

# Improved Compound Multiphase Waveforms with Additional Amplitude Modulation (periodic mode) for Marine Radars

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## Abstract

This paper has presented the basis of a compound multiphase waveform design with additional amplitude modulation, capable of controlling a waveform pick-factor, suitable for use with marine radar. The waveform shows good Doppler tolerance, with the low side-lobes performance maintained over the central zone of an ambiguity function. A clear waveform design procedure has been presented that does not require the implementation of numerical optimization procedures. It has been shown that a compromise between side-lobes suppression and the value of pick-factor can be found. These waveforms allow achieving better results as compared to compound multiphase waveforms without additional amplitude modulation under mismatched weighting filtering. In this article we considered periodic mode of Radar.

**Keywords:** “Composite signal, Ambiguity function, Cross-ambiguity function, Amplitude modulation, Periodic mode”

## 1. Introduction

The compound multiphase signal consists of multiplication of two sequences; each of them is a signal with a quadratic variation of phases. Expressions for complex amplitudes for the base (snB) and external (snV) signal and final multiphase compound signal's sequence (sn) are follows [1,2]:

$$s_n^B = \exp \left\{ j \frac{\pi}{4} \alpha' \left[ 2 \left( n - N_B E \left[ \frac{n}{N_B} \right] + 1 \right) - (N_B + \mu_{0B}) \right]^2 \right\} \quad (1)$$

$$s_n^V = \exp \left\{ j \frac{\pi}{4} \beta' \left[ 2 \left( E \left[ \frac{n}{N_{B_1}} \right] - N_V E \left[ \frac{E \left[ \frac{n}{N_{B_1}} \right]}{N_V} \right] + 1 \right) - (N_V + \mu_{0V}) \right]^2 \right\}, n = \overline{0, N-1} \quad (2)$$

$$s_n = s_n^V \cdot s_n^B \quad (3)$$

where  $\alpha' = \alpha T_0^2$ ;  $\beta' = \beta (T_0 N_B)^2$ ;  $\alpha, \beta, \mu_{0B}, \mu_{0V}, N_{B_1}$  – parameters of phase modulation;  $T_0$  – duration of one pulse;  $N_B T_0$  – period of the base sequence;  $N_V T_0$  – period of the external sequence;  $N T_0 = N_B N_V T_0$  – period of the sequence;  $E[x]$  – integer part of  $x$ . If  $N_B$  is even so  $\mu_{0B} = 0$ , if  $N_B$  is no even so  $\mu_{0B} = 1$ .  $\mu_{0V}$  is formed by the same way. As opposite to previous prepared article (we considered aperiodic mode of Radar) [3], in this article we considered periodic mode, which is important for building continuous wave radar. For illustration three types of periodic compound multiphase signals were considered. The length of sequence is  $N=324$  ( $N_B=18, N_V=18$ ),  $\mu_{0B}=\mu_{0V}=0$  with parameters:

1.  $\alpha' = -1/N_B, \beta' = 1/N_B^2$ . The signal with such parameters has a maximum ratio of the free zone (FZ) area around the central peak (CP) to the CP topographic section area (on the zero level) [2].
2.  $\alpha' = -1/N_B, \beta' = 1/N_B$ . The side-lobes level is increased in the FZ region for these parameters in comparative to the case 1). But the Autocorrelation Function (ACF) has the lowest side-lobes level for its entire length. It should also be noted that FZ decreases around CP of Ambiguity Function (AF) [4].

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$$\alpha' = -1/N_B, \beta' = 0.$$

At the same time the periodic compound multiphase signals with the length of sequence  $N=306$  ( $N_B=18, N_V=17$ ),  $\alpha'=1/N_B, \beta' = 2/N_V, \mu_{0B}=0, \mu_{0V}=1$  were considered..

