced by Shi & Eberhart (1999).

Initialization: Positions (X_k) and velocities (V_k) for each particle k (k=1,2,…,pn) are randomly determined.

Evaluation Function: Determines p-best ($pbest_k$) and g-best(gbest) particles based on their performance.

Parameters Update: At each iteration, parameters such as inertia (w), cognitive coefficient (c_1), and social coefficient (c_2) are updated according to predefined intervals.

Velocity and Position Update: Update equations for velocity (v_{id}^{k+1}) and position (x_{id}^{k+1}) are applied using the current parameters and random values.

Iteration: Steps 3 to 5 are repeated until a maximum number of iterations (*maxt*) is reached.

PSO is utilized for optimization problems where the goal is to find the best solution by iteratively adjusting the particles' positions within a search space.

The Modified ANFIS Model

The Adaptive Neuro-Fuzzy Inference System (ANFIS) learns membership functions and rules using data. In the system, circular shapes represent fixed nodes, while square nodes denote parameters (Jin, 2013). Both input and output parameters were trained by Jang (1991, 1992). Sen (2004, 2009) utilized a combination of artificial neural networks (ANNs) and the Sugeno approach. When applying the Sugeno inference system in ANFIS, only the input section of the dataset is fuzzified (Takagi & Sugeno, 1985; Sugeno, 1985). The number of rules corresponds to the number of membership functions, known as a one-input-one-output system. In the Sugeno fuzzy system, weights "w1" and "w2" range between [0, 1]. The output variable, y, is calculated as a weighted average of inputs, aiming to minimize errors during training. In the Sugeno fuzzy system, rules apply only to "low normal" cases, but the ANFIS approach can be applied to four rules covering the entire domain, such as high, normal-high, normal, and low, to determine the best model (Güclü et al., 2014). The Modified ANFIS model was first introduced by Egrioglu et al. and redesigned for time series forecasting. When using ANFIS for time series modeling, the input variables must be lagged variables, and time series data should be clustered before constructing the input-output relationship. Clustering facilitates the generation of rules, similar to various fuzzy time series approaches. The parameters of the consequent part are trained by the improved PSO algorithm.

The widely used measure, RMSE, was employed to evaluate the effectiveness of the forecasting techniques.

$$MSE = \frac{1}{ntrain}\sum_{i=1}^{ntrain}(x_t - \hat{x}_t)^2 \tag{1}$$

$$RMSE = \sqrt{\frac{1}{ntrain}\sum_{i=1}^{ntrain}(x_t - \hat{x}_t)^2} \tag{2}$$

where "ntrain" in the denominator denotes the length of the training data set, $\hat{x}_t$ is called the forecast for t^{th} observation of the time series data. Finally, an algorithm is provided below to explain how to compute the output of the recommended ANFIS. The output value of the recommended ANFIS can be calculated by following the steps outlined in the algorithm below.

The Modified ANFIS

The computation of the output of the new ANFIS. The architecture of the modified ANFIS (for two inputs: x_{t-1}, x_{t-2} and two rules: I*f* x_{t-1} is L_1 and x_{t-2} is L_2 then $f_1 = p_1 x_{t-1} + q_1 x_{t-2} + r_1$, *if* x_{t-1} is L_2 and x_{t-2} is L_1 then $f_2 = p_2 x_{t-1} + q_2 x_{t-2} + r_2$)

Step 1. Membership values are calculated based on fuzzy c-means clustering. The notations are expressed by

$\mu_{L_j}(x_{t-j})$: The membership value for the j[th] lagged variable to i[th] fuzzy set.

$\mu_{L_1}(x_{t-1}), \mu_{L_2}(x_{t-1}), \mu_{L_1}(x_{t-2}), \mu_{L_2}(x_{t-2})$. Membership values are calculated for the new ANFIS.

Step 2. The computation of weights is defined by

$$w_1 = \min\left(\mu_{L_1}(x_{t-1}), \mu_{L_2}(x_{t-2})\right), w_2 = \min((\mu_{L_2}(x_{t-1}), \mu_{L_1}(x_{t-2}))$$

Step 3. Normalization

$$\overline{w_i} = \frac{w_i}{w_1+w_2},\ i = 1,2 \tag{3}$$

Step 4. The computation of the output of each rule is defined by

$$\overline{w_i}f_i = \overline{w_i}(p_i x_{t-1} + q_i x_{t-2} + r_i) \tag{4}$$

Step 5. The combined output of all rules is defined by

$$\widehat{x_t} = \sum_i \overline{w_i}\, f_i \tag{5}$$

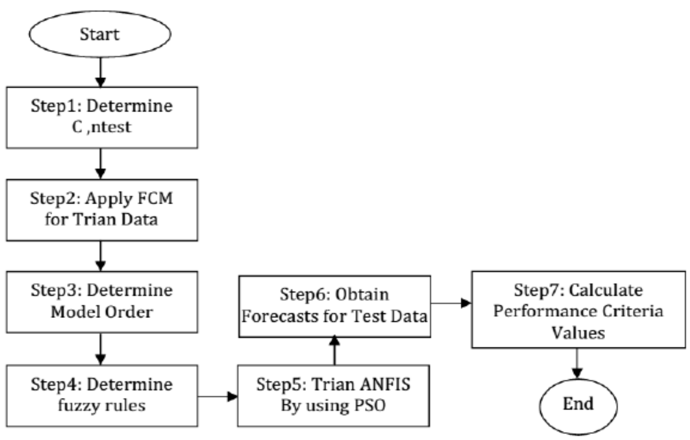

Figure 1. Depicts the Flow Chart of The Algorithm (Egrioglu, et al. 2014)

