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ASSOCIATED PROPERTIES OF α-$\pi g\alpha$-CLOSED FUNCTIONS

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Abstract – The concept of α-open sets was introduced by [17]. The primary purpose of this paper is to introduce and study pre-$\pi g\alpha$-closed functions by using α-open sets.

Keywords – *α-open sets, pre-α-closed function, pre-$\pi g\alpha$-closed functions, α-$\pi g\alpha$-closed functions.*

1 Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc by utilizing generalized open sets. One of the most well known notions and also an inspiration source is the notion of α-open sets introduced in [17].

In 1970, Levine [9] defined and studied generalized closed sets in topological spaces. In 1982, Malghan [15] defined generalized closed functions and obtained some preservation theorems of normality and regularity. In 1990, Arya and Nour [5] defined generalized semi-open sets and used them to obtain characterizations of s-normal spaces due to Maheshwari and Prasad [10]. In 1993, Devi et.al. [6] defined and studied generalized semi-closed functions and showed that the continuous generalized semi-closed surjective image of a normal space is s-normal. In 1998, Noiri et.al. [18] defined generalized pre closed sets and introduced generalized pre closed functions and showed that the

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continuous generalized pre closed surjective image of normal space is prenormal [20](or p-normal [21]). Recently, Tahiliani [23] has defined generalized β-closed functions and has shown that the continuous generalized β-closed surjective images of normal (resp. regular) spaces are β-normal [11] (resp. β-regular [2]). Further, it has shown that β-regularity is preserved under continuous pre-β-open [11] β-$g\beta$-closed [23] surjections. Recently, Arockiarani et.al [4] has defined $\pi g\alpha$-closed sets and studied properties and characterizations of them.

2 Preliminaries

Throughout this paper, X and Y refer always to topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, $cl(A)$ and $int(A)$ denote the closure of A and the interior of A in X, respectively.

A subset A of X is said to be regular open [22] (resp. regular closed [22]) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). The finite union of regular open sets is said to be π-open [24]. The complement of a π-open set is said to be π-closed [24].

A subset A of X is said to be β-open [1] (= semi pre-open [3]) if $A \subseteq cl(int(cl(A)))$.

A subset A of X is said to be α-open [17] if $A \subseteq int(cl(int(A)))$.

The complement of α-open (resp. regular open) set is called α-closed (resp. regular closed).

The intersection of all α-closed sets of X containing A is called the α-closure of A and is denoted by $\alpha cl(A)$.

It is evident that a set A is α-closed if and only if $\alpha cl(A) = A$.

The α-interior of A, $\alpha int(A)$, is the union of all α-open sets contained in A.

A subset A of X is said to be α-clopen if it is α-open and α-closed.

The family of all α-open (resp. α-closed, α-clopen, β-open, regular open, regular closed) sets of X is denoted by $\alpha O(X)$ (resp. $\alpha C(X)$, $\alpha CO(X)$, $\beta O(X)$, $RO(X)$, $RC(X)$).

The family of all α-open sets of X containing a point $x \in X$ is denoted by $\alpha O(X, x)$.

A subset A of a topological space (X, τ) is called $\pi g\alpha$-closed [4] set of X if $\alpha cl(A) \subseteq U$ holds whenever $A \subseteq U$ and U is π-open in X.

A will be called $\pi g\alpha$-open if $X \backslash A$ is $\pi g\alpha$-closed.

Theorem 2.1. *[3] For any subset A of a topological space X, the following conditions are equivalent:*

1. $A \in \beta O(X)$;
2. $A \subseteq cl(int(cl(A)))$;
3. $cl(A) \in RC(X)$.