Signals with an additional amplitude modulation (AM) can be described by the following expression:

$$s_n^{am} = s_n * v_n \quad (4)$$

$$v_n = v_n^b \cdot v_n^v \cdot E\left[\frac{n}{N_V}\right]_{+1}, n = 0 \div N-1,$$

where  $s_n^{am}$  - compound multiphase signal with an additional AM;  $s_n$  - complex envelope of the signal;  $v_n^b, v_n^v$  - weighting coefficients for the base and external sequences.

The expression of the AF for the periodic compound multiphase signal with an additional AM has the form:

$$\chi_{ss}(k, l) = \sum_{n=0}^{N-1} s_n^{am} * s_{(n+k)}^{am} \cdot e^{i \frac{2\pi l n}{4N}} \quad (5)$$

where  $k$  - discrete values of time delay with the selected step  $T_0$ ;  $l$  - discrete values of frequency.

It's convenient to use the pick-factor parameter for describing behaviour of AM modulation [5]. It is defined as the ratio:

$$\xi = \frac{|s_{n\_max}^{am}|^2 N}{\sum_{n=0}^{N-1} |s_n^{am}|^2} \quad (6)$$

The law of the changing weighting function sin is considered in the paper. This function allows controlling the behaviour of AF. Its advantage is possibility to control the signal peak-factor and the side-lobes level of AF.

$$v_n^B = \sin\left[\pi\left(\frac{y_B}{N_B+1} + \frac{n}{N_B+z_B}\right)\right], n = 0 \div N_B-1 \quad (7)$$

$$v_n^V = \sin\left[\pi\left(\frac{y_V}{N_V+1} + \frac{n}{N_V+z_V}\right)\right], n = 0 \div N_V-1$$

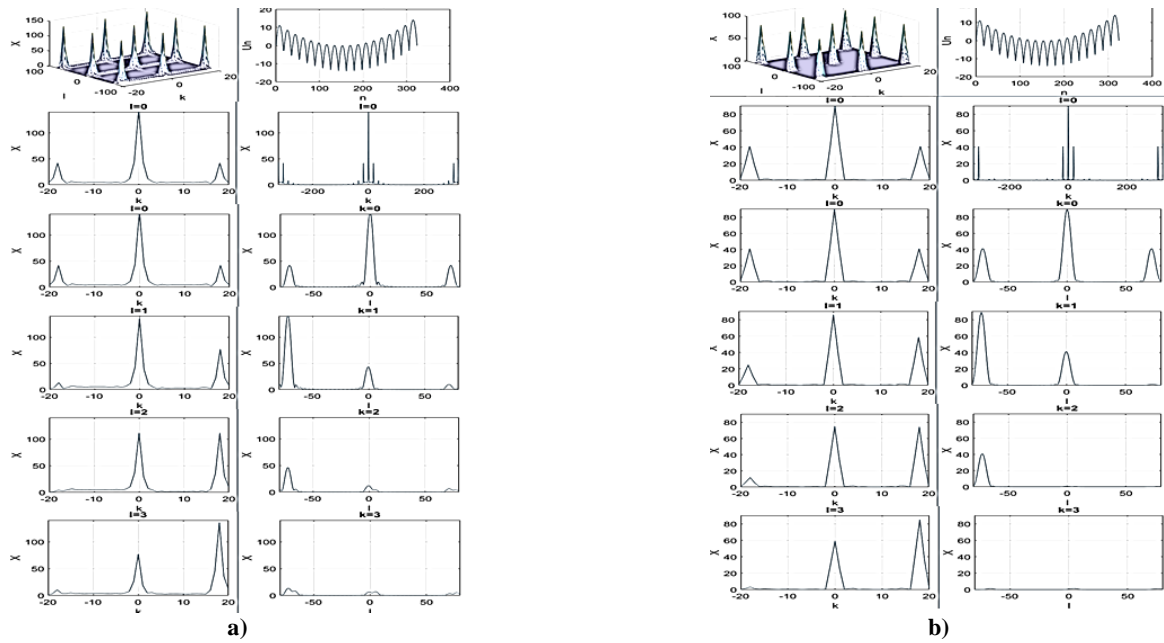
where  $y_B, y_V$  - the positive numbers for the base and external sequences, respectively, varies in the range  $1 \leq y_B < \frac{N_B+1}{2}$ ,  $1 \leq y_V < \frac{N_V+1}{2}$ ;  $z_B, z_V$  - variables for the basic and external sequences are  $z_B = \frac{2y_B N_B - (N_B+1)}{N_B+1-2y_B}$ ,  $z_V = \frac{2y_V N_V - (N_V+1)}{N_V+1-2y_V}$ .