Fuzzy C-Means Clustering

The fuzzy c-means clustering (FCM) algorithm is used for calculating the membership values of inputs. The main purpose is to minimize the distance between centers of clusters and observations so more homogenous clusters can be generated. The FCM needs to assign the number of clusters, c, in advance. X= $\{x_1, x_2, ..., x_n\}$ the observation values, (v_i) the cluster center vector, (U) the membership value matrix, ($m \in (1, \infty)$) the fuzziness parameter, (n) the number of observations, (A) the norm matrix, (μ_{ik}) the membership value, (V) the matrix of cluster centers and are all set. The objective function is defined by

$$\min J(X;U;V) = \sum_{i=1}^{c}\sum_{k=1}^{n}(\mu_{ik})^m d^2(x_k, v_i)_A \tag{6}$$

Firstly, c and m are determined. In general, m is chosen where $1.5 \le m \le 3$ as a rule of thumb for Eqs. (7)-(9) are used in the FCM algorithm.

$$v_i^t = \frac{(\sum_{k=1}^{n} (\mu_{ik}^{(t)})^m x_k)}{\sum_{k=1}^{n} (\mu_{ik}^{(t)})^m}, \forall i = 1, \ldots, c \tag{7}$$

$$\mu_{ik}^{(t)} = \left[\sum_{j=1}^{c} (\frac{d(x_k, v_i^{(t-1)})}{d(x_k, v_i^{(t-1)})})^{\frac{2}{m-1}} \right]^{-1} \tag{8}$$

$$E_t = \sum_{i=1}^{c} \left\| v_{i,(t+1)} - v_{i,t} \right\| \tag{9}$$

Algorithm 3. Fuzzy C-Means

Initialize the membership matrix U randomly.2) Initialize c, m, and ε. 3) Calculate the vector of the cluster center v_i by using Eq. (7). 4) Calculate the membership values by using Eq. (8). 5) Calculate E_t by using Eq. (9). 6) If $E_t \leq \varepsilon$ so, then stop. Otherwise, return to Step 3.

ERNN Model

The ERNN, similar to a three-layer feed-forward neural network, is a simple recurrent neural network proposed by Elman (1990). It consists of three primary components: an input layer, a hidden layer, and an output layer. Additionally, it includes a context layer that provides feedback from the hidden layer outputs of previous time steps. Neurons in each layer transmit information to the next layer (Chandra, 2012; Zhang, 2012; Cacciola, 2012; Megali, 2012; Pellicano, 2012; Morabito, 2012).

$$S_i(t) = g(\sum_{k=1}^{K} V_{ik} S_k(t-1) + \sum_{j=1}^{J} W_{ij} I_j(t-1)) \tag{10}$$

$S_k(t)$ and $I_j(t)$ denote the output of the context state and input neurons. v_{ik} and w_{ij} denote their corresponding weights. g(.) denotes the logistic transfer function or linear transfer function. Figure 2 depicts a typical structure of an Elman network (Wang et al, 2014).

The Proposed Method: ERNN- MANFIS

Egrioglu et al. (2014) proposed a modified ANFIS (MANFIS) to improve predictive accuracy in time series forecasting by employing fuzzy rules. Both ANFIS and MANFIS are tailored to model non-linear relationships between inputs and outputs. Linear models are commonly used in time series estimation because they effectively capture straightforward input-output relationships, but they do not inherently identify whether relationships among lagged variables are linear or nonlinear.

In this manuscript, a hybrid strategy combining ERNN and MANFIS is proposed as an enhanced approach for time series forecasting. The method integrates membership values derived systematically from fuzzy c-means clustering during the fuzzification of input values. In the training phase, ERNN is employed, while MANFIS is optimized using particle swarm optimization (PSO). The training of ERNN-MANFIS involves sequential steps with PSO to achieve improved performance across all computational processes. The hybridization of ELMAN and ANFIS in a single stage introduces a novel approach, termed ERNN-MANFIS.

Step 1. Membership values for fuzzy sets are computed based on the cluster centers determined by Fuzzy C-Means (FCM). Eq. (17) is used to calculate these membership values. Assuming two entries

(x_{t-1}, x_{t-2})

$$O_i^1 = \mu_t(x), i = 1,2 \tag{11}$$

$$O_i^1 = \mu_{L_{t-2}}(x), i = 3,4 \tag{12}$$

Step 2. Each firing strength rule corresponds to the outputs of the second layer, which are computed based on the membership values. The outputs of the second layer are determined as follows:

$$O_1^2 = w_1 = \min(O_1^1, O_4^1) \tag{13}$$

$$O_2^2 = w_2 = \min(O_2^1, O_3^1) \tag{14}$$

In this layer, the number of outputs is equal to the number of rules.

