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ON RANKING OF TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS AND ITS APPLICATION TO MULTI ATTRIBUTE GROUP DECISION MAKING

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Abstract – This paper focuses on the study of two characteristics of trapezoidal intuitionistic fuzzy number (TRIFN), viz., Value index and Ambiguity index. Based on these two indexes, we develop an algorithm for ranking of trapezoidal intuitionistic fuzzy number (TRIFN). Furthermore, we present an application of this ranking method in multi attribute group decision making problem. An illustrative numerical example demonstrate our approach to multi attribute group decision making problem.

Keywords – *Value, ambiguity, ranking, trapezoidal intuitionistic fuzzy numbers, multi attribute group decision making.*

1 Introduction

The theory of intuitionistic fuzzy sets were introduced by Atanassov [1] as a generalization of fuzzy set theory proposed by Zadeh [14]. The notion of fuzzy numbers were extended to develop the concept of intuitionistic fuzzy numbers by adding an additional non-membership function which is able to express more abundant and flexible information as compared to fuzzy numbers. Various definitions of intuitionistic fuzzy numbers and ranking methods have been proposed over last few years. Mitchell [7] introduced a ranking method for intuitionistic fuzzy number considering intuitionistic fuzzy numbers as an ensemble of fuzzy numbers. Chen and Hwang [2] introduced a ranking method based on scorings of intuitionistic fuzzy numbers. The concept of Chen and Hwang

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have been later generalized by Nayagam et.al [4] to formulate a new process of ranking called method of IF scorning. Wang [8] gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic fuzzy number. Further Wang and Zhang [9] defined the trapezoidal intuitionistic fuzzy numbers and gave a ranking method which transformed the ranking of trapezoidal intuitionistic fuzzy number in to ranking of interval numbers. Li [6] developed a ratio ranking method for triangular intuitionistic fuzzy numbers and applied to multi attribute decision making. The application of trapezoidal intuitionistic fuzzy numbers in decision making problems are abundant in literature [[5],[6],[8]- [11]]. Since ranking of alternative plays an efficient role in decision making problems,ranking of trapezoidal intuitionistic fuzzy number has become a task of outmost importance when we deal with decision making problems based on intuitionistic fuzzy information. In this article we have paid attention to the formulation of a ranking algorithm based on linear sum of value and ambiguity indexes and the procedure has been applied to rank the alternatives in multi attribute group decision making (MAGDM) problems. However, to solve the MAGDM problem, we have adopted the method suggested by Wu and Cao [10] which is based on intuitionistic trapezoidal fuzzy weighted geometric operator (ITFWG) and intuitionistic trapezoidal fuzzy hybrid geometric operator (ITFHG).

The rest of the paper is set out as follows: Section 2 includes basic definitions and operations of trapezoidal intutionistic fuzzy numbers. Section 3 consist of the algorithm which have been developed for ranking of trapezoidal intutionistic fuzzy numbers. In section 4 and 5, application of the formulated algorithm have been illustrated by giving a suitable numerical example. Section 6 contains the conclusion of this article.

2 Preliminaries

We collect some basic definitions and notations related to trapezoidal intitionistic fuzzy number.

Definition 2.1. [8] A TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a special Intuitionistic Fuzzy set on a set of real number $\mathbb{R}$, whose membership function and non membership function are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} w_{\tilde{a}} & a_1 \le x \le a_2 \\ w_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)} w_{\tilde{a}} & a_3 \le x \le a_4 \\ 0 & a_4 < x or a_1 > x \end{cases} \tag{1}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2-x)+u_{\tilde{a}}(x-a_1)}{(a_2-a_1)} & a_1 \le x \le a_2 \\ u_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{(x-a_3)+u_{\tilde{a}}(a_4-x)}{(a_4-a_3)} & a_3 \le x \le a_4 \\ 1 & a_4 < x or a_1 > x \end{cases} \tag{2}$$

respectively.

The values $w_{\tilde{a}}$ and $u_{\tilde{a}}$ represents the maximum degree of membership and minimum degree of non membership, respectively, such that the conditions $0 \leq w_{\tilde{a}} \leq 1$, $0 \leq u_{\tilde{a}} \leq 1$ and $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$ are satisfied. The parameters $w_{\tilde{a}}$ and $u_{\tilde{a}}$ reflects the confidence level and non confidence level of the TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$,respectively.