The formula (8) describes other one of the possible weighting functions for the signals (3):

$$v_n^B = \left(\sin\left[\pi\left(\frac{y_B}{N_B+1} + \frac{n}{N_B+z_B}\right)\right]\right)^2, n = 0 \div N_B-1 \quad (8)$$

$$v_n^V = \left(\sin\left[\pi\left(\frac{y_V}{N_V+1} + \frac{n}{N_V+z_V}\right)\right]\right)^2, n = 0 \div N_V-1$$

The structure of AF of multiphase compound signals with an additional AM is compared with Cross-ambiguity function (CAF) compound multiphase signals without AM, but with mismatched treatment using weighing functions sin (7) and sin<sup>2</sup> (8). On the left upper part of the Figure the body of CAF in the central peak area is represented. Lower the sections of CAF along axe of range for different discrete values of Doppler frequency ( $l=0, l=1, l=2, l=3, l=4$ ) in FZ (central peak area) are represented. On the right upper part of the Figure the law of the phase modulation of the composite multiphase signal is illustrated. Lower the Cross correlation function (section of CAF by the plane  $l=0$ ) on all it's length is presented. And more lower the sections of CAF along Doppler axe by the planes ( $k=0; k=1; k=2; k=3$ ).

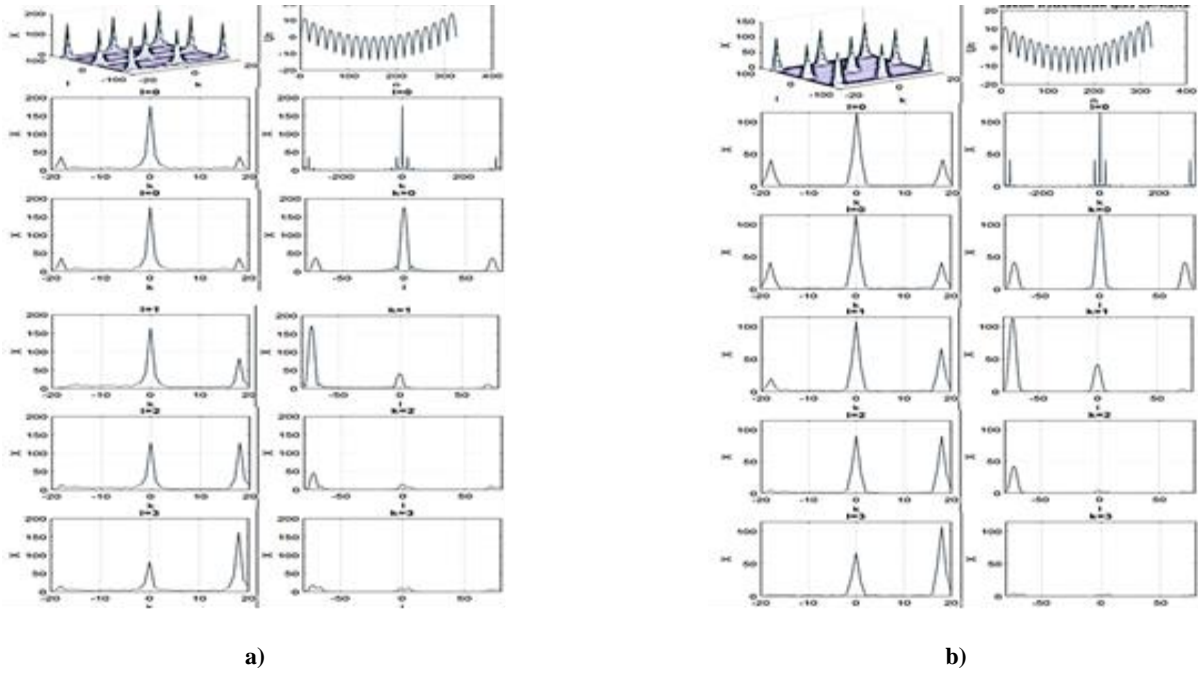


**Fig 1. a)** The body of the CAF of the periodic multiphase compound signal without an additional AM,  $N=324$  ( $N_B=18$ ,  $N_V=18$ )  $\alpha'=-1/N_B$ ,  $\beta'=1/N_B$ ,  $\mu_{0V} = \mu_{0B} = 0$ ,  $N_{B1} = N_B$ , with the weighting function  $\sin$  ( $y_B = y_V = 1$ ), phases of the signal, sections of the body of the CAF; **b)** the body of the AF periodic multiphase compound signal with an additional AM,  $N=324$  ( $N_B=18$ ,  $N_V=18$ )  $\alpha'=-1/N_B$ ,  $\beta'=1/N_B$ ,  $\mu_{0V} = \mu_{0B} = 0$ ,  $N_{B1} = N_B$ , with the weighting coefficients  $\sin$  ( $y_B = y_V = 1$ ), phases of the signal, sections of the body of the AF.

