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THE RESTRICT AND EXTEND OF SOFT SET

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Abstract – In 1999, Molodtsov [1] introduced the concept of soft sets. In 2003, Maji et al. [2] presented a detailed theoretical study of soft sets which includes soft subset of a soft set, equality of soft sets and operations on soft sets such as union and intersection. In 2009, Ali et al. [3] studied and discussed the basic properties of these operations and defined some new operations in soft set theory as restricted soft intersection, restricted soft union and extended soft intersection. In this paper, we have introduced new concepts of soft sets: restrict of soft set, extent of soft set and mutual soft sets and studied some relations between them and operations on soft sets

Keywords – *Soft Sets, Soft Subsets, Soft Equal, Restrict of Soft Sets, Extent of Soft Sets, Mutual Soft Sets.*

1 Introduction

Dealing with uncertainties is a major problem in many areas such as economics, engineering, environmental science, medical science and social sciences. These kinds of problems cannot be dealt with by classical methods, because classical methods have inherent difficulties. To overcome these kinds of difficulties, Molodtsov [1] proposed a completely new approach, which is called soft set theory, for modeling uncertainty. Then Maji et al. [2] introduced several operations on soft sets. The main purpose of this paper is to introduce new concepts in soft set theory (restrict, extend) and studied their relations with restricted soft intersection, restricted soft union, extended soft intersection and extended soft union. Also we have defined mutual soft sets and found an equivalent condition to exist a unique soft union of two soft sets over a common universe and we have generalized that of a non-empty family of soft sets over a common universe.

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2 Preliminaries

In this section, we give some basic definitions for soft sets. Throughout this paper, U denotes an initial universe set and E is a set of parameters, the power set of U is denoted by $P(U)$ and $A \neq \emptyset$ is a subset of E.

Definition 2.1. [1] A pair (F,A) is called a *soft set* over U, where F is a mapping given by $F: A \to P(U)$.

Definition 2.2. Let (F,A) and (G,B) be two soft sets over a common universe U. Then (G,B) is called a *soft subset* of (F,A), denote by $(G,B) \tilde{\subseteq} (F,A)$, if it satisfies the following:
1) $B \subseteq A$,
2) $G(x) \subseteq F(x)$ For all $x \in B$.

Definition 2.3. [2] Two soft sets (F,A) and (G,B) over a common universe U are called *soft equal*, denote by $(F,A) = (G,B)$, if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A). i.e. $G(x) = F(x)\ \forall x \in A = B$.

Definition 2.4. Let (F,A) be a soft set over U and $\emptyset \neq B \subseteq A$. The *restrict* of (F,A) on B is defined as the soft set (F_B, B) where F_B is restrict of F on B. i.e. $F_B(x) = F(x)\ \forall x \in B$. It is clear that $(F_B, B) \tilde{\subseteq} (F,A)$.

This definition is equivalent to definition of soft subset at [2].

Definition 2.5. Let (F,A) and (G,B) be two soft sets over a common universe U. The *extend* of (F,A) by (G,B) is defined as the soft set $(\bar{F}_G, A \cup B)$ where,

$$\bar{F}_G(x) = \begin{cases} F(x)\ ;\ x \in A \\ G(x)\ ;\ x \in B - A \end{cases}$$

It is clear that:

1) $(F,A) \tilde{\subseteq} (\bar{F}_G, A \cup B)$.
2) If $B \subseteq A$, then $\bar{F}_G(x) = F(x)\ \ \forall x \in A$. Thus $(\bar{F}_G, A) = (F,A)$.
3) $\left(\left(\overline{\bar{F}_G}\right)_G, A \cup B\right) = (\bar{F}_G, A \cup B)$.

Definition 2.6. [2] A soft set (F,A) over U is called a *null soft set*, denoted by (Φ, A), if $F(x) = \Phi(x) = \emptyset\ \ \forall x \in A$.

Definition 2.7. [2] A soft set (F,A) over U is called an *absolute soft set*, denoted by (Ω, A), if $F(x) = \Omega(x) = U\ \ \forall x \in A$.

