
**POLİTEKNİK DERGİSİ**

***JOURNAL of POLYTECHNIC***

ISSN: 1302-0900 (PRINT), ISSN: 2147-9429 (ONLINE)
URL: http://dergipark.gov.tr/politeknik

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Bu makaleye şu şekilde atıfta bulunabilirsiniz(To cite to this article): Khajishvili M., Gomidze N., Jabnidze I., Makharadze K. and Chikhladze M., "Evalution bandwidth of optical signal via statistical moments of random phase screen", ***Politeknik Dergisi***, 21(1): 123-128, (2018).

Erişim linki (To link to this article): http://dergipark.gov.tr/politeknik/archive

DOI: **10.2339/politeknik.379641**

# Evalution Bandwidth of Optical Signal via Statistical Moments of Random Phase Screen

*Araştırma Makalesi / Research Article*

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(Received : 14.01.2017 ; Accepted : 16.02.2017)

**ABSTRACT**

The majority of the works are dedicated to the specific problem, for instance, scattering from the rough surface although, the general theoretical bases of the statistical and spectral characteristics of the scattered light for the light of non-Gaussian statistics have not been studied so far. The signals with such statistics are produced when the characteristic scales of the refraction factor is in comparison with the size of illuminated region. In the present paper, we discuss the method of signal processing for analyzing first and second order statistical moments of electromagnetic radiation.

**Keywords: Phase screen, laser radiation, statistical moments.**

## 1. INTRODUCTION

The superhigh frequency range the analogical methods of signal processing are well known. Analogical methods were first applied in 1950s of the last century in visible part of the spectrum [1]. But since the laser sources of the spectrum have been invented the analogical methods were widely applied in spectroscopy as well. As for the numerical methods of signal processing, which are discussed in the present project, are one of the actual problems of modern society.

Nowadays, there is not a universal analytical method for solving such problems and therefore, we can get the information on the scattered radiation by applying the figural methods or from the experiment itself.

Via using lasers, an important light field can be made on large distances from the device and distance sounding to be carried out [2]. In short impulses (≈10-8-10-9 c) the light concentration enables us to do lab analyses without darkening (on the expenses of the receiver's gating). High spectral brightness of laser radiation provides the high sensibility of laser fluorometer and we can make an express-analysis of natural waters sample (some cm3) without their concentration, we can also register the Raman-scattering signal of water molecules and use it as a comparative signal - internal Rapper.

In the work [3] are presented graphics of spectrum samples of relative intensity of the fluorescence of Hexane, distilants, drinking water, sea water and oil extract taken by our modernized laser-spectroscopy equipment.

We can evaluate the fluctuations of light wave in the randomly inhomogeneous media by means fluctuations of intensity and phase that is why, the statistic characteristics of the field are defined by the statistic characteristics of intensity and phase [4].

The source of the ray is laser equipment because phase screen is randomly inhomogeneous media. That is why, it is obvious that the inclination of the ray from the straight direction takes place. We should discuss the propagation of the ray in the randomly inhomogeneous media as a stochastic process, and we can use Einstein-Fokker-Kolmogorov's equation for an angular separation of rays [5].

The classification of wave phenomena is definitely connected to dynamic problem statement. Rather wide class of wave problems is formulated as following: say, the body (or system of bodies) covered with surface is placed in homogeneous or inhomogeneous media, in which the waves of this or that nature are propagated (electromagnetic, acoustic, flexible, spin waves, etc.). Respectively, indicate the linear, differential or integro-differential wave operator with . In the area of wave equation which is free from the wave sources, we will have the following expression [5].

$$\hat{L}u = 0,$$

Where, - is a wave field, which may be scalar or vector. In case when it is vector, operator will be a tensor. The primary wave is created either with real sources, or virtual sources. For instance, the primary wave can be , which in most cases is a flat wave, and we have to find a scattered field. Besides the sources and scattering body shape certain boundary condition for limited surface of scattering body and the radiation condition are required (or the condition in infinity). In the determined task we would have primary field on the surface. The primary field statistics is determined only by the second order moment [6,7]