As a result (Fig.1,2,3), side-lobes level is significantly reduced after using the periodic compound multiphase signals with an additional AM. CP area isn't expanded significantly. By increasing the  $y_B$  and  $y_V$ , the value of peak-factor is decreasing. By changing the parameters  $y_B$  and  $y_V$  we can control the peak-factor of the signal. Table 1 shows these values:

**Table 1. The peak-factor dependence from parameters  $y_B=y_V$  for periodic multiphase compound signal with an additional AM,  $N=324$  ( $N_B=18$ ,  $N_V=18$ )  $\alpha'=-1/N_B$ ,  $\beta'=1/N_B$ ,  $\mu_{0V} = \mu_{0B} = 0$ ,  $N_{B1} = N_B$ , (matched processing) with the weighting coefficients  $\sin$**

$y_B=y_V$	with the weighting	with the weighting
	coefficients $\sin$	coefficients $\sin^2$
$\xi$		
1	3.59	6.38
2	2.83	4.97
3	2.24	3.77
5	1.5	2,09
6	1.28	1,6
7	1.13	1,28
8	1.05	1.09
9	1.01	1.01



**Fig 2. a) The body of the CAF of the periodic multiphase compound signal without an additional AM,  $N=324$  ( $N_B=18, N_V=18$ )  $\alpha'=-1/N_B, \beta'=1/N_B, \mu_{0V} = \mu_{0B} = 0, N_{B1} = N_B$ , with the weighting function  $\sin(y_B=y_V=2)$ , phases of the signal, sections of the body of the CAF; b) the body of the AF periodic multiphase compound signal with an additional AM,  $N=324$  ( $N_B=18, N_V=18$ )  $\alpha'=-1/N_B, \beta'=1/N_B, \mu_{0V} = \mu_{0B} = 0, N_{B1} = N_B$ , with the weighting coefficients  $\sin(y_B=y_V=2)$ , phases of the signal, sections of the body of the AF**