Definition 2.8. [3] Let (F,A) be a soft set over U. The *soft complement* of (F,A) is defined as the soft set (F^c,A) where $F^c(x)=U-F(x)\ \forall x\in A$.

It is clear that $\left(F^{c^c},A\right)=(F,A)$.

Definition 2.9. [4] Let (F,A) be a soft set over U and $f:U\to U'$ be a mapping of sets. Then we can define a soft set $(f(F),A)$ over U' where $f(F):A\to P(U')$ is defined as $(f(F))(x)=f(F(x))$ for all $x\in A$.

Definition 2.10. Let (F,A) be a soft set over U' and $f:U\to U'$ be a mapping of sets. Then we can define a soft set $(f^{-1}(F),A)$ over U where $f^{-1}(F):A\to P(U)$ is defined as $\left(f^{-1}(F)\right)(x)=f^{-1}(F(x))$ for all $x\in A$.

Proposition 2.11. Let (F,A) be a soft set over U and $\varnothing\neq B\subseteq A$. Then:

$$\left((F^c)_B,B\right)=\left((F_B)^c,B\right)$$

Proof. Let x be an element of B. Then:

$$(F^c)_B(x)=F^c(x)=U-F(x)=U-F_B(x)=(F_B)^c(x)$$

Proposition 2.12. Let (F,A) and (G,B) be two soft sets over a common universe U. Then:

$$\left((\bar{F}_G)^c,A\cup B\right)=\left((\overline{F^c})_{G^c},A\cup B\right)$$

Proof. Let x be an element of $A\cup B$. Then:

$$\begin{aligned}(\bar{F}_G)^c(x)=U-(\bar{F}_G)(x)&=\begin{cases}U-F(x)\ ;\ x\in A\\ U-G(x)\ ;\ x\in B-A\end{cases}\\&=\begin{cases}F^c(x)\ ;\ x\in A\\ G^c(x)\ ;\ x\in B-A\end{cases}\\&=\left((\overline{F^c})_{G^c}\right)(x)\end{aligned}$$

Proposition 2.13. Let (F,A) be a soft set over U, and $f:U\to U'$ be a mapping of sets. If $\varnothing\neq B\subseteq A$, Then:

$$\left(f(F_B),B\right)=\left((f(F))_B,B\right)$$

Proof. Let x be an element of B. Then:

$$\left(f(F_B)\right)(x)=f(F_B(x))=f(F(x))=(f(F))(x)=(f(F))_B(x)$$

Proposition 2.14. Let (F,A) and (G,B) be two soft sets over a common universe U and $f:U \to U'$ be a mapping of sets. Then:

$$\left(f\left(\bar{F}_G\right), A \cup B\right) = \left(\left(\overline{f(F)}\right)_{f(G)}, A \cup B\right)$$

Proof. Let x be an element of $A \cup B$. Then:

$$\left(f\left(\bar{F}_G\right)\right)(x) = f\left(\bar{F}_G(x)\right) = \begin{cases} f\left(F(x)\right) \ ; \ x \in A \\ f\left(G(x)\right) \ ; \ x \in B - A \end{cases}$$
$$= \begin{cases} \left(f(F)\right)(x) \ ; \ x \in A \\ \left(f(G)\right)(x) \ ; \ x \in B - A \end{cases}$$
$$= \left(\left(\overline{f(F)}\right)_{f(G)}\right)(x)$$

Proposition 2.15. Let (F,A) be a soft set over U', and $f:U \to U'$ be a mapping of sets. If $\varnothing \neq B \subseteq A$, Then:

$$\left(f^{-1}(F_B), B\right) = \left(\left(f^{-1}(F)\right)_B, B\right)$$

Proof. Let x be an element of B. Then:

$$\left(f^{-1}(F_B)\right)(x) = f^{-1}\left(F_B(x)\right) = f^{-1}\left(F(x)\right) = \left(f^{-1}(F)\right)(x) = \left(f^{-1}(F)\right)_B(x)$$