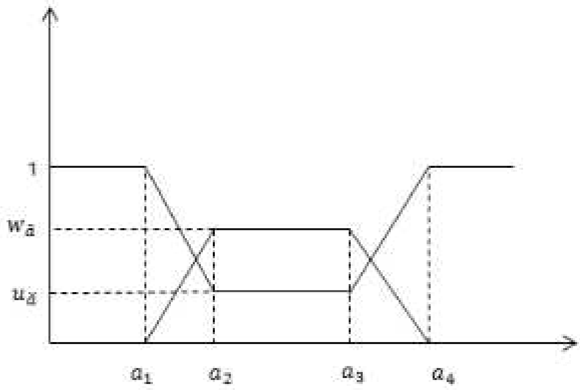

Figure 1: Trapezoidal Intuitionistic fuzzy numbers(TRIFN)

The function $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x)$ is called an IF index of an element x in $\tilde{a}$.It is the degree of the indeterminacy membership of the element x in $\tilde{a}$.

Arithmatical operations of trapezoidal intuitionistic fuzzy number

Definition 2.2. [9] Let $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); w_{\tilde{b}}, u_{\tilde{b}} \rangle$ be two TRIFNs and λ be a real number. The arithmetical operations are listed as follows:

- $\tilde{a} \oplus \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); w_{\tilde{a}} + w_{\tilde{b}} - w_{\tilde{a}} w_{\tilde{b}}, u_{\tilde{a}} u_{\tilde{b}} \rangle$
- $\tilde{a} \otimes \tilde{b} = \langle (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); w_{\tilde{a}} w_{\tilde{b}}, u_{\tilde{a}} + u_{\tilde{b}} - u_{\tilde{a}} u_{\tilde{b}} \rangle$
- $\lambda \tilde{a} = \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); 1 - (1 - w_{\tilde{a}})^{\lambda}, u_{\tilde{a}}^{\lambda} \rangle$
- $\tilde{a}^{\lambda} = \langle (a_1^{\lambda}, a_2^{\lambda}, a_3^{\lambda}, a_4^{\lambda}); w_{\tilde{a}}^{\lambda}, 1 - (1 - u_{\tilde{a}})^{\lambda} \rangle$

Definition 2.3. A α-cut set of a TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a crisp sub set of $\mathbb{R}$, denoted and defined as $\tilde{a}_{\alpha} = \{x | \mu_{\tilde{a}}(x) \geq \alpha\}$ where $0 \leq \alpha \leq w_{\tilde{a}}$. The α-cut set of a TRIFN $\tilde{a}$ can be represented as the closed interval $[L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)] = [a_1 + \dfrac{\alpha(a_2 - a_1)}{w_{\tilde{a}}}, a_4 - \dfrac{\alpha(a_4 - a_3)}{w_{\tilde{a}}}]$.

Definition 2.4. A β-cut set of a TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a crisp sub set of $\mathbb{R}$, denoted and defined as $\tilde{a}_{\beta} = \{x | \nu_{\tilde{a}}(x) \leq \beta\}$ where $u_{\tilde{a}} \leq \beta \leq 1$. The β-cut set of a TRIFN $\tilde{a}$ can be represented as the closed interval$[L_{\tilde{a}}(\beta), R_{\tilde{a}}(\beta)] = [\dfrac{(1-\beta)a_2 + (\beta - u_{\tilde{a}})a_1}{1 - u_{\tilde{a}}}, \dfrac{(1-\beta)a_3 + (\beta - u_{\tilde{a}})a_4}{1 - u_{\tilde{a}}}]$.

Value and ambiguity of a trapezoidal intuitionistic fuzzy number

The value and ambiguity of a trapezoidal intuitionistic fuzzy number can be defined similarly to those of a triangular intuitionistic fuzzy number(TIFNs) introduced by D.F.Li [6].