Step 3. The rule of fire strengths is normalized. The normalized rule fire strengths for the network, the structure is calculated as follows:

$$O_i^3 = \overline{w}_i = \frac{w_i}{w_1 + w_2}, i = 1,2 \tag{15}$$

Step 4. Output values of the linear functions are given as follows:

$$\hat{x}_t^1 = p_1 x_{t-1} + q_1 x_{t-2} + r_1 \tag{16}$$

$$\hat{x}_t^2 = p_2 x_{t-1} + q_2 x_{t-2} + r_2 \tag{17}$$

The set of the parameters $\{p_1, p_2, q_1, q_2, r_1, r_2\}$ in Eq. (15) and Eq. (16) will be denoted as consequent parameters. Step 4. The output of the MANFIS is calculated using Eq. (17).

$$\hat{x}_{t.ANFIS} = \overline{w}_1 \hat{x}_t^1 + \overline{w}_2 \hat{x}_t^2 \tag{18}$$

Step 5. The output of ERNN is calculated as follows:

$$\hat{x}_{t.ERNN} = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} \tag{19}$$

Step 6. The output of ERNN-MANFIS is calculated as follows:

$$\hat{x}_t = v_1 \hat{x}_{t.ANFIS} + v_2 \hat{x}_{t.ERNN} + v_3 \tag{20}$$

The mathematical expression of the hybrid method is presented with a closed form in Eq. (20). The ERNN-MANFIS is a linear combination of ERNN and MANFIS models, but the combination weights are estimated with other model parameters in the single optimization processes by employing PSO.

Training ERNN-MANFIS with PSO

1. **Initialization of PSO Parameters**: Define social, cognitive, and inertia parameters, along with the number of particles and maximum iterations.
2. **Random Initialization**: Initialize velocity values and the initial positions of particles randomly, corresponding to parameters in the ERNN-MANFIS.
3. **Evaluation Function**: Calculate the Mean Square Error (MSE) for each particle using the formula provided in Eq. (10).
4. **Initialization of Best Positions**: Establish initial values for personal best (pbest) and global best (gbest). Update pbest and gbest iteratively during each iteration, checking termination criteria.
5. **Update Parameters**: Calculate cognitive, social, and inertia parameters using Eqs. (5)-(7) respectively. Update indices of failure and success.
6. **Update Velocity and Positions**: Determine new velocity and position values for particles using Eqs. (8) -(9). Recalculate evaluation values for the new particles as described in Step 3, and return to Step 4.

These steps outline the process of the Particle Swarm Optimization (PSO) method within the context of optimizing parameters in the ERNN-MANFIS framework for forecasting interval time series data.

The main advantages of the recommended method can be summarized as follows:

1. Implementing the fuzzy clustering method in the fuzzification step leads not to dealing with subjective judgments.
2. By implementing EANN in the identification of fuzzy relations, complex fuzzy relation tables, and complex matrix operations are not needed. Besides, the advantage of the flexible modeling of EANN is obtained.
3. By implementing the recommended model called ERNN-MANFIS, there is no need to use fuzzy numbers for input memberships, as the parameters of the output membership function are determined using the particle swarm optimization method.
4. The proposed method also removes both seasonality and trend.
5. In the recommended method, membership values were systematically obtained using the fuzzy c-means clustering method during the fuzzification step.

Application

In this study, seven different time series data in the form of interval representations were analyzed by using the MATLAB program to assess the forecasting performances of the recommended method called ERNN-MANFIS. All-time series comprise daily data from 2013 to 2020 with the lowest and the highest values. Time series are S&P 500, NASDAQ 100, NIKKEI 225, DAX, FTSE 100 and DOW 30, US Index. The index data was obtained from the Yahoo Finance website. The FTSE 100 index comprises stocks traded on the London Stock Exchange, the NIKKEI 225 index includes stocks traded on the Tokyo Stock Exchange, and the S&P 500 index consists of stocks traded on the New York Stock Exchange. The London Stock Exchange represents Europe's largest stock exchange, the Tokyo Stock Exchange represents Asia's largest, and the New York Stock Exchange is the largest in the world. Weekly closing prices of the NASDAQ 100 index are used, and the data was sourced from the Refinitiv Eikon database. The DAX index, from the Frankfurt Stock Exchange (Germany) (DAX Classic All Share), represents mid-efficient market types. The Dow 30 index represents the United States stock market, while the US Index refers to the US Dollar Index.

