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COTANGENT SIMILARITY MEASURE OF ROUGH NEUTROSOPHIC SETS AND ITS APPLICATION TO MEDICAL DIAGNOSIS

Surapati Pramanik[1,*] <sura_pati@yahoo.co.in>
Kalyan Mondal[2] <kalyanmathematic@gmail.com>

[1]*Department of Mathematics, Nandalal Ghosh B. T. College, Panpur, PO- Narayanpur, and District: North 24 Parganas, Pincode: 743126, West Bengal, India.*

[2]*Department of Mathematics, West Bengal State University, Barasat, North 24 Paraganas***,** *Berunanpukuria, P.O. Malikapur, North 24 Parganas, Pincode: 700126, West Bengal, India.*

Abstract – Similarity measure plays an important role in medical diagnosis. In this paper, a new rough cotangent similarity measure between two rough neutrosophic sets is proposed. The notion of rough neutrosophic set is used as vector representations in 3D-vector space. The rating of all elements in rough neutrosophic set is expressed with the upper and lower approximation operator and the pair of neutrosophic sets which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. A numerical example of the medical diagnosis is provided to show the effectiveness and flexibility of the proposed method.

Keywords – *Rough cotangent similarity measure, Rough sets, Neutrosophic sets, Indeterminacy Membership degree, 3D vector space.*

1 Introduction

Similarity measure is an important research topic in the current fuzzy, rough, neutrosophic and differrnt hybrid environments. In 1965, Zadeh [48] introduced the concept of fuzzy set to deal with informational (epistemic) vagueness. Fuzzy set is capable of formalizing and reasoning of intangible internal characteristics, typically natural language-based and visual image information, as well as incomplete, unreliable, imprecise and vague performance and priority data. However, while focusing on the degree of membership of vague parameters or events, fuzzy set fails to deal with indeterminacy magnitudes of measured responses. In 1986, Atanassov [1] developed the concept of intuitionistic fuzzy set (IFS) which considers degree of membership (acceptance) and degree of non-membership (rejection) simultaneously. However, IFS cannot deal with all types of uncertainties, particularly

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[*]*Corresponding Author.*

paradoxes. One of the interesting generalizations of the theory of Cantor set [11], fuzzy set [48] and intuitionistic fuzzy set [1] is the theory of neutrosophic sets [37] introduced by Smarandache in the late 1990s. Neutrosophic sets [38], [39] and their specific sub-class of single-valued neutrosophic set (SVNS) [43] are characterized by the three independent functions, namely membership (truth) function, non-membership (falsity) function and indeterminacy function. Smarandache [39] stated that such formulation enables modeling of the most general ambiguity cases, including paradoxes. In the literature, some interesting applications of neutrosophic logic, neutrosophic sets and single valued neutrosophic sets are reported in different fields such as decision making [3, 4, 5, 6, 8, 20, 44, 45, 46], education [23, 25, 32], image processing [12, 16, 49], medical diagnosis [19], conflict resolution [2, 35], Robotics [40], social problem [22, 33, 41], etc.

In 1982, Pawlak [31] introduced the notion of rough set theory as the extension of the Cantor set theory [11]. Broumi et al. [10] comment that the concept of rough set is a formal tool for modeling and processing incomplete information in information systems. Rough set theory [31] is very useful to study of intelligent systems characterized by uncertain or insufficient information. Main mathematical basis of rough set theory is formed by two basic components namely, crisp set and equivalence relation. Rough set is the approximation of a pair of sets known as the lower approximation and the upper approximation. Here, the lower and upper approximation operators are equivalence relation.

In 2014, Broumi et al. [9, 10] introdced the concept of rough neutrosophic set. It is a new hybrid intelligent structure. İt is developed based on the concept of rough set theory [31] and single valued neutrosophic set theory [43] Rough neutrosophic set theory [9, 10] is the generalization of rough fuzzy sets [15, 29, 30], and rough intuitionistic fuzzy sets [42]. While the concept of single valued neutrosophic set [43] is a powerful tool to deal with the situations with indeterminacy and inconsistancy, the theory of rough neutrosophic sets [9, 10] is also a powerful mathematical tool to deal with incompleteness.