3 Pre-$\pi g\alpha$-closed Functions

Lemma 3.1. *A subset A of a space X is $\pi g\alpha$-open in X if and only if $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and F is π-closed in X.*

Remark 3.2. *Every α-open set is $\pi g\alpha$-open but not conversely.*

Example 3.3. *Let* $X = \{a, b, c\}$ *and* $\tau = \{\phi, X, \{a\}, \{a, c\}\}$. *Then* $\{b\}$ *is* $\pi g\alpha$*-open set but not* α*-open.*

Definition 3.4. *A function* $f : X \to Y$ *is said to be pre-*$\pi g\alpha$*-closed (=* α*-*$\pi g\alpha$*-closed) (resp. regular* $\pi g\alpha$*-closed, almost* $\pi g\alpha$*-closed) if for each* $F \in \alpha C(X)$ *(resp.* $F \in \alpha CO(X)$, $F \in RC(X)$*),* $f(F)$ *is* $\pi g\alpha$*-closed in* Y.

Definition 3.5. *A function* $f : X \to Y$ *is said to be* $\pi g\alpha$*-closed if for each closed set* F *of* X, $f(F)$ *is* $\pi g\alpha$*-closed in* Y.

Remark 3.6. *From the above definitions, we obtain the following diagram:*

$$\begin{array}{ccc} \textbf{pre-}\pi g\alpha\textbf{-closed} & \longrightarrow & \textbf{regular } \pi g\alpha\textbf{-closed} \\ \downarrow & & \uparrow \\ \pi g\alpha\textbf{-closed} & \longrightarrow & \textbf{almost } \pi g\alpha\textbf{-closed} \end{array}$$

None of all implications in the above diagram is reversible as the following examples show.

Example 3.7. *Let* $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{c\}\}$ *and* $\sigma = \{\phi, Y, \{b\}, \{c\}, \{b, c\}\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is regular* $\pi g\alpha$*-closed but it is not pre-*$\pi g\alpha$*-closed.*

Example 3.8. *Let* $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{b, c\}\}$ *and* $\sigma = \{\phi, Y, \{b\}, \{c\}, \{b, c\}\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $\pi g\alpha$*-closed but not pre-*$\pi g\alpha$*-closed.*

Example 3.9. *Let* $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a, b\}\}$ *and* $\sigma = \{\phi, Y, \{b\}, \{c\}, \{b, c\}\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is almost* $\pi g\alpha$*-closed but not* $\pi g\alpha$*-closed.*

Example 3.10. *Let* $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ *and* $\sigma = \{\phi, Y, \{b\}, \{c\}, \{b, c\}\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is regular* $\pi g\alpha$*-closed but not almost* $\pi g\alpha$*-closed.*

The proof of the following Lemma follows using a standard technique and thus omitted.

Lemma 3.11. *A surjective function* $f : X \to Y$ *is pre-*$\pi g\alpha$*-closed (resp. regular* $\pi g\alpha$*-closed) if and only if for each subset B of Y and each* $U \in \alpha O(X)$ *(resp.* $U \in \alpha CO(X)$*) containing* $f^{-1}(B)$*, there exists an* $\pi g\alpha$*-open set V of Y such that* $B \subseteq V$ *and* $f^{-1}(V) \subseteq U$*.*

Corollary 3.12. *If a surjective function* $f : X \to Y$ *is pre-*$\pi g\alpha$*-closed (resp. regular* $\pi g\alpha$*-closed), then for each* π*-closed set* K *of* Y *and each* $U \in \alpha O(X)$ *(resp.*$U \in \alpha CO(X)$*) containing* $f^{-1}(K)$*, there exists* $V \in \alpha O(Y)$ *containing K such that* $f^{-1}(V) \subseteq U$*.*

Proof. Suppose that $f : X \to Y$ is pre-$\pi g\alpha$-closed (resp. regular $\pi g\alpha$-closed). Let K be any π-closed set of Y and $U \in \alpha O(X)$ (resp. $U \in \alpha CO(X)$) containing $f^{-1}(K)$. By Lemma 3.11, there exists an $\pi g\alpha$-open set G of Y such that $K \subseteq G$ and $f^{-1}(G) \subseteq U$. Since K is π-closed, by Lemma 3.1, $K \subseteq \alpha int(G)$. Put $V = \alpha int(G)$. Then, $K \subseteq V \in \alpha O(Y)$ and $f^{-1}(V) \subseteq U$.

