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INTUITIONISTIC FUZZY STRONGLY α-GENERALIZED SEMI CLOSED SETS

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Abstaract − In this paper, intuitionistic fuzzy strongly α-generalized semi closed sets and intuitionistic fuzzy strongly α-generalized open sets are introduced. Some of their properties are discussed with existing intuitionistic fuzzy generalized closed and intuitionistic fuzzy generalized open sets.

Keywords − *Fuzzy sets, Intuitionistic fuzzy sets, Intuitionistic fuzzy topological spaces, Intuitionistic fuzzy α-generalized semi closed sets.*

1 Introduction

Zadeh [12] introduced the concept of fuzzy sets and later Atanassov [1] generalized this idea to the new class of intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Levine(1970) introduced the notion of generalized closed (briefly g-closed) sets in topological spaces. The aim of this paper is to introduce and study stronger form of alpha generalized semi closed sets in intuitionistic fuzzy topological spaces for which we introduce the concepts of intuitionistic fuzzy strongly α-generalized semi closed sets and intuitionistic fuzzy strongly α-generalized semi open sets. Moreover, We study their properties.

2 Preliminary

Definition 2.1. [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

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$$A = \{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle \mid x \in X\}$$

where the function $\mu_A(x)$:X $\rightarrow$ [0,1] denotes the degree of membership(namely $\mu_A(x)$) and the function $\nu_A(x)$:X $\rightarrow$ [0,1] denotes the degree of non-membership(namely $\nu_A(x)$) of each element x $\in$ X to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x $\in$ X.

IFS(X) denotes the set of all intuitionistic fuzzy sets in X.

Definition 2.2. [1] Let A and B be IFSs of the form
A=$\{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle \mid x \in X\}$ and B=$\{\langle\ x,\ \mu_B(x),\ \nu_B(x)\ \rangle \mid x \in X\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x $\in$ X,

2. A = B if and only if $A \subseteq B$ and $B \subseteq A$,

3. $A^c = \{\langle\ x,\ \nu_A(x),\ \mu_A(x)\ \rangle \mid x \in X\}$,

4. $A \cap B = \{\langle\ x,\ \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x)\ \rangle \mid x \in X\}$,

5. $A \cup B = \{\langle\ x,\ \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x)\ \rangle \mid x \in X\}$.

For sake of simplicity, we shall use the notation A=$\langle\ x,\ \mu_A,\ \nu_A\ \rangle$ instead of
$A = \{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle \mid x \in X\}$.

Definition 2.3. [1] The intuitionistic fuzzy sets $0_\sim = \{\langle\ x,\ 0,\ 1\ \rangle : x \in X\}$ and $1_\sim = \{\langle\ x,\ 1,\ 0\ \rangle : x \in X\}$ are the empty set and the whole set of X respectively.

Definition 2.4. [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

1. $0_\sim,\ 1_\sim \in \tau$,

2. $G_1 \cap G_2 \in \tau$ for any $G_1,\ G_2 \in \tau$,

3. $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.
The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.5. [2] Let (X, τ) be an IFTS and A = $\langle\ x,\ \mu_A,\ \nu_A\ \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:

1. $int(A) = \cup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$,

2. $cl(A) = \cap\{K \mid K$ is an IFCS in X and $A \subseteq K\}$.

Proposition 2.6. [2] For any IFSs A and B in (X, τ), we have

1. $int(A) \subseteq A$,

2. $A \subseteq cl(A)$,

3. A $\subseteq$ B $\Rightarrow$ int(A) $\subseteq$ int(B) and cl(A) $\subseteq$ cl(B),
4. int(int(A)) = int(A),
5. cl(cl(A)) = cl(A),
6. cl(A$\cup$B) = cl(A)$\cup$cl(B),
7. int(A$\cap$B) = int(A)$\cap$int(B).

Proposition 2.7. [2] For any IFS A in (X, τ), we have

1. int($0_\sim$)= $0_\sim$ and cl($0_\sim$)= $0_\sim$,
2. int($1_\sim$)= $1_\sim$ and cl($1_\sim$)= $1_\sim$,
3. $(\text{int}(A))^c = \text{cl}(A^c)$,
4. $(\text{cl}(A))^c = \text{int}(A^c)$.

Proposition 2.8. [3] If A is an IFCS in X then cl(A) = A and if A is an IFOS in X then int(A) = A. The arbitrary union of IFCSs is an IFCS in X.