Many methods have been proposed in the literature to measure the degree of similarity between neutrosophic sets. Broumi and Smarandache [7] studied the Hausdorff distance [17] between neutrosophic sets, some distance based similarity measures and set theoretic approach and matching functions. Majumdar and Smanta [21] studied several similarity measures of SNVSs based on distance, membership grades, a matching function, and then proposed an entropy measure for a SVNS. Ye [44] proposed the distance-based similarity measure of SVNSs and applied it to the group decision making problems with single valued neutrosophic information. Ye [46] also proposed three vector similarity measure, an instance of SVNS and interval valued neutrosophic set, including the Jaccard [18], Dice [14], and cosine similarity [36] and applied them to multi-attribute decision-making problems under simplified neutrosophic environment. Ye [47] studied improved cosine similarity measures of SNSs based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures and provided medical diagnosis method based on the improved cosine similarity measures. Recently, Mondal and Pramanik [28] proposed a neutrosophic similarity measure based on tangent function. Mondal and Pramanik [26] also proposed neutrosophic refined similarity measure based on cotangent function. Biswas et al. [5] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers.

Literature rewview reflects that a few studies related to decision making under rough neutrosophic environment are done. Mondal and Pramanik [24] proposed rough neutrosophic multi-attribute decision-making based on grey relational analysis [13]. Pramanik and Mondal [34] proposed cosine similarity measure under rough neutrosophic environment. Mondal and Pramanik [27] also proposed rough neutrosophic multi-attribute decision-making based on accuracy score function.

Realistic practical problems consist of more uncertainty and complexity. So, it is necessary to employ more flexible tool which can deal uncertain situation easily. In this situation, rough neutrosophic set [10] is very useful tool to uncertainty and incompleteness. In this paper, we propose cotangent similarity measure of rough neutrosophic sets and establish some of its properties. Finally, a numerical example of medical diagnosis is presented to demonstrate the applicability and effectiveness of the proposed approach.

The rest of the paper is organized as follows: In section 2, some basic definitions of single valued neutrosophic sets and rough neutrosophic sets are preseneted. Section 3 is devoted to present rough neutrosophic cotangent similarity measure and proofs of some its basic properties. In section 4, numerical example is provided to show the applicability of the proposed approach to medical diagnosis. Section 5 presents the concluding remarks.

2 Mathematical Preliminaries

Definition 2.1.1 [43] Let X be a universal space of points (objects) with a generic element of X denoted by x.

A single valued neutrosophic set [43] S is characterized by a truth membership function $T_S(x)$, a falsity membership function $F_S(x)$ and indeterminacy function $I_S(x)$ with $T_S(y), F_S(x), I_S(x) \in [0,1]$ for all x in X.

When X is continuous, a SNVS S can be written as follows:

$$S = \int_x \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in X$$

and when X is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in X$$

It should be observed that for a SVNS S,

$$0 \leq \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \leq 3, \quad \forall x \in X$$

Definition 2.1.2 [43] The complement of a single valued neutrosophic set S [43] is denoted by S^c and is defined as

$$T_S{}^c(x) = F_S(x); \; I_S{}^c(x) = 1 - I_S(x); \; F_S{}^c(x) = T_S(x)$$

Definition 2.1.3 [43] A SVNS S_N is contained in the other SVNS [43] S_P, denoted as

$$S_N \subseteq S_P \text{ iff } T_{S_N}(x) \leq T_{S_P}(x); \; I_{S_N}(x) \geq I_{S_P}(x); \; F_{S_N}(x) \geq F_{S_P}(x), \; \forall x \in X .$$