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$$\Gamma_u^0(1,2) = \langle u_0(1) u_0^*(2) \rangle ,$$

In optics it is called the function of the second order coherence. We may have the higher order moments of the primary field. Obviously, the complete statistic description of the $u$ field is obtained through the densities of probability of $n$ $(n = 1,2...)$ order. In the conditions of the given task, random can be: a) real $q$ sources; b) virtual sources ($u_0/S_0$ ; c) $S$ boundary form and condition; d) media property, i. e. $\hat{L}$ operator itself. Of course, the problem statement does not imply the determination of the correspondent methods – approximate methods. Actually, the fluctuations of this or that parameter or function can be little and large, slow, fast or sharply expressed. Correlation can be strong or weak, and so on. This kind of differences of physical nature of tasks cause requires new methods and approaches. This is where the multiple secondary schemes are originated from, which are already connected not to the problem statement, but to their solvation. The multiplicity of the secondary schemes makes it difficult to orientate in the issues of wave statistics.

The present article deals with the methods of signal processing for the analyses of spectral characteristics of electromagnetic radiation. Analogical methods of signal processing are well known in the super high frequency range. Analogical methods were first applied in the visible spectrum range in the 1950s of the last century [8]. But since the invention of the laser sources of the light, the analogical methods have been widely applied in the spectroscopy. The numerical methods of signal processing take their origin from the experiment of photon counting, which was conducted for studying statistical properties of different laser sources [9]. On the basis of these methods we have worked out the highly effective numerical, rapid autocorrelator, which was working in the real-time mode and enabled us to make measurements in the broad frequency range 1 – 108 GHz. The purpose of the autocorrelator was to broaden the frequency range lower than 1 MHz. this problem has been resolved not long ago with Fabry–Perot's interferometer [10]. Creation of the numerical generator for broadening the frequency range of the signal in the super high range is one of the profound tasks for us. We have developed the algorithm of the correlator construction, described the principle of correlator operation during changing the breadth of spectral lines. We have also discussed the cases of Gaussian and non-Gaussian signals as well as the experimental and theoretical outcomes about scattering systems when a signal has a non-Gaussian shape.

## 2. PROBLEM STATEMENT

From the above described four types of primary schemes, we will only discuss the fourth type of task. Our goal is to study the distribution of statistical moments of the scattered laser radiation on the basis of the model of random phase screen.

Assume, that on $z = 0$ plain the primary $E_0$ field statistics are given, i.e. its moments are given (coherence functions). We have to find out how are these functions changed via removing $z = 0$ plain, if the field is transformed on the way (for example, the wave moves through the diaphragm, lens, etc.).