Definition 2.5. [3] Let $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ be an α-cut set and a β-cut set of a trapezoidal intuitionistic fuzzy number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$, respectively. Then the values of the membership function $\mu_{\tilde{a}}(x)$ and the non-membership function $\nu_{\tilde{a}}(x)$ for the trapezoidal intuitionistic fuzzy number $\tilde{a}$ are defined as follows:

$$V_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} \frac{L_{\tilde{a}}(\alpha) + R_{\tilde{a}}(\alpha)}{2} f(\alpha) d\alpha \tag{3}$$

$$V_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^1 \frac{L_{\tilde{a}}(\beta) + R_{\tilde{a}}(\beta)}{2} g(\beta) d\beta \tag{4}$$

respectively,where the function $f(\alpha)$ is a non-negative and non-decreasing function on the interval $[0, w_{\tilde{a}}]$ with $f(0) = 0$ and $\int_0^{w_{\tilde{a}}} f(\alpha)d\alpha = w_{\tilde{a}}$; the function $g(\beta)$ is a non-negative and non-increasing function on the interval $[u_{\tilde{a}}, 1]$ with $g(1) = 0$ and $\int_{u_{\tilde{a}}}^1 g(\beta)d\beta = 1 - u_{\tilde{a}}$.

Definition 2.6. [3] Let $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ be an α-cut set and a β-cut set of a trapezoidal intuitionistic fuzzy number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$, respectively. Then the ambiguities of the membership function $\mu_{\tilde{a}}(x)$ and the non-membership function $\nu_{\tilde{a}}(x)$ for the trapezoidal intuitionistic fuzzy number $\tilde{a}$ are defined as follows:

$$A_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} (R_{\tilde{a}}(\alpha) - L_{\tilde{a}}(\alpha)) f(\alpha) d\alpha \tag{5}$$

$$A_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^1 (R_{\tilde{a}}(\beta) - L_{\tilde{a}}(\beta)) g(\beta) d\beta \tag{6}$$

respectively.

Remark 2.7. The weight functions $f(\alpha)$ and $g(\beta)$ can be chosen according to decision maker's choice. We shall choose $f(\alpha) = \alpha/w_{\tilde{a}}$ and $g(\beta) = (1 - \beta)/(1 - u_{\tilde{a}})$ in this paper. The value of membership and non-membership can be calculated substituting these $f(\alpha)$ and $g(\beta)$ in equation (2) and (3) as follows:

$$\begin{aligned}
V_\mu(\tilde{a}) &= \int_0^{w_{\tilde{a}}} \left[a_1 + \frac{\alpha(a_2 - a_1)}{w_{\tilde{a}}} + a_4 - \frac{\alpha(a_4 - a_3)}{w_{\tilde{a}}} \right] \frac{\alpha}{2w_{\tilde{a}}} d\alpha \\
&= \left[\frac{a_1 + a_4}{4w_{\tilde{a}}} \alpha^2 \right]_0^{w_{\tilde{a}}} + \left[\frac{(a_2 - a_1 - a_4 + a_3)}{6(w_{\tilde{a}})^2} \alpha^3 \right]_0^{w_{\tilde{a}}} \\
&= \frac{a_1 + a_4 + 2(a_2 + a_3)}{12} w_{\tilde{a}}
\end{aligned}$$

$$
\begin{aligned}
V_\nu(\tilde{a}) &= \int_{u_{\tilde{a}}}^{1}\left[\frac{(1-\beta)a_2+(\beta-u_{\tilde{a}})a_1}{1-u_{\tilde{a}}}+\frac{(1-\beta)a_3+(\beta-u_{\tilde{a}})a_4}{1-u_{\tilde{a}}}\right]\frac{(1-\beta)}{2(1-u_{\tilde{a}})}d\beta \\
&= \int_{u_{\tilde{a}}}^{1}\frac{(a_2+a_3-a_1-a_4)(1-\beta)^2+(1-u_{\tilde{a}})(a_1+a_4)(1-\beta)}{2(1-u_{\tilde{a}})}d\beta \\
&= -\left[\frac{(a_2+a_3-a_1-a_4)(1-\beta)^3}{6(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1}-\left[\frac{(a_1+a_4)(1-u_{\tilde{a}})(1-\beta)^2}{4(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1} \\
&= \frac{[a_1+a_4+2(a_2+a_3)](1-u_{\tilde{a}})}{12}
\end{aligned}
$$