Definition 3.13. *[7] A function $f : X \to Y$ is said to be*

1. *π-irresolute if $f^{-1}(F)$ is π-closed in X for every π-closed set F of Y.*

2. *m-π-closed if $f(F)$ is π-closed in Y for every π-closed set F of X.*

Lemma 3.14. *A function $f : X \to Y$ is π-irresolute if and only if $f^{-1}(F)$ is π-open in X for every π-open set F of Y.*

Theorem 3.15. *If $f : X \to Y$ is π-irresolute pre-$\pi g\alpha$-closed bijection, then $f(H)$ is $\pi g\alpha$-closed in Y for each $\pi g\alpha$-closed set H of X.*

Proof. Let H be any $\pi g\alpha$-closed set of X and V an π-open set of Y containing $f(H)$. Since $f^{-1}(V)$ is an π-open set of X containing H, $\alpha cl(H) \subseteq f^{-1}(V)$ and hence $f(\alpha cl(H)) \subseteq V$. Since f is pre-$\pi g\alpha$-closed and $\alpha cl(H) \in \alpha C(X)$, $f(\alpha cl(H))$ is $\pi g\alpha$-closed in Y. We have $\alpha cl(f(H)) \subseteq \alpha cl(f(\alpha cl(H))) \subseteq V$. Therefore, $f(H)$ is $\pi g\alpha$-closed in Y.

Definition 3.16. *A function $f : X \to Y$ is said to be pre-$\pi g\alpha$-continuous or α-$\pi g\alpha$-continuous if $f^{-1}(K)$ is $\pi g\alpha$-closed in X for every $K \in \alpha C(Y)$.*

It is obvious that a function $f : X \to Y$ is pre-$\pi g\alpha$-continuous if and only if $f^{-1}(V)$ is $\pi g\alpha$-open in X for every $V \in \alpha O(Y)$.

Theorem 3.17. *If $f : X \to Y$ is m-π-closed pre-$\pi g\alpha$-continuous bijection, then $f^{-1}(K)$ is $\pi g\alpha$-closed in X for each $\pi g\alpha$-closed set K of Y.*

Proof. Let K be $\pi g\alpha$-closed set of Y and U an π-open set of X containing $f^{-1}(K)$. Put $V = Y - f(X - U)$, then V is an π-open in Y, $K \subseteq V$ and $f^{-1}(V) \subseteq U$. Therefore, we have $\alpha cl(K) \subseteq V$ and hence $f^{-1}(K) \subseteq f^{-1}(\alpha cl(K)) \subseteq f^{-1}(V) \subseteq U$. Since f is pre-$\pi g\alpha$-continuous and $\alpha cl(K)$ is α-closed in Y, $f^{-1}(\alpha cl(K))$ is $\pi g\alpha$-closed in X and hence $\alpha cl(f^{-1}(K)) \subseteq \alpha cl(f^{-1}(\alpha cl(K))) \subseteq U$. This shows that $f^{-1}(K)$ is $\pi g\alpha$-closed in X.

Recall that a function $f : X \to Y$ is said to be α-irresolute [14] if $f^{-1}(V) \in \alpha O(X)$ for every $V \in \alpha O(Y)$.

Remark 3.18. *Every α-irresolute function is pre-$\pi g\alpha$-continuous but not conversely.*

Proof. Let $A \in \alpha O(Y)$. Since f is α-irresolute, $f^{-1}(A) \in \alpha O(X)$. Since α-open set is $\pi g\alpha$-open, $f^{-1}(A)$ is $\pi g\alpha$-open in X. Hence f is pre-$\pi g\alpha$-continuous.

Example 3.19. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}, \{c\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is pre-$\pi g\alpha$-continuous but not α-irresolute.*

Corollary 3.20. *If $f : X \to Y$ is m-π-closed α-irresolute bijection, then $f^{-1}(K)$ is $\pi g\alpha$-closed in X for each $\pi g\alpha$-closed set K of Y.*

Proof. It is obtained from Theorem 3.17 and Remark 3.18.

