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CHARACTERIZATIONS OF FUZZY SOFT PRE SEPARATION AXIOMS

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Abstract – The notions of fuzzy pre open soft sets and fuzzy pre closed soft sets were introduced by Abd El-latif et al. [2]. In this paper, we continue the study on fuzzy soft topological spaces and investigate the properties of fuzzy pre open soft sets, fuzzy pre closed soft sets and study various properties and notions related to these structures. In particular, we study the relationship between fuzzy pre soft interior fuzzy pre soft closure. Moreover, we study the properties of fuzzy soft pre regular spaces and fuzzy soft pre normal spaces, which are basic for further research on fuzzy soft topology and will fortify the footing of the theory of fuzzy soft topological space.

Keywords – *Fuzzy soft topological space, Fuzzy pre open soft, Fuzzy pre closed soft, Fuzzy pre continuous soft functions, Fuzzy soft pre separation axioms, Fuzzy soft pre regular, Fuzzy soft pre normal.*

1 Introduction

In real life situation, the problems in economics, engineering, social sciences, medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainties. But, all these theories have their inherent difficulties. The reason for these difficulties Molodtsov [35] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. In [35, 36], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [33], the properties and applications of soft set theory have been studied increasingly [7, 28, 36].

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Xiao et al. [46] and Pei and Miao [39] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [5, 6, 9, 17, 26, 31, 32, 33, 34, 36, 37, 49]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [10].
Recently, in 2011, Shabir and Naz [43] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [45] investigate some properties of these soft separation axioms. In [18], Kandil et. al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [25] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al.[21]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al. [19, 20, 22, 23, 24, 27]. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [13]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The notion of $b-$open soft sets was initiated by El-sheikh and Abd El-latif [12] and extended in [40]. An applications on $b-$open soft sets were introduced in [3, 14].
Maji et. al. [31] initiated the study involving both fuzzy sets and soft sets. In [8], the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then, many scientists such as X. Yang et. al. [47], improved the concept of fuzziness of soft sets. In [4], Karal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Chang [11], introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathfrak{T}$ of fuzzy subsets of X. Tanay et. al. [44] introduced the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta [42] gave the definition of fuzzy soft topology over the initial universe set. Some fuzzy soft topological properties based on fuzzy semi (resp. β-) open soft sets, were introduced in [1, 16, 17, 26].

In the present paper, we investigate more properties of the concepts of fuzzy pre open soft sets, fuzzy pre closed soft sets, fuzzy pre soft interior, fuzzy pre soft closure and fuzzy soft pre separation axioms in fuzzy soft topological spaces. In particular, we study the relationship between fuzzy pre soft interior and fuzzy pre soft closure. Also, we study the properties of fuzzy soft pre regular spaces and fuzzy soft pre normal spaces. Moreover, we show that if every fuzzy soft point f_e is fuzzy pre closed soft set in a fuzzy soft topological space $(X, \mathfrak{T}, E)$, then $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_1- (resp. T_2-) space. We hope that, the findings in this paper will help researcher enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life.

2 Preliminary

In this section, we present the basic definitions and Theorems related to fuzzy soft set theory.

Definition 2.1. [48] A fuzzy set A in a non-empty set X is characterized by a membership function $\mu_A : X \longrightarrow [0,1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$. The family of all fuzzy sets is denoted by I^X.

Definition 2.2. [31] Let $A \subseteq E$. A pair (f, A), denoted by f_A, is called a fuzzy soft set over X , where f is a mapping given by $f : A \rightarrow I^X$ defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A} = \overline{0}$ if $e \notin A$ and $\mu^e_{f_A} \neq \overline{0}$ if $e \in A$ where $\overline{0}(e) = 0 \; \forall \; x \in X$. The family of all these fuzzy soft sets over X denoted by $FSS(X)_A$.

Definition 2.3. [41]. Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then $\mathfrak{T}$ is called a fuzzy soft topology on X if

(1) $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$, where $\tilde{0}_E(e) = \overline{0}$ and $\tilde{1}_E(e) = \overline{1}$, $\forall e \in E$,

(2) The union of any members of $\mathfrak{T}$, belongs to $\mathfrak{T}$,

(3) The intersection of any two members of $\mathfrak{T}$, belongs to $\mathfrak{T}$.

The triplet $(X, \mathfrak{T}, E)$ is called a fuzzy soft topological space over X. Also, each member of $\mathfrak{T}$ is called a fuzzy open soft in $(X, \mathfrak{T}, E)$. We denote the set of all fuzzy open soft sets by $FOS(X, \mathfrak{T}, E)$, or $FOS(X)$.

Definition 2.4. [41] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in X, if its relative complement f^c_A is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by $FCS(X, \mathfrak{T}, E)$, or $FCS(X)$.

Definition 2.5. [38] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. The fuzzy soft closure of f_A, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of f_A. i.e.,

$Fcl(f_A) = \sqcap\{h_D : h_D \; is \; fuzzy \; closed \; soft \; set \; and \; f_A \sqsubseteq h_D\}$.

The fuzzy soft interior of g_B, denoted by $Fint(g_B)$ is the fuzzy soft union of all fuzzy open soft subsets of g_B.i.e.,

$Fint(g_B) = \sqcup\{h_D : h_D \; is \; fuzzy \; open \; soft \; set \; and \; h_D \sqsubseteq g_B\}$.

Definition 2.6. [30] The fuzzy soft set $f_A \in FSS(X)_E$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha$ $(0 < \alpha \leq 1)$ and $\mu^e_{f_A}(y) = \overline{0}$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.

Definition 2.7. [30] The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set (g, A), denoted by $x^e_\alpha \tilde{\in} (g, A)$, if for the element $e \in A$, $\alpha \leq \mu^e_{g_A}(x)$.

Theorem 2.1. [30] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and f_e be a fuzzy soft point. Then, the following properties hold:

(1) If $f_e \tilde{\in} g_A$, then $f_e \tilde{\notin} g^c_A$;

(2) $f_e \tilde{\in} g_A \nRightarrow f_e^c \tilde{\in} g_A^c$;

(3) Every non-null fuzzy soft set f_A can be expressed as the union of all the fuzzy soft points belonging to f_A.

