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# NEW CONTINUOUS AND OPEN FUNCTIONS IN TOPOLOGICAL SPACES

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**Abstract** – The aim of this paper is to introduce new continuous and open functions called somewhat $\theta$gs-continuous and somewhat $\theta$gs-open functions using $\theta$gs-open sets.Its various characterisations and properties are established.

***Keywords*** – *$\theta$gs-open set, Somewhat $\theta$gs-continuous, Somewhat $\theta$gs-irresolute , Somewhat $\theta$gs-open functoin.*

# 1 Introduction

Levine [6] introduced the notion of generalized closed set. This notion has been studied extensively in recent years by many toplogists. The investigation of generalized closed sets had led to several new and interesting concepts. Recently in [7] the notion of of $\theta$-generalized semi closed (briefly,$\theta$gs-closed)set was introduced by G.B.Navalagi et al. Gentry and Hoyle[4] introduced and studied the conepts of somewhat continuous and somewhat open functions.In [11] the notion of somewhat $\omega\alpha$-continous and somewhat $\omega\alpha$-open functions are introduced.
In this paper, we will continue the study of related functions with $\theta$gs-closed and $\theta$gs-open sets. We introduce and charcterize the concept of somewhat $\theta$gs-continuous and somewhat $\theta$gs-irresolute and somewhat $\theta$gs-open functions.

# 2 Preliminary

Throughout this paper (X, $\tau$), (Y, $\sigma$)(or simply $X, Y$)denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset $A$ of a space $X$ the closure and interior of $A$ with respect to $\tau$ are denoted by $Cl(A)$ and $Int(A)$ respectively.

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**Definition 2.1.** A subset $A$ of a space $X$ is called
(1) a semi-open set [5] if $A \subset Cl(Int(A))$.
(2) a semi-closed set [2] if $Int(Cl(Int(A))) \subset A$.

**Definition 2.2.** [3] $A$ point $x \in X$ is called a semi-$\theta$-cluster point of $A$ if $sCl(U) \cap A \neq \phi$, for each semi-open set $U$ containing $x$. The set of all semi-$\theta$-cluster point of $A$ is called semi-$\theta$-closure of $A$ and is denoted by $sCl_\theta(A)$. A subset $A$ is called semi-$\theta$-closed set if $sCl_\theta(A) = A$. The complement of semi-$\theta$-closed set is semi-$\theta$-open set.

**Definition 2.3.** [7] A subset $A$ of $X$ is $\theta$generalized semi-closed(briefly, $\theta$gs-closed)set if $sCl_\theta(A) \subset U$ whenever $A \subset U$ and $U$ is open in $X$. The complement of $\theta$gs-closed set is $\theta$generalized-semi open (briefly,$\theta$gs-open).The family of all $\theta$gs-closed sets of $X$ is denoted by $\theta$GSC(X,$\tau$) and $\theta$gs-open sets by $\theta$GSO(X,$\tau$).

**Definition 2.4.** A function $f\colon (X,\tau) \to (Y,\sigma)$ is called:
(i) $\theta$-generalized semi-irresolute (briefly,$\theta$gs-irresolute)[8] if $f^{-1}(F)$ is $\theta$gs-closed set in $X$ for every $\theta$gs-closed set $F$ of $Y$
(ii) $\theta$-generalized semi-continuous (briefly,$\theta$gs-continuous)[8] if $f^{-1}(F)$ is $\theta$gs-closed set in $X$ for every closed set $F$ of $Y$.
(iii) somewhat continuous [4] if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists an open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.

**Definition 2.5.** [9] *A function $f\colon X \to Y$ is said to be $\theta$gs-open (resp., $\theta$gs-closed) if $f(V)$ is $\theta$gs-open (resp., $\theta$gs-closed) in $Y$ for every open set (resp., closed) $V$ in $X$.*

# 3 Somewhat $\theta$gs-Continuous Functions

**Definition 3.1.** A function $f : (X,\tau) \to (Y,\sigma)$ is said to be somewhat $\theta$gs-continuous function if for every $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists a $\theta$gs-open set V in X such that and $V \neq \phi$ and $V \subseteq f^{-1}(U)$.