For the composition of pre-$\pi g\alpha$-closed functions, we have the following Theorems.

Theorem 3.21. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the composition $gof : X \to Z$ is pre-$\pi g\alpha$-closed if f is pre-$\pi g\alpha$-closed and g is π-irresolute pre-$\pi g\alpha$-closed bijection.*

Proof. The proof follows immediately from Theorem 3.15.

Theorem 3.22. *Let $f : X \to Y$ and $g : Y \to Z$ be functions and let the composition $gof : X \to Z$ be pre-$\pi g\alpha$-closed. Then the following hold:*

1. *If f is an α-irresolute surjection, then g is pre-$\pi g\alpha$-closed;*
2. *If g is a m-π-closed pre-$\pi g\alpha$-continuous injection, then f is pre-$\pi g\alpha$-closed.*

Proof. (1) Let $K \in \alpha C(Y)$. Since f is α-irresolute and surjective, $f^{-1}(K) \in \alpha C(X)$ and $(gof)(f^{-1}(K)) = g(K)$. Therefore, $g(K)$ is $\pi g\alpha$-closed in Z and hence g is pre-$\pi g\alpha$-closed.

(2) Let $H \in \alpha C(X)$. Then $(gof)(H)$ is $\pi g\alpha$-closed in Z and $g^{-1}((gof)(H)) = f(H)$. By Theorem 3.17, $f(H)$ is $\pi g\alpha$-closed in Y and hence f is pre-$\pi g\alpha$-closed.

The following Lemma is analogous to Lemma 3.11, the straightforward proof is omitted.

Lemma 3.23. *A surjective function $f : X \to Y$ is almost $\pi g\alpha$-closed if and only if for each subset B of Y and each $U \in RO(X)$ containing $f^{-1}(B)$, there exists an $\pi g\alpha$-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$.*

Corollary 3.24. *If a surjective function $f : X \to Y$ is almost $\pi g\alpha$-closed, then for each π-closed set K of Y and each $U \in RO(X)$ containing $f^{-1}(K)$, there exists $V \in \alpha O(Y)$ such that $K \subseteq V$ and $f^{-1}(V) \subseteq U$.*

Proof. The proof is similar to that of Corollary 3.12.

Recall that a topological space (X, τ) is said to be quasi-normal [24] if for every disjoint π-closed sets A and B of X, there exist disjoint sets $U, V \in \tau$ such that $A \subseteq U$ and $B \subseteq V$.

Definition 3.25. *A topological space (X, τ) is said to be quasi-α-normal if for every disjoint π-closed sets A and B of X, there exist disjoint sets U, $V \in \alpha O(X)$ such that $A \subseteq U$ and $B \subseteq V$.*

Theorem 3.26. *Let $f : X \to Y$ be a π-irresolute almost $\pi g\alpha$-closed surjection. If X is quasi-normal, then Y is quasi-α-normal.*

Proof. Let K_1 and K_2 be any disjoint π-closed sets of Y. Since f is π-irresolute, $f^{-1}(K_1)$ and $f^{-1}(K_2)$ are disjoint π-closed sets of X. By the quasi-normality of X, there exist disjoint open sets U_1 and U_2 such that $f^{-1}(K_i) \subseteq U_i$, where $i = 1, 2$. Now, put $G_i = int(cl(U_i))$ for $i = 1, 2$, then $G_i \in RO(X)$, $f^{-1}(K_i) \subseteq U_i \subseteq G_i$ and $G_1 \cap G_2 = \phi$. By Corollary 3.24, there exists $V_i \in \alpha O(Y)$ such that $K_i \subseteq V_i$ and $f^{-1}(V_i) \subseteq G_i$, $i = 1, 2$. Since $G_1 \cap G_2 = \phi$, f is surjective we have $V_1 \cap V_2 = \phi$. This shows that Y is quasi-α-normal.