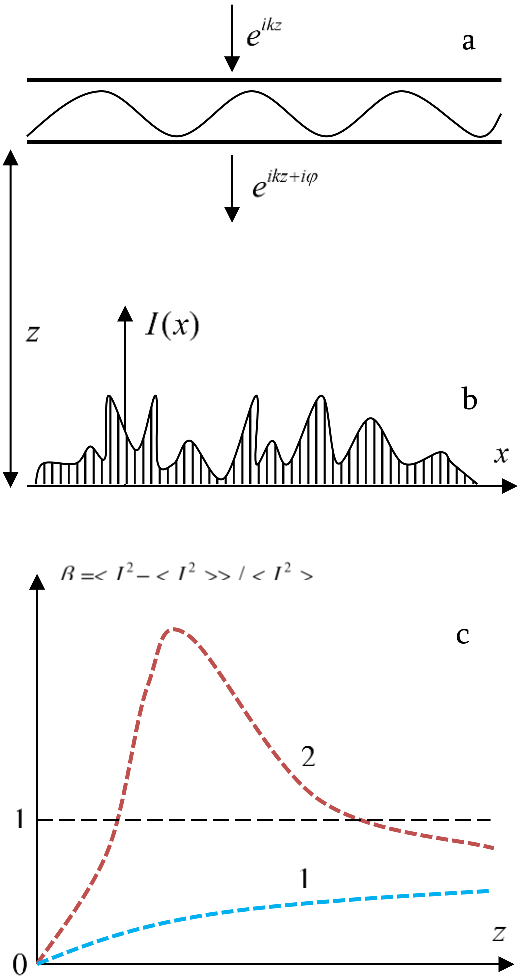


**Figure 1**. a) model of phase screen, b) distribution of intensity after phase screen, c) qualitative image for scintillation index. 1– week fluctuation $< \psi^2 ><< 1$ , 2 –strong fluctuation $< \psi^2 >>> 1$ .

Formally this problem can be solved quite easily: if we know how the determined wave is altered, (totally coherent) then it is enough to average by assemble the determined solvation of $E_0$ field. But this way, as a rule, reduced to toughly calculated integrals. For instance, while calculating the fluctuations through the ratio of intensities, $\beta =< I^2 - < I^2 >> / < I^2 >$ (same as a scintillation index), it is required to calculate the octal integral, which is practically impossible to be calculated even in case of facilitated model. Facilitated limited model implies that the flat wave falls down on the layer $e^{ikz}$, directly on the reverse side of the screen $E_0 = e^{ikz+i\psi}$, where $\psi(x,y)$ - is a random phase. Knowing the statistics of the screen phase causes the determination of the field statistics on $z=0$ plain. The "system" which transforms the field coming out of the screen, in this case, is simply free space. As a result of the diffraction the wave which moves through the chaotic phase screen is fluctuating (fig. 1, b). Though, the intensity on the phase screen is constant (fig. 1, a).

In this particular case, it is possible to calculate the scintillation index $\beta$ for the phase screen during weak fluctuations of the phase $\langle\psi^2\rangle << 1$. In case of strong fluctuations, $\langle\psi^2\rangle \geq 1$ - for calculating $\beta$ at small $z$ – distances we can use excitation method (we should take into account that the intensity fluctuations beyond the screen is small), and for $z$ distances we can apply field normalization method. The normal law of distributing probabilities implies that during long $z$ distances in the observation point multiple non-correlated wave creation takes place from different regions of the screen. We can find field asymptotics in the focused area while existing big phase fluctuations, i.e. when $\langle\psi^2\rangle >> 1$. As a result we get $\beta(z)$ curves, the qualitative image of which is shown on fig.1 c.

## 3. STATISTICS OF SIGNALS

In order to describe the statistical process, let us represent the basic mathematical formalism which will be used for further discussions [15].

Say that any of the time dependent process is described with the variable $s(t)$. In spite of the fact that the functional dependence of $s$ signal on time can be random, the measurement outcomes can be represented by the function $W(s(t))$ of the density distribution of one dimensional probability, which determines the probability of the fact that the random volume receives $s$ value in $t$ time. Analogically, two dimensional distributions $W(s(t_1), s(t_2))$ describe the probability of the fact that as a result of measuring, the amplitudes of the random variables in $t_1$ and $t_2$ moments receive $s_1$ and $s_2$ values. In multi-dimensional case the function of density distribution of the probabilities will be $W(s(t_i))$ and it.

describes the comparatively complete statistics of the random $s(t)$ function. The statistic moments contain equivalent information:

$$\langle s^n(t)\rangle = \int_{-\infty}^{+\infty} s^n W(s)ds\,, \qquad (3.1)$$

And correlative functions:

$$\left\langle \prod_i s^{n_i}(t_i)\right\rangle = \int_{-\infty}^{+\infty} W(s_i)\prod_i s_i^{n_i} ds_i\,. \qquad (3.2)$$