Proposition 2.16. Let (F,A) and (G,B) be two soft sets over a common universe U' and $f:U \to U'$ be a mapping of sets. Then:

$$\left(f^{-1}\left(\bar{F}_G\right), A \cup B\right) = \left(\left(\overline{f^{-1}(F)}\right)_{f^{-1}(G)}, A \cup B\right)$$

Proof. Let x be an element of $A \cup B$. Then:

$$\left(f^{-1}\left(\bar{F}_G\right)\right)(x) = f^{-1}\left(\bar{F}_G(x)\right) = \begin{cases} f^{-1}\left(F(x)\right) \ ; \ x \in A \\ f^{-1}\left(G(x)\right) \ ; \ x \in B - A \end{cases}$$
$$= \begin{cases} \left(f^{-1}(F)\right)(x) \ ; \ x \in A \\ \left(f^{-1}(G)\right)(x) \ ; \ x \in B - A \end{cases}$$
$$= \left(\left(\overline{f^{-1}(F)}\right)_{f^{-1}(G)}\right)(x)$$

3 Relations Between Restrict and Extend of Soft Sets and Operations on Soft Sets

Definition 3.1. [3] Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\varnothing$. The *restricted soft intersection* of (F,A) and (G,B) is defined as the soft set $\left(F\,\tilde{\cap}_r\,G, A\cap B\right)$ where,

$$\left(F\,\tilde{\cap}_r\,G\right)(x)=F(x)\cap G(x)\ \ \forall x\in A\cap B$$

It is clear that:

$$\left(F\,\tilde{\cap}_r\,G, A\cap B\right)\tilde{\subseteq}\left(F,A\right)\&\left(F\,\tilde{\cap}_r\,G, A\cap B\right)\tilde{\subseteq}\left(G,B\right)\&\left(F\,\tilde{\cap}_r\,F^c, A\right)=(\Phi,A)$$

Definition 3.2. [3] The *extended soft intersection* of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set $\left(F\,\tilde{\cap}_e\,G, A\cup B\right)$ where,

$$\left(F\,\tilde{\cap}_e\,G\right)(x)=\begin{cases} F(x) & ;x\in A-B\\ G(x) & ;x\in B-A\\ F(x)\cap G(x) & ;x\in A\cap B\end{cases}$$

It is clear that:

1) If $A\cap B=\varnothing$, then $\left(F\,\tilde{\cap}_e\,G, A\cup B\right)=\left(\bar{F}_G, A\cup B\right)=\left(\bar{G}_F, A\cup B\right)$.
2) If $A=B$, then $\left(F\,\tilde{\cap}_e\,G, A\right)=\left(F\,\tilde{\cap}_r\,G, A\right)$.

Definition 3.3. [3] Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\varnothing$. The *restricted soft union* of (F,A) and (G,B) is defined as the soft set $\left(F\,\tilde{\cup}_r\,G, A\cap B\right)$ where,

$$\left(F\,\tilde{\cup}_r\,G\right)(x)=F(x)\cup G(x)\ \ \forall x\in A\cap B$$

Definition 3.4. [2] The *extended soft union* (it is called as union in [2]) of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set $\left(F\,\tilde{\cup}_e\,G, A\cup B\right)$ where,

$$\left(F\,\tilde{\cup}_e\,G\right)(x)=\begin{cases} F(x) & ;x\in A-B\\ G(x) & ;x\in B-A\\ F(x)\cup G(x) & ;x\in A\cap B\end{cases}$$

It is clear that:

1) $\left(F,A\right)\tilde{\subseteq}\left(F\,\tilde{\cup}_e\,G, A\cup B\right)\&\left(G,B\right)\tilde{\subseteq}\left(F\,\tilde{\cup}_e\,G, A\cup B\right)\&\left(F\,\tilde{\cup}_e\,F^c, A\right)=(\Omega,A)$.
2) If $A\cap B=\varnothing$, then $\left(F\,\tilde{\cup}_e\,G, A\cup B\right)=\left(F\,\tilde{\cap}_e\,G, A\cup B\right)=\left(\bar{F}_G, A\cup B\right)=\left(\bar{G}_F, A\cup B\right)$.