Similarly, the ambiguity of membership and non-membership can be calculated by substituting the values of $f(\alpha)$ and $g(\beta)$ in equation (4) and (5) as follows:

$$
\begin{aligned}
A_\mu(\tilde{a}) &= \int_{0}^{w_{\tilde{a}}}\left[a_4-\frac{\alpha(a_4-a_3)}{w_{\tilde{a}}}-a_1-\frac{\alpha(a_2-a_1)}{w_{\tilde{a}}}\right]\frac{\alpha}{w_{\tilde{a}}}d\alpha \\
&= \left[\frac{a_4-a_1}{w_{\tilde{a}}}\alpha^2\right]_0^{2w_{\tilde{a}}}-\left[\frac{(a_2-a_1+a_4-a_3)}{3(w_{\tilde{a}})^2}\alpha^3\right]_0^{w_{\tilde{a}}} \\
&= \frac{(a_4-a_1)-2(a_2-a_3)}{6}w_{\tilde{a}}
\end{aligned}
$$

$$
\begin{aligned}
A_\nu(\tilde{a}) &= \int_{u_{\tilde{a}}}^{1}\left[\frac{(1-\beta)a_3+(\beta-u_{\tilde{a}})a_4}{1-u_{\tilde{a}}}-\frac{(1-\beta)a_2+(\beta-u_{\tilde{a}})a_1}{1-u_{\tilde{a}}}\right]\frac{(1-\beta)}{1-u_{\tilde{a}}}d\beta \\
&= \int_{u_{\tilde{a}}}^{1}\frac{[-(a_2-a_3-a_1+a_4)(1-\beta)^2+(1-u_{\tilde{a}})(a_4-a_1)(1-\beta)]}{(1-u_{\tilde{a}})^2}d\beta \\
&= \left[\frac{(a_4-a_1+a_2-a_3)(1-\beta)^3}{3(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1}-\left[\frac{(a_4-a_1)(1-u_{\tilde{a}})(1-\beta)^2}{2(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1} \\
&= \frac{(a_4-a_1)-2(a_2-a_3)}{6}(1-u_{\tilde{a}})
\end{aligned}
$$

3 Ranking of Trapezoidal Intuitionistic Fuzzy Number

The algorithm for ranking of trapezoidal intuitionistic fuzzy numbers is as follows:

- **Step-1:** Compute value and ambiguity of a trapezoidal intuitionistic fuzzy number as follows:

  (i) Evaluate value of membership $V_\mu(\tilde{a})$ and value of non-membership $V_\nu(\tilde{a})$ of a trapezoidal intutionistic fuzzy number using the following formulas:

$$V_\mu(\tilde{a}) = \frac{(a_1+a_4)+2(a_2+a_3)}{12}w_{\tilde{a}}$$

$$V_\nu(\tilde{a}) = \frac{(a_1 + a_4) + 2(a_2 + a_3)}{12}(1 - u_{\tilde{a}})$$

With the condition that $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$,it follows that $V_\mu(\tilde{a}) \leq V_\nu(\tilde{a})$. Thus, the values of membership and non-membership functions of a TRIFN $\tilde{a}$ may be concisely expressed as an interval $[V_\mu(\tilde{a}), V_\nu(\tilde{a})]$.

(ii) The ambiguity of membership $A_\mu(\tilde{a})$ and ambiguity of non-membership $A_\nu(\tilde{a})$ of a trapezoidal intutionistic fuzzy number using the following formulas:

$$A_\mu(\tilde{a}) = \frac{(a_4 - a_1) - 2(a_2 - a_3)}{6} w_{\tilde{a}}$$

$$A_\nu(\tilde{a}) = \frac{(a_4 - a_1) - 2(a_2 - a_3)}{6}(1 - u_{\tilde{a}})$$

With the condition that $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$,it follows that $A_\mu(\tilde{a}) \leq A_\nu(\tilde{a})$. Thus, the ambiguities of membership and non-membership functions of a TRIFN $\tilde{a}$ may be concisely expressed as an interval $[A_\mu(\tilde{a}), A_\nu(\tilde{a})]$.

- **Step-2:** Compute value and ambiguity indices of a TRIFN given by the formulae:

$$V(\tilde{a}) = \frac{V_\mu(\tilde{a}) + V_\nu(\tilde{a})}{2}$$

$$A(\tilde{a}) = \frac{A_\mu(\tilde{a}) + A_\nu(\tilde{a})}{2}$$

- **Step-3:** Next we define ranking function for trapezoidal intuitionistic fuzzy number $\tilde{a}$ as

$$R(\tilde{a}) = V(\tilde{a}) + A(\tilde{a})$$

4 Application of Ranking Method to Multi Attribute Group Decision Making (MAGDM)

In this section we shall apply the above discussed ranking method to MAGDM using trapezoidal intuitionistic fuzzy numbers. To solve MAGDM problem we shall employ the method based on ITFWG and ITFHG operators defined by Wu and Cao [10].