Definition 2.8. [30] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $Y \subseteq X$. Let h_E^Y be a fuzzy soft set over (Y, E) such that $h_E^Y : E \to I^Y$ such that $h_E^Y(e) = \mu^e_{h_E^Y}$,

$$\mu^e_{h_E^Y}(x) = \begin{cases} 1, & x \in Y, \\ 0, & x \notin Y. \end{cases}$$

Let $\mathfrak{T}_Y = \{h_E^Y \sqcap g_B : g_B \in \mathfrak{T}\}$, then the fuzzy soft topology $\mathfrak{T}_Y$ on $(Y, E)$is called fuzzy soft subspace topology for (Y, E) and $(Y, \mathfrak{T}_Y, E)$ is called fuzzy soft subspace of $(X, \mathfrak{T}, E)$. If $h_E^Y \in \mathfrak{T}$ (resp. $h_E^Y \in \mathfrak{T}^c$), then $(Y, \mathfrak{T}_Y, E)$ is called fuzzy open (resp. closed) soft subspace of $(X, \mathfrak{T}, E)$.

Definition 2.9. [38] Let $FSS(X)_E$ and $FSS(Y)_K$ be families of fuzzy soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then, the map f_{pu} is called a fuzzy soft mapping from X to Y and denoted by $f_{pu} : FSS(X)_E \to FSS(Y)_K$ such that,

(1) If $f_A \in FSS(X)_E$. Then, the image of f_A under the fuzzy soft mapping f_{pu} is the fuzzy soft set over Y defined by $f_{pu}(f_A)$, where $\forall\, k \in p(E)$, $\forall\, y \in Y$,

$$f_{pu}(f_A)(k)(y) = \begin{cases} \bigvee_{u(x)=y} \ [\vee_{p(e)=k}(f_A(e))](x) & if\ x \in u^{-1}(y), \\ 0 & otherwise. \end{cases}$$

(2) If $g_B \in FSS(Y)_K$, then the pre-image of g_B under the fuzzy soft mapping f_{pu} is the fuzzy soft set over X defined by $f_{pu}^{-1}(g_B)$, where $\forall\, e \in p^{-1}(K)$, $\forall\, x \in X$,

$$f_{pu}^{-1}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & for\ p(e) \in B, \\ 0 & otherwise. \end{cases}$$

The fuzzy soft mapping f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective), also it is said to be constant if p and u are constant.

Definition 2.10. [38] Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be two fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a fuzzy soft mapping. Then, f_{pu} is called

(1) Fuzzy continuous soft if $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1 \ \forall\, (g_B) \in \mathfrak{T}_2$.

(2) Fuzzy open soft if $f_{pu}(g_A) \in \mathfrak{T}_2 \forall\, (g_A) \in \mathfrak{T}_1$.

Theorem 2.2. [4] Let $FSS(X)_E$ and $FSS(Y)_K$ be two families of fuzzy soft sets. For the fuzzy soft function $f_{pu} : FSS(X)_E \to FSS(Y)_K$, the following statements hold,

(a) $f_{pu}^{-1}((g, B)^c) = (f_{pu}^{-1}(g, B))^c \forall\, (g, B) \in FSS(Y)_K$.

(b) $f_{pu}(f_{pu}^{-1}((g, B))) \sqsubseteq (g, B) \forall\, (g, B) \in FSS(Y)_K$. If f_{pu} is surjective, then the equality holds.

(c) $(f, A) \sqsubseteq f_{pu}^{-1}(f_{pu}((f, A))) \forall\, (f, A) \in FSS(X)_E$. If f_{pu} is injective, then the equality holds.

(d) $f_{pu}(\tilde{0}_E) = \tilde{0}_K$, $f_{pu}(\tilde{1}_E) \sqsubseteq \tilde{1}_K$. If f_{pu} is surjective, then the equality holds.

(e) $f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ and $f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$.

(f) If $(f, A) \sqsubseteq (g, A)$, then $f_{pu}(f, A) \sqsubseteq f_{pu}(g, A)$.

(g) If $(f, B) \sqsubseteq (g, B)$, then $f_{pu}^{-1}(f, B) \sqsubseteq f_{pu}^{-1}(g, B)\ \forall\ (f, B), (g, B) \in FSS(Y)_K$.

(h) $f_{pu}^{-1}(\sqcup_{j\in J}(f, B)_j) = \sqcup_{j\in J} f_{pu}^{-1}(f, B)_j$ and $f_{pu}^{-1}(\sqcap_{j\in J}(f, B)_j) = \sqcap_{j\in J} f_{pu}^{-1}(f, B)_j, \forall\ (f, B)_j \in FSS(Y)_K$.

(I) $f_{pu}(\sqcup_{j\in J}(f, A)_j) = \sqcup_{j\in J} f_{pu}(f, A)_j$ and $f_{pu}(\sqcap_{j\in J}(f, A)_j) \sqsubseteq \sqcap_{j\in J} f_{pu}(f, A)_j\ \forall\ (f, A)_j \in FSS(X)_E$. If f_{pu} is injective, then the equality holds.

Theorem 2.3. [26] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then:

(1) $f_A \in FSOS(X)$ if and only if $Fcl(f_A) = Fcl(Fint(f_A))$.

(2) If $g_B \in \mathfrak{T}$, then $g_B \sqcap Fcl(f_A) \sqsubseteq Fcl(g_B \sqcap g_B)$.

Definition 2.11. [18] Let (X, τ, E) be a soft topological space and $F_A \in SS(X)_E$. If $F_A \tilde{\subseteq} int(cl(F_A))$, then F_A is called pre open soft set. We denote the set of all pre open soft sets by $POS(X, \tau, E)$, or $POS(X)$ and the set of all pre closed soft sets by $PCS(X, \tau, E)$, or $PCS(X)$.

Definition 2.12. [2] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. If $f_A \sqsubseteq Fint(Fcl(f_A))$, then f_A is called fuzzy pre open soft set. We denote the set of all fuzzy pre open soft sets by $FPOS(X, \mathfrak{T}, E)$, or $FPOS(X)$ and the set of all fuzzy pre closed soft sets by $FPCS(X, \mathfrak{T}, E)$, or $FPCS(X)$.