3) If $A=B$, then $\left(F\,\tilde{\bigcup}_e\,G,A\right)=\left(F\,\tilde{\bigcup}_r\,G,A\right)$.

Proposition 3.5. Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\bigcap B\neq\varnothing$. Then we have the following:

1) $\left(\left(F\,\tilde{\bigcap}_e\,G\right)_{A\bigcap B},A\bigcap B\right)=\left(F\,\tilde{\bigcap}_r\,G,A\bigcap B\right)$.

2) $\left(\left(F\,\tilde{\bigcup}_e\,G\right)_{A\bigcap B},A\bigcap B\right)=\left(F\,\tilde{\bigcup}_r\,G,A\bigcap B\right)$.

Proof. Proof is straightforward.

Proposition 3.6. Let (F,A) and (G,B) be two soft sets over a common universe U . Then we have the following:

1) $\left(\left(\overline{F^c}\right)_G\,\tilde{\bigcap}_r\,\overline{F}_{G^c},A\bigcup B\right)=\left(\left(\overline{F^c}\right)_G\,\tilde{\bigcap}_e\,\overline{F}_{G^c},A\bigcup B\right)=(\Phi,A\bigcup B)$.

2) $\left(\left(\overline{F^c}\right)_G\,\tilde{\bigcup}_r\,\overline{F}_{G^c},A\bigcup B\right)=\left(\left(\overline{F^c}\right)_G\,\tilde{\bigcup}_e\,\overline{F}_{G^c},A\bigcup B\right)=(\Omega,A\bigcup B)$.

Proof. 1) Let x be an element of $A\bigcup B$. Then:

$$\left(\left(\overline{F^c}\right)_G\,\tilde{\bigcap}_r\,\overline{F}_{G^c}\right)(x)=\left(\overline{F^c}\right)_G(x)\bigcap\left(\overline{F}_{G^c}\right)(x)=\begin{cases}F^c(x)\bigcap F(x)=\varnothing & ;x\in A\\ G(x)\bigcap G^c(x)=\varnothing & ;x\in B-A\end{cases}$$
$$=\Phi(x)$$

2) Let x be an element of $A\bigcup B$. Then:

$$\left(\left(\overline{F^c}\right)_G\,\tilde{\bigcup}_r\,\overline{F}_{G^c}\right)(x)=\left(\overline{F^c}\right)_G(x)\bigcup\left(\overline{F}_{G^c}\right)(x)=\begin{cases}F^c(x)\bigcup F(x)=U & ;\ x\in A\\ G(x)\bigcup G^c(x)=U & ;x\in B-A\end{cases}$$
$$=\Omega(x)$$

Theorem. 3.7. Let (F,A) and (G,B) be two soft sets over a common universe U . Then we have the following:

1) $\left(\left(F\,\tilde{\bigcap}_e\,G\right)_A,A\right)\,\tilde{\subseteq}\,(F,A)$ & $\left(\left(F\,\tilde{\bigcap}_e\,G\right)_B,B\right)\,\tilde{\subseteq}\,(G,B)$.

2) $\left(F\,\tilde{\bigcap}_e\,G,A\bigcup B\right)$ is the largest soft set which satisfies the two soft inclusions in (1).