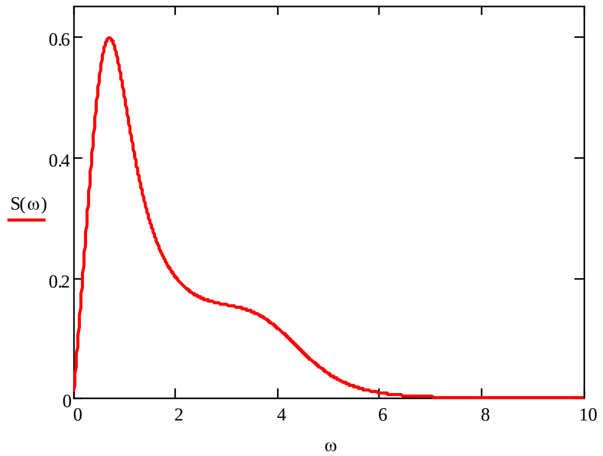


**Figure 2.** Distribution of Lorenthz signal density

In case of stationary statistical process, the distribution of probabilities does not depend on time, i. e.:

$$\langle W(t)W(t+\tau)\rangle = \langle W(0)W(\tau)\rangle\,, \qquad (3.4)$$

Onsager's hypothesis, the fluctuation processes in water can be described with the macroscopic hydrodynamic rules. And fluctuation attenuation is described with time auto-correlative function, the Fourier-transform of which gives the analytical expression of the intensity of the optical spectrum, according to **Wiener-Khintchin** theorem. This theorem connects the density of signal capacity $S(\omega)$ to its autocorrelation function $G(\tau)$.

$$S(\omega) = \lim_{T\to\infty}\left\langle \frac{1}{T}\left|\int_{-T/2}^{T/2} s(t)e^{-i\omega t}dt\right|^2\right\rangle =$$

$$= \int_{-\infty}^{\infty} G(\tau)e^{-i\omega\tau}d\tau$$

Thus, the signal capacity spectrum and its autocorrelation function are mutually related to each other with Fourier transform. For instance, if $s(t)$ has a Lorenz form with a half width of the spectrum $\Gamma$, then the autocorrelation function of this signal represents a reducing exponent ( $\tau_c = \Gamma^{-1}$) given on the fig.2.

## 4. OPTICALLY SOLID RANDOM PHASE SCREEN

Let us discuss the electromagnetic wave propagation process on a random optically solid phase screen. Through this screen, during electromagnetic wave propagation, the random wave displacement will be produced which is a random function of coordinates.

The majority of the works is dedicated to the problem of concreteness, for instance, scattering off the irregular surface, though the general theoretical principles of the statistical and spectral characteristics of the scattered light are not completely processed for the light of non-Gaussian statistics. The signals with such statistics will be created in case when the scales characteristics to the refraction index are comparable with the size of lighting area.

Let us discuss the simple scheme of the experiment. During normal falling down the laser beam is focused on the phase screen. The phase screen is not thick and the directly scattered radiation is registered by means of photomultiplier. The axis of the photomultiplier makes a $\theta$ angle towards the laser beam (fig. 3).

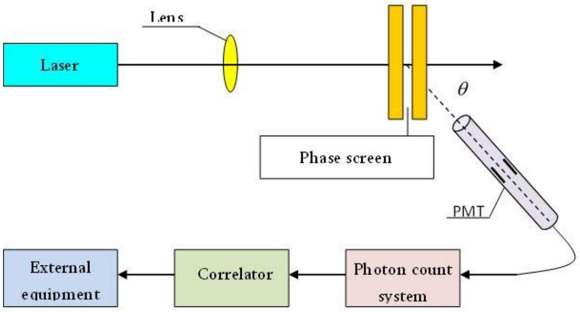


**Figure 3.** The scheme of the device in order to study the light scattered from the phase screen

The positive-frequency part of the electric field of the light wave, which moves through the phase screen, can be expressed as following:

$$\xi^{+}(\vec{r},0;t)=E_0 e^{i[\varphi(\vec{r},t)-\omega_0 t]-\frac{r^2}{W_0^2}} \tag{4.1}$$

where $\varphi(\vec{r},t)$ - is the random phase displacement caused by the screen dependent on the space coordinates. (4.1) expression is recorded in the cylindrical coordinate system, with the coordinates ( $\vec{r},\phi,z$ ). The phase screen is located in the $z=0$ plain. $W_0$ is the width of the Gaussian distribution of the stream intensity and it is characterized to the size of the illuminated area. According to the Helmholtz formula the field in $V\equiv(\vec{r},\phi,z)$ point is determined by the following expression:

$$\xi^{+}(\vec{R};t)=E_0 e^{-i\omega_0 t}\int_{-\infty}^{+\infty} d^2\vec{r}' e^{ik\left|\vec{r}-\vec{R}\right|} e^{i\varphi(\vec{r},t)} e^{-\frac{r'^2}{W_0^2}} \tag{4.2}$$

where, $\vec{k}$ is a wave vector of light and the integration is carried out according to the illuminated area. Let's make the following expression:

$$\left|\vec{r}-\vec{R}'\right|=\sqrt{r^2+z^2+r'^2-2(\vec{r}\vec{r}')}\approx\sqrt{r^2+z^2}\left(1-\frac{(\vec{r}\vec{r}')}{r^2+z^2}\right)$$

The last expression is fair in the far zone when the distance to the source is longer than the linear size of the illuminated area, therefore:

$$\xi^{+}(\vec{R};t)\cong E_0 e^{-i\omega_0 t}\int_{-\infty}^{+\infty} d^2\vec{r}' e^{ik\left|\vec{R}\right|(1-|r'|\sin\theta\cos\psi')} e^{i\varphi(\vec{r},t)} e^{-\frac{r'^2}{W_0^2}} \tag{4.3}$$

where $\psi'$ - $\vec{r}$ - is a polar angle of the vector.

In order to calculate the statistical and spectral characteristics of such field, it is required to find out the properties of $\varphi$ phase function. For facilitating the calculations, consider that $\varphi$ has Gaussian statistics. During the light propagation, in randomly inhomogeneous media like an atmosphere, for instance, or liquid crystal, it is difficult to consider the statistics of $\varphi$ as Gaussian, but the necessity of the further analysis requires considering the statistics of $\varphi$ as Gaussian:

$$<\exp\left[-i\sum_i\varphi_i\right]>=\exp\left[-\frac{1}{2}<\left(\sum_i\varphi_i\right)^2>\right] \tag{4.4}$$

In the concrete case, consider that the space correlation structure of the phase function can be written as following: $\rho(\vec{r})=<\varphi(0,t)\varphi(\vec{r},t)>/<\varphi^2>$. The average intensity of light in the distant zone can be found on the basis of (4.3) and (4.4) formulas:

$$<I(\theta;t)>=<I>=\left|E_0\right|^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d^2r'd^2r''\times$$
$$\times e^{ik\sin\theta(r'\cos\psi'-r''\cos\psi'')} e^{-\overline{\varphi}^2(1-\rho(r'-r''))} e^{-\frac{r'^2-r''^2}{W_0^2}} \tag{4.5}$$

Similarly, we can calculate the second moment of the fluctuation distribution of the intensity:

$$<I^2>=W_0^2\left|E_0\right|^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d^2r'd^2r''d^2r''' e^{2ikr''\sin\theta\cos\beta-\frac{r'^2-r''^2+r'''^2}{W_0^2}}\times$$
$$\times e^{-\overline{\varphi}^2(2-\rho(r''+r''')-\rho(r''+r''')-\rho(r'+r'')\rho(r'-r'')+\rho(r'+r''')+\rho(r'-r'''))} \tag{4.6}$$

Where $\beta$ represents the polar angle of $\vec{r}''$ vector.