Definition 2.13. [2] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and $f_e \tilde{\in} FSS(X)_E$. Then,

(1) f_e is called fuzzy pre interior soft point of f_A if $\exists\ g_B \in FPOS(X)$ such that $f_e \tilde{\in} g_B \sqsubseteq f_A$. The set of all fuzzy pre interior soft points of f_A is called the fuzzy pre soft interior of f_A and is denoted by $FPint(f_A)$ consequently, $FPint(f_A) = \sqcup\{g_B : g_B \sqsubseteq f_A,\ g_B \in FPOS(X)\}$.

(2) f_e is called fuzzy pre closure soft point of f_A if $f_A \sqcap h_C \neq \tilde{0}_E\ \forall\ h_D \in FPOS(X)$. The set of all fuzzy pre closure soft points of f_A is called fuzzy pre soft closure of f_A and denoted by $FPcl(f_A)$. Consequently, $FPcl(f_A) = \sqcap\{h_D : h_D \in FPCS(X),\ f_A \sqsubseteq h_D\}$.

3 Fuzzy Pre Open (Closed) Soft Sets

The notions of fuzzy pre open soft sets and fuzzy pre closed soft sets were introduced by Abd El-latif et al. [2]. In this section, we investigate more properties of the notions of fuzzy pre open soft sets, fuzzy pre closed soft sets and study various properties and notions related to these structures.

Theorem 3.1. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FPOS(X)$.Then

(1) Arbitrary fuzzy soft union of fuzzy pre open soft sets is fuzzy pre open soft.

(2) Arbitrary fuzzy soft intersection of fuzzy pre closed soft sets is fuzzy pre closed soft.

Proof.

(1) Let $\{(f, A)_j : j \in J\} \subseteq FPOS(X)$. Then, $\forall\ j \in J$, $(f, A)_j \sqsubseteq Fint(Fcl((f, A)_j))$. It follows that, $\sqcup_j (f, A)_j \sqsubseteq \sqcup_j (Fint(Fcl((f, A)_j))) \sqsubseteq Fint(\sqcup_j Fcl(f, A)_j) = Fint(Fcl(\sqcup_j (f, A)_j)$. Hence, $\sqcup_j (f, A)_j \in FPOS(X)\ \forall\ j \in J$.

(2) By a similar way.

Theorem 3.2. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then, $f_A \in FPOS(X)$ if and only if $Fcl(f_A) = Fint(Fcl(f_A))$.

Proof. Immediate.

Theorem 3.3. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A, g_B \in FSS(X)_E$. Then, the following properties are satisfied for the fuzzy pre interior operator, denoted by $FPint$.

(1) $FPint(\tilde{1}_E) = \tilde{1}_E$ and $FPint(\tilde{0}_E)) = \tilde{0}_E$.

(2) $FPint(f_A) \sqsubseteq (f_A)$.

(3) $FPint(f_A)$ is the largest fuzzy pre open soft set contained in f_A.

(4) If $f_A \sqsubseteq g_B$, then $FPint(f_A) \sqsubseteq FPint(g_B)$.

(5) $FPint(FPint(f_A)) = FPint(f_A)$.

(6) $FPint(f_A) \sqcup FPint(g_B) \sqsubseteq FPint[(f_A) \sqcup (g_B)]$.

(7) $FPint[(f_A) \sqcap (g_B)] \sqsubseteq FPint(f_A) \sqcap FPint(g_B)$.

Proof. Obvious.

Theorem 3.4. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A, g_B \in FSS(X)_E$. Then, the following properties are satisfied for the fuzzy pre closure operator, denoted by $FPcl$.

(1) $FPcl(\tilde{1}_E) = \tilde{1}_E$ and $FPcl(\tilde{0}_E) = \tilde{0}_E$.

(2) $(f_A) \sqsubseteq FPcl(f_A)$.

(3) $FPcl(f_A)$ is the smallest fuzzy pre closed soft set contains f_A.

(4) If $f_A \sqsubseteq g_B$, then $FPcl(f_A) \sqsubseteq FPcl(g_B)$.

(5) $FPcl(FPcl(f_A)) = FPcl(f_A)$.

(6) $FPcl(f_A) \sqcup FPcl(g_B) \sqsubseteq FPcl[(f_A) \sqcup (g_B)]$.

(7) $FPcl[(f_A) \sqcap (g_B)] \sqsubseteq FPcl(f_A) \sqcap FPcl(g_B)$.

Proof. Immediate.

Lemma 3.1. Every fuzzy open (resp. closed) soft set in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is fuzzy pre open (resp. closed) soft.

Proof. Let $f_A \in FOS(X)$. Then, $Fint(f_A) = f_A$. Since $f_A \sqsubseteq Fcl(f_A)$, then $f_A \sqsubseteq Fint(Fcl(f_A))$. Thus, $f_A \in FPOS(X)$.

Remark 3.1. The converse of Lemma 3.1 is not true in general as shown in the following example.

Example 3.1. Let $X = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $A, B, C, D \subseteq E$ where $A = \{e_1, e_2\}$, $B = \{e_2, e_3\}$, $C = \{e_1, e_3\}$ and $D = \{e_2\}$. Let $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_{1A}, f_{2B}, f_{3D}, f_{4E}, f_{5B}, f_{6D}\}$ where $f_{1A}, f_{2B}, f_{3D}, f_{4E}, f_{5B}, f_{6D}$ are fuzzy soft sets over X defined as follows:

$\mu^{e_1}_{f_{1A}} = \{a_{0.5}, b_{0.75}, c_{0.4}\}$, $\mu^{e_2}_{f_{1A}} = \{a_{0.3}, b_{0.8}, c_{0.7}\}$,
$\mu^{e_2}_{f_{2B}} = \{a_{0.4}, b_{0.6}, c_{0.3}\}$, $\mu^{e_3}_{f_{2B}} = \{a_{0.2}, b_{0.4}, c_{0.45}\}$,
$\mu^{e_2}_{f_{3D}} = \{a_{0.3}, b_{0.6}, c_{0.3}\}$,
$\mu^{e_1}_{f_{4E}} = \{a_{0.5}, b_{0.75}, c_{0.4}\}$, $\mu^{e_2}_{f_{4E}} = \{a_{0.4}, b_{0.8}, c_{0.7}\}$, $\mu^{e_3}_{f_{4E}} = \{a_{0.2}, b_{0.4}, c_{0.45}\}$,
$\mu^{e_2}_{f_{5B}} = \{a_{0.4}, b_{0.8}, c_{0.7}\}$, $\mu^{e_3}_{f_{5B}} = \{a_{0.2}, b_{0.4}, c_{0.45}\}$,
$\mu^{e_2}_{f_{6D}} = \{a_{0.3}, b_{0.8}, c_{0.7}\}$.