Definition 2.9. An IFS A = $\langle$ x, μ_A, ν_A $\rangle$ in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int(A)) [3],
2. intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) $\subseteq$ A [5],
3. intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) $\subseteq$ A [3],
4. intuitionistic fuzzy pre closed set (IFPCS in short) if cl(int(A)) $\subseteq$ A [3],
5. intuitionistic fuzzy γ-closed set (IFγCS in short) if cl(int(A))$\cap$int(cl(A)) $\subseteq$ A [4],
6. intuitionistic fuzzy semipreclosed set (IFSPCS in short) if there exists an IF-PCS B such that int(B) $\subseteq$ A $\subseteq$ B [7].

Definition 2.10. An IFS A = $\langle$ x, μ_A, ν_A $\rangle$ in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)) [3],
2. intuitionistic fuzzy α-open set (IFαOS in short) if A $\subseteq$ int(cl(int(A))) [5],
3. intuitionistic fuzzy semiopen set (IFSOS in short)if A $\subseteq$ cl(int(A)) [3],
4. intuitionistic fuzzy preopen set (IFPOS in short) if A $\subseteq$ int(cl(A)) [3],
5. intuitionistic fuzzy γ-open set (IFγOS in short) if A $\subseteq$ int(cl(A))$\cup$cl(int(A)) [4],
6. intuitionistic fuzzy semipreopen set (IFSPOS in short) if there exists an IFPOS B such that B $\subseteq$ A $\subseteq$ cl(B)[7].

Definition 2.11. Let A be an IFS of an IFTS (X, τ). Then

1. αcl(A) = $\cap$\{K | K is an IFαCS in X and A $\subseteq$ K\}[8],

2. αint(A) = $\cup$\{K | K is an IFαOS in X and K $\subseteq$ A\}[8],
3. sint(A) = $\cup$\{K | K is an IFSOS in X and K $\subseteq$ A\}[11],
4. scl(A) = $\cap$\{K | K is an IFSCS in X and A $\subseteq$ K\}[11].

Result 2.12. Every IFαCS in (X, τ) is an IFαGSCS in (X, τ) but not conversely [6].

Definition 2.13. An IFS A of an IFTS (X, τ) is an

1. intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in X [10],
2. intuitionistic fuzzy generalized semiclosed set (IFGSCS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in X [9].

The complements of the above mentioned intuitionistic fuzzy generalized closed sets are called their respective intuitionistic fuzzy generalized open sets.

Definition 2.14. [6] An IFS A in (X, τ) is said to be an intuitionistic fuzzy α-generalized semi-closed set (IFαGSCS in short) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSOS in (X, τ). The complement A^c of an IFαGSCS A in an IFTS (X, τ) is called an intuitionistic fuzzy α generalized semi open set (IFαGSOS in short) in X.

Remark 2.15. [8] Let A be an IFS in (X, τ). Then

1. αcl(A) = A$\cup$cl(int(cl(A))),
2. αint(A) = A$\cap$int(cl(int(A))).

Definition 2.16. [10] Two IFSs are said to be q-coincident (A q B in short) if and only if there exists an element x $\in$ X such that μ_A(x) $>$ $\nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. For any two IFSs A and B of X, A $\bar{q}$ B if and only if A $\subseteq$ B^c.

3 Intuitionistic Fuzzy Strongly α-generalized Semi-closed Sets

In this section we introduce intuitionistic fuzzy strongly α-generalized semi-closed sets and study some of its properties.

Definition 3.1. An IFS A in (X, τ) is said to be an intuitionistic fuzzy strongly α-generalized semi-closed set (IFsαGSCS in short) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFGSOS in (X, τ) and the family of all IFsαGSCS of an IFTS (X, τ) is denoted by IFsαGSC(X).

Example 3.2. Let X=\{a, b\}. Let τ=\{$0_\sim$, G, $1_\sim$\} be an IFT on X, where G = $\langle$ x, (0.7, 0.6), (0.3, 0.4)$\rangle$. Then the IFS A = $\langle$ x, (0.2, 0.3), (0.8, 0.7)$\rangle$ is an IFsαGSCS in (X, τ).

Theorem 3.3. Every IFCS in (X, τ) is an IFsαGSCS but not conversely.

