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ON SOFT $b-I-$OPEN SETS WITH RESPECT TO SOFT IDEAL

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Abstaract — The aim of this work is to present and learning a novel set of soft I-open sets, namel soft b-open sets and acquire some of their features. Then debated the relations among soft semi-I-open sets, soft pre-I-open sets, soft $\beta-I$-open sets and soft $b-I$-open sets. We also researched the new notions of soft $b-I$-continuous functions and soft $b-I$-open (soft $b-I$-closed) functions.

Keywords — *Soft sets, soft $b-I-$open sets, soft $b-I-$closed sets, soft $b-I-$continuity, soft $b-I$-open functions, soft ideal.*

1 Introduction

Kuratowski [1] studied and introduced the concept of ideal topological spaces. The concept of I-open sets in topological spaces was presented by Jankovic and Hamlet [2], which formed via ideals. And in 1999, a Russian scientist Molodtsov [3] introduced the concept of soft sets. He excellently implemented the soft set theory. Later, Maji et al. [4,5] defined some operations on soft sets. On the other hand, Aktas and Cagman [6] compared soft sets with fuzzy sets and rough sets. Chang [7] studied the topological structures of set theories dealing with ambiguities first time. Then, Shabir and Naz [8] presented the concept of soft topological spaces that are described over an original universe with a fixed set of parameters. Additionally the soft separation axioms were presented for soft topological spaces by Shabir and Naz [8]. Zorlutuna et al.[9] presented the notion of soft continuity of functions and some of its features were studied. Then Aygunoglu and Aygun [10] continued to study continuous soft functions. Lately, Kharal and Ahmad [11] defined the concept of a function on soft grades and reviewed several features of images and reverse images of soft sets. Furthermore, these concepts were applied in medical by they Akdag and Ozkan [12] introduced the soft b-sets and soft b-continuous functions. Then the

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definition of soft ideal was gave by Kale and Guler [13] and they also presented the features of soft ideal topological space. Furthermore, the notion of soft I-regularity and soft I-normality were introduced by they. Later, Akdag and Erol [14] defined soft I-open sets and soft I-continuity of functions. They [15] also defined soft semi I-open sets and soft semi I-continuity of functions.

The aim of this work is to acquaint the notion of soft $b-I$-open sets, soft $b-I$-continuous functions, soft $b-I$-open functions and soft $b-I$-closed functions and to get some characterizations and fundamental features of this sets and functions. We debated the intercourses soft semi I-open sets, soft pre I-open sets, soft $\beta-I$-open sets and soft $b-I$-open sets. We also studied the relationships among soft $b-I$-continuous functions, soft semi-I-continuous functions, soft pre$-I$-continuous functions and soft $\beta-I$-continuous functions.

2 Preliminaries

In the valid part we will shortly recollection some fundamental descriptions and lemmas for soft sets.

Definition 1. [3] *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in P(X)\}$, where $f_A : E \to P(X)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.*

Note that the set of all soft sets over X will be denoted by $S(X)$.

Definition 2. [4] *Let F_A and F_B be soft sets over a common universe X. Then F_A is said to be a soft subset of F_B if $f_A(e) \subset f_B(e)$, for all $e \in A$ and this relation is denoted by $F_A \widetilde{\subseteq} F_B$. Also, F_A is said to be a soft equal to F_B if $f_A(e) = f_B(e)$, for all $e \in A$ and this relation is denoted by $F_A = F_B$.*

Definition 3. [19] *The complement of a soft set F_A denoted by F_A^c is defined by $f_A^c : A \to P(X)$ is a mapping given by $f_A^c(e) = X - f_A(e)$, $\forall e \in A$. f_A^c is called the soft complement function of f_A. Clearly, $(f_A^c)^c$ is the same as f_A and $((F_A)^c)^c = F_A$.*

Definition 4. [4] *A soft set F_A over X is said to be a null soft set denoted by $\widetilde{\emptyset}$, if $\forall e \in A$, $f_A(e) = \emptyset$.*

Definition 5. [4] *A soft set F_A over X is said to be an absolute soft set denoted by $\widetilde{X}$, if $\forall e \in E$, $f_A(e) = X$.*

Clearly, $\widetilde{X}^c = \widetilde{\emptyset}$ and $\widetilde{\emptyset}^c = \widetilde{X}$.