Then $\mathfrak{T}$ defines a fuzzy soft topology on X. Then, the fuzzy soft set k_E where:

$\mu^{e_1}_{k_E} = \{a_{0.5}, b_{0.75}, c_{0.45}\}$, $\mu^{e_2}_{k_E} = \{a_{0.9}, b_{0.8}, c_{0.7}\}$, $\mu^{e_3}_{k_E} = \{a_{0.25}, b_{0.7}, c_1\}$.

is fuzzy pre open soft set of $(X, \mathfrak{T}, E)$, but it is not fuzzy open soft.

Theorem 3.5. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)$. Then,

(1) $FPint(f^c_A) = \tilde{1} - [FPcl(f_A)]$.

(2) $FPcl(f^c_A) = \tilde{1} - [FPint(f_A)]$.

Proof.

(1) Since $FPcl(f_A) = \sqcap\{h_D : h_D \in FPCS(X),\ f_A \sqsubseteq h_D\}$. Then, $\tilde{1} - FPcl(f_A) = \sqcup\{h^c_D : h^c_D \in FPOS(X),\ h^c_D \sqsubseteq f^c_A\} = FPint(f^c_A)$.

(2) By a similar way.

Theorem 3.6. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, $f_A \in FOS(X)$ and $g_B \in FPOS(X)$. Then, $f_A \sqcap g_B \in FPOS(X)$.

Proof. Let $f_A \in FOS(X)$ and $g_B \in FPOS(X)$. Then, $f_A \sqcap g_B \sqsubseteq Fint(f_A) \sqcap Fint(Fcl(g_B)) = Fint[Fcl(f_A) \sqcap (g_B)] \sqsubseteq Fint(Fcl[(f_A) \sqcap (g_B)))$ from Theorem 2.3 (2). Hence, $f_A \sqcap g_B \in FPOS(X)$.

Theorem 3.7. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then, $f_A \in FPCS(X)$ if and only if $Fcl(Fint(f_A)) \sqsubseteq f_A$.

Proof. Obvious.

Corollary 3.1. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then, $f_A \in FPCS(X)$ if and only if $f_A = f_A \sqcup Fcl(Fint(f_A))$.

4 Fuzzy Pre Continuous Soft Functions

In [4], Karal et al. defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Kandil et al. [25] introduced some types of soft function in soft topological spaces. Here, we introduce the notions of fuzzy pre soft function in fuzzy soft topological spaces and study its basic properties.

Definition 4.1. Let $(X, \mathfrak{T}_1, E)$, $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ be a fuzzy soft function. Then, the function f_{pu} is called;

(1) Fuzzy pre continuous soft if $f_{pu}^{-1}(g_B) \in FPOS(X) \forall\, g_B \in \mathfrak{T}_2$.

(2) Fuzzy pre open soft if $f_{pu}(g_A) \in FPOS(Y) \forall\, g_A \in \mathfrak{T}_1$.

(3) Fuzzy pre closed soft if $f_{pu}(f_A) \in FPCS(Y) \forall\, f_A \in \mathfrak{T}_1^c$.

(4) Fuzzy pre irresolute soft if $f_{pu}^{-1}(g_B) \in FPOS(X) \forall\, g_B \in FPOS(Y)$.

(5) Fuzzy pre irresolute open soft if $f_{pu}(g_A) \in FPOS(Y) \forall\, g_A \in FPOS(X)$.

(6) Fuzzy pre irresolute closed soft if $f_{pu}(f_A) \in FPCS(Y) \forall\, f_A \in FPCS(Y)$.

Example 4.1. Let $X = Y = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $A \subseteq E$ where $A = \{e_1, e_2\}$. Let $f_{pu} : (X, \mathfrak{T}_1, E) \rightarrow (Y, \mathfrak{T}_2, K)$ be the constant soft mapping where $\mathfrak{T}_1$ is the indiscrete fuzzy soft topology and $\mathfrak{T}_2$ is the discrete fuzzy soft topology such that $u(x) = a \;\forall\, x \in X$ and $p(e) = e_1 \;\forall\, e \in E$. Let f_A be fuzzy soft set over Y defined as follows:

$\mu_{f_A}^{e_1} = \{a_{0.1}, b_{0.5}, c_{0.6}\}$, $\mu_{f_A}^{e_2} = \{a_{0.6}, b_{0.2}, c_{0.5}\}$.

Then $f_A \in \mathfrak{T}_2$. Now, we find $f_{pu}^{-1}(f_A)$ as follows:

$f_{pu}^{-1}(f_A)(e_1)(a) = f_A(p(e_1))(u(a)) = f_A(e_1)(a) = 0.5,$
$f_{pu}^{-1}(f_A)(e_1)(b) = f_A(p(e_1))(u(b)) = f_A(e_1)(a) = 0.5,$
$f_{pu}^{-1}(f_A)(e_1)(c) = f_A(p(e_1))(u(c)) = f_A(e_1)(a) = 0.5,$
$f_{pu}^{-1}(f_A)(e_2)(a) = f_A(p(e_2))(u(a)) = f_A(e_1)(a) = 0.5,$
$f_{pu}^{-1}(f_A)(e_2)(b) = f_A(p(e_2))(u(b)) = f_A(e_1)(a) = 0.5,$
$f_{pu}^{-1}(f_A)(e_2)(c) = f_A(p(e_2))(u(c)) = f_A(e_1)(a) = 0.5,$
$f_{pu}^{-1}(f_A)(e_3)(a) = f_A(p(e_3))(u(a)) = f_A(e_1)(a) = 0.5,$
$f_{pu}^{-1}(f_A)(e_3)(b) = f_A(p(e_3))(u(b)) = f_A(e_1)(a) = 0.5,$

$f_{pu}^{-1}(f_A)(e_3)(c) = f_A(p(e_3))(u(c)) = f_A(e_1)(a) = 0.5.$

Hence, $f_{pu}^{-1}(f_A) \notin FPOS(X)$. Therefore, f_{pu} is not fuzzy pre continuous soft function.