Proof. Assume that A is an IFCS in (X, τ). Let us consider an IFS A $\subseteq$ U where U is an IFGSOS in X. Since αcl(A) $\subseteq$ cl(A) and A is an IFCS in X, αcl(A) $\subseteq$ cl(A)= A $\subseteq$ U and U is IFGSOS. That is αcl(A) $\subseteq$ U. Therefore A is IFsαGSCS in X.

Example 3.4. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.8, 0.6), (0.2, 0.4)$\rangle$. Then the IFS A = $\langle$x, (0.1, 0.3), (0.9, 0.7)$\rangle$ is IFsαGSCS but not an IFCS in X.

Theorem 3.5. Every IFαCS in (X, τ) is an IFsαGSCS in (X, τ) but not conversely.

Proof. Let A be an IFαCS in X. Let us consider an IFS A $\subseteq$ U where U is an IFGSOS in (X, τ). Since A is an IFαCS, αcl(A) = A. Hence αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is IFGSOS. Therefore A is an IFsαGSCS in X.

Example 3.6. Let X = {a, b}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where G_1 = $\langle$x, (0.4, 0.3), (0.5, 0.6)$\rangle$ and G_2 = $\langle$x, (0.3, 0.2), (0.6, 0.7)$\rangle$. Consider an IFS A = $\langle$ x, (0.8, 0.8), (0.1, 0.1)$\rangle$ which is IFsαGSCS but not IFαCS, since cl(int(cl(A))) = $1_\sim \nsubseteq$ A.

Theorem 3.7. Every IFRCS in (X, τ) is an IFsαGSCS in (X, τ) but not conversely.

Proof. Let A be an IFRCS in (X, τ). Since every IFRCS is an IFCS, A is an IFCS in X. Hence by Theorem 3.3, A is an IFsαGSCS in X.

Example 3.8. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.6, 0.6), (0.4, 0.4)$\rangle$. Consider an IFS A = $\langle$ x, (0.3, 0.2), (0.7, 0.8)$\rangle$ which is an IFsαGSCS but not IFRCS in X as cl(int(A))= $0_\sim \neq$ A.

Theorem 3.9. Every IFsαGSCS in (X, τ) is an IFαGSCS in (X, τ) but not conversely.

Proof. Assume that A is an IFsαGSCS in (X, τ). Let us consider IFS A $\subseteq$ U where U is an IFSOS in X. Since every IFSOS is an IFGSOS and by hypothesis αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFGSOS in X. We have αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSOS in X. Hence A is an IFαGSCS in X.

Example 3.10. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.8), (0.4, 0.2)$\rangle$. Then the IFS A = $\langle$x, (0.5, 0.3), (0.2, 0.3)$\rangle$ is an IFαGSCS but not an IFsαGSCS in X.

Remark 3.11. An IFP closedness is independent of IFsαGS closedness.

Example 3.12. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.8, 0.7), (0.2, 0.3)$\rangle$. Then the IFS A = $\langle$x, (0.4, 0.5), (0.5, 0.4)$\rangle$ is IFPCS but not IFsαGSCS.

Example 3.13. Let X = {a, b}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.6), (0.7, 0.4)$\rangle$. Then the IFS A = $\langle$x, (0.6, 0.3), (0.4, 0.7)$\rangle$ is IFsαGSCS but not an IFPCS.

Remark 3.14. An IFSP closedness is independent of IFsαGS closedness.

Example 3.15. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.6, 0.3), (0.2, 0.2)$\rangle$. Then the IFS A = $\langle$x, (0.7, 0.4), (0.1, 0.1)$\rangle$ is IFSPCS but not IFsαGSCS.

Example 3.16. Let X = {a, b}. Let $\tau = \{0_\sim, \text{G}, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.6, 0.3), (0.2, 0.2)⟩. Then the IFS A = ⟨x, (0.1, 0.1), (0.7, 0.5)⟩ is IFsαGSCS but not IFSPCS.

Remark 3.17. An IFγCS in (X, τ) need not be an IFsαGSCS.

Example 3.18. Let X = {a, b}. Let $\tau = \{0_\sim, \text{G}, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩. Then the IFS A = ⟨x, (0.4, 0.4), (0.5, 0.4)⟩ is IFγCS but not IFsαGSCS.