Definition 6. [9] *The soft set F_A is called a soft point if there exists a $x \in X$ and $A \subset E$ such that $f_A(e) = \{x\}$, for all $e \in A$ and $f_A(e) = \emptyset$;, for all $e \in E - A$. A soft point is denoted by F_A^x .The soft point F_E^x is called absolute soft point. A soft point F_A^x is said to belong to a soft set G_B if $x \in g_B(e)$, for each $e \in A$, and symbolically denoted by $F_A^x \widetilde{\in} G_B$.*

Definition 7. [3] *The union of two soft sets of F_A and G_B over the common universe X is the soft set H_C,where $C = A \cup B$ and for all $e \in C$,*

$$h_C(e) = \begin{cases} f_A(e), & \text{if } e \in A - B, \\ g_B(e), & \text{if } e \in B - A, \\ f_A(e) \cup g_B(e), & \text{if } e \in A \cup B. \end{cases}$$

We write $F_A \widetilde{\cup} G_B = H_C$.

Definition 8. [3] *The intersection of two soft sets F_A and G_B over the common universe X is the soft set H_C, where $C = A \cap B$ and for all $e \in C$, $h_C(e) = f_A(e) \cap g_B(e)$. This relationship is written as $F_A \widetilde{\cap} G_B = H_C$.*

Definition 9. [20] *Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if,*
(a) $\widetilde{\emptyset}$, $\widetilde{X} \widetilde{\in} \tau$
(b) the intersection of any two soft sets in τ belongs to τ
(c) the union of any number of soft sets in τ belongs to τ.

The triple (X, τ, E) is called a soft topological space over X. Every member of τ is called soft open in (X, τ, E). If complement of any soft set belongs to τ, then it is called soft closed set (X, τ, E).

Definition 10. [20, 9] *Let (X, τ, A) be a soft topological space over X and F_A be a soft set over X. The soft closure of F_A denoted by $cl(F_A)$ is the intersection of all closed soft super sets of F_A. The soft interior of F_A denoted by $int(F_A)$ is the union of all open soft subsets of F_A.*

Definition 11. [8] *A soft set F_A in a soft topological space (X, τ, A) is called a soft neighborhood (briefly: nbd) of the soft point $x_G \widetilde{\in} \widetilde{X}$ if there exists a soft open set H_A such that $x_G \widetilde{\in} H_A \widetilde{\subset} F_A$.*

Definition 12. [8] *Let F_A be a soft set over X and Y be a nonempty subset of X. Then the sub soft set of F_A over Y denoted by ${}^Y F_A$ is defined as ${}^Y F_A(e) = Y \cap f_A(e)$, for each $e \in A$. In other word ${}^Y F_A = \widetilde{Y} \widetilde{\cap} F_A$.*

Definition 13. [13] *A soft ideal I is a nonempty collection of soft sets over X if;*
(a) $F_A \widetilde{\in} I$, $G_A \widetilde{\subset} F_A$ implies $G_A \widetilde{\in} I$.
(b) $F_A \widetilde{\in} I$, $G_A \widetilde{\in} I$ implies $F_A \widetilde{\cup} G_A \widetilde{\in} I$.
A soft topological space (X, τ, A) with a soft ideal I called soft ideal topological space and denoted by (X, τ, A, I).

Definition 14. [13] *Let F_A be a soft set in a soft ideal topological space (X, τ, A, I) and $(.)^*$ be a soft operator from $S(X)$ to $S(X)$. Then the soft local mapping of F_A defined by $F_A^*(I, \tau) = \left\{ F_A^x : X_A \widetilde{\cap} F_A \widetilde{\notin} I \text{ for every } X_A \widetilde{\in} \nu(F_A^x) \right\}$ denoted by F_A^* simply.*

Lemma 1. [13] *Let (X, τ, A, I) be a soft ideal topological space and F_A, G_A be two soft sets. Then*
(a) $F_A \widetilde{\subset} G_A$ implies $F_A^* \widetilde{\subset} G_A^*$ and $\left(F_A \widetilde{\cup} G_A\right)^* = F_A^* \widetilde{\cup} G_A^*$.
(b) $F_A^* \widetilde{\subset} cl(F_A)$ and $(F_A^*)^* \widetilde{\subset} F_A^*$.
(c) If F_A is soft open $F_A \widetilde{\cap} G_A \widetilde{\in} I$ implies $F_A \widetilde{\cap} G_A^* = \widetilde{\emptyset}$
(d) F_A^* is soft closed.
(e) If F_A is soft closed then $F_A^* \widetilde{\subset} F_A$.