Table 1. Performance evaluations of MANFIS-ERNN utilizing seven different data sets based on RMSE

	DAX	DOW 30	NASDAQ 100	S&P 500	NIKKEI 225	FTSE 100	US Index
The Lowest	0,02069	0,02727	0,0315	0,00155	0,02443	0,0165	0,26547
The Highest	0,14384	0,03361	0,03345	0,00425	0,03229	0,01395	0,25931

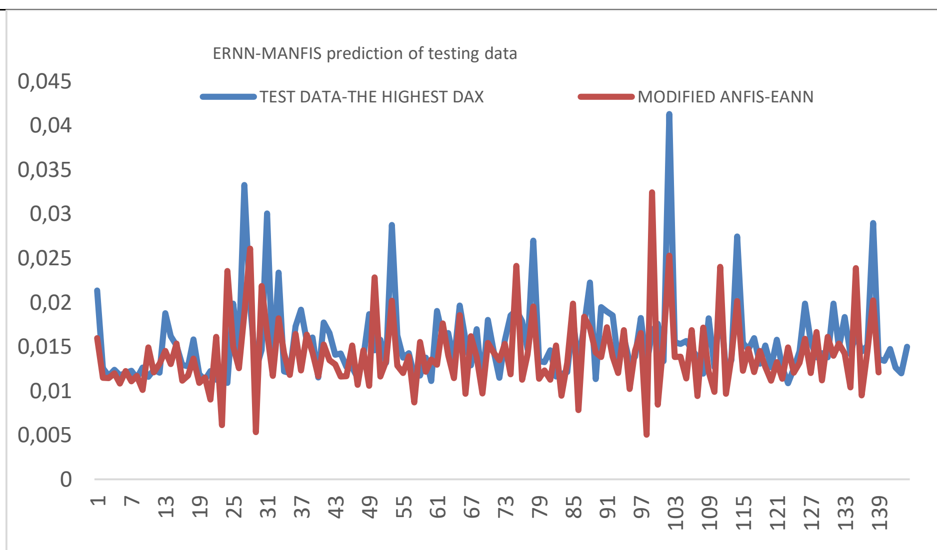

Figure 2. Time series graph of the test data and ERNN-MANFIS forecasts for the highest DAX series.

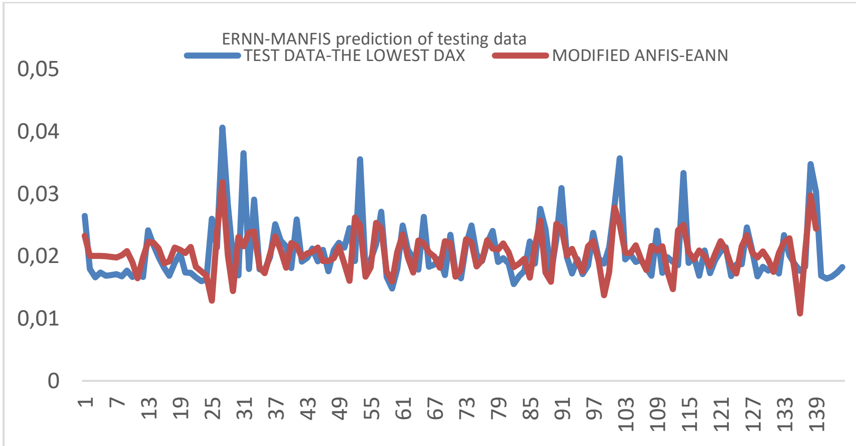

Figure 3. Time series graph of the test data and ERNN-MANFIS forecasts for the lowest DAX series

Table 2. Details of the ERNN-MANFIS for predicting time series

Parameters	Description/Value
Number of Clusters	5
Number of Particles for Each Population	30
Number of Iterations	100
Test data for the Modified ANFIS	15
Number of Input Layers	12
Number of Hidden Layers	12
Test Data for Elman Artificial Neural Network	144
Test Data for ERNN-MANFIS	139
Hybrid Optimization Method	ERNN and The Modified ANFIS
Activation function of the input layer	Logarithmic
Activation Function of The Hidden Layer	Linear

Table 3. The comparison of the results for the lowest DAX index data

Methods	Song & Chissom (1994)	Chen (1996)	Hurang et al. (1998)	ANFIS	Song & Chissom (1993b)	The Proposed Method
MSE	0.5114	0.0370	0.0055	0.0588	0.0776	1,24617E-06

Table 4. The comparison of the results for the highest DAX index data

Methods	Song & Chissom (1994)	Chen (1996)	Huarng et al. (1998)	ANFIS	Song & Chissom (1993b)	The Proposed Method
MSE	1,058	1,026	1,056	1,245	1,0045	0,0085

Discussion and Conclusions

A novel method called ERNN-MANFIS is proposed to improve forecasting accuracy when the time series data are in the form of ITS. Many datasets generated across various disciplines naturally take the form of interval data, which are often forced to be transformed into other forms, especially real numbers, leading to inevitable information loss. By directly addressing interval data, the proposed method offers an efficient alternative for processing such datasets, enabling deeper insights and more accurate analyses. The hybrid ERNN-MANFIS method demonstrates significant potential in terms of solution quality and efficiency.