Definition 2.1.4 [43] Two single valued neutrosophic sets [43] S_N and S_P are equal, i.e.

$$S_N = S_P, \text{ iff, } S_N \subseteq S_P \text{ and } S_N \supseteq S_P$$

Definition 2.1.5 [43] The union of two SVNSs [43] S_N and S_P is a SVNS S_Q , written as

$$S_Q = S_N \cup S_P .$$

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_N and S_P by the following equation

$$T_{S_Q}(x) = \max\left(T_{S_N}(x), T_{S_P}(x)\right);$$
$$I_{S_Q}(x) = \max\left(I_{S_N}(x), I_{S_P}(x)\right);$$
$$F_{S_Q}(x) = \min\left(F_{S_N}(x), F_{S_P}(x)\right)$$

for all x in X.

Definition 2.1.6 [43] The intersection of two SVNSs [43] N and P is a SVNS Q, written as $Q = N \cap P$. Its truth membership, indeterminacy membership and falsity membership functions are related to N an P by the following equation

$$T_{S_Q}(x) = \min\left(T_{S_N}(x), T_{S_P}(x)\right);$$
$$I_{S_Q}(x) = \max\left(I_{S_N}(x), I_{S_P}(x)\right);$$
$$F_{S_Q}(x) = \max\left(F_{S_N}(x), F_{S_P}(x)\right), \forall x \in X$$

Distance Between Two Neutrosophic Sets

The general SVNS can be presented in the follow form

$$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in X\}$$

Finite SVNSs can be represented as follows:

$$S = \{(x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, (x_m/(T_S(x_m), I_S(x_m), F_S(x_m)))\}, \forall x \in X \quad (1)$$

Definition 2.1.7 [21] Let

$$S_N = \{(x_1/(T_{S_N}(x_1), I_{S_N}(x_1), F_{S_N}(x_1))), \cdots, (x_n/(T_{S_N}(x_n), I_{S_N}(x_n), F_{S_N}(x_n)))\} \quad (2)$$
$$S_P = \{(x_1/(T_{S_P}(x_1), I_{S_P}(x_1), F_{S_P}(x_1))), \cdots, (x_n/(T_{S_P}(x_n), I_{S_P}(x_n), F_{S_P}(x_n)))\} \quad (3)$$

be two single-valued neutrosophic sets, then the Hamming distance [21] between two SNVS N and P is defined as follows:

$$d_S(S_N, S_P) = \sum_{i=1}^{n} \left\langle \left| T_{S_N}(x) - T_{S_P}(x) \right| + \left| I_{S_N}(x) - I_{S_P}(x) \right| + \left| F_{S_N}(x) - F_{S_P}(x) \right| \right\rangle \quad (4)$$

and normalized Hamming distance [21] between two SNVSs S_N and S_P is defined as follows:

$$ {}^{N}d_S(S_N, S_P) = \frac{1}{3n}\sum_{i=1}^{n}\left\langle \left|T_{S_N}(x) - T_{S_P}(x)\right| + \left|I_{S_N}(x) - I_{S_P}(x)\right| + \left|F_{S_N}(x) - F_{S_P}(x)\right| \right\rangle \quad (5) $$

with the following properties

1. $0 \le d_S(S_N, S_P) \le 3n$ (6)
2. $0 \le {}^{N}d_S(S_N, S_P) \le 1$ (7)

2.2. Definitions

[9, 10] Rough set theory [9, 10] consists of two basic components namely, crisp set and equivalence relation. The basic idea of rough set is based on the approximation of sets by a couple of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation.