Definition 15. [13] *Let (X, τ, A, I) be a soft ideal topological space. The soft set operator cl^* is called a soft$^*-$closure and is defined as $cl^*(F_A) = F_A \widetilde{\cup} F_A^*$ for a soft subset F_A.*

Proposition 1. [13] *Let (X, τ, A, I) be a soft ideal topological space and F_A, G_A be two soft sets. Then*
(a) $cl^*\left(\widetilde{\emptyset}\right) = \widetilde{\emptyset}$ and $cl^*\left(\widetilde{X}\right) = \widetilde{X}$.
(b) $F_A \widetilde{\subset} cl^*(F_A)$ and $cl^*(cl^*(F_A)) = cl^*(F_A)$.
(c) If $F_A \widetilde{\subset} G_A$ then $cl^*(F_A) \widetilde{\subset} cl^*(G_A)$.
(d) $cl^*(F_A) \widetilde{\cup} cl^*(G_A) = cl^*\left(F_A \widetilde{\cup} G_A\right)$.

Lemma 2. [13] *Let (X, τ, A, I) be a soft ideal topological space.*
(a) If $I = \left\{\widetilde{\emptyset}\right\}$, then $F_A^* = cl(F_A)$
(b) If $I = S(X)$, then $F_A^* = \widetilde{\emptyset}$.

Definition 16. [11] *Let X_E and Y_K be soft classes. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then a mapping $f : X_E \to Y_K$ is defined as: for a soft set F_A in X_E, $(f(F_A), B)$, $B = p(A) \subset K$ is a soft set in Y_K given by $f(F_A)(\beta) = u\left(\underset{\alpha \in p^{-1}(\beta) \cap A}{\cup} f(\alpha)\right)$ for $\beta \in K$. $(f(F_A), B)$ is called a soft image of a soft set F_A. If $B = K$, then we shall write $(f(F_A, K)$ as $f(F_A)$.*

Definition 17. [11] *Let $f : X_E \to Y_K$ be a mapping from a soft class X_E to another soft class Y_K, and G_C a soft set in soft class Y_K, where $C \subset K$. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then $(f^{-1}(G_C), D)$, $D = p^{-1}(C)$ is a soft set in the soft classes X_E defined as: $f^{-1}(G_C)(\alpha) = u^{-1}(g(p(\alpha)))$ for $\alpha \in D \subset E$. $(f^{-1}(G_C), D)$ is called a soft inverse image of G_C. Hereafter we shall write $(f^{-1}(G_C), E)$ as $f^{-1}(G_C)$.*

3 Soft $b - I$-Open Sets and Soft $b - I$-Closed Sets

Definition 18. *Let (X, τ, A, I) be a soft ideal topological space and a soft subset F_A in X. Then F_A is said;*
(a) [15] soft semi-$I-$open set if $F_A \widetilde{\subset} cl^*(int(F_A))$.
(b) soft pre-$I-$open set if $F_A \widetilde{\subset} int(cl^*(F_A))$.
(c) soft $\beta - I$-open set if $F_A \widetilde{\subset} cl(int(cl^*(F_A)))$.
(d) soft $b - I$-open set if $F_A \widetilde{\subset} cl^*(int(F_A)) \widetilde{\cup} int(cl^*(F_A))$.

By $SIO(X, \tau, A, I)$ (resp. $SSIO(X, \tau, A, I)$, $SPIO(X, \tau, A, I)$, $SbIO(X, \tau, A, I)$, $S\beta IO(X, \tau, A, I)$) we denote the family of all soft $I-$open (resp. soft semi-$I-$open, soft pre-$I-$open, soft $b - I-$open, soft $\beta - I-$open) sets of a soft topological space (X, τ, A, I).

Remark 1. *In following example indicatesed that every soft semi$-I-$open set is soft $b - I-$open set but the reverse is generally not true.*

Example 1. *Let $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,*
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}\right\}$ and $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_1\})\}$ is soft $b - I-$open set but is not soft semi$-I$-open set.