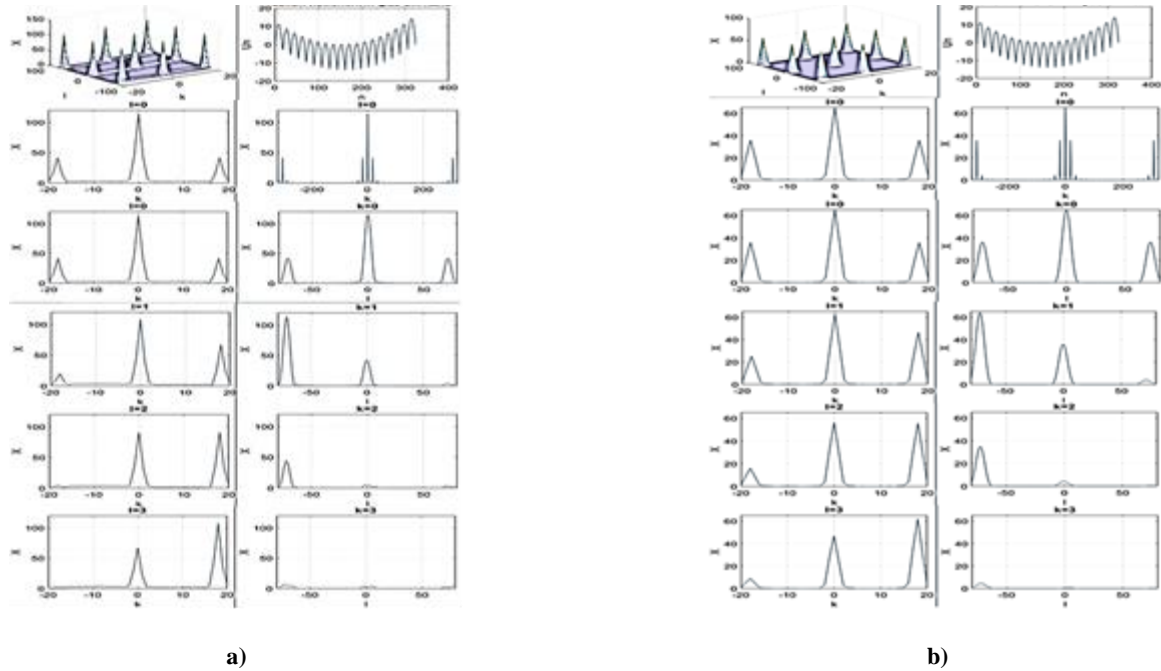
The calculations were made for three types of the signals, as indicated above:

$$\alpha'=-1/N_B, \beta'=1/N_B^2, N=324 (N_B=18, N_V=18).$$

$$\alpha'=-1/N_B, \beta'=1/N_B, N=324 (N_B=18, N_V=18),$$

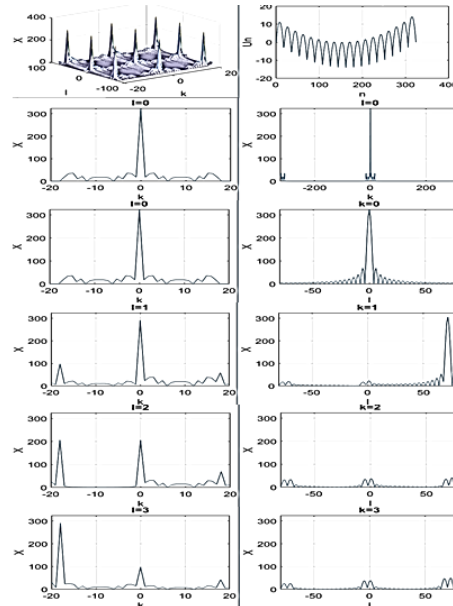
$$\alpha'=-1/N_B, \beta'=0, N=324 (N_B=18, N_V=18).$$

The three kinds of signals with different sets of parameters were compared. The compound multiphase signal with an additional AM  $N=324$  ( $N_B=18, N_V=18$ ),  $\alpha'=-1/N_B, \beta'=1/N_B, \mu_{0V} = \mu_{0B} = 0, N_{B1} = N_B$ , (matched processing) with the weighting function  $\sin$  shows the highest ratio of the CP to the side-lobes level. The signal with  $\beta'=1/N_B^2$  shows almost the same. For example, we demonstrated the cases of signals with parameters  $\alpha'=-1/N_B, \beta'=1/N_B, N=324$  ( $N_B=18, N_V=18$ ),  $y_B=y_V=1,2$  (Fig.1,2). They have the highest side-lobe level in the FZ region.