In general case, the (4.6) integral is a difficult solution, although, when $\overline{\varphi}^2>>1$, the integral can be calculated through the saddle point approximation:

$$\exp[\overline{\varphi}^2\rho(r)]\approx 1+\exp[\overline{\varphi}^2-1]\exp[-\overline{\varphi}^2 r^2/\lambda^2] \tag{4.7}$$

This approximation is fair in the area where $f$ significantly depends on $r$. Considering (4.7) in (4.5) gives the following expression:

$$<I^2>=\pi^2 W_0^2\left|E_0\right|^2\left\{W_0^2 e^{-\overline{\varphi}^2} e^{\frac{1}{2}k_0^2W_0^2\sin^2\theta}+\frac{1-e^{-\overline{\varphi}^2}}{\frac{1}{W_0^2}+\frac{2\overline{\varphi}^2}{\lambda^2}} e^{-\frac{1}{2}\frac{k^2\sin^2\theta}{\frac{1}{W_0^2}+\frac{2\overline{\varphi}^2}{\lambda^2}}}\right\} \tag{4.8}$$

The first member of the right side corresponds to the diffraction on the "aperture", i.e. in our case it corresponds to the diffraction during falling laser beam on the sample. The second member is completely conditioned by the phase fluctuations. For great values of

$\overline{\varphi}^2$, we can ignore $\exp(-\overline{\varphi}^2)$ members, in this case, (4.8) gives the following expression:

$$< I >= \frac{\pi^2 W_0^2 |E_0|^2}{2\overline{\varphi}^2} \exp\left[ -\frac{k^2 \lambda^2 \sin^2\theta}{4\overline{\varphi}^2} \right] \quad (4.9)$$

In the abovementioned approximation we can calculate (4.6) integral as well, which determines the second moment of the radiation. After a rather long refraction we obtain:

$$\frac{< I^2 >}{< I >^2} = 2 - \frac{2\lambda^2}{W_0^2} + \frac{\lambda^2 \overline{\varphi}^2}{4W_0^2} \exp\left[ -\frac{k^2\lambda^2 \sin^2\theta}{4\overline{\varphi}^2} \right] \quad (4.10)$$

This expression aspires towards 2 when $\lambda / W_0 \to 0$, which is not characterized to Gaussian statistics. In case of the condition $\lambda << W_0$, for example, the second moment can be more than 2 if $\overline{\varphi}^2$ is a great enough value. In addition, this effect can increase by increasing $\theta$ . (4.9) average intensity is distinguished with controversial properties since in the expression of exponent „–“ sign appears.

It is more difficult to calculate the higher quality moments through the above described methods. However, we can obtain more general results in the approximation discussed for “micro area” model. Assume, that the basic illuminated area is composed of $V$ area of $N$ amount. $V$ micro areas in the distant wave zone gives statistically independent components. This model enables us to neglect $\exp(-\overline{\varphi}^2)$ members. Therefore:

$$\xi^+(\theta;t) = \sum_{j=1}^{N} a_j(\theta,t) \exp\left(i\psi_j\right), \quad (4.11)$$

$$a_j^2(\theta,t) = |E_0|^2 \int_V \int_V e^{ik|\vec{r}-\vec{r}'|\sin\theta\cos\beta + \phi_j(\vec{r};t) - \phi_j(\vec{r}';t)} d^2r d^2r' \quad (4.12)$$

Where $\beta$ is an angular variable, which corresponds to $|\vec{r} - \vec{r}'|$ polar vector. (4.11) expression describes a random state of finite value on the complex $\xi^+$ plain.