Definition 2.2.1 [9, 10] Let Y be a non-null set and R be an equivalence relation on Y. Let P be neutrosophic set in Y with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Y, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$ \underline{N}(P) = \left\langle < x,\ T_{\underline{N}(P)}(x),\ I_{\underline{N}(P)}(x),\ F_{\underline{N}(P)}(x) > / Y \in [x]_R, x \in Y \right\rangle \quad (8) $$

$$ \overline{N}(P) = \left\langle < x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x),\ F_{\overline{N}(P)}(x) > / Y \in [x]_R, x \in Y \right\rangle \quad (9) $$

Here,

$T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, T_P(Y)$,

$I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, I_P(Y)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, F_P(Y)$,

$T_{\overline{N}(P)}(x) = \vee_Y \in [x]_R\, T_P(Y)$, $I_{\overline{N}(P)}(x) = \vee_Y \in [x]_R\, T_P(Y)$,

$F_{\overline{N}(P)}(x) = \vee_Y \in [x]_R\, I_P(Y)$

So,

$0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$

$0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

Here $\vee$ and $\wedge$ indicate “max” and “min’’ operators respectively, $T_P(Y)$, $I_P(Y)$ and $F_P(Y)$ are the membership , indeterminacy and non-membership of Y with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Y.

Thus NS mappings $\underline{N}, \overline{N} : N(Y) \rightarrow N(Y)$ are, respectively, refered to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (Y, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence classes of R if $\underline{N}(P) = \overline{N}(P)$; i.e.

$$T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x),\ I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x),\ F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x).$$

For any x $\in Y$, P is said to be a definable neutrosophic set in the approximation (Y, R). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 2.2.2 [9, 10] If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Y, R), the rough complement of $N(P)$ is the rough neutrosophic set denoted $\sim N(P) = (\underline{N}(P)^c, \overline{N}(P)^c)$,where $\underline{N}(P)^c, \overline{N}(P)^c$ are the complements of neutrosophic sets of $\underline{N}(P), \overline{N}(P)$ respectively.

$$\underline{N}(P)^c = \left\langle < x, T_{\underline{N}(P)}(x), 1 - I_{\underline{N}(P)}(x),\ F_{\underline{N}(P)}(x) > /\, , x \in Y \right\rangle,$$

and

$$\overline{N}(P)^c = \left\langle < x, T_{\underline{N}(P)}(x),\ 1 - I_{\overline{N}(P)}(x),\ F_{\overline{N}(P)}(x) > /\, , x \in Y \right\rangle \tag{10}$$

Definition 2.2.3 [9, 10] If $N(P)$ *and* $N(Q)$ are two rough neutrosophic sets of the neutrosophic sets respectively in Y, then the following definitions holds.

$$N(P) = N(Q) \Leftrightarrow \underline{N}(P) = \underline{N}(Q) \wedge \overline{N}(P) = \overline{N}(Q)$$
$$N(P) \subseteq N(Q) \Leftrightarrow \underline{N}(P) \subseteq \underline{N}(Q) \wedge \overline{N}(P) \subseteq \overline{N}(Q)$$
$$N(P) \bigcup N(Q) = < \underline{N}(P) \bigcup \underline{N}(Q),\ \overline{N}(P) \bigcup \overline{N}(Q) >$$
$$N(P) \bigcap N(Q) = < \underline{N}(P) \bigcap \underline{N}(Q),\ \overline{N}(P) \bigcap \overline{N}(Q) >$$
$$N(P) + N(Q) = < \underline{N}(P) + \underline{N}(Q),\ \overline{N}(P) + \overline{N}(Q) >$$
$$N(P)\ .N(Q) = < \underline{N}(P)\ .\ \underline{N}(Q),\ \overline{N}(P)\ .\overline{N}(Q) >$$

If A, B, C are rough neutrosophic sets in (Y, R), then the following proposition are stated from definitions

Proposition I [9, 10]

1. $\sim A(\sim A) = A$
2. $A \bigcup B = B \bigcup A,\ \ A \bigcup B = B \bigcup A$
3. $(A \bigcup B) \bigcup C = A \bigcup (B \bigcup C),\ \ (A \bigcap B) \bigcap C = A \bigcap (B \bigcap C)$
4. $(A \bigcup B) \bigcap C = (A \bigcup B) \bigcap (A \bigcup C),\ \ (A \bigcap B) \bigcup C = (A \bigcap B) \bigcup (A \bigcap C)$