Proof. 1) Let x be an element of A . Then:

$$\left.\begin{array}{ll}if\ \ x\in A-B & \Rightarrow\left(F\,\tilde{\bigcap}_e\,G\right)(x)=F(x)\\ if\ \ x\in A\bigcap B & \Rightarrow\left(F\,\tilde{\bigcap}_e\,G\right)(x)=F(x)\bigcap G(x)\subseteq F(x)\end{array}\right\}\Rightarrow$$
$$\Rightarrow\left(F\,\tilde{\bigcap}_e\,G\right)(x)\subseteq F(x)\quad\forall x\in A$$
$$\Rightarrow\left(F\,\tilde{\bigcap}_e\,G\right)_A(x)\subseteq F(x)\quad\forall x\in A$$

Thus:

$$\left(\left(F \tilde{\cap}_e G\right)_A, A\right) \tilde{\subseteq} (F, A)$$

Let x be an element of B. Then:

$$\left.\begin{array}{ll} \text{if } x \in B - A & \Rightarrow \left(F \tilde{\cap}_e G\right)(x) = G(x) \\ \text{if } x \in A \cap B & \Rightarrow \left(F \tilde{\cap}_e G\right)(x) = F(x) \cap G(x) \subseteq G(x) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left(F \tilde{\cap}_e G\right)(x) \subseteq G(x) \quad \forall x \in B$$

$$\Rightarrow \left(F \tilde{\cap}_e G\right)_B (x) \subseteq G(x) \quad \forall x \in B$$

Thus:

$$\left(\left(F \tilde{\cap}_e G\right)_B, B\right) \tilde{\subseteq} (G, B)$$

2) Let $(H, A \cup B)$ be a soft set such that $\left(H_A, A\right) \tilde{\subseteq} (F, A)$ & $\left(H_B, B\right) \tilde{\subseteq} (G, B)$ and we will prove that:

$$(H, A \cup B) \tilde{\subseteq} \left(F \tilde{\cap}_e G, A \cup B\right)$$

Let x be an element of $A \cup B$. Then:

$$\left.\begin{array}{ll} \text{if } x \in A - B & \Rightarrow \left(F \tilde{\cap}_e G\right)(x) = F(x) \supseteq H_A(x) = H(x) \\ \text{if } x \in B - A & \Rightarrow \left(F \tilde{\cap}_e G\right)(x) = G(x) \supseteq H_B(x) = H(x) \\ \text{if } x \in A \cap B & \Rightarrow \left(F \tilde{\cap}_e G\right)(x) = F(x) \cap G(x) \supseteq H_A(x) \cap H_B(x) = H(x) \end{array}\right\} \Rightarrow$$

$$\Rightarrow H(x) \subseteq \left(F \tilde{\cap}_e G\right)(x) \quad \forall x \in A \cup B$$

Thus:

$$(H, A \cup B) \tilde{\subseteq} \left(F \tilde{\cap}_e G, A \cup B\right)$$

Theorem. 3.8. Let (F, A) and (G, B) be two soft sets over a common universe U. Then we have the following:

1) $(F, A) \tilde{\subseteq} \left(\left(F \tilde{\cup}_e G\right)_A, A\right)$ & $(G, B) \tilde{\subseteq} \left(\left(F \tilde{\cup}_e G\right)_B, B\right)$.

2) $\left(F \tilde{\cup}_e G, A \cup B\right)$ is the smallest soft set which satisfies the two soft inclusions in (1).

Proof. 1) Let x be an element of A. Then:

$$\left.\begin{array}{ll} if \quad x \in A - B & \Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) = F(x) \\ if \quad x \in A \bigcap B & \Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) = F(x) \bigcup G(x) \supseteq F(x) \end{array}\right\} \Rightarrow$$

$$\Rightarrow F(x) \subseteq \left(F \tilde{\bigcup}_e G\right)(x) \quad \forall x \in A$$

$$\Rightarrow F(x) \subseteq \left(F \tilde{\bigcup}_e G\right)_A(x) \quad \forall x \in A$$

Thus:

$$(F, A) \tilde{\subseteq} \left(\left(F \tilde{\bigcup}_e G\right)_A, A\right)$$

Let x be an element of B. Then:

$$\left.\begin{array}{ll} if \quad x \in B - A & \Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) = G(x) \\ if \quad x \in A \bigcap B & \Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) = F(x) \bigcup G(x) \supseteq G(x) \end{array}\right\} \Rightarrow$$

$$\Rightarrow G(x) \subseteq \left(F \tilde{\bigcup}_e G\right)(x) \quad \forall x \in B$$

$$\Rightarrow G(x) \subseteq \left(F \tilde{\bigcup}_e G\right)_B(x) \quad \forall x \in B$$