Theorem 4.1. Every fuzzy continuous soft function is fuzzy pre continuous soft.

Proof. Immediate from Lemma 3.1.

Theorem 4.2. Let $(X, \mathfrak{T}_1, E)$, $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and f_{pu} be a soft function such that $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$. Then, the following are equivalent:

(1) f_{pu} is a fuzzy pre continuous soft function.

(2) $f_{pu}^{-1}(h_B) \in FPCS(X)\ \forall\ h_B \in FCS(Y)$.

(3) $f_{pu}(FPcl(g_A) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(g_A))\ \forall\ g_A \in FSS(X)_E$.

(4) $FPcl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B))\ \forall\ h_B \in FSS(Y)_K$.

(5) $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) \sqsubseteq FPint(f_{pu}^{-1}(h_B))\ \forall\ h_B \in FSS(Y)_K$.

Proof.

(1) $\Rightarrow$ (2) Let h_B be a fuzzy closed soft set over Y. Then, $h_B^c \in FOS(Y)$ and $f_{pu}^{-1}(h_B^c) \in FPOS(X)$ from Definition 4.1. Since $f_{pu}^{-1}(h_B^c) = (f_{pu}^{-1}(h_B))^c$ from Theorem 2.2. Thus, $f_{pu}^{-1}(h_B) \in FPCS(X)$.

(2) $\Rightarrow$ (3) Let $g_A \in FSS(X)_E$. Since $g_A \sqsubseteq f_{pu}^{-1}(f_{pu}(g_A)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(g_A))) \in FPCS(X)$ from (2) and Theorem 2.2. Then, $g_A \sqsubseteq FPcl(g_A) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(g_A)))$. Hence, $f_{pu}(FPcl(g_A)) \sqsubseteq f_{pu}(f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(g_A)))) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(g_A)))$ from Theorem 2.2. Thus, $f_{pu}(FPcl(g_A)) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(g_A))$.

(3) $\Rightarrow$ (4) Let $h_B \in FSS(Y)_K$ and $g_A = f_{pu}^{-1}(h_B)$. Then, $f_{pu}(FPcl f_{pu}^{-1}(h_B)) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(f_{pu}^{-1}(h_B)))$ From (3). Hence, $FPcl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(f_{pu}(FPcl(f_{pu}^{-1}(h_B))))$ $\sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(f_{pu}^{-1}(h_B)))) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B))$ from Theorem 2.2. Thus, $FPcl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B))$.

(4) $\Rightarrow$ (2) Let h_B be a fuzzy closed soft set over Y. Then,
$FPcl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B))\ \forall\ h_B \in FSS(Y)_K$ from (4). But clearly, $f_{pu}^{-1}(h_B) \sqsubseteq FPcl(f_{pu}^{-1}(h_B))$. This means that, $f_{pu}^{-1}(h_B) = FPcl(f_{pu}^{-1}(h_B))$, and consequently $f_{pu}^{-1}(h_B) \in FPCS(X)$.

(1) $\Rightarrow$ (5) Let $h_B \in FSS(Y)_K$. Then, $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) \in FPOS(X)$ from (1). Hence, $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) = FPint(f_{pu}^{-1} Fint_{\mathfrak{T}_2}(h_B)) \sqsubseteq FPint(f_{pu}^{-1}(h_B))$. Thus, $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) \sqsubseteq FPint(f_{pu}^{-1}(h_B))$.

(5) $\Rightarrow$ (1) Let h_B be a fuzzy open soft set over Y. Then, $Fint_{\mathfrak{T}_2}(h_B) = h_B$ and $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) = f_{pu}^{-1}((h_B)) \sqsubseteq FPint(f_{pu}^{-1}(h_B))$ from (5). But, we have
$FPint(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(h_B)$. This means that, $FPint(f_{pu}^{-1}(h_B)) = f_{pu}^{-1}(h_B) \in FPOS(X)$. Thus, f_{pu} is a fuzzy pre continuous soft function.

Theorem 4.3. Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and f_{pu} be a soft function such that $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$. Then, the following are equivalent,

(1) f_{pu} is a fuzzy pre open soft function.

(2) $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) \sqsubseteq FPint(f_{pu}(g_A)) \; \forall \, g_A \in FSS(X)_E$.

Proof.

(1) $\Rightarrow$ (2) Let $g_A \in FSS(X)_E$. Since $Fint_{\mathfrak{T}_1}(g_A) \in \mathfrak{T}_1$. Then, $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) \in FPOS(Y) \; \forall \, g_A \in \mathfrak{T}_1$ by (1). It follow that,

$$f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) = FPint(f_{pu}Fint_{\mathfrak{T}_1}(g_A)) \sqsubseteq FPint(f_{pu}(g_A))$$

Therefore, $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) \sqsubseteq FPint(f_{pu}(g_A)) \; \forall \, g_A \in FSS(X)_E$.

(2) $\Rightarrow$ (1) Let $g_A \in \mathfrak{T}_1$. By hypothesis, $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) = f_{pu}(g_A) \sqsubseteq FPint(f_{pu}(g_A)) \in FPOS(Y)$, but $FPint(f_{pu}(g_A)) \sqsubseteq f_{pu}(g_A)$. So, $FPint(f_{pu}(g_A)) = f_{pu}(g_A) \in FPOS(Y) \; \forall \, g_A \in \mathfrak{T}_1$. Hence, f_{pu} is a fuzzy pre open soft function.