**Theorem 3.2.** Every somewhat continuous function is somewhat $\theta$gs-continuous funtion.

**Proof:** Let $f : X \to Y$ is somewhat $\theta$gs-continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \phi$. Since f is somewhat continuous function, there exists an open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$. Since every open set is $\theta$gs-open set ,there exists $\theta$gs-open set V such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$, which implies f is somewhat $\theta$gs-continuous function.

**Remark 3.3.** . Converse of the above theorem need not be true in general which follows from the following example.

**Example 3.4.** . Let X = Y = $\{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{X, \phi, \{a\}, \{b, c\}\}$. We have $\theta$GSO(X) = $\{X, \phi, \{a\}, \{b, c\}\}$. Then the identity function is somewhat $\theta$gs-continuous function but not somewhat continuous function.

**Theorem 3.5.** . If $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be any two functions. If f is somewhat $\theta$gs-continuous function and g is contonuous function, then $g \circ f$ is somewhat $\theta$gs-continuous funtion.

**Proof:** Let U be any open set in Z. Suppose that $g^{-1}(U) \neq \phi$. Since $U \in \eta$ and g is continuous function, $g^{-1}(U) \in \eta$. Suppose that $f^{-1}(g^{-1}(U)) \neq \phi$. By hypothesis f is somewhat $\theta$gs-continuous function, there exists a $\theta$gs-open set in $V$ in X such that $V \neq \phi$ and $V \subseteq f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$, which implies that $V \subseteq (g \circ f)^{-1}(U)$. Therefore $g \circ f$ is somewhat $\theta$gs-continuous funtion.

**Definition 3.6.** . A subset M of a topological space X is said to be $\theta$gs-dense in X if there is no proper $\theta$gs-closed set F in X such that $M \subset F \subset X$.

**Theorem 3.7.** . If $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent;
(i) f is somewhat $\theta$gs-continuous function.
(ii) If F is a closed subset of Y such that $f^{-1}(F) \neq X$, then there is proper $\theta$gs-closed subset D of X such that $f^{-1}(F) \subset D$.
(iii) If M is a $\theta$gs-dense subset of X, then f(M) is a dense subset of Y.

**Proof:** (i)$\Rightarrow$(ii):Let F be a closed subset of Y such that $f^{-1}(F) \neq X$. Then $f^{-1}(Y - F) = X - f^{-1}(F) \neq \phi$. By hypothesis (i) there exists a $\theta$gs-open set V in X such that $V \neq \phi$ and $V \subset f^{-1}(Y - F) = X - f^{-1}(F)$. This imples $f^{-1}(F) \subset X - V$ and X-V = D is a $\theta$gs-closed set in X.Therefore(ii) holds.

(ii)$\Rightarrow$ (i): Let U be an open set in Y and $f^{-1}(U) \neq \phi$. Then $f^{-1}(Y - U) = X - f^{-1}(U) = \phi$. By hypothesis, there exists a proper $\theta$gs-closed set D such that $f^{-1}(Y - U) \subset D$. This implies that $X - D \subset f^{-1}(U)$ and X-D is $\theta$gs-open and $X - D \neq \phi$.

(ii)$\Rightarrow$(iii): Let M be any $\theta$gs-dense set in X. Suppose f(M) is not dense subset of Y, then there exists a proper $\theta$gs-closed set D such that $M \subset f^{-1}(F) \subset D \subset X$. This contradicts the fact that M is a $\theta$gs-dense set in X. Therefore (iii) holds.