We collect some basic notations and definitions of different types of operators.

Definition 4.1. [10] Let $\tilde{\alpha}_j (j = 1, 2, ..., n)$ be a collection of trapezoidal intuitionistic fuzzy numbers, and let ITFWG:$\Omega^n \rightarrow \Omega$, if

$$ITFWG_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \tilde{\alpha}_1^{\omega_1} \otimes \tilde{\alpha}_2^{\omega_2} ...... \otimes \tilde{\alpha}_n^{\omega_n}$$

then ITFWG is called intuitionistic trapezoidal fuzzy weighted geometric operator of dimension n, where $\omega = (\omega_1, \omega_2, ...\omega_n)$ is the weight vector of $\tilde{\alpha}_j (j = 1, 2, ..., n)$, with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$. Especially, if $\omega = (1/n, 1/n, ..., 1/n)^T$, then the ITFWG operator is reduced to an intuitionistic trapezoidal fuzzy heometric averaging(ITFGA) operator of dimension n. which is defined as follows:

$$ITFGA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = (\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 ...... \otimes \tilde{\alpha}_n)^{1/n}$$

Definition 4.2. [10] Let $\tilde{\alpha}_j (j = 1, 2, ..., n)$ be a collection of trapezoidal intuitionistic fuzzy numbers. An intuitionistic trapezoidal fuzzy hybrid geometric (ITFHG) operator of dimension n is a mapping ITFHG:$\Omega^n \rightarrow \Omega$, that has an associated vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ such that $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

$$ITFHG_{\omega,w}(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \tilde{\alpha}_{\sigma(1)}^{\omega_1} \otimes \tilde{\alpha}_{\sigma(2)}^{\omega_2} ...... \otimes \tilde{\alpha}_{\sigma(n)}^{\omega_n}$$

where $\tilde{\alpha}_{\sigma(j)}$ is the largest of the weighted intuitionistic trapezoidal fuzzy numbers $\dddot{\tilde{\alpha}}_j (\dddot{\tilde{\alpha}}_j = \tilde{\alpha}_j^{nw_j}, j = 1, 2, ..., n)$. $w = (w_1, w_2, ..., w_n)^T$ is the weight vector of the $\tilde{\alpha}_j$ with $w_j in$ [0,1] and $\sum_{j=1}^{n} w_j = 1$, and n is the balancing coefficient, which plays a role of balance in a such a case, if the vector $w = (w_1, w_2, ..., w_n)^T$ approaches $(1/n, 1/n, ..., 1/n)^T$, then the vector $(\tilde{\alpha}_1^{nw_1}, \tilde{\alpha}_2^{nw_2}, ..., \tilde{\alpha}_n^{nw_n})^T$ approaches $(\tilde{\alpha}_1, \tilde{\alpha}_2,$ ....., $\tilde{\alpha}_n)^T$.

Illustration of MAGDM problem

Let $A = \{A_1, A_2, ..., A_m\}$ be a discrete set of alternatives, and $U = \{U_1, U_2, ..., U_n\}$ be the set of attributes, $\psi = \{\psi_1, \psi_2, ..., \psi_n\}$ is the weighting vector of the attribute $U_j (j = 1, 2, ..., n)$, where $\psi > 0$, $\sum_{j=1}^{n} \psi_j = 1$. Let $D = \{d_1, d_2, ..., d_t\}$ be the set of decision makers, $w = (w_1, w_2, ..., w_t)^T$ be the weight vector of decision makers, with $w_k \in$ [0,1] and $\sum_{j=1}^{n} w_k = 1$. Suppose that $\tilde{R}^{(k)} = \left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}$ $=\left(\left[a_{1_{ij}}^{(k)}, a_{2_{ij}}^{(k)}, a_{3_{ij}}^{(k)}, a_{4_{ij}}^{(k)}\right]; w_{\tilde{a}_{ij}}^{(k)}, u_{\tilde{a}_{ij}}^{(k)}\right)$ is the intuitionistic trapezoidal fuzzy decision matrix, $w_{\tilde{a}_{ij}}^{(k)} \subset$ [0,1],$u_{\tilde{a}_{ij}}^{(k)} \subset$ [0,1], $w_{\tilde{a}_{ij}}^{(k)} + u_{\tilde{a}_{ij}}^{(k)} \leq 1$,$j$=1,2,....,n, i=1,2,....,m, k= 1,2,....,t.