Definition 3.27. *A function $f : X \to Y$ is said to be α-open [16] (resp. α-closed [19]), if $f(U) \in \alpha O(Y)$ (resp. $f(U) \in \alpha C(Y)$) for every open (resp. closed) set U of X.*

Definition 3.28. *[13] A subset A of X is said to be $g\alpha$-closed if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.*

Definition 3.29. *[12] A function $f : X \to Y$ is said to be $g\alpha$-closed if $f(U)$ is $g\alpha$-closed in Y for every closed set U of X.*

Remark 3.30. *For a function of a topological space, the following hold:*

$$\textbf{closed} \longrightarrow \alpha\textbf{-closed} \longrightarrow g\alpha\textbf{-closed} \longrightarrow \pi g\alpha\textbf{-closed}$$

The reverse implications are not true.

The following four Corollaries are immediate consequences of Theorem 3.26.

Corollary 3.31. *If $f : X \to Y$ is a π-irresolute $\pi g\alpha$-closed surjection and X is quasi-normal, then Y is quasi-α-normal.*

Corollary 3.32. *If $f : X \to Y$ is a π-irresolute $g\alpha$-closed surjection and X is quasi-normal, then Y is quasi-α-normal.*

Corollary 3.33. *If $f : X \to Y$ is a π-irresolute α-closed surjection and X is quasi-normal, then Y is quasi-α-normal.*

Corollary 3.34. *If $f : X \to Y$ is a π-irresolute closed surjection and X is quasi-normal, then Y is quasi-α-normal.*

Definition 3.35. *[4, 17] A function $f : X \to Y$ is said to be pre-α-closed (resp. pre-α-open) if for each $F \in \alpha C(X)$ (resp. $F \in \alpha O(X)$), $f(F) \in \alpha C(Y)$ (resp. $f(F) \in \alpha O(Y)$).*

Remark 3.36. *Every pre-α-closed function is α-closed but not conversely.*

Proof. Let A be a closed set of X. Then A is α-closed set of X. Since f is pre-α-closed, f(A) $\in \alpha$C(Y). Hence f is α-closed.

Example 3.37. *In Example 3.19, f is α-closed but not pre-α-closed.*

Remark 3.38. *Every pre-α-closed function is pre-$\pi g\alpha$-closed but not conversely.*

Proof. Let $F \in \alpha C(X)$. Since f is pre-α-closed, $f(F) \in \alpha C(Y)$. Since α-closed set is $\pi g\alpha$-closed, $f(F)$ is $\pi g\alpha$-closed in Y. Hence f is pre-$\pi g\alpha$-closed.

Example 3.39. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is pre-$\pi g\alpha$-closed but not pre-α-closed.*

Theorem 3.40. *Let $f : X \to Y$ be a m-π-closed pre-$\pi g\alpha$-continuous injection. If Y is quasi-α-normal, then X is quasi-α-normal.*

Proof. Let H_1 and H_2 be any disjoint π-closed sets of X. Since f is a m-π-closed injection, $f(H_1)$ and $f(H_2)$ are disjoint π-closed sets of Y. By the quasi-α-normality of Y, there exist disjoint sets $V_1, V_2 \in \alpha O(Y)$ such that $f(H_i) \subseteq V_i$, for $i = 1, 2$. Since f is pre-$\pi g\alpha$-continuous $f^{-1}(V_i)$ and $f^{-1}(V_i)$ are disjoint $\pi g\alpha$-open sets of X and $H_i \subseteq f^{-1}(V_i)$ for $i = 1, 2$. Now, put $U_i = \alpha int(f^{-1}(V_i))$ for $i = 1, 2$. Then $U_i \in \alpha O(X)$, $H_i \subseteq U_i$ and $U_1 \cap U_2 = \phi$. This shows that X is quasi-α-normal.

Corollary 3.41. *If $f : X \to Y$ is a m-π-closed α-irresolute injection and Y is quasi-α-normal, then X is quasi-α-normal.*

Proof. This is an immediate consequence of Theorem 3.40, since every α-irresolute function is pre-$\pi g\alpha$-continuous.