An adopted hybrid strategy combines ERNN and MANFIS to obtain time series forecasting. The method integrates membership values derived systematically from fuzzy c-means clustering during the fuzzification of input values. In the training phase, ERNN is employed, while MANFIS is optimized using particle swarm optimization (PSO). The training of ERNN-MANFIS involves sequential steps with PSO to achieve improved performance across all computational processes. The hybridization of ELMAN and ANFIS in a single stage introduces a novel approach.

The advantages of the ERNN-MANFIS model include systematic derivation of membership values, elimination of complex fuzzy relationship tables, and elimination of effects such as seasonality, trend, etc. This model offers a flexible and efficient approach to increasing the forecast accuracy in interval-valued time series.

The proposed method used the seven different time series data in the form of interval representations and compared them with the most implemented methods in the literature, namely, Song & Chissom (1993b, 1994), Chen (1996), Hurang et al. (1998), ANFIS (2015). The results suggest that a fraction of the MSE score is attained by the proposed method when compared to highly implemented methods.

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Statements of Publication Ethics

We hereby declare that the study has not unethical issues and that research and publication ethics have been observed carefully.

Researchers' Contribution Rate

The study was conducted and reported with equal collaboration among the researchers.

Ethics Committee Approval Information

I hereby declare that I did not collect data from participants using any survey, interview, focus group study, observation, experiment or other interview techniques within the scope of the study whose information is given below, that I did not conduct any experiments on humans or animals, etc., that I did not violate the personal data protection law, and that I informed the other authors about the completion of this document as the responsible author; I declare as the responsible author that this study is one of the studies that does not require ethics committee approval.

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Ekler
MATLAB Codes
Song & Chissom (1994)

```
function [Ong, Defong,Ongg1]=songchissom(la,salt,x,ss,U)
% la:aralık uzunluğu
% salt: Evrensel kümenin alt sınırı
% x: zaman serisi
% ss: sınıf sayısı
n=length(x); % n:veri büyüklüðü
for i=1:ss
   ualt(i)=salt+(i-1)*la;
   uust(i)=salt+i*la;
   mb(i)=(ualt(i)+uust(i))/2;
end
% A'ları tanımlıyor.
A=eye(ss);% ss*ss lik birim matris oluşturur.
A(1,2)=0.5;A(ss,ss-1)=0.5;
for i=2:(ss-1)
   A(i,i+1)=0.5;
   A(i,i-1)=0.5;
end
fs=umax(U');% veriyi fuzzifike ettik.
fs1=fs(1:(n-1));
fs2=fs(2:n);
for j=1:ss
   for i=1:ss
      fr(i,j)=0;
   end
end
for j=1:ss
   for i=1:(n-1)
      if fs1(i)==j
         fr(fs1(i),fs2(i))=1;
      end
   end
end
k=0;
for i=1:ss
   for j=1:ss
      if fr(i,j)==1
         k=k+1;
         if k==1
            Q=minop(A(i,:),A(j,:));
         else
            R=minop(A(i,:),A(j,:));
            Q=matmax(Q,R);
         end
      end
   end
end
% Bulanýk deðerleri buluyoruz.
Ong=U(1,:);
```

```
for i=1:n
   ong=bileske(U(i,:),Q);
   Ong=[Ong;ong];
end
%Defuzzyifike adýmý
Ong=Ong(2:(n+1),:);
Sumong=sum(Ong');
for i=1:n
   for j=1:ss
      Song(i,j)=Ong(i,j)/Sumong(i);
   end
end
Defong=mb*Song';
mb
size(Defong)
e=Defong'-x;
MSE=sum(e.*e)/n
Ong1=umax(Ong')
for i=1:n
   Ongg1(i)=mb(Ong1(i));
end
RMSE=MSE^0.5
```

Chen(1996)

```
function [rmsetest,mapetest,rmseegt,mapeegt]=chen5(x,salt,ss,ntest)
%This program can be used to apply Chen (1996) method.
%Inputs:
% la=length of interval
% x = all data of time series (training and test set, it should be nx1
%dimension)
% salt= Lower bound for universe of discourse
% ss= number of fuzzy sets, it should be given 100 at least. Actual number
%of fuzzy sets are computed in program. ss should be given as very big.
% ntest = test set length, the test data are taken by end of the time
% series.
%Outputs:
% ongegt: Forecats for training set
% ongtest: Forecasts for test set
% rmseegt and rmsetest: Root of mean square error values for train and test
% sets, respectively.
% fr: The matrix for fuzzy logic group relation table
% fs: fuzzy sets
la=(max(x)-min(x))/ss;
n1=size(x,1);
n=n1-ntest;
xegt=x(2:n);
xtest=x((n+1):n1);
for i=1:ss
   ualt(i)=salt+(i-1)*la;
   uust(i)=salt+i*la;
   mb(i)=(ualt(i)+uust(i))/2;
end
```

```
for i=1:n
    for j=1:ss
        if ((x(i)<=uust(j))&(x(i)>=ualt(j)))
            fs(i)=j;
        end
    end
end
fs1=fs(1:(n-1));
fs2=fs(2:n);
for j=1:ss
for i=1:ss
fr(i,j)=0;
end
end
for j=1:ss
for i=1:(n-1)
    if fs1(i)==j
      fr(fs1(i),fs2(i))=1;
    end
end
end
ongoru=fr*mb';
agr=sum(fr');
for i=1:ss
    if agr(i)==0
        ong(i)=mb(i);
    else
        ong(i)=ongoru(i)/agr(i);
    end
end
ong2(1)=nan;
for i=2:n
    ong2(i)=ong(fs(i-1));
end
for i=(n+1):n1
    for j=1:ss
        if ((x(i-1)<=uust(j))&&(x(i-1)>=ualt(j)))
            fs(i-1)=j;
        end
    end
    ong2(i)=ong(fs(i-1));
end
ongegt=ong2(2:n);
ongtest=ong2((n+1):n1);
mseegt=(xegt'-ongegt)*(xegt'-ongegt)'/(n-1);
msetest=(xtest'-ongtest)*(xtest'-ongtest)'/(ntest);
rmseegt=power(mseegt,0.5);
rmsetest=power(msetest,0.5);
mapeegt=mean(abs((xegt'-ongegt)/xegt'));
mapetest=mean(abs((xtest'-ongtest)/xtest'));
end
```