The algorithm for solving MAGDM problem [10] is as follows:

1. First we apply the weights of attribute, and the ITFWG operator

$$\tilde{r}_i^{(k)} = ITFWG\left(\tilde{r}_{i1}^{(k)}, \tilde{r}_{i2}^{(k)}, ..., \tilde{r}_{in}^{(k)}\right), i = 1, 2, ..., m, k = 1, 2, ..., t,$$

   to derive the individual overall preference intuitionistic trapezoidal fuzzy values $\tilde{r}_i^{(k)}$ of the alternative A_i.

2. Utilizing the ITFHG operator we derive colletive overall preference intuitionistic trapezoidal fuzzy values $\tilde{r}_i$ $(i = 1, 2, ..., m)$ of the alternative A_i:

$$\tilde{r}_i = ([a_{1_i}, a_{2_i}, a_{3_i}, a_{4_i}]; w_{\tilde{r}_i}, u_{\tilde{r}_i}) = ITFHG_{\omega,W}\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, ..., \tilde{r}_{ij}^{(t)}\right)$$

   where $w = (w_1, w_2, ..., w_t)^T$ is the weight vector of decision makers, with $w_k \in$ [0,1] and $\sum_{j=1}^{n} w_k = 1$; $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the associated weight vector of the ITFHG operator,$\omega_k \in$ [0,1] and $\sum_{j=1}^{n} \omega_k = 1$.

3. Next we shall calculate value and ambiguity indices using (3.1) and (3.2), and evaluate R(.) for each alternative.

4. The best one will be choosen among the alternatives depending on their respective ranks. The greater the value of $R(\tilde{r}_i)$, the better the alternative A_i (i=1,2,....,m) will be.

5 Numerical Example

We shall consider a numerical example [10] with four alternatives $A_i (i = 1, 2, 3, 4)$ and four attributes U_1,U_2, U_3, U_4 have been considered with weighting vector $\psi = (0.22, 0.20, 0.28, 0.30)^T$. A group of four decision makers $D = \{d_1, d_2, d_3, d_4\}$ with weight vector $w = (0.20, 0.30, 0.35, 0.15)^T$ has been assigned to find the best alternative. The initial decision matrices $\tilde{R}^{(k)} = \left(\tilde{r}_{ij}^{(k)}\right)_{4\times4}$ (k=1,2,3,4) are as follows:

$$\tilde{R}^{(1)} = \begin{bmatrix} ([0.6, 0.7, 0.8, 0.9]; 0.6, 0.2) & ([0.3, 0.4, 0.5, 0.6]; 0.6, 0.4) & ([0.5, 0.6, 0.7, 0.9]; 0.3, 0.4) & ([0.6, 0.7, 0.8, 0.9]; 0.5, 0.3) \\ ([0.7, 0.8; 0.9, 1.0]; 0.7, 0.3) & ([0.5, 0.6, 0.7, 0.8]; 0.5, 0.3) & ([0.4, 0.5, 0.7, 0.8]; 0.7, 0.3) & ([0.5, 0.7, 0.8, 0.9]; 0.4, 0.6) \\ ([0.4, 0.5, 0.6, 0.7]; 0.4, 0.2) & ([0.3, 0.4, 0.5, 0.7]; 0.6, 0.3) & ([0.2, 0.4, 0.5, 0.6]; 0.5, 0.3) & ([0.6, 0.7, 0.8, 0.9]; 0.5, 0.2) \\ ([0.1, 0.2, 0.4, 0.5]; 0.6, 0.2) & ([0.3, 0.4, 0.6, 0.7]; 0.5, 0.2) & ([0.5, 0.6, 0.7, 0.8]; 0.4, 0.3) & ([0.4, 0.5, 0.6, 0.7]; 0.5, 0.1) \end{bmatrix}$$