Remark 2. *In following example shown that every soft pre−I-open set is soft $b-I$-open set but the inverse is usually not true.*

Example 2. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$ *and*
$\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_{A_1}, F_{A_2}, F_{A_3}\}$, *where*
$F_{A_1}, F_{A_2}, F_{A_3}$ *are soft sets over* X, *defined as follows:*
$F_{A_1} = \{(e_1, \{h_1\})\}$,
$F_{A_2} = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$,
$F_{A_3} = \{(e_1, X), (e_2, \{h_2\})\}$,
Then τ *defines a soft topology on* X, *and thus* (X, τ, A, I) *is a soft ideal topological space, where* $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$;
is soft $b-I$*-open set but not soft pre-*I*-open set.*

Remark 3. *In following example shown that every soft* $b-I$*-open set is soft* $\beta-I$*-open set but the inverse is usually not true.*

Example 3. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$ *and* $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$ *is soft* $\beta-I-$*open set but is not soft* $b-I$*-open set.*

Remark 4. *In following example shows that every soft open set is soft* $b-I-$*open set but not usually reverse.*

Example 4. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}\right\}$ *and* $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_2\})\}$ *is soft* $b-I-$*open set but is not soft-open set.*

Definition 19. [12] *A soft subset* F_A *of a soft topological space* (X, τ, A) *is said to be soft b-open set if* $F_A \widetilde{\subset} cl(int(F_A)) \widetilde{\cup} int(cl(F_A))$.

The collection of all soft $b-$open sets in (X, τ, A) is denoted $SbO(X)$.

Remark 5. *In following example shows that every soft* $b-I-$*open set is soft* $b-$*open set but not usually reverse.*

Example 5. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$ *and* $I = S(X)$.
Then $F_A = \{(e_1, X), (e_2, \{h_1\})\}$ *is soft* $b-I-$*open set but is not soft* $b-$*open set.*

Definition 20. *A soft subset* F_A *of a soft ideal topological space* (X, τ, A, I) *is said to be soft*-perfect if* $F_A = F_A^*$

Theorem 1. *For a soft subset* F_A *of a soft ideal topological space* (X, τ, A, I) *the following are true:*
(a) If $I = \{\widetilde{\emptyset}\}$ *and* F_A *is soft pre-open set, then* F_A *is soft* $b-I$*-open set.*
(b) If $I = S(X)$ *and* F_A *is soft* $b-I$*-open set, then* F_A *is soft-open set.*
(c) If F_A *is soft*-perfect and* F_A *is soft* $b-I$*-open set, then* F_A *is soft semi-*$I-$*open set.*

Proof. (a) Let F_A be a soft pre-open set. By Lemma 2, since $I = \{\widetilde{\emptyset}\}$, then $F_A^* = cl(F_A)$.

Therefore, $F_A \widetilde{\subset} int(cl(F_A)) = int(F_A^*) = int(cl^*(F_A))$
$\widetilde{\subset} int(cl^*(F_A)) \widetilde{\cup} cl^*(int(F_A))$. Hence F_A is soft $b - I$-open set.
(b) Let F_A be a soft $b - I$-open set. By Lemma 2, since
$I = S(X)$, then $F_A^* = \left\{\widetilde{\emptyset}\right\}$ and $cl^*(F_A) = F_A^* \widetilde{\cup} F_A = F_A$.
Thus $F_A \widetilde{\subset} cl^*(int(F_A)) \widetilde{\cup} int(cl^*(F_A))$
$= int(F_A) \widetilde{\cup} int(F_A) = int(F_A)$. Hence $F_A = int(F_A)$.
Therefore F_A is soft open set.
(c) Let F_A is soft*-perfect then $cl^*(F_A) = F_A \widetilde{\cup} F_A^* = F_A$.
Since F_A is soft $b - I$-open set then $F_A \widetilde{\subset} int(cl^*(F_A)) \widetilde{\cup} cl^*(int(F_A))$
$= int(F_A) \widetilde{\cup} int(F_A \widetilde{\cup} (intF_A)^*) = int(F_A) \widetilde{\cup} (intF_A)^* = cl^*(int(F_A))$.
Thus F_A is soft semi-I-open set. □