**Fig 3. a)** The body of the CAF of the periodic multiphase compound signal without an additional AM,  $N=324$  ( $N_B=18, N_V=18$ )  $\alpha'=-1/N_B, \beta'=1/N_B, \mu_{0V}=\mu_{0B}=0, N_{B1}=N_B$ , with the weighting function  $\sin^2$  ( $y_B=y_V=2$ ) for (8), phases of the signal, sections of the body of the CAF; **b)** the body of the AF periodic multiphase compound signal with an additional AM,  $N=324$  ( $N_B=18, N_V=18$ )  $\alpha'=-1/N_B, \beta'=1/N_B, \mu_{0V}=\mu_{0B}=0, N_{B1}=N_B$ , with the weighting coefficients  $\sin^2$  ( $y_B=y_V=2$ ) for (8), phases of the signal, sections of the body of the AF

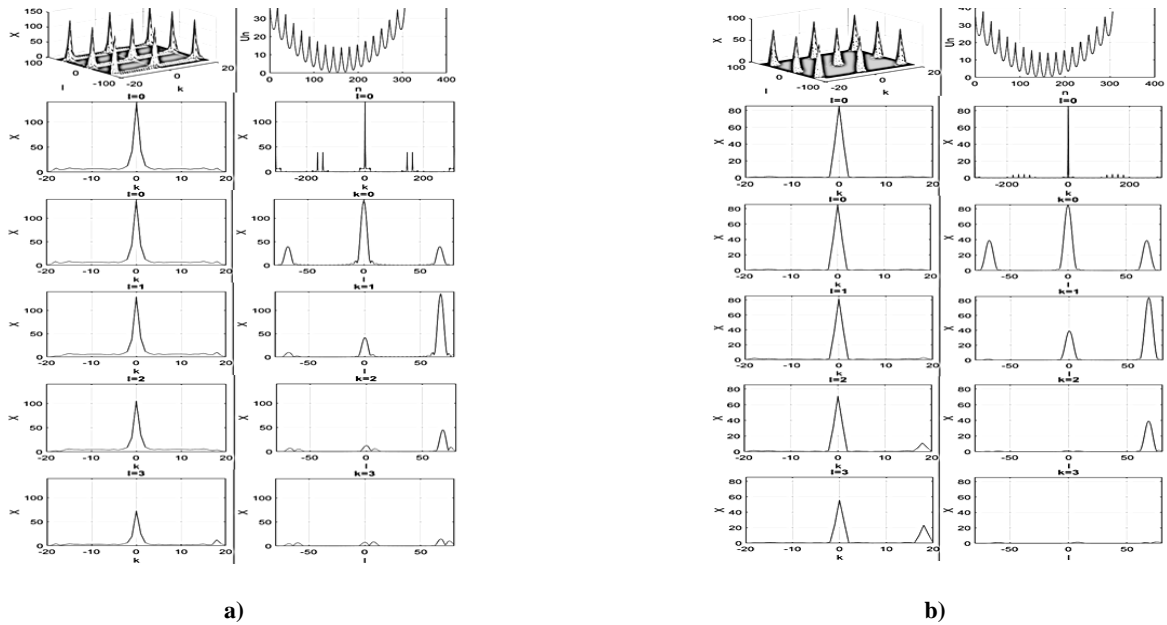
Thus, the lowest side-lobes level was given by signals with peak-factor  $\zeta = 3,59$  and  $\zeta = 2,83$  for weighting function  $\sin$ , while  $y_V=y_B=1,2$ . Peak-factor for signals with lowest side-lobes level were  $\zeta=6,38$  and  $\zeta=4,97$  (for weighting function  $\sin^2$ ). For comparison AF body for rectangular shape of compound signals is presented on Figure 4.