$$\tilde{R}^{(2)} = \begin{bmatrix} ([0, 1, 0.3, 0.4, 0.5], 0.7, 0.2) & ([0.2, 0.4, 0.6, 0.8]; 0.5, 0.2) & ([0.2, 0.5, 0.6, 0.8]; 0.4, 0.3) & ([0.1, 0.4, 0.5, 0.6]; 0.7, 0.1) \\ ([0.2, 0.4, 0.6, 0.8]; 0.6, 0.3) & ([0.4, 0.6, 0.8, 0.9]; 0.7, 0.2) & ([0.4, 0.6, 0.8, 1.0]; 0.5, 0.3) & ([0.3, 0.5, 0.6, 0.7]; 0.8, 0.1) \\ ([0.2, 0.4, 0.6, 1.0]; 0.6, 0.2) & ([0.2, 0.4, 0.6, 0.8]; 0.8, 0.2) & ([0.1, 0.2, 0.6, 0.8]; 0.6, 0.2) & ([0.1, 0.2, 0.3, 0.5]; 0.6, 0.4) \\ ([0.2, 0.3, 0.4, 0.7]; 0.5, 0.3) & ([0.1, 0.2, 0.3, 0.5]; 0.6, 0.2) & ([0.1, 0.3, 0.5, 0.7]; 0.7, 0.2) & ([0.1, 0.2, 0.4, 0.5]; 0.5, 0.3) \end{bmatrix}$$

$$\tilde{R}^{(3)} = \begin{bmatrix} ([0.6, 0.7, 0.8, 0.9]; 0.7, 0.2) & ([0.1, 0.6, 0.4, 0.3]; 0.5, 0.2) & ([0.3, 0.5, 0.6, 0.7]; 0.4, 0.3) & ([0.1, 0.2, 0.4, 0.5]; 0.7, 0.1) \\ ([0.5, 0.7, 0.8, 0.9]; 0.3, 0.5) & ([0.4, 0.5, 0.6, 0.8]; 0.7, 0.3) & ([0.4, 0.6, 0.7, 0.8]; 0.3, 0.1) & ([0.3, 0.5, 0.6, 0.8]; 0.5, 0.3) \\ ([0.7, 0.8, 0.9, 1.0]; 0.6, 0.2) & ([0.3, 0.4, 0.6, 0.7]; 0.5, 0.2) & ([0.1, 0.2, 0.6, 0.8]; 0.5, 0.3) & ([0.1, 0.2, 0.4, 0.5]; 0.6, 0.3) \\ ([0.4, 0.5, 0.7, 0.9]; 0.5, 0.3) & ([0.1, 0.2, 0.3, 0.4]; 0.4, 0.1) & ([0.1, 0.3, 0.5, 0.6]; 0.6, 0.2) & ([0.1, 0.2, 0.3, 0, 5]; 0.5, 0.2) \end{bmatrix}$$

$$\tilde{R}^{(4)} = \begin{bmatrix} ([0.4, 0.5, 0.7, 0.8]; 0.4, 0.5) & ([0.4, 0.5, 0.6, 0.7]; 0.6, 0.4) & ([0.5, 0.6, 0.7, 0.9]; 0.3, 0.4) & ([0.4, 0.7, 0.8, 0.9]; 0.3, 0.6) \\ ([0.5, 0.6, 0.7, 0.9]; 0.3, 0.5) & ([0.5, 0.6, 0.7, 0.8]; 0.4, 0.3) & ([0.4, 0.5, 0.7, 0.8]; 0.7, 0.3) & ([0.5, 0.6, 0.8, 0.9]; 0.5, 0.6) \\ ([0.3, 0.5, 0.6, 0.8]; 0.4, 0.2) & ([0.2, 0.4, 0.5, 0.8]; 0.6, 0.2) & ([0.2, 0.4, 0.5, 0.6]; 0.5, 0.3) & ([0.3, 0.5, 0.6, 0.8]; 0.4, 0.2) \\ ([0.1, 0.2, 0.4, 0.6]; 0.6, 0.3) & ([0.3, 0.5, 0.6, 0.7]; 0.5, 0.1) & ([0.5, 0.6, 0.7, 0.8]; 0.4, 0.3) & ([0.2, 0.4, 0.6, 0.7]; 0.5, 0.1) \end{bmatrix}$$

Solution of the given MAGDM problem consist of following steps:

- Based on the information given in the decision matrix and utilizing ITFWG operator we first evaluate the individual overall preference intuitionistic trapeziodal fuzzy numbers $\tilde{r}_i^{(k)}$ of the alternatives A_i.