Proposition II [9, 10]
De Morgan's Laws are satisfied for rough neutrosophic sets

1. $\sim(N(P)\cup N(Q)) = (\sim N(P))\cap(\sim N(Q))$
2. $\sim(N(P)\cap N(Q)) = (\sim N(P))\cup(\sim N(Q))$

Proposition III [9, 10]
If P and Q are two rough neutrosophic sets in U such that $P\subseteq Q, then\ N(P)\subseteq N(Q)$

1. $N(P\cap Q)\subseteq N(P)\cap N(Q)$
2. $N(P\cup Q)\supseteq N(P)\cup N(Q)$

Proposition IV[9, 10]
1. $\underline{N}(P) = \sim\overline{N}(\sim P)$
2. $\overline{N}(P) = \sim\underline{N}(\sim P)$
3. $\underline{N}(P)\subseteq \overline{N}(P)$

3 Cotangent Similarity Measures of Rough Neutrosophic Sets

Let $M=<\left\langle\left(\underline{T}_M(x_i), \underline{I}_M(x_i), \underline{F}_M(x_i)\right),\left(\overline{T}_M(x_i), \overline{I}_M(x_i), \overline{F}_M(x_i)\right)\right\rangle>$ and

$N=<\left\langle\left(\underline{T}_N(x_i), \underline{I}_N(x_i), \underline{F}_N(x_i)\right),\left(\overline{T}_N(x_i), \overline{I}_N(x_i), \overline{F}_N(x_i)\right)\right\rangle>$ be two rough neutrosophic numbers. Now rough cotangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them. Therefore, a new cotangent similarity measure between rough neutrosophic sets is proposed in 3-D vector space.

Definition 3.1 Rough cotangent similarity measure

Assume that there are two rough neutrosophic sets
$M=\left\langle\left(\underline{T}_M(x_i), \underline{I}_M(x_i), \underline{F}_M(x_i)\right),\left(\overline{T}_M(x_i), \overline{I}_M(x_i), \overline{F}_M(x_i)\right)\right\rangle$ and
$N=\left\langle\left(\underline{T}_N(x_i), \underline{I}_N(x_i), \underline{F}_N(x_i)\right),\left(\overline{T}_N(x_i), \overline{I}_N(x_i), \overline{F}_N(x_i)\right)\right\rangle$ in $X = \{x_1, x_2, \ldots, x_n\}$. A cotangent similarity measure between rough neutrosophic sets M and N is proposed as follows:

$$COT_{RNS}(M,N)=$$
$$\left[\frac{1}{n}\sum_{i=1}^{n}\left\langle\cot\left(\frac{\pi}{12}\left(3+\left|\delta T_M(x_i)-\delta T_N(x_i)\right|+\left|\delta I_M(x_i)-\delta I_N(x_i)\right|+\left|\delta F_M(x_i)-\delta F_N(x_i)\right|\right)\right)\right\rangle\right] \tag{11}$$

Here,

$$\delta T_M(x_i)=\left(\frac{\underline{T}_M(x_i)+\overline{T}_M(x_i)}{2}\right),\ \delta T_N(x_i)=\left(\frac{\underline{T}_N(x_i)+\overline{T}_N(x_i)}{2}\right),\ \delta I_M(x_i)=\left(\frac{\underline{I}_M(x_i)+\overline{I}_M(x_i)}{2}\right),$$

$\delta I_N(x_i)=\left(\dfrac{\underline{I}_N(x_i)+\overline{I}_N(x_i)}{2}\right)$, $\delta F_M(x_i)=\left(\dfrac{\underline{F}_M(x_i)+\overline{F}_M(x_i)}{2}\right)$, $\delta F_N(x_i)=\left(\dfrac{\underline{F}_N(x_i)+\overline{F}_N(x_i)}{2}\right)$.