Huarng et al. (1998)

```
function[RMSEtest,MAPEtest,DAtest,RMSEegt,MAPEegt,DAegt,Ong,Defegt,Deftest,yegt,ytest]=hwang98tv(
w,y,ss,ntest)
% Hwag et al.(1998)
% la:aralık uzunluğu
% salt: Evrensel kümenin alt sınırı
% x: zaman serisi
% ss: sýnýf sayýsý
x=diff(y);
la=(max(x)-min(x))/ss;
salt=min(x);
n=length(x)-ntest; % n:veri büyüklüğü
for i=1:ss
   ualt(i)=salt+(i-1)*la;
   uust(i)=salt+i*la;
   mb(i)=(ualt(i)+uust(i))/2;
end
for i=1:(n+ntest)
for j=1:ss
   if ((x(i)<=uust(j))&&(x(i)>=ualt(j)))
      fs(i)=j;
   end
end
end
for i=1:(n+ntest)
   if fs(i)==1
      U(i,:)=zeros(1,ss);
      U(i,1)=1;
      U(i,2)=0.5;
   elseif fs(i)==ss
      U(i,:)=zeros(1,ss);
      U(i,ss-1)=0.5;
      U(i,ss)=1;
   else
      U(i,:)=zeros(1,ss);
      U(i,fs(i)+1)=0.5;
      U(i,fs(i))=1;
      U(i,fs(i)-1)=0.5;
   end
end
A=eye(ss);
A(1,2)=0.5;A(ss,ss-1)=0.5;
for i=2:(ss-1)
   A(i,i+1)=0.5;
   A(i,i-1)=0.5;
end
n=n+ntest;
Ong=U(1,:);
for itv=w+1:n
   fs1=fs(itv-w:itv-2);
   OM=zeros(1,ss);
   for i1=1:w-1
```

```
        if i1==1
            OM(i1,:)=A(fs1(w-i1),:);
        end
        OM=[OM;A(fs1(w-i1),:)];
    end
    CM=A(fs(itv-1),:);
    CM2=CM;
    for i=1:w-1
        CM2=[CM2;CM];
    end
    R=OM.*CM2;
    ong=max(R);
    Ong=[Ong;ong];
end
Ong=Ong(2:end,:);
[Defong]=matrmax3(Ong,mb);
n=n-ntest;
Defegt=Defong(1:(n-w));
Deftest=Defong(n-w+1:(n+ntest-w));
yegt=Defegt+y(w:n-1)';
ytest=Deftest+y(n:n+ntest-1)';
n=length(y);
y1=y(w+2:n-ntest);
y2=y(n-ntest+1:n);
[RMSEegt,MAPEegt,DAegt]=kriter(yegt,y1);
[RMSEtest,MAPEtest,DAtest]=kriter(ytest,y2);
End
```