Definition 3.42. *A topological space X is said to be quasi-regular if for each π-closed set F and each point $x \in X - F$, there exist disjoint U, $V \in \tau$ such that $x \in U$ and $F \subseteq V$.*

Theorem 3.43. *For a topological space X, the following properties are equivalent:*

1. *X is quasi-regular;*
2. *For each π-open set U in X and each $x \in U$, there exists $V \in \tau$ such that $x \in V \subseteq cl(V) \subseteq U$;*
3. *For each π-open set U in X and each $x \in U$, there exists a clopen set V such that $x \in V \subseteq U$.*

Proof. (1) $\Rightarrow$ (2): Let U be an π-open set of X containing x. Then $X \backslash U$ is a π-closed set not containing x. By (1), there exist disjoint $X \backslash cl(V)$, V $\in \tau$ such that x $\in$ V and X$\backslash$U $\subseteq$ X$\backslash$cl(V). Then we have V $\in \tau$ such that x $\in$ V $\subseteq$ cl(V) $\subseteq$ U.

(2) $\Rightarrow$ (3): Let U be an π-open set of X containing x. By (2), there exists V $\in \tau$ such that x $\in$ V $\subseteq$ cl(V) $\subseteq$ U. Take V = cl(V). Thus V is closed and so V is clopen. Hence we have V is clopen set such that x $\in$ V $\subseteq$ U.

(3) $\Rightarrow$ (1): Let F = X$\backslash$U be a π-closed set not containing x. Then U is an π-open set of X containing x. By (3), there exists a clopen set V such that x $\in$ V $\subseteq$ U. Then there exist disjoint G = X$\backslash$V, V $\in \tau$ such that x $\in$ V and F = X$\backslash$U $\subseteq$ G = X$\backslash$V. Hence X is quasi-regular.

Definition 3.44. *A topological space X is said to be quasi-α-regular if for each π-closed set F and each point $x \in X - F$, there exist disjoint U, $V \in \alpha O(X)$ such that $x \in U$ and $F \subseteq V$.*

Theorem 3.45. *For a topological space X, the following properties are equivalent:*

1. *X is quasi-α-regular;*
2. *For each π-open set U in X and each $x \in U$, there exists $V \in \alpha O(X)$ such that $x \in V \subseteq \alpha cl(V) \subseteq U$;*
3. *For each π-open set U in X and each $x \in U$, there exists $V \in \alpha CO(X)$ such that $x \in V \subseteq U$.*

Theorem 3.46. *Let $f : X \to Y$ be an π-irresolute α-open almost $\pi g\alpha$-closed surjection. If X is quasi-regular, then Y is quasi-α-regular.*

Proof. Let $y \in Y$ and V be an π-open neighbourhood of y. Take a point $x \in f^{-1}(y)$. Then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is π-open in X. By the quasi-regularity of X, there exists an π-open set U of X such that $x \in U \subseteq cl(U) \subseteq f^{-1}(V)$. Then $y \in f(U) \subseteq f(cl(U)) \subseteq V$. Also, since U is open set of X and f is α-open, $f(U) \in \alpha O(Y)$. Moreover, since U is β-open, by Theorem 2.1, $cl(U)$ is regular closed set of X. Since f is almost $\pi g\alpha$-closed, $f(cl(U))$ is $\pi g\alpha$-closed in Y. Therefore, we obtain $y \in f(U) \subseteq \alpha cl(f(U)) \subseteq \alpha cl(f(cl(U))) \subseteq V$. It follows from Theorem 3.45 that Y is quasi-α-regular.

Corollary 3.47. *If $f : X \to Y$ is an π-irresolute α-open $\pi g\alpha$-closed surjection and X is quasi-regular, then Y is quasi-α-regular.*

Corollary 3.48. *If $f : X \to Y$ is an π-irresolute α-open α-closed surjection and X is quasi-regular, then Y is quasi-α-regular.*

4 Conclusion

Topology is an area of Mathematics concerned with the properties of space that are preserved under continuous deformations including stretching and bending, but not tearing. By the middle of the 20th century, topology had become a major branch of Mathematics.