Proposition V
Let M and N be rough neutrosophic sets then

1. $0 \le COT_{RNS}(M,N) \le 1$
2. $COT_{RNS}(M,N) = COT_{RNS}(N,M)$
3. $COT_{RNS}(M, N) = 1$, iff $M = N$
4. If P is a RNS in Y and $M \subset N \subset P$ then, $COT_{RNS}(M, P) \le COT_{RNS}(M, N)$, and $COT_{RNS}(M, P) \le COT_{RNS}(N, P)$

Proof :

1.Since, $\dfrac{\pi}{4} \le \left(\dfrac{\pi}{12}\left(3+|\delta T_M(x_i)-\delta T_N(x_i)|+|\delta I_M(x_i)-\delta I_N(x_i)|+|\delta F_M(x_i)-\delta F_N(x_i)|\right)\right) \le \dfrac{\pi}{2}$, it is obvious that the cotangent function $COT_{RNS}(M,N)$ are within 0 and 1.

2. It is obvious that the proposition is true.

3. When $M = N$, then obviously $COT_{RNS}(M, N) = 1$. On the other hand if $COT_{RNS}(M, N) =1$

then,
$\delta T_M(x_i) = \delta T_N(x_i)$, $\delta I_M(x_i) = \delta I_N(x_i)$, $\delta F_M(x_i) = \delta F_N(x_i)$ *ie,*
$\underline{T}_M(x_i)=\underline{T}_N(x_i)$, $\overline{T}_M(x_i)=\overline{T}_N(x_i)$, $\underline{I}_M(x_i)=\underline{I}_N(x_i)$, $\overline{I}_M(x_i)=\overline{I}_N(x_i)$, $\underline{F}_M(x_i)=\underline{F}_N(x_i)$,
$\overline{F}_M(x_i)=\overline{F}_N(x_i)$

This implies that $M = N$.

4. If $M \subset N \subset P$ then we can write $\underline{T}_M(x_i)\le\underline{T}_N(x_i)\le\underline{T}_P(x_i)$, $\overline{T}_M(x_i)\le\overline{T}_N(x_i)\le\overline{T}_P(x_i)$,
$\underline{I}_M(x_i)\ge\underline{I}_N(x_i)\ge\underline{I}_P(x_i)$, $\overline{I}_M(x_i)\ge\overline{I}_N(x_i)\ge\overline{I}_P(x_i)$, $\underline{F}_M(x_i)\ge\underline{F}_N(x_i)\ge\underline{F}_P(x_i)$,
$\overline{F}_M(x_i)\ge\overline{F}_N(x_i)\ge\overline{F}_P(x_i)$.

The cotangent function is decreasing function within the interval $\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right]$. Hence we can write $COT_{RNS}(M, P) \le COT_{RNS}(M, N)$, and $COT_{RNS}(M, P) \le COT_{RNS}(N, P)$.

Definition 3.3 Weighted rough cotangent similarity measure

If we consider the weights of each element x_i, a weighted rough cotangent similarity measure between rough neutrosophic sets A and B can be defined as follows:

$$COT_{WRNS}(M,N) =$$
$$\left[\frac{1}{n}\sum_{i=1}^{n} w_i \left\langle \cot\left(\frac{\pi}{12}\left(3+|\delta T_M(x_i)-\delta T_N(x_i)|+|\delta I_M(x_i)-\delta I_N(x_i)|+|\delta F_M(x_i)-\delta F_N(x_i)|\right)\right)\right\rangle\right]$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \dfrac{1}{n}$, i = 1, 2,…, n, then $COT_{WRNS}(M, N) = COT_{RNS}(M, N)$

Proposition VI: The weighted rough cotangent similarity measure $COT_{WRNS}(M, N)$ between two rough neutrosophic sets M and N also satisfies the following properties:

1. $0 \le COT_{WRNS}(M, N) \le 1$
2. $COT_{WRNS}(M, N) = COT_{WRNS}(N, M)$
3. $COT_{WRNS}(M, N) = 1$, iff $M = N$
4. If P is a WRNS in Y and $M \subset N \subset P$ then, $COT_{WRNS}(M, P) \le COT_{WRNS}(M, N)$, and $COT_{WRNS}(M, P) \le COT_{WRNS}(N, P)$

Proof :

1.Since, $\frac{\pi}{4} \le \left(\frac{\pi}{12}\left(3 + \left| \delta T_M(x_i) - \delta T_N(x_i) \right| + \left| \delta I_M(x_i) - \delta I_N(x_i) \right| + \left| \delta F_M(x_i) - \delta F_N(x_i) \right| \right) \right) \le \frac{\pi}{2}$ and $\sum_{i=1}^{n} w_i = 1$, it is obvious that the weighted cotangent function are within 0 and 1 ie, $0 \le COT_{WRNS}(M, N) \le 1.$

2. It is obvious that the proposition is true.

3. Here, $\sum_{i=1}^{n} w_i = 1.$ When $M = N$, then obviously $COT_{WRNS}(M, N) = 1$. On the other hand if $COT_{WRNS}(M, N) = 1$ then,

$\delta T_M(x_i) = \delta T_N(x_i)$, $\delta I_M(x_i) = \delta I_N(x_i)$, $\delta F_M(x_i) = \delta F_N(x_i)$ *ie,*
$\underline{T}_M(x_i) = \underline{T}_N(x_i)$, $\overline{T}_M(x_i) = \overline{T}_N(x_i)$, $\underline{I}_M(x_i) = \underline{I}_N(x_i)$, $\overline{I}_M(x_i) = \overline{I}_N(x_i)$, $\underline{F}_M(x_i) = \underline{F}_N(x_i)$,
$\overline{F}_M(x_i) = \overline{F}_N(x_i)$

This implies that $M = N$.

4. If $M \subset N \subset P$ then we can write $\underline{T}_M(x_i) \le \underline{T}_N(x_i) \le \underline{T}_P(x_i)$, $\overline{T}_M(x_i) \le \overline{T}_N(x_i) \le \overline{T}_P(x_i)$,
$\underline{I}_M(x_i) \ge \underline{I}_N(x_i) \ge \underline{I}_P(x_i)$, $\overline{I}_M(x_i) \ge \overline{I}_N(x_i) \ge \overline{I}_P(x_i)$, $\underline{F}_M(x_i) \ge \underline{F}_N(x_i) \ge \underline{F}_P(x_i)$,
$\overline{F}_M(x_i) \ge \overline{F}_N(x_i) \ge \overline{F}_P(x_i)$.

The cotangent function is decreasing function within the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$. Here, $\sum_{i=1}^{n} w_i = 1.$ Hence we can write $COT_{WRNS}(M, P) \le COT_{WRNS}(M, N)$, and $COT_{WRNS}(M, P) \le COT_{WRNS}(N, P)$.

4 Examples on Medical Diagnosis

We consider a medical diagnosis problem from practical point of view for illustration of the proposed approach. Medical diagnosis comprises of uncertainties and increased volume of information available to physicians from new medical technologies. The process of classifying different set of symptoms under a single name of a disease is very difficult task. In some practical situations, there exists possibility of each element within a lower and an upper approximation of neutrosophic sets. It can deal with the medical diagnosis involving more indeterminacy. Actually this approach is more flexible and easy to use. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will provide the proper medical diagnosis. The main feature of this proposed approach is that it

considers truth membership, indeterminate and false membership of each element between two approximations of neutrosophic sets by taking one time inspection for diagnosis.
Now, an example of a medical diagnosis is presented. Let P = {P_1, P_2, P_3} be a set of patients, D = {Viral Fever, Malaria, Stomach problem, Chest problem} be a set of diseases and S = {Temperature, Headache, Stomach pain, Cough, Chest pain.} be a set of symptoms. Our task is to examine the patient and to determine the disease of the patient in rough neutrosophic environment.