Thus:

$$(G, B) \tilde{\subseteq} \left(\left(F \tilde{\bigcup}_e G\right)_B, B\right)$$

2) Let $(H, A \bigcup B)$ be a soft set such that $\left(F, A\right) \tilde{\subseteq} \left(H_A, A\right) \& \left(G, B\right) \tilde{\subseteq} \left(H_B, B\right)$ and we will prove that:

$$\left(F \tilde{\bigcup}_e G, A \cup B\right) \tilde{\subseteq} \left(H, A \cup B\right)$$

Let x be an element of $A \bigcup B$. Then:

$$\left.\begin{array}{ll} if \quad x \in A - B & \Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) = F(x) \subseteq H_A(x) = H(x) \\ if \quad x \in B - A & \Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) = G(x) \subseteq H_B(x) = H(x) \\ if \quad x \in A \bigcap B & \Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) = F(x) \bigcup G(x) \subseteq H_A(x) \bigcup H_B(x) = H(x) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left(F \tilde{\bigcup}_e G\right)(x) \subseteq H(x) \ \forall x \in A \bigcup B$$

Thus:

$$\left(F \tilde{\bigcup}_e G, A \cup B\right) \tilde{\subseteq} \left(H, A \cup B\right)$$

Theorem. 3.9. Let (F, A) and (G, B) be two soft sets over a common universe U. Then there is an unique soft set $(H, A \bigcup B)$ over U such that:

$$\left(H_A, A\right) = (F, A) \& \left(H_B, B\right) = (G, B)$$

if and only if $F(x) = G(x) \ \forall x \in A \bigcap B$.

Proof. Suppose that $\left(H, A \cup B\right)$ is a soft set over U such that:

$$\left(H_A, A\right) = (F, A) \ \& \ \left(H_B, B\right) = (G, B)$$

Then:

$$\forall x \in A \cap B \ \Rightarrow F(x) = H_A(x) = H(x) \ \& \ G(x) = H_B(x) = H(x) \Rightarrow F(x) = G(x)$$

Conversely, suppose that $F(x) = G(x) \ \forall x \in A \cap B$.

Indeed the soft set $\left(H, A \cup B\right) = \left(F \tilde{\cap}_e G, A \cup B\right)$ where,

$$H(x) = \left(F \tilde{\cap}_e G\right)(x) = \begin{cases} F(x) \ ; \ x \in A \\ G(x) \ ; \ x \in B \end{cases}$$

And holds:

$$\forall x \in A \Rightarrow H_A(x) = H(x) = F(x) \Rightarrow \left(H_A, A\right) = \left(F, A\right)$$
$$\forall x \in B \Rightarrow H_B(x) = H(x) = G(x) \Rightarrow \left(H_B, B\right) = \left(G, B\right)$$

Now, let $\left(H', A \cup B\right)$ be a soft set such that $\left(H'_A, A\right) = \left(F, A\right) \& \left(H'_B, B\right) = \left(G, B\right)$ and we will prove that:

$$\left(H', A \cup B\right) = \left(H, A \cup B\right)$$

Let x be an element of $A \cup B$. Then:

$$\left.\begin{aligned} &if \ \ x \in A \ \Rightarrow H'(x) = H'_A(x) = F(x) = H_A(x) = H(x) \\ &if \ \ x \in B \ \Rightarrow H'(x) = H'_B(x) = G(x) = H_B(x) = H(x) \end{aligned}\right\} \Rightarrow$$
$$\Rightarrow H'(x) = H(x) \ \forall x \in A \cup B$$

Thus:

$$\left(H', A \cup B\right) = \left(H, A \cup B\right)$$

Definition 3.10. Two soft sets (F, A) and (G, B) over a common universe U are called *mutual soft sets* if $F(x) = G(x) \ \forall x \in A \cap B$.