The relations between various types of intuitionistic fuzzy closed sets are given in the following diagram.

$$\begin{tikzcd}
 & \text{IF}\alpha\text{CS} \arrow[d] \arrow[dr] & \\
\text{IFCS} \arrow[ur] \arrow[r] & \text{IFs}\alpha\text{GSCS} \arrow[r] & \text{IF}\alpha\text{GSCS} \\
 & \text{IFRCS} \arrow[ul] \arrow[u] \arrow[ur] &
\end{tikzcd}$$

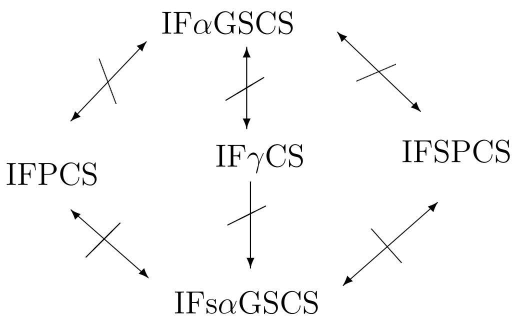
The reverse implications are not true in general.

Remark 3.19. The intersection of any two IFsαGSCS is not an IFsαGSCS in general as can be seen in the following Example.

Example 3.20. Let X = {a, b}. Let $\tau = \{0_\sim, \text{G}, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.2, 0.4),(0.8, 0.6)⟩. Then the IFS A = ⟨x, (0.3, 0.7), (0.6, 0.3)⟩ and B = ⟨x, (0.9, 0.4), (0.1, 0.5)⟩ are IFsαGSCS in X. Now A∩B = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩ $\subseteq$ U = ⟨x, (0.4, 0.5), (0.3, 0.4)⟩ and U is IFGSOS in X. But αcl(A∩B) = $1_\sim \nsubseteq$ U. Therefore A∩B is not an IFsαGSCS in X.

Theorem 3.21. Let (X, τ) be an IFTS. Then for every A $\in$ IFsαGSC(X) and for every IFS B in X, A$\subseteq$ B $\subseteq$ αcl(A) implies B$\in$ IFsαGSC(X).

Proof. Let B$\subseteq$U where U is an IFGSOS in X. Since A $\subseteq$ B, A $\subseteq$ U. Since A is an IFsαGSCS in X, αcl(A) $\subseteq$ U. By hypothesis B $\subseteq$ αcl(A). This implies αcl(B) $\subseteq$ αcl(A) $\subseteq$ U. Therefore αcl(B) $\subseteq$ U. Hence B is an IFsαGSCS in X. The independent relations between various types of intuitionistic fuzzy closed sets are given in the following diagram.



In this diagram, A $\nleftrightarrow$ B denotes A and B are independent and A $\nrightarrow$ B denoted A need not be B.

Theorem 3.22. If A is an IFGSOS and an IFsαGSCS, then A is an IFαCS in X.

Proof. Let A be an IFGSOS in X. Since A $\subseteq$ A, by hypothesis αcl(A) $\subseteq$ A. But always A $\subseteq$ αcl(A). Therefore αcl(A) = A. Hence A is an IFαCS in X.

Theorem 3.23. Let (X, τ) be an IFTS. Then A is an IFsαGSCS if and only if A $\overline{q}$ F implies αcl(A) $\overline{q}$ F for every IFGSCS F of X.

Proof. Necessary Part: Let F be an IFGSCS and A$\overline{q}$F. Then A $\subseteq F'$ where F' is an IFGSOS in X. By assumption αcl(A) $\subseteq F'$. Hence αcl(A) $\overline{q}$ F.
Sufficient Part: Let F be IFGSCS in X such that A $\subseteq F'$. By hypothesis, A $\overline{q}$ F implies αcl(A) $\overline{q}$ F. This implies αcl(A) $\subseteq F'$ whenever A $\subseteq F'$ and F' is an IFGSOS in X. Hence A is an IFsαGSCS in X.

4 Intuitionistic Fuzzy Strongly α-generalized Semi-open Sets

In this section we introduce intuitionistic fuzzy strongly α-generalized semi-open sets and study some of its properties.

Definition 4.1. An IFS A is said to be intuitionistic fuzzy strongly α-generalized semi-open set (IFsαGSOS in short) in (X, τ) if the complement A^c is an IFsαGSCS in X. The family of all IFsαGSOS of an IFTS (X, τ) is denoted by IFsαGSO(X).

Theorem 4.2. For any IFTS (X, τ), every IFOS is an IFsαGSOS but not conversely.