(iii)$\Rightarrow$ (ii): Suppose (iii) is not true. Then there exists a closed set F in Y such that $f^{-1}(F) \neq X$. But there is no proper $\theta$gs-closed set that $f^{-1}(F) \subset D$. This means that $f^{-1}(F)$ is $\theta$gs-dense in $X$. But from hypothesis $f(f^{-1}(F)) = F$ must be dense in Y, which is contradiction to the choice of F. Hence (ii) hold.

**Theorem 3.8.** .If $f : (X, \tau) \to (Y, \sigma)$ be a function and $X = A \cup B$, A and B are open subsets of X such that $(f/A)$ and $(f/B)$ are somewhat $\theta$gs-continuous functions then f is somewhat $\theta$gs-continuous function.

**Proof:**Let U be an open set in Y such that $f^{-1}(U) \neq \phi$. Then $(f/A)^{-1}(U) \neq \phi$ of $(f/B)^{-1}(U) \neq \phi$ or both $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$.

Case(i): Suppose $(f/A)^{-1}(U) \neq \phi$. Since $(f/A)$ is somewhat $\theta$gs-continuous, then there exists a $\theta$gs-open set V in A such that $V \neq \phi$ and $V \subset (f/A)^{-1}(U) \subseteq f^{-1}(U)$. Since V is $\theta$gs-open set in A and A is open in X,V is $\theta$gs-open in X. Hence f is somewhat $\theta$gs-continuous function.

Case(ii): Suppose $(f/B)^{-1}(U) \neq \phi$. Since $(f/B)$ is somewhat $\theta$gs-continuous function, then there exists a $\theta$gs-open set V in B such that $V \neq \phi$ and $V \subset (f/B)^{-1}(U) \subset f^{-1}(U)$. Since V is $\theta$gs-open in B and B is open in X, V is $\theta$gs-open in X. Hence f is somewhat $\theta$gs-continuous function.

Case(iii): Suppose $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$. Follows from case(i) and case(ii).

**Definition 3.9.** A topological space X is said to be $\theta$gs-separable if there exists a countable subset B of X which is $\theta$gs-dense in X.

**Theorem 3.10.** . If f is somewhat $\theta$gs-continuous function from X onto Y and if X is $\theta$gs-separable, then Y is separable.

**Proof:** Let $f : (X, \tau) \to (Y, \sigma)$ be somewhat $\theta$gs-continuous function such that X is $\theta$gs-separable. Then by definition there exists a countable subset B of X which is $\theta$gs-dense in X. Then by Theorem 3.7, f(B) is dense in Y. Since B is countable f(B) is also countable which is dense in Y, ehich implies that Y is separable.

# 4 Somewhat $\theta$gs-irresolute Function

**Definition 4.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat $\theta$gs-irresolute function if for $U \in \theta GSO(\sigma)$ and $f^{-1}(U) \neq \phi$ there exists a non-empty $\theta$gs-open set V in X such that $V \subset f^{-1}(U)$.

**Theorem 4.2.** . If f is somewhat $\theta$gs-irresolute function and g is $\theta$gs-irresolute function, then $g \circ f$ is somewhat $\theta$gs-irresolute function.

**Proof:** Let $U \in \theta GSO(\eta)$. Suppose that $g^{-1}(U) \neq \phi$. Since $U \in \theta GSO(\eta)$ and g is somewhat $\theta$gs-irresolute function,there exists a $\theta$gs-open set V in $X$ such that $V \neq \phi$ and $V \subseteq f^{-1}(g^{-1}(U))$. But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$, which implies that $V \subseteq (g \circ f)^{-1}(U)$. Therefore $g \circ f$ is somewhat $\theta$gs-irresolute function.

**Theorem 4.3.** . If $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent;
(i) f is somewhat $\theta$gs-irresolute function.
(ii) If F is a closed subset of Y such that $f^{-1}(F) \neq X$, then there is proper $\theta$gs-closed subset D of X such that $f^{-1}(F) \subset D$.
(iii) If M is a $\theta$gs-dense subset of X, then f(M) is a dense subset of Y.