Proposition 3.11. Two soft sets (F, A) and (G, B) over a common universe U are *mutual soft sets* if and only if $\left(\bar{F}_G, A \cup B\right) = \left(\bar{G}_F, A \cup B\right)$.

Proof. Suppose that (F, A) and (G, B) are mutual soft sets. Then:

$$F(x) = G(x) \ \forall x \in A \cap B$$

Thus:

$$\bar{F}_G(x) = \begin{cases} F(x) \ ; x \in A \\ G(x) \ ; \ x \in B - A \end{cases}$$

$$= \begin{cases} F(x) & ; x \in A - B \\ F(x) = G(x) & ; x \in A \cap B \\ G(x) & ; x \in B - A \end{cases}$$

$$= \begin{cases} F(x) \ ; \ x \in A - B \\ G(x) \ ; \ x \in B \end{cases}$$

$$= \bar{G}_F(x)$$

Conversely, suppose that $\left(\bar{F}_G, A \cup B\right) = \left(\bar{G}_F, A \cup B\right)$. Then:

$$\forall x \in A \cap B \Rightarrow F(x) = \bar{F}_G(x) = \bar{G}_F(x) = G(x) \Rightarrow F(x) = G(x) \ \forall x \in A \cap B$$

Theorem. 3.12. Let $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ be a non-empty family of soft sets over a common universe U . Then there is an unique soft set $\left(H, \bigcup_{i \in I} A_i\right)$ over U such that:

$$\left(H_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$$

if and only if $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ are pairwise mutual soft sets

Proof. Suppose that $\left(H, \bigcup_{i \in I} A_i\right)$ is a soft set over U such that:

$$\left(H_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$$

Then:

$$\forall \alpha, \beta \in I, \forall x \in A_\alpha \cap A_\beta \Rightarrow F_\alpha(x) = H_{A_\alpha}(x) = H(x) \ \& \ F_\beta(x) = H_{A_\beta}(x) = H(x)$$

$$\Rightarrow F_\alpha(x) = F_\beta(x) \ \forall x \in A_\alpha \cap A_\beta \ , \ \forall \alpha, \beta \in I$$

Thus $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ are pairwise mutual soft sets.

Conversely, suppose that $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ are pairwise mutual soft sets. Then:

$$F_i(x) = F_j(x) \ \forall x \in A_i \cap A_j \ , \ \forall i, j \in I$$

We will define $\left(H, \bigcup_{i\in I} A_i\right)$ as: $\forall x \in \bigcup_{i\in I} A_i \Rightarrow \exists \alpha \in I; x \in A_\alpha$ we set $H(x) = F_\alpha(x)$.

H is well define because if $\beta \in I$ such that $x \in A_\beta$, then $x \in A_\alpha \cap A_\beta$, Thus:

$$H(x) = F_\alpha(x) = F_\beta(x).$$

H holds:

$$\forall \alpha \in I, \forall x \in A_\alpha \Rightarrow H_{A_\alpha}(x) = H(x) = F_\alpha(x) \Rightarrow \left(H_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$$

Now, let $\left(H', \bigcup_{i\in I} A_i\right)$ be a soft set such that $\left(H'_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$.

Then:

$$\forall x \in \bigcup_{i\in I} A_i \Rightarrow \exists \alpha \in I; x \in A_\alpha$$

$$\Rightarrow H'(x) = H'_{A_\alpha}(x) = F_\alpha(x) = H(x)$$

Thus:

$$H'(x) = H(x) \ \ \forall x \in \bigcup_{i\in I} A_i$$

Hence:

$$\left(H', \bigcup_{i\in I} A_i\right) = \left(H, \bigcup_{i\in I} A_i\right)$$

4 Conclusions

In this paper, we have introduced new concepts in soft set theory: restrict of soft set, extent of soft set and mutual soft sets. And we studied their relations with soft complement, restricted soft intersection, restricted soft union, extended soft intersection and extended soft union. To extend this work, one could extend study these concepts and relations of soft sets in other algebraic structures such as soft groups, soft rings, etc.

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