  $\tilde{r}_1^{(1)} = ([0.50, 0.60, 0.70, 0.83]; 0.47, 0.33)$
  $\tilde{r}_1^{(2)} = ([0.14, 0.40, 0.52, 0.66]; 0.56, 0.20)$
  $\tilde{r}_1^{(3)} = ([0.20, 0.37, 0.55, 0.65]; 0.50, 0.22)$
  $\tilde{r}_1^{(4)} = ([0.43, 0.58, 0.71, 0.83]; 0.37, 0.65)$
  $\tilde{r}_2^{(1)} = ([0.51, 0.64, 0.77, 0.87]; 0.55, 0.48)$
  $\tilde{r}_2^{(2)} = ([0.32, 0.52, 0.69, 0.84]; 0.64, 0.22)$
  $\tilde{r}_2^{(3)} = ([0.39, 0.57, 0.67, 0.82]; 0.44, 0.23)$
  $\tilde{r}_2^{(4)} = ([0.47, 0.57, 0.73, 0.85]; 0.47, 0.56)$
  $\tilde{r}_3^{(1)} = ([0.35, 0.50, 0.60, 0.72]; 0.49, 0.25)$
  $\tilde{r}_3^{(2)} = ([0.13, 0.27, 0.49, 0.73]; 0.64, 0.27)$
  $\tilde{r}_3^{(3)} = ([0.19, 0.31, 0.58, 0.71]; 0.47, 0.24)$
  $\tilde{r}_3^{(4)} = ([0.25, 0.45, 0.55, 0.74]; 0.46, 0.57)$
  $\tilde{r}_4^{(1)} = ([0.30, 0.41, 0.57, 0.67]; 0.49, 0.20)$
  $\tilde{r}_4^{(2)} = ([0.12, 0.24, 0.40, 0.73]; 0.57, 0.25)$
  $\tilde{r}_4^{(3)} = ([0.47, 0.57, 0.73, 0.85]; 0.47, 0.56)$
  $\tilde{r}_4^{(4)} = ([0.24, 0.40, 0.57, 0.70]; 0.49, 0.71)$

- Applying ITFHG operator we evaluate the collective overall preference intuitionistic trapeziodal fuzzy numbers $\tilde{r}_i$ and let $\omega = (0.155, 0.345, 0.345, 0.155)^T$.

  $\tilde{r}_1 = ([0.24, 0.46, 0.60, 0.72]; 0.50, 0.34)$
  $\tilde{r}_2 = ([0.39, 0.43, 0.71, 0.81]; 0.55, 0.34)$
  $\tilde{r}_3 = ([0.20, 0.35, 0.55, 0.74]; 0.55, 0.33)$
  $\tilde{r}_4 = ([0.19, 0.31, 0.47, 0.63]; 0.55, 0.37)$

- Next we evaluate value and ambiguity indices of $\tilde{r}_i$ as follows:

  $V(\tilde{r}_1) = 0.1488 \quad A(\tilde{r}_1) = 0.0734 \quad R(\tilde{r}_1) = 0.2223$
  $V(\tilde{r}_2) = 0.1754 \quad A(\tilde{r}_2) = 0.0988 \quad R(\tilde{r}_2) = 0.2733$
  $V(\tilde{r}_3) = 0.1392 \quad A(\tilde{r}_3) = 0.0955 \quad R(\tilde{r}_3) = 0.2347$
  $V(\tilde{r}_4) = 0.1152 \quad A(\tilde{r}_4) = 0.0747 \quad R(\tilde{r}_4) = 0.1899$

- Rank all the alternatives $A_i (i = 1, 2, 3, 4)$ according to the descending order of $R(\tilde{r}_i)$ i.e.,$A_2 \succ A_3 \succ A_1 \succ A_4$ and thus most desirable alternative is A_2.

6 Conclusion

In the present article we studied two characteristics of trapezoidal intuitionistic fuzzy number (TRIFN), viz., Value index and Ambiguity index. An algorithm for ranking of TRIFNs has been developed in this paper which is based on these two characteristics. The application of the above method have been stated through a Multi attribute group decision making problem where the alternatives derived in the decision making process have been ranked using the proposed method. This ranking method can be applied to the computation of shortest path in a fuzzy weighted network characterized by intuitionistic fuzzy numbers (IFNs) Transportation Problem, Assignment Problem, Multi objective optimization problems etc., where the ranking of alternatives(or variables) plays a significant role.

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