Topology as a branch of Mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of transformation especially those properties that are invariant under a certain kind of equivalence and it is the study of those properties of geometric configurations which remain invariant when these configurations are subjected to one-to-one bicontinuous transformations or homeomorphisms. Topology operates with more general concepts than analysis. Differential properties of a given transformation are nonessential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of problems to which analysis cannot give the answer.

In this paper, the concept of α-open sets introduced by [17] is used to introduce and study pre-$\pi g\alpha$-closed functions. The associated functions of pre-$\pi g\alpha$-closed functions are widely investigated.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ., 12(1983), 77-90.

[2] M. E. Abd El-Monsef, A. N. Geaisa and R. A. Mahmoud, *β-regular spaces*, Proc. Math. Phys. Soc. Egypt., 60(1985), 47-52.

[3] D. Andrijevic, *Semi-preopen sets*, Mat. Vesnik, 38(1986), 24-32.

[4] I. Arockiarani, K. Balachandran and C. Janaki, *On contra-$\pi g\alpha$-continuous functions*, Kochi J. Math., 3(2008), 201-209.

[5] S. P. Arya and T. M. Nour, *Characterizations of s-normal spaces*, Indian J. Pure Appl. Math., 21(8)(1990), 717-719.

[6] R. Devi, K. Balachandran and H. Maki, *Semi-generalized closed maps and generalized semi-closed maps*, Mem. Fac. Kochi Univ. Ser. A. Math., 14(1993), 41-54.

[7] E. Ekici and C. W. Baker, *On πg-closed sets and continuity*, Kochi J. Math., 2(2007), 35-42.

[8] S. Jafari, *Rare α-continuity*, Bull. Malays. Math. Sci. Soc., (2)28(2)(2005), 157-161.

[9] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, 19(2)(1970), 89-96.

[10] S. N. Maheshwari and R. Prasad, *On s-normal spaces*, Bull. Math. Soc. Sci. Math. R. S. Roumanie, 22(70)(1978), 27-29.

[11] R. A. Mahmoud and M. E. Abd El-Monsef, *β-irresolute and β-topological invariant*, Proc. Math. Pakistan. Acad. Sci., 27(1990), 285-296.

[12] H. Maki, R. Devi and K. Balachandran, *Associated topologies of generalized α-closed sets and α-generalized closed sets*, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 15(1994), 51-63.

[13] H. Maki, R. Devi and K. Balachandran, *Generalized α-closed sets in topology*, Bull. Fukuoka Univ. Ed. Part III, 42(1993), 13-21.

[14] S. N. Maheshwari and S. S. Thakur, *On α-irresolute mappings*, Tamkang J. Math., 11(1980), 209-214.

[15] S. R. Malghan, *Generalized closed maps*, J. Karnataka Univ. Sci., 27(1982), 82-88.

[16] A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, *α-continuous and α-open mappings*, Acta Math. Hungar., 41(1983), 213-218.

[17] O. Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15(1965), 961-970.

[18] T. Noiri, H. Maki and J. Umehara, *Generalized preclosed functions*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 19(1998), 13-20.

[19] T. Noiri, *Almost continuity and some separation axioms*, Glasnik Math., 9(29)(1974), 131-135.

[20] T. M. J. Nour, *Contributions to the theory of bitopological spaces*, Ph. D Thesis, Delhi University, India, 1989.

[21] Paul and Bhattacharyya, *On p-normal spaces*, Soochow J. Math., 21(3)(1995), 273-289.

[22] M. H. Stone, *Applications of the theory of Boolean rings to general topology*, Trans. Amer. Math. Soc., 41(1937), 375-481.

[23] S. Tahiliani, *Generalized β-closed functions*, Bull. Cal. Math. Soc., 98(4)(2006), 367-376.

[24] V. Zaitsev, *On certain classes of topological spaces and their bicompactifications*, Dokl. Akad. Nauk. SSSR, 178(1968), 778-779.