Song & Chissom (1993b)

```
function
[RMSEegt,MAPEegt,DAegt,RMSEtest,MAPEtest,DAtest,Defong,Ong,mb,RR1,Q]=songch1993b(x,ss,ntest)
%This program is written for Song & Chissom (1993b) method
% Inputs:
% x: Time Series
% ss: Number of Fuzzy Sets
% ntest: test set length, the test data are taken by end of the time
% series.
salt=min(x);
la=(max(x)-min(x))/ss;
n=length(x)-ntest;
for i=1:ss
    ualt(i)=salt+(i-1)*la;
    uust(i)=salt+i*la;
    mb(i)=(ualt(i)+uust(i))/2;
end
for i=1:(n+ntest)
for j=1:ss
    if ((x(i)<=uust(j))&&(x(i)>=ualt(j)))
        fs(i)=j;
    end
end
end
```

```
for i=1:(n+ntest)
    if fs(i)==1
        U(i,:)=zeros(1,ss);
        U(i,1)=1;
        U(i,2)=0.5;
    elseif fs(i)==ss
        U(i,:)=zeros(1,ss);
        U(i,ss-1)=0.5;
        U(i,ss)=1;
    else
        U(i,:)=zeros(1,ss);
        U(i,fs(i)+1)=0.5;
        U(i,fs(i))=1;
        U(i,fs(i)-1)=0.5;
    end
end
A=eye(ss);
A(1,2)=0.5;A(ss,ss-1)=0.5;
for i=2:(ss-1)
    A(i,i+1)=0.5;
    A(i,i-1)=0.5;
end
fs1=fs(1:(n-1));
fs2=fs(2:n);
for j=1:ss
    for i=1:ss
        fr(i,j)=0;
    end
end
for j=1:ss
    for i=1:(n-1)
        if fs1(i)==j
            fr(fs1(i),fs2(i))=1;
        end
    end
end
k=0;
for i=1:ss
    for j=1:ss
        if fr(i,j)==1
            k=k+1;
            if k==1
                Q=minop(A(i,:),A(j,:));
            else
                R=minop(A(i,:),A(j,:));
                RR1{k,1}=R;
                Q=matmax(Q,R);
            end
        end
    end
end
n=n+ntest;
```

```
Ong=U(1,:);
for i=1:n
   ong=bileske(U(i,:),Q);
   Ong=[Ong;ong];
end
Ong=Ong(2:(n+1),:);
[Defong]=matrmax2(Ong,mb);
n=n-ntest;
xegt=x(2:n);
xtest=x(n+1:(n+ntest));
Defegt=Defong(1:(n-1));
Deftest=Defong(n:(n+ntest-1));
[RMSEegt,MAPEegt,DAegt]=kriter(Defegt,xegt);
[RMSEtest,MAPEtest,DAtest]=kriter(Deftest,xtest);
```

ANFIS

```
function
[RMSEtest1,MAPEtest1,RMSEtest2,MAPEtest2,RMSEtest3,MAPEtest3,RMSEtest4,MAPEtest4,Ong]=ANFI
Scoz(xy,m,ntest)
x=(xy-min(xy))/(max(xy)-min(xy));
M=lagmatrix(x,m);
ns=size(M,1);
nsut=size(M,2);
M=M(max(m)+1:ns,:);
ns=size(M,1);
Megt=M(1:ns-ntest,:);
Mtest=M(ns-ntest+1:ns,:);
Mtest(:,nsut)=Mtest(:,nsut)*(max(xy)-min(xy))+min(xy);
numMFs=[2 3 4 5];
for i=1:4
   % 'gbellmf'
   in_fis = genfis1(Megt,numMFs(i),'gbellmf');
   out_fis = anfis(Megt,in_fis,50);
   Ong{i,1}=evalfis(Mtest(:,1:nsut-1),out_fis);
   Ong{i,1}=Ong{i,1}*(max(xy)-min(xy))+min(xy);
   [RMSEtest1(i),MAPEtest1(i)]=kriter(Ong{i,1},Mtest(:,nsut)');
   %'gaussmf'
   in_fis = genfis1(Megt,numMFs(i),'gaussmf');
   out_fis = anfis(Megt,in_fis,50);
   Ong{i,2}=evalfis(Mtest(:,1:nsut-1),out_fis);
   Ong{i,2}=Ong{i,2}*(max(xy)-min(xy))+min(xy);
   [RMSEtest2(i),MAPEtest2(i)]=kriter(Ong{i,2},Mtest(:,nsut)');
   %trapmf
   in_fis = genfis1(Megt,numMFs(i),'trapmf');
   out_fis = anfis(Megt,in_fis,50);
   Ong{i,3}=evalfis(Mtest(:,1:nsut-1),out_fis);
   Ong{i,3}=Ong{i,3}*(max(xy)-min(xy))+min(xy);
   [RMSEtest3(i),MAPEtest3(i)]=kriter(Ong{i,3},Mtest(:,nsut)');
   %trimf
   in_fis = genfis1(Megt,numMFs(i),'trimf');
   out_fis = anfis(Megt,in_fis,50);
   Ong{i,4}=evalfis(Mtest(:,1:nsut-1),out_fis);
```

```
  Ong{i,4}=Ong{i,4}*(max(xy)-min(xy))+min(xy);
  [RMSEtest4(i),MAPEtest4(i)]=kriter(Ong{i,4},Mtest(:,nsut)');
end
```