Theorem 4.4. Let $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ be a fuzzy pre open soft function. If $k_D \in FSS(Y)_K$ and $l_C \in \mathfrak{T}_1^c$ such that $f_{pu}^{-1}(k_D) \sqsubseteq l_C$, then there exists $h_B \in FPCS(Y)$ such that $k_D \sqsubseteq h_B$ and $f_{pu}^{-1}(h_B) \sqsubseteq l_C$.

Proof. Let $k_D \in FSS(Y)_K$ and $l_C \in \mathfrak{T}_1^c$ such that $f_{pu}^{-1}(k_D) \sqsubseteq l_C$. Then, $f_{pu}(l'_C) \sqsubseteq k_D^c$ from Theorem 2.2 where $l_C^c \in \mathfrak{T}_1$. Since f_{pu} is fuzzy pre open soft function. Then, $f_{pu}(l_C^c) \in FPOS(Y)$. Take $h_B = [f_{pu}(l_C^c)]^c$. Hence, $h_B \in FPCS(Y)$ such that $k_D \sqsubseteq h_B$ and $f_{pu}^{-1}(h_B) = f_{pu}^{-1}([f_{pu}(l_C^c)]^c) \sqsubseteq f_{pu}^{-1}(k_D^c)^c = f_{pu}^{-1}(k_D) \sqsubseteq l_C$. This completes the proof.

Theorem 4.5. Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and f_{pu} be a soft function such that $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$. Then, the following are equivalent:

(1) f_{pu} is a fuzzy pre closed soft function.

(2) $FPcl(f_{pu}(h_A)) \sqsubseteq f_{pu}(Fcl_{\mathfrak{T}_1}(h_A)) \; \forall \, h_A \in FSS(X)_E$.

Proof. It follows immediately from Theorem 4.3.

5 Fuzzy Soft Pre Separation Axioms

Soft separation axioms in soft topological spaces were introduced by Shabir et al. [43]. Kandil et al. [25] introduced and studied the notions of soft semi separation axioms in soft topological spaces. Here, we introduce the notions of fuzzy soft pre separation axioms in fuzzy soft topological spaces and study some of its basic properties in detail.

Definition 5.1. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be a fuzzy soft pre T_o-space if for every pair of distinct fuzzy soft points f_e, g_e there exists a fuzzy pre open soft set containing one of the points but not the other.

Examples 5.1. **(1)** Let $X = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $\mathfrak{T}$ be the discrete fuzzy soft topology on X. Then, $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_o-space.

(2) Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and $\mathfrak{T}$ be the indiscrete fuzzy soft topology on X. Then, $\mathfrak{T}$ is not fuzzy soft pre T_o-space.

Theorem 5.1. A soft subspace $(Y, \mathfrak{T}_Y, E)$ of a fuzzy soft pre T_o-space $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_o.

Proof. Let h_e, g_e be two distinct fuzzy soft points in (Y, E). Then, these fuzzy soft points are also in (X, E). Hence, there exists a fuzzy pre open soft set f_A in $\mathfrak{T}$ containing one of the fuzzy soft points but not the other. Thus, $h_E^Y \sqcap f_A$ is a fuzzy pre open soft set in $(Y, \mathfrak{T}_Y, E)$ containing one of the fuzzy soft points but not the other from Definition 2.8. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft pre T_o.

Definition 5.2. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be a fuzzy soft pre T_1-space if for every pair of distinct fuzzy soft points f_e, g_e there exist fuzzy pre open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$, $g_e \tilde{\notin} f_A$; and $f_e \tilde{\notin} g_B$, $g_e \tilde{\in} g_B$.

Example 5.1. Let $X = \{a, b\}$, $E = \{e_1, e_2, e_3\}$ and $\mathfrak{T}$ be the discrete fuzzy soft topology on X. Then, $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_1-space.

Theorem 5.2. A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of a fuzzy soft pre T_1-space $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_1.

Proof. It is similar to the proof of Theorem 5.1.

Theorem 5.3. If every fuzzy soft point of a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is fuzzy pre closed soft, then $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_1.

Proof. Suppose that f_e and g_e be two distinct fuzzy soft points of (X, E). By hypothesis, f_e and g_e are fuzzy pre closed soft sets. Hence, f_e^c and g_e^c are distinct fuzzy pre open soft sets where $f_e \tilde{\in} g_e^c$, $g_e \tilde{\notin} g_e^c$; and $f_e \tilde{\notin} f_e^c$, $g_e \tilde{\in} f_e^c$. Therefore, $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_1.

Definition 5.3. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be a fuzzy soft pre T_2-space if for every pair of distinct fuzzy soft points f_e, g_e there exist disjoint fuzzy pre open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$ and $g_e \tilde{\in} g_B$.

Example 5.2. Let $X = \{a, b, c, d\}$, $E = \{e_1, e_2\}$ and $\mathfrak{T}$ be the discrete fuzzy soft topology on X. Then, $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_2-space.

Proposition 5.1. For a fuzzy soft topological space $(X, \mathfrak{T}, E)$ we have:
fuzzy soft pre T_2-space $\Rightarrow$ fuzzy soft pre T_1-space $\Rightarrow$ fuzzy soft pre T_o-space.

Proof.

(1) Let $(X, \mathfrak{T}, E)$ be a fuzzy soft pre T_2-space and f_e, g_e be two distinct fuzzy soft points. Then, there exist disjoint fuzzy pre open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$ and $g_e \tilde{\in} g_B$. Since $f_A \sqcap g_B = \tilde{0}_E$. Then, $f_e \tilde{\notin} g_B$ and $g_e \tilde{\notin} f_A$. Therefore, there exist fuzzy pre open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$, $g_e \tilde{\notin} f_A$; and $f_e \tilde{\notin} g_B$, $g_e \tilde{\in} g_B$. Thus, $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_1-space.

(2) Let $(X, \mathfrak{T}, E)$ be a fuzzy soft pre T_1-space and f_e, g_e be two distinct fuzzy soft points. Then, there exist fuzzy pre open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$, $g_e \tilde{\notin} f_A$; and $f_e \tilde{\notin} g_B$, $g_e \tilde{\in} g_B$. Then, we have a fuzzy pre open soft set containing one of the fuzzy soft point but not the other. Thus, $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_o-space.