**Proof:** Obvious.

**Theorem 4.4.** .If $f : (X, \tau) \to (Y, \sigma)$ be a function and $X = A \cup B$, A and B are open subsets of X such that $(f/A)$ and $(f/B)$ are somewhat $\theta$gs-irresolute function then f is somewhat $\theta$gs-irresolute function.

**Proof:** Obvious.

**Definition 4.5.** .[1] If X is a set and $\tau$ and $\sigma$ are topologies for X, then $\tau$ is equivalent to $\sigma$ provided if $U \in \tau$ and $U \neq \phi$, then there is a open set V in $(X, \sigma)$ such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$ then there is an open set V in $(X, \tau)$ such that $V \neq \phi$ and $V \subset U$.

**Definition 4.6.** If X is a set and $\tau$ and $\sigma$ are topologies for X, then $\tau$ is said to be $\theta$gs-equivalent to $\sigma$ provided if $U \in \tau$ and $U \neq \phi$, then there is a $\theta$gs-open set V in $(X, \sigma)$ such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$ then there is $\theta$gs-open set V in $(X, \tau)$ such that $V \neq \phi$ and $V \subset U$.

**Theorem 4.7.** . Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat continuous function and let $\tau^*$ be a topology for X, which is $\theta$gs-equivalent to $\tau$ then the function $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat $\theta$gs-continuous function.

**Proof:** Let U be any open set in $(Y, \sigma)$ such that $f^{-1}(U) \neq \phi$. Since by hypothesis $f : (X, \tau) \to (Y, \sigma)$ is somewhat continuous function by definition there exists an open set in O in $(X, \tau)$ such that $O \neq \phi$ and $O \subseteq f^{-1}(U)$. Since O is an open set in $(X, \tau)$ such that $O \neq \phi$ and since by hypothesis $\tau$ is $\theta$gs-equivalent to $\tau^*$ by definition there exists a $\theta$gs-open set V in $(X, \tau^*)$ such that $V \neq \phi$ and $V \subset O \subset f^{-1}(U)$. Hence $O \subset f^{-1}(U)$. Thus for any open set U in $(Y, \sigma)$ such that $f^{-1}(U) \neq \phi$ there exists a $\theta$gs-open set V in $(X, \tau^*)$ such that $V \subset f^{-1}(U)$. So $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat $\theta$gs-continuous function.

**Theorem 4.8.** . Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat $\theta$gs-continuous function and let $\sigma^*$ be a topology for Y, which is equivalent to $\sigma$. Then the function $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat $\theta$gs-continuous function.

**Proof:** Let U be any open set in $(Y, \sigma^*)$ such that $f^{-1}(U) \neq \phi$ which implies $U \neq \phi$. Since $\sigma$ and $\sigma^*$ are equivalent, then there exists an open set W in $(Y, \sigma)$ such that $W \neq \phi$ and $W \subset U$. Now, W is open set such that $W \neq \phi$, which implies $f^{-1}(W) \neq \phi$. Now by hypothesis $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\theta$gs-continuous function.Therefore there exists a $\theta$gs-open set in V in $(X, \tau)$ such that $V \subseteq f^{-1}(W)$. Now $W \subset U$ implies $f^{-1}(W) \subset f^{-1}(U)$. This implies $V \subset f^{-1}(W) \subset f^{-1}(U)$. So, we have $V \subset f^{-1}(U)$, which implies that $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat $\theta$gs-continuous function.

**Theorem 4.9.** . Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat $\theta$gs-irresolute surjection and let $\tau^*$ be a topology for X, which is $\theta$gs-equivalent to $\tau$ then the function $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat $\theta$gs-irresolute function.