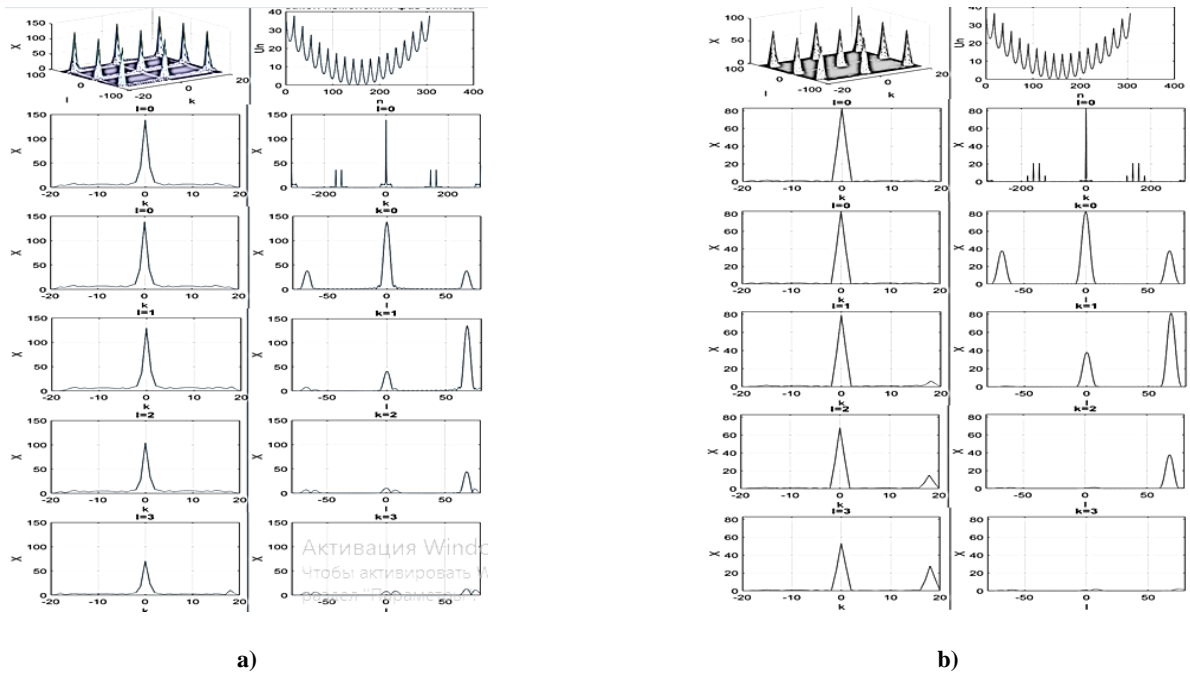
**Fig 4. The body of the AF of the periodic multiphase compound signal,  $N=324$  ( $N_B=18, N_V=18$ ),  $\alpha'=-1/N_B, \beta'=1/N_B, \mu_{0V}=\mu_{0B}=0, N_{B1}=N_B$ , phases of the signal, sections of the body of the AF**

Undoubtedly, the compound signal on Fig. 4 has the highest side-lobes level in the FZ region. This is a big drawback of this type processing.

The signals with different pulses' quantity of base and external sequence ( $N_B=18, N_V=17$ ) are considered. Parameters of signal are  $\alpha'=1/N_B, \beta'=2/N_V, \mu_{0V}=1, \mu_{0B}=0, N_{B1}=N_B$ .



**Fig 5. a)** The body of the CAF of the periodic multiphase compound signal without an additional AM,  $N=306$  ( $N_B=18, N_V=17$ )  $\alpha'=1/N_B, \beta'=2/N_V, \mu_{0V}=1, \mu_{0B}=0, N_{B1}=N_B$ , with the weighting function  $\sin$  ( $y_B=y_V=1$ ), phases of the signal, sections of the body of the CAF; **b)** the body of the AF periodic multiphase compound signal with an additional AM,  $N=306$  ( $N_B=18, N_V=17$ )  $\alpha'=1/N_B, \beta'=2/N_V, \mu_{0V}=1, \mu_{0B}=0, N_{B1}=N_B$ , with the weighting coefficients  $\sin$  ( $y_B=y_V=1$ ), phases of the signal, sections of the AF