Theorem 5.4. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space . If $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_2-space, then for every pair of distinct fuzzy soft points f_e, g_e there exists a fuzzy pre closed soft set k_A such that containing one of the fuzzy soft points $g_e \tilde{\in} k_A$, but not the other $f_e \tilde{\notin} k_A$ and $g_e \tilde{\notin} FPcl(k_A)$.

Proof. Let f_e, g_e be two distinct fuzzy soft points. By assumption, there exists disjoint fuzzy pre open soft sets b_A and h_B such that $f_e \tilde{\in} b_A$, $g_e \tilde{\in} h_B$. Hence, $g_e \tilde{\in} b_A^c$ and $f_e \tilde{\notin} b_A^c$ from Theorem 2.1. Thus, b_A^c is a fuzzy pre closed soft set containing g_e but not f_e and $f_e \tilde{\notin} FPcl(b_A^c) = b_A^c$.

Theorem 5.5. A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of fuzzy soft pre T_2-space $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_2.

Proof. Let j_e, k_e be two distinct fuzzy soft points in (Y, E). Then, these fuzzy soft points are also in (X, E). Hence, there exist disjoint fuzzy pre open soft sets f_A and g_B in $\mathfrak{T}$ such that $j_e \in f_A$ and $k_e \in g_B$. Thus, $h_E^Y \sqcap f_A$ and $h_E^Y \sqcap g_B$ are disjoint fuzzy pre open soft sets in $\mathfrak{T}_Y$ such that $j_e \in h_E^Y \sqcap f_A$ and $k_e \in h_E^Y \sqcap g_B$. So, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft pre T_2.

Theorem 5.6. If every fuzzy soft point of a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is fuzzy pre closed soft, then $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_2.

Proof. It similar to the proof of Theorem 5.3.

Definition 5.4. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, h_C be a fuzzy pre closed soft set and g_e be a fuzzy soft point such that $g_e \tilde{\notin} h_C$. If there exist disjoint fuzzy pre open soft sets f_S and f_W such that $g_e \tilde{\in} f_S$ and $g_B \sqsubseteq f_W$. Then, $(X, \mathfrak{T}, E)$ is called fuzzy soft pre regular space. A fuzzy soft pre regular T_1-space is called a fuzzy soft pre T_3-space.

Proposition 5.2. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, h_C be a fuzzy pre closed soft set and g_e be a fuzzy soft point such that $g_e \tilde{\notin} h_C$. If $(X, \mathfrak{T}, E)$ is fuzzy soft pre regular space, then there exists a fuzzy pre open soft set f_A such that $g_e \tilde{\in} f_A$ and $f_A \sqcap h_C = \tilde{0}_E$.

Proof. Obvious from Definition 5.4.

Theorem 5.7. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft pre regular space and be a fuzzy pre open soft set g_B such that $f_e \tilde{\in} g_B$. Then, there exists a fuzzy pre open soft set f_S such that $f_e \tilde{\in} f_S$ and $FPcl(f_S) \sqsubseteq g_B$.

Proof. Let g_B be a fuzzy pre open soft set containing a fuzzy soft point f_e in a fuzzy soft pre regular space $(X, \mathfrak{T}, E)$. Then, g_B^c is a fuzzy pre closed soft such that $f_e \tilde{\notin} g_B^c$ from Theorem 2.1. By hypothesis, there exist disjoint fuzzy pre open soft sets f_S and f_W such that $f_e \tilde{\in} f_S$ and $g_B^c \sqsubseteq f_W$. It follows that, $f_W^c \sqsubseteq g_B$ and $f_S \sqsubseteq f_W^c$. Thus, $FPcl(f_S) \sqsubseteq f_W^c \sqsubseteq g_B$. So, we have a fuzzy pre open soft set f_S containing f_e such that $FPcl(f_S) \sqsubseteq g_B$.

Theorem 5.8. Every fuzzy soft pre T_3-space, in which every fuzzy soft point is fuzzy pre closed soft, is fuzzy soft pre T_2-space.

Proof. Let f_e, g_e be two distinct fuzzy soft points of a fuzzy soft pre T_3-space $(X, \mathfrak{T}, E)$. By hypothesis, g_e is fuzzy pre closed soft set and $f_e \tilde{\notin} g_e$. From the fuzzy soft pre regularity, there exist disjoint fuzzy pre open soft sets k_A and h_B such that $f_e \tilde{\in} k_A$ and $g_e \sqsubseteq h_B$. Thus, $f_e \tilde{\in} k_A$ and $g_e \tilde{\in} h_B$. Therefore, $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_2-space.

Theorem 5.9. A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of a fuzzy soft pre T_3-space $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_3.

Proof. By Theorem 5.2, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft pre T_1-space. Now, we want to prove that $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft pre regular space. Let k_E be a fuzzy pre closed soft set in (Y, E) and g_e be a fuzzy soft point in (Y, E) such that $g_e \tilde{\notin} k_E$. Then, $k_E = h_E^Y \sqcap g_B$ for some fuzzy pre closed soft set g_B in (X, E). Hence, $g_e \tilde{\notin} h_E^Y \sqcap g_B$. But $g_e \tilde{\in} h_E^Y$, so $g_e \tilde{\notin} g_B$. Since $(X, \mathfrak{T}, E)$ is fuzzy soft pre T_3. Then, there exist disjoint fuzzy pre open soft sets f_S and f_W in $\mathfrak{T}$ such that $g_e \tilde{\in} f_S$ and $g_B \sqsubseteq f_W$. It follows that, $h_E^Y \sqcap f_S$ and $h_E^Y \sqcap f_W$ are disjoint fuzzy pre open soft sets in $\mathfrak{T}_Y$ such that $g_e \tilde{\in} h_E^Y \sqcap f_S$ and $k_E \sqsubseteq h_E^Y \sqcap f_W$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft pre T_3.