**Proof:** Let U be any open set in $(Y, \sigma)$ such that $f^{-1}(U) \neq \phi$. Since by hypothesis $f : (X, \tau) \rightarrow (Y, \sigma)$ is somewhat $\theta$gs-irresolute, by definition there exists a $\theta$gs-open set in O in $(X, \tau)$ such that $O \neq \phi$ and $O \subseteq f^{-1}(U)$. Since O is a $\theta$gs-open set in $(X, \tau)$ such that $O \neq \phi$ and since by hypothesis $\tau$ is $\theta$gs-equivalent to $\tau^*$ by definition there exists a $\theta$gs-open set V in $(X, \tau^*)$ such that $V \neq \phi$ and $V \subset O \subset f^{-1}(U)$. Hence $O \subset f^{-1}(U)$. Thus for any open set U in $(Y, \sigma)$ such that $f^{-1}(U) \neq \phi$ there exists a $\theta$gs-open set V in $(X, \tau^*)$ such that $V \subset f^{-1}(U)$. So $f : (X, \tau^*) \rightarrow (Y, \sigma)$ is somewhat $\theta$gs-irresolute function.

**Theorem 4.10.** . Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a somewhat $\theta$gs-irresolute surjection function and let $\sigma^*$ be a topology for Y, which is equivalent to $\sigma$. Then the function $f : (X, \tau) \rightarrow (Y, \sigma^*)$ is somewhat $\theta$gs-continuous function.

**Proof:** Let U be any open set in $(Y, \sigma^*)$ such that $f^{-1}(U) \neq \phi$ which implies $U \neq \phi$. Since $\sigma$ and $\sigma^*$ are equivalent, then there exists an open set W in $(Y, \sigma)$ such that $W \neq \phi$ and $W \subset U$. Now, W is open set such that $W \neq \phi$, which implies $f^{-1}(W) \neq \phi$. Now by hypothesis $f : (X, \tau) \rightarrow (Y, \sigma)$ is somewhat $\theta$gs-irresolute function. Therefore there exists a $\theta$gs-open set in V in $X$ such that $V \subseteq f^{-1}(W)$. Now $W \subset U$ implies $f^{-1}(W) \subset f^{-1}(U)$. This implies $V \subset f^{-1}(W) \subset f^{-1}(U)$. So, we have $V \subset f^{-1}(U)$, which implies that $f : (X, \tau) \rightarrow (Y, \sigma^*)$ is somewhat $\theta$gs-irresolute function.

# 5 Somewhat $\theta$gs-Open Functions

**Definition 5.1.** A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be somewhat $\theta$gs-open function provided that for every $U \in \tau$ and $U \neq \phi$ there exists a $\theta$gs-open set V in Y such that and $V \neq \phi$ and $V \subseteq f(U)$.

**Theorem 5.2.** Every somewhat open function is somewhat $\theta$gs-open function.

**Proof:** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be somewhat open function. Let $U \in \tau$ and $U \neq \phi$. Since f is somewhat open function, there exists an open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$. But every open is $\theta$gs-open. So there exists a $\theta$gs-open set V in $Y$ such that $V \neq \phi$. Thus f is somewhat $\theta$gs-open function.

**Remark 5.3.** .Converse of the above theorem need not be true in general, which follows from the following example.

**Example 5.4.** .Let X = Y = $\{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$, $\sigma = \{Y, \phi, \{b\}, \{c\}, \{a\}, \{a, c\}\}$. We have $\theta$GSO(Y) = $\{Y, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$. Then the identity function is somewhat $\theta$gs-open function but not somewhat open function.

**Theorem 5.5.** . If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an open function and $g : (Y, \sigma) \rightarrow (Z, \eta)$ somewhat $\theta$gs-open function then $g \circ f$ is somewhat $\theta$gs-open funtion.