Table 1: (Relation-1) The relation between Patients and Symptoms

Relation-1	**Temperature**	**Headache**	**Stomach pain**	**cough**	**Chest pain**
P_1	$\left\langle \begin{matrix}(0.6,0.3,0.3),\\(0.8,0.3,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.4,0.4,0.3),\\(0.6,0.2,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.4,0.2),\\(0.7,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.6,0.3,0.3),\\(0.8,0.1,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.4,0.4),\\(0.5,0.2,0.2)\end{matrix}\right\rangle$
P_2	$\left\langle \begin{matrix}(0.5,0.4,0.3),\\(0.7,0.2,0.3)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.5),\\(0.7,0.3,0.3)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.2,0.4),\\(0.7,0.0,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.5),\\(0.9,0.3,0.3)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.5,0.3),\\(0.7,0.3,0.3)\end{matrix}\right\rangle$
P_3	$\left\langle \begin{matrix}(0.7,0.4,0.2),\\(0.9,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.2),\\(0.7,0.1,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.6,0.5,0.4),\\(0.8,0.3,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.6,0.3,0.4),\\(0.8,0.1,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.5,0.3),\\(0.7,0.3,0.1)\end{matrix}\right\rangle$

Table 2: (Relation-2) The relation among Symptoms and Diseases

Relation-2	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
Temperature	$\left\langle \begin{matrix}(0.6,0.5,0.4),\\(0.8,0.5,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.3,0.4,0.5),\\(0.5,0.2,0.3)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.3,0.3,0.4),\\(0.5,0.1,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.2,0.4,0.5),\\(0.4,0.4,0.3)\end{matrix}\right\rangle$
Headache	$\left\langle \begin{matrix}(0.5,0.4,0.4),\\(0.7,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.4,0.3,0.5),\\(0.6,0.3,0.3)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.2,0.4,0.4),\\(0.4,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.3,0.5,0.4),\\(0.5,0.3,0.2)\end{matrix}\right\rangle$
Stomach pain	$\left\langle \begin{matrix}(0.2,0.3,0.3),\\(0.4,0.3,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.3),\\(0.3,0.2,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.4,0.4,0.4),\\(0.6,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.6),\\(0.3,0.2,0.2)\end{matrix}\right\rangle$
Cough	$\left\langle \begin{matrix}(0.4,0.3,0.4),\\(0.6,0.1,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.3,0.3,0.3),\\(0.5,0.3,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.6,0.6),\\(0.3,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.4,0.3),\\(0.7,0.2,0.1)\end{matrix}\right\rangle$
Chest pain	$\left\langle \begin{matrix}(0.2,0.4,0.3),\\(0.6,0.2,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.3,0.4),\\(0.3,0.1,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.2,0.4,0.4),\\(0.4,0.2,0.4)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.3,0.4,0.3),\\(0.5,0.2,0.3)\end{matrix}\right\rangle$

Table 3: The Correlation Measure between Relation-1 and Relation-2

Rough cotangent similarity measure	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
P_1	**0.8726**	0.8194	0.7977	0.8235
P_2	**0.8298**	0.7968	0.8024	0.7857
P_3	**0.8382**	0.7356	0.7448	0.7536

The highest correlation measure (see the Table 3) reflects the proper medical diagnosis. Therefore, all three patients P_1, P_2, P_3 suffer from viral fever.

5. Conclusion

In this paper, we have proposed rough cotangent similarity measure of rough neutrosophic sets and proved some of their basic properties. We have presented an application of rough cotangent similarity measure of rough neutrosophic sets in medical diagnosis problems. We hope that the proposed concept can be applied in solving realistic multi-attribute decision making problems.

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