Proof. Let A be an IFOS in X. Then A^c is an IFCS in X. By Theorem 3.3, A^c is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Example 4.3. Let X = {a, b}. Let τ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = $\langle$x, (0.8, 0.6), (0.2, 0.4)$\rangle$. Consider the IFS A = $\langle$x, (0.9, 0.7), (0.1, 0.3)$\rangle$. Since A^c is an IFsαGSCS, A is an IFsαGSOS but not IFOS in X.

Theorem 4.4. In any IFTS (X, τ) every IFαOS is an IFsαGSOS but not conversely.

Proof. Let A be an IFαOS in X. Then A^c is an IFαCS in X. By Theorem 3.5, A^c is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Example 4.5. Let X = {a, b}. Let τ = {$0_\sim$, G_1, G_2, $1_\sim$} be an IFT on X, where G_1 = $\langle$x, (0.4, 0.3), (0.5, 0.6)$\rangle$ and G_2 = $\langle$x, (0.3, 0.2), (0.6, 0.7)$\rangle$. Then the IFS A = $\langle$x, (0.1, 0.1), (0.8, 0.8)$\rangle$ is an IFsαGSOS in X but not an IFαOS in X.

Theorem 4.6. In any IFTS (X, τ), every IFROS is an IFsαGSOS but not conversely.

Proof. Let A be an IFROS in X. Then A^c is an IFRCS in X. By Theorem 3.7, A^c is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Example 4.7. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.6, 0.6), (0.4, 0.4)$\rangle$. Then the IFS A = $\langle$x, (0.7, 0.8), (0.3, 0.2)$\rangle$ is an IFsαGSOS in X but not an IFROS in X.

Theorem 4.8. In any IFTS (X, τ), every IFsαGSOS is an IFαGSOS but not conversely.

Proof. Let A be an IFsαGSOS in X. Then A^c is an IFsαGSCS in X. By Theorem 3.9, A^c is an IFαGSCS in X. Hence A is an IFαGSOS in X.

Example 4.9. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.8), (0.4, 0.2)$\rangle$. Then the IFS A = $\langle$x, (0.2, 0.3), (0.5, 0.3)$\rangle$ is an IFαGSOS in X but not an IFsαGSOS in X.

Remark 4.10. The union of any two IFsαGSOS is not an IFsαGSOS in general.

Example 4.11. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.2, 0.4),(0.8, 0.6)$\rangle$. Then the IFS A = $\langle$x, (0.6, 0.3), (0.3, 0.7)$\rangle$ and B = $\langle$x, (0.1, 0.5), (0.9, 0.4)$\rangle$ are IFsαGSOS in X. Now A$\cup$B = $\langle$x, (0.6, 0.5), (0.3, 0.4) $\rangle$ is not an IFsαGSOS in X.

Theorem 4.12. An IFS A of an IFTS (X, τ) is an IFsαGSOS if and only if F $\subseteq$ αint(A) whenever F is an IFGSCS in X and F $\subseteq$ A.

Proof. Necessary Part: Let A be an IFsαGSOS in X. Let F be an IFGSCS in X and F $\subseteq$ A. Then $F^{'}$ is an IFGSOS in X such that $A^{'} \subseteq F^{'}$. Since $A^{'}$ is an IFsαGSCS, we have αcl($A^{'}$) $\subseteq F^{'}$. Hence (αint(A))$^{'} \subseteq F^{'}$. Therefore F $\subseteq$ αint(A).
Sufficient Part: Let A be an IFS in X and let F $\subseteq$ αint(A) whenever F is an IFGSCS in X and F $\subseteq$ A. Then $A^{'} \subseteq F^{'}$ and $F^{'}$ is an IFGSOS. By hypothesis, (αint(A))$^{'} \subseteq F^{'}$, which implies αcl($A^{'}$) $\subseteq F^{'}$. Therefore $A^{'}$ is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Theorem 4.13. If A is an IFsαGSOS in (X, τ), then A is an IFGSOS in (X, τ).

Proof. Let A be an IFsαGSOS in X. This implies A is an IFαGSOS in X. Since every IFαGSOS is an IFGSOS, A is an IFGSOS in X.

5 Conclusion

In this paper we introduced the stronger form of intuitionistic fuzzy α- generalized semi closed set and some of its properties are discussed with existing intuitionistic fuzzy generalized closed sets. Also some comparable examples are given and some important notions of Intuitionistic fuzzy strongly α- generalized semi open sets are discussed.

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