**Proof:** Let $U \in \tau$. Suppose $U \neq \phi$. Since f is an open function, f(U) is open and $f(U) \neq \phi$. Thus $f(U) \in \sigma$ and $f(U) \neq \phi$. Since g is somewhat $\theta$gs-open function and $f(U) \in \sigma$ such that $f(U) \neq \phi$ there exists a $\theta$gs-open set in $V \in \eta$, $V \subset g(f(U))$, which implies $g \circ f$ is somewhat $\theta$gs-open funtion.

**Theorem 5.6.** . If $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bejective function. Then the following are equivalent;
(i) f is somewhat $\theta$gs-open function.
(ii) If F is a closed subset of Y such that $f(F) \neq Y$, then there exists a $\theta$gs-closed subset D of Y such that $D \neq \phi$ and $f(F) \subset D$.

**Proof:** (i)⇒(ii):Let F be a closed subset of Y such that $f(F) \neq Y$. From (i), there exists a $\theta$gs-open set V in X such that $V \neq \phi$ such that $V \subset f(X - F)$. Put D = Y-V. Clearly D is a $\theta$gs-closed set in Y and we claim that $D \neq \phi$. If $D = Y$, then $V = \phi$ which is a contradiction. Since $V \subset f(X - F)$, $D = Y - V \subset Y - [f(X - F)] = f(F)$.

(ii)⇒ (i): Let U be any non empty open set in X.Put $F = X - U$. Then F is a closed subset of X and $f(X - U) = f(F) = Y - f(U)$ whicg implies $f(F) \neq \phi$. Therefore by (ii), there is a $\theta$gs-closed subset D of Y such that $f(U) \subset D$. Put V = X-D, clearly V is $\theta$gs-open set and $V \neq \phi$.Further $V = X - D \subset Y - f(F) = Y - [Y - f(F)] = f(U)$.

**Theorem 5.7.** . If $f : (X, \tau) \rightarrow (Y, \sigma)$ be somewhat $\theta$gs-open function and A be any open subset of X. Then $f/A : (A, \tau/A) \rightarrow (Y, \sigma)$ is also somewhat $\theta$gs-open funtion.

**Proof:** Let $U \in \tau/A$ such that $U \neq \phi$. Since U is open in A and A is open in $(X, \tau)$, U is open in $(X, \tau)$ and since by hypothesis f is somewhat $\theta$gs-open function, then there exists a $\theta$gs-open set in $V$ in Y, such that $V \subset f(U)$. Thus, for any open set U in $(A, \tau/A)$ with $U \neq \phi$, there exists a $\theta$gs-open set V in Y such that $V \subset f(U)$ which implies $f/A$ is somewhat $\theta$gs-open funtion.

**Theorem 5.8.** . Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function such that f/A and f/B are somewhat $\theta$gs-open, then f is somewhat $\theta$gs-open funtion, where $X = A \cup B$, A and B are open subsets of X.

**Proof:**Let U be an open set in X such that $U \neq \phi$. Since $X = A \cup B$ , either $A \cap U \neq \phi$ or $B \cap U \neq \phi$ or both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. Since U is opn in X, U is open in both Then $A, \tau/A$ and $B, \tau/B$.

Case(i): Suppose $U \cap A \neq \phi$ where $U \cap A$ is open in $A, \tau/A)$. Since by hypothesis f/A is somewhat $\theta$gs-open function, then there exists a $\theta$gs-open set V in Y such that $V \subset f(U \cap A) \subset f(U)$,which implies f is somewhat $\theta$gs-open function.

Case(ii): Suppose $U \cap B \neq \phi$ where $U \cap B$ is open in $B, \tau/B)$. Since by hypothesis f/B is somewhat $\theta$gs-open function, then there exists a $\theta$gs-open set V in Y such that $V \subset f(U \cap B) \subset f(U)$,which implies f is somewhat $\theta$gs-open function.

Case(iii): Suppose that $U \cap A \neq \phi$ and $U \cap B \neq \phi$. Then obviously f is somewhat $\theta$gs-open function from case(i) and case(ii). Thus f is somewhat $\theta$gs-open function.

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