## 5. OUTCOMES OF NUMERICAL EXPERIMENT

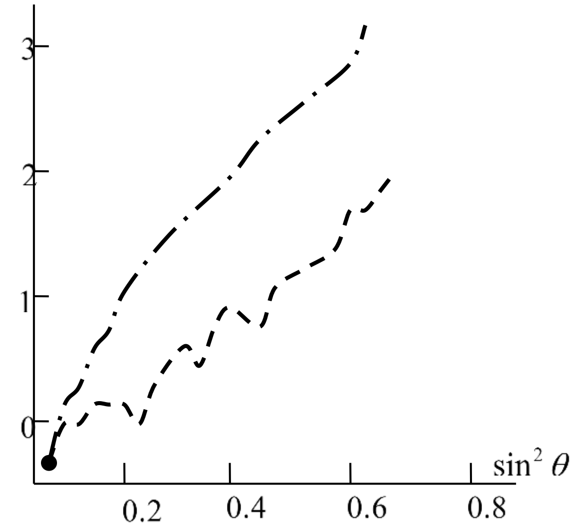


**Figure 4.** Angular distribution of the first and second order moments of the intensity of the light scattered by liquid crystal phase screen of 25 μm, when the electro-hydrodynamic turbulence on the liquid crystal phase screen is created in conditions of 20 V. The area of the lighting area during an experiment was $W_0 = 10.5$ μm, the length of the liquid Chrystal - $l = 2,6$ μm, $\overline{\varphi}^2$ ~36.

The dependencies of (4.9) and (4.10) intensities on $W_0$ and $\theta$ parameters were checked in an experimental way during studying light scattering from the thin (25 μm) liquid chrystal surface, on which electro-hydrodynamic turbulence was created (fig. 4). In our case, we considered that in certain conditions similar system acts like an optically solid phase screen. Though, the existence of the slight depolarization of the scattered light, besides the phase fluctuations, causes the creation of amplitude fluctuations. The existence of the amplitude fluctuations may cause slight deviation from the angular dependence of the intensity. These deviations are determined by (4.9) and (4.10) formulations. The dependence of the sample on the finite thickness is revealed during the dependence of the intensity on $W_0$ parameter. For a big $W_0$ parameter, this effect is small and can be ignored, though, when $W_0$ and the sample thicknesses are the values of one and the same order, we may have a significant distortion of the wave front.

## 6. CONCLUSION

Thus, we have constructed the model of light scattering on the random phase screen, in which, it is considered that the scattered light phase is changed in a linear way in two dimensions. The model is fair in case of slightly illuminated area. The characteristic angle of the inclination of the scattered wave front is determined with the value $(\overline{\varphi}^2)^{1/2}/k\lambda$, which is close to the value of angular width of the average intensity distribution determined in the expression (4.6). This effect in fact, becomes measurable in an experimental way. It should be noted that in the liquid-crystal systems this effect can be observed without additional means of techniques even when the sample is illuminated with halogen light. As we can see, this circumstance depends on the fact that the difference of the ray course from different areas of the thin structure of the sample equals to at least several wave length, i.e. it belongs to the order of the white light coherency length.

Non-Gaussian member causes only the distortion of the form of the spectrum, but at the same time the correlation time is increased up to time order characterized to phase fluctuation. It is well known that the direct dimension of the phase spectrum is available in conditions of non-Gaussian statistics.

In the concluding part it should be noted that the study of statistical characteristics of the laser radiation scattered from the systems having non-Gaussian statistics has not started recently and naturally, we cannot have any claims that we will be able to create perfect, adequate theoretical model, even though, in case of optically solid phase screen for instance. But we can say that the outcomes of analytical and numerical experiment conducted by us will be valuable by all means in the point of view of analyzing statistical characteristics of the signals having non-Gaussian statistics.

## ACKNOWLEDGEMENT

Work it was spent within the limits of the scientific projects:

“Quantitative analysis of fluorescence characteristics of optically solid, random phase screen and spectral analysis of the statistical moments of the correlation function of the intensity of scattered laser radiation“ (FR/152/9-240/14 scientific supervisor Prof. Nugzar Gomidze), financed by SRNSF (Shota Rustaveli National Scientific Found, Georgia).

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