GENİŞLETİLMİŞ ÖZET

Aralık değerli zaman serileri, verilerin zamanla belirli aralıklar halinde toplandığı ve özellikle finans, mühendislik, tıp ve meteoroloji gibi birçok alanda önemli uygulamalara sahip bir tekniktir. İlk olarak Bock ve Diday (2000) tarafından tanıtılan bu kavram, klasik veri analizini ve istatistiksel yöntemleri aralık değerli veriler gibi sembolik verilere genişleterek, yapay zeka ve desen tanıma alanlarında daha kapsamlı analizler yapılmasına olanak tanır. Aralık değerli serilerin analizi için çeşitli yöntemler geliştirilmiştir. Bunlar arasında aralık değerli zaman serileri histogramları (De Carvalho, 1995), temel bileşen analizi (Lauro & Palumbo, 2000; Cazes et al., 1997) ve merkezi eğilim ile yayılma ölçüleri (Bertrand & Goupil, 2000; Billard & Diday, 2003) bulunmaktadır. Ayrıca, Maia ve ark. (2008) tarafından önerilen dört farklı yaklaşım, aralık değerli zaman serilerinin modellenmesi ve tahmin edilmesi için AR, ARIMA, yapay sinir ağları (ANN) ve hibrit modelleri kullanmaktadır. Yapay sinir ağları (ANN) gibi modeller, borsa tahmini gibi uygulamalarda yaygın olarak kullanılmış ve üstel düzeltme modellerinin üstünlüğü vurgulanmıştır. Bu çalışma ERNN-MANFIS adı verilen yeni bir hibrit model önermektedir. Bu model Parçacık Sürü Optimizasyonu (PSO) ile optimize edilmiş, Adaptif Neuro-Fuzzy Çıkarım Sistemi (ANFIS) ve Elman Tekrarlayan Sinir Ağı (ERNN) birleşimi ile oluşturulmuştur. ERNN, zaman serisi verilerinin zamansal desenlerini yakalamaya olanak tanırken, MANFIS bulanık mantığı kullanarak doğrusal olmayan ilişkileri modellemektedir. Bu hibrit model, verilerdeki karmaşık ilişkileri daha iyi anlamak ve tahmin doğruluğunu artırmak amacıyla PSO ile optimize edilmektedir. ERNN-MANFIS modelinin avantajları arasında üyelik değerlerinin sistematik bir şekilde türetilmesi, karmaşık bulanık ilişki tablolarının ortadan kaldırılması ve mevsimsellik, trend gibi etkilerin giderilmesi yer almaktadır. Bu model, aralık değerli zaman serilerindeki tahmin doğruluğunu artırmak için esnek ve verimli bir yaklaşım sunmaktadır. Model, S&P500, NASDAQ100, NIKKEI225 gibi finansal endeksler dahil olmak üzere yedi farklı veri setine uygulanmış ve sonuçlar, ERNN-MANFIS'in mevcut yöntemlere göre daha iyi öngörü doğruluğu sağladığını göstermiştir. Ortalama Karekök Hatası (RMSE) kullanılarak yapılan performans değerlendirmesi, önerilen modelin doğruluğunun arttığını ortaya koymuştur. Çalışmada, aralık değerli zaman serilerinin doğru tahmin edilmesinin, planlama ve strateji oluşturma gibi çeşitli alanlarda kritik öneme sahip olduğu vurgulanmaktadır. Literatür, tahmin için stokastik ve stokastik olmayan birçok yöntem sunmakta olup, yöntem seçimi genellikle verilerin niteliğine bağlıdır. Aralık değerli zaman serileri, kesin değerler yerine aralıklar olarak toplanan verilerle ilgili bir tekniktir. Klasik zaman serisi yöntemleri bu tür verilere uygulanabilse de, aralık verilerini tek nokta verilerine dönüştürmek bilgi kaybına yol açabilir. Aralık değerli veriler, özellikle finansal fiyat aralıkları, mühendislikte voltaj aralıkları, tıpta kan basıncı aralıkları ve meteorolojide yağış aralıkları gibi uygulamalara sahiptir.

Ichino ve arkadaşlarının (1996) erken çalışmaları, aralık verileri için bölge odaklı metodolojiyi kullanarak sembolik sınıflandırıcılar geliştirmiştir. Sonraki yıllarda, aralık zaman serileri tahmini için yapay sinir ağları (ANN), ARIMA ve hibrit modeller kullanılarak daha kapsamlı yöntemler geliştirilmiştir. Bu çalışmada, ERNN-MANFIS adı verilen yeni bir hibrit model önerilmektedir. Bu model, Elman Tekrarlayan Sinir Ağı (ERNN) ile Değiştirilmiş Adaptif Neuro-Fuzzy Çıkarım Sistemi (MANFIS)'ni birleştirerek, aralık değerli zaman serilerini modellemekte daha etkili bir yöntem sunmaktadır. MANFIS modeli, ERNN ile birleştirilerek aralık değerli zaman serileri tahmini için yeni bir yaklaşım getirilmiştir. Hibrit model, bulanık c-ortalamalar kümeleme tekniği kullanarak üyelik değerlerini sistematik olarak türetir, bu da tahmin doğruluğunu artırır. Eğitim aşamasında, PSO ile optimize edilen MANFIS parametreleri modelin genel performansını artırmaktadır. Çalışma, aralık değerli zaman serilerinin tahmini için geliştirilen bu yeni hibrit modeli farklı veri setlerinde test etmekte ve bu yöntemin doğruluğunu mevcut yöntemlerle karşılaştırmaktadır. Son olarak, çalışmanın sonuçları ve gelecekteki araştırma alanlarına dair tartışmalar yer almaktadır. Aralık değerli zaman serileri, sembolik veri analizi kapsamında önemli bir yer tutar ve zaman içindeki veri değişkenliğini basitçe temsil eder.