Definition 5.5. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and h_C, g_B be disjoint fuzzy pre closed soft sets. If there exist disjoint fuzzy pre open soft sets f_S and f_W such that $h_C \sqsubseteq f_S$, $g_B \sqsubseteq f_W$. Then, $(X, \mathfrak{T}, E)$ is called fuzzy soft pre normal space. A fuzzy soft pre normal T_1-space is called a fuzzy soft pre T_4-space.

Theorem 5.10. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space. Then, the following are equivalent:

(1) $(X, \mathfrak{T}, E)$ is a fuzzy soft pre normal space.

(2) For every fuzzy pre closed soft set h_C and fuzzy pre open soft set g_B such that $h_C \sqsubseteq g_B$, there exists a fuzzy pre open soft set f_S such that $h_C \sqsubseteq f_S$, $FPcl(f_S) \sqsubseteq g_B$.

Proof.

(1) ⇒ (2) Let h_C be a pre closed soft set and g_B be a fuzzy pre open soft set such that $h_C \sqsubseteq g_B$. Then, h_C, g_B^c are disjoint fuzzy pre closed soft sets. It follows by (1), there exist disjoint fuzzy pre open soft sets f_S and f_W such that $h_C \sqsubseteq f_S$, $g_B^c \sqsubseteq f_W$. Now, $f_S \sqsubseteq f_W^c$, so $FPcl(f_S) \sqsubseteq FPcl f_W^c = f_W^c$, where g_B is fuzzy pre open soft set. Also, $f_W^c \sqsubseteq g_B$. Hence, $FPcl(f_S^c) \sqsubseteq f_W^c \sqsubseteq g_B$. Thus, $h_C \sqsubseteq f_S$, $FPcl(f_S) \sqsubseteq g_B$.

(2) ⇒ (1) Let g_A and g_B be disjoint fuzzy pre closed soft sets. Then, $g_A \sqsubseteq g_B^c$. By hypothesis, there exists a fuzzy pre open soft set f_S such that $g_A \sqsubseteq f_S$, $FPcl(f_S) \sqsubseteq g_B^c$. So $g_B \sqsubseteq [FPcl(f_S)]^c$, $g_A \sqsubseteq f_S$ and $[FPcl(f_S)]^c \sqcap f_S = \tilde{0}_E$, where f_S and $[FPcl(f_S)]^c$ are fuzzy pre open soft sets. Thus, $(X, \mathfrak{T}, E)$ is fuzzy soft pre normal space.

Theorem 5.11. A fuzzy pre closed fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of a fuzzy soft pre normal space $(X, \mathfrak{T}, E)$ is fuzzy soft pre normal.

Proof. Let g_A and g_B be disjoint fuzzy pre closed soft sets in $\mathfrak{T}_Y$. Then, $g_A = h_E^Y \sqcap f_C$ and $g_B = h_E^Y \sqcap f_D$ for some fuzzy pre closed soft sets f_C, f_D in (X, E). Hence, f_C, f_D are disjoint fuzzy pre closed soft sets in $\mathfrak{T}$. Since $(X, \mathfrak{T}, E)$ is fuzzy soft pre normal. Then, there exist disjoint fuzzy pre open soft sets f_S and f_W in $\mathfrak{T}$ such that $f_C \sqsubseteq f_S$, $f_D \sqsubseteq f_W$. It follows that, $h_E^Y \sqcap f_S$ and $h_E^Y \sqcap f_W$ are disjoint fuzzy pre open soft sets in $\mathfrak{T}_Y$ such that $g_A = h_E^Y \sqcap f_C \sqsubseteq h_E^Y \sqcap f_S$ and $g_B = h_E^Y \sqcap f_D \sqsubseteq h_E^Y \sqcap f_W$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft pre normal.

Theorem 5.12. Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : SS(X)_E \rightarrow SS(Y)_K$ be a fuzzy soft function which is bijective, fuzzy pre irresolute soft and fuzzy pre irresolute open soft. If $(X, \mathfrak{T}_1, E)$ is a fuzzy soft pre normal space, then $(Y, \mathfrak{T}_2, K)$ is also a fuzzy soft pre normal space.

Proof. Let f_A, g_B be disjoint fuzzy pre closed soft sets in Y. Since f_{pu} is fuzzy pre irresolute soft, then $f_{pu}^{-1}(f_A)$ and $f_{pu}^{-1}(g_B)$ are fuzzy pre closed soft set in X such that $f_{pu}^{-1}(f_A) \sqcap f_{pu}^{-1}(g_B) = f_{pu}^{-1}[f_A \sqcap g_B] = f_{pu}^{-1}[\tilde{0}_K] = \tilde{0}_E$ from Theorem 2.2. By hypothesis, there exist disjoint fuzzy pre open soft sets k_C and h_D in X such that $f_{pu}^{-1}(f_A) \sqsubseteq k_C$ and $f_{pu}^{-1}(g_B) \sqsubseteq h_D$. It follows that, $f_A = f_{pu}[f_{pu}^{-1}(f_A)] \sqsubseteq f_{pu}(k_C)$,$g_B = f_{pu}[f_{pu}^{-1}(g_B)] \sqsubseteq f_{pu}(h_D)$ from Theorem 2.2 and $f_{pu}(k_C) \sqcap f_{pu}(h_D) = f_{pu}[k_C \sqcap h_D] = f_{pu}[\tilde{0}_E] = \tilde{0}_K$ from Theorem 2.2. Since f_{pu} is fuzzy pre irresolute open soft function. Then, $f_{pu}(k_C), f_{pu}(h_D)$ are fuzzy pre open soft sets in Y. Thus, $(Y, \mathfrak{T}_2, K)$ is a fuzzy soft pre normal space.

6 Conclusion

Therefore, we introduce some properties of the notions of fuzzy pre open soft sets, fuzzy pre closed soft sets, fuzzy pre soft interior, fuzzy pre soft closure and fuzzy pre separation axioms and have established several interesting properties. Since the authors introduced topological structures on fuzzy soft sets [8, 15, 44], so the pre topological properties is generalized here to the fuzzy soft sets which will be useful in the fuzzy systems. Because there exists compact connections between soft sets and information systems [46, 39], we can use the results deducted from the studies on fuzzy soft topological space to improve these kinds of connections. We hope that the findings in this paper will help researcher enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life.

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