**Fig 6. a)** The body of the CAF of the periodic multiphase compound signal without an additional AM,  $N=306$  ( $N_B=18, N_V=17$ )  $\alpha'=1/N_B, \beta'=2/N_V, \mu_{0V}=1, \mu_{0B}=0, N_{B1}=N_B$ , with the weighting function  $\sin^2$  ( $y_B=y_V=3$ ) for (8), phases of the signal, sections of the body of the CAF; **b)** the body of the AF periodic multiphase compound signal with an additional AM,  $N=306$  ( $N_B=18, N_V=17$ )  $\alpha'=1/N_B, \beta'=2/N_V, \mu_{0V}=1, \mu_{0B}=0, N_{B1}=N_B$ , with the weighting coefficients  $\sin^2$  ( $y_B=y_V=3$ ) for (8), phases of the signal, sections of the body of the AF

## 2. Results

The results of research for these cases are presented in Table 2:

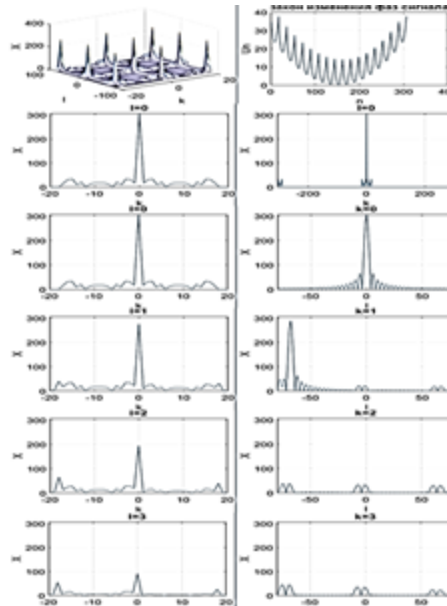
**Table 2. The peak-factor dependence from parameters  $y_B=y_V$  for periodic multiphase compound signal with an additional AM,  $N=306$  ( $N_B=18, N_V=17$ )  $\alpha'=1/N_B, \beta'=2/N_V, \mu_{0V}=1, \mu_{0B}=0, N_{B1}=N_B$ , (matched processing) with the weighting coefficients  $\sin$  and  $\sin^2$**

$y_B=y_V$	with the weighting	with the weighting
	coefficients $\sin$	coefficients $\sin^2$
	$\zeta$	
1	3.58	6.36
2	2.80	4.92
3	2.21	3.69
5	1.46	2.01
6	1.25	1.53
7	1.11	1.24
8	1.03	1.07
9	1.00	1.01

As we can see, if we reduce quantity of pulses in the signal the pick-factor is reduced. The side-lobes level in the FZ region does not hesitate in large limits if we use  $N=306$  or  $N=324$ . But periodic multiphase compound signal with an additional AM with  $N=306$  have lower side-lobes level on all sections of the body of the AF (for  $l=0$ ), without high side peak.

To summarize, periodic multiphase compound signal with an additional AM have the lowest side-lobes level. A matched processing was used, so we can say this type of signal does not have signal to noise losses.

These results can be used not only for the compound multiphase signals, but also for equivalent compound LFM signals [6,7,8].



**Fig 7. The body of the AF of the periodic multiphase compound signal,  $N=306$  ( $N_B=18, N_V=17$ )  $\alpha'=1/N_B, \beta'=2/N_V, \mu_{0V}=1, \mu_{0B}=0, N_{B1}=N_B$ , phases of the signal, sections of the body of the AF**

To summarize, periodic multiphase compound signals with an additional AM have the lowest side-lobes level in FZ and have good Correlation functions with low sensitivity to the Doppler shifts. A matched processing was used, so this type of signal does not have signal to noise ratio losses.

These results can be used not only for the compound multiphase signals, but also for equivalent compound LFM signals [6,7,8].

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