Proposition 2. *The union of two soft $b - I-$open sets in a soft ideal topological space (X, τ, A, I) is soft $b - I$-open set.*

Proof. Let F_A and G_A be two soft $b - I-$open sets. Then
$F_A \widetilde{\cup} G_A \widetilde{\subset} [cl^*(int(F_A)) \widetilde{\cup} int(cl^*(F_A))] \widetilde{\cup}$
$[cl^*(int(G_A)) \widetilde{\cup} int(cl^*(G_A)]$
$= [cl^*(int(F_A)) \widetilde{\cup} cl^*(int(G_A))] \widetilde{\cup}$
$[int(cl^*(F_A)) \widetilde{\cup} int(cl^*(G_A))]$
$\widetilde{\subset} cl^*[int(F_A) \widetilde{\cup} int(G_A)] \widetilde{\cup} int[cl^*(F_A) \widetilde{\cup} cl^*(G_A)]$
$\widetilde{\subset} cl^*(int[F_A \widetilde{\cup} (G_A)]) \widetilde{\cup} int(cl^*[F_A \widetilde{\cup} G_A])$.
Thus $F_A \widetilde{\cup} G_A$ is soft $b - I$-open set. □

Conclusion 1. *Let $\{(F_{A_i}) : i \in \Delta\}$ be a family of soft $b - I-$open sets. Then $\widetilde{\cup}_{i \in \Delta} (F_{A_i})$ is soft $b - I-$open set.*

Proof. Let $\{(F_{A_i})\}$ be a family of soft $b - I-$open sets. Then for each i,
$(F_{A_i}) \widetilde{\subset} cl^*(int (F_{A_i})) \widetilde{\cup} int(cl^* (F_{A_i}))$. Now
$\widetilde{\cup} (F_{A_i}) \widetilde{\subset} \widetilde{\cup} [cl^*(int((F_{A_i}))) \widetilde{\cup} cl^*(int(F_{A_i})]$
$\widetilde{\subset} [cl^*(int(\widetilde{\cup}(F, A)_\alpha))) \widetilde{\cup} int(cl^*(\widetilde{\cup}(F, A)_\alpha)))]$.
Therefore $\widetilde{\cup}_{i \in \Delta} (F_{A_i})$ is a soft $b - I-$open set. □

Remark 6. *The intersection of two soft $b - I-$open sets in a soft ideal topological space (X, τ, A, I) is not soft $b-I$-open in general as shown by the following example.*

Example 6. *Let $X = \{h_1, h_2\}$, $A = \{e_1, e_2, e_3\}$,*
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$
and $I = \left\{\widetilde{\emptyset}\right\}$. Then $F_A = \{(e_1, \{h_1\}), (e_3, \{h_3\})\}$ and
$G_A = \{(e_2, \{h_2\}), (e_3, \{h_3\})\}$ are soft $b - I-$open sets
but $F_A \widetilde{\cap} G_A = \{(e_3, \{h_3\})\}$is not soft $b - I$-open set.

Theorem 2. *Let F_A and G_A be two soft subsets in a soft ideal topological space (X, τ, A, I). Then the following statements are hold:*

(a) If F_A is soft $b - I-$open set and G_A is soft open set then $F_A \widetilde{\cap} G_A$ is soft $b - I-$open set.

(b) If F_A is soft $b - I-$open set and G_A is soft $\alpha - I-$open set then $F_A \widetilde{\cap} G_A$ is soft $b - I-$open set.

Proof. (a) Let F_A is soft $b-I-$open set and G_A is soft open set, then
$F_A\widetilde{\cap}G_A\widetilde{\subset}[cl^*(int(F_A))\widetilde{\cup}int(cl^*(F_A))]\widetilde{\cap}G_A$
$= [cl^*(int(F_A))\widetilde{\cap}G_A]\widetilde{\cup}[int(cl^*(F_A))\widetilde{\cap}G_A]$
$= [(int(F_A)\widetilde{\cup}(int(F_A))^*)\widetilde{\cap}G_A]\widetilde{\cup}[int(F_A\widetilde{\cup}F_A^*)\widetilde{\cap}G_A]$
$= [(intF_A\widetilde{\cap}G_A)\widetilde{\cup}((int(F_A))^*\widetilde{\cap}G_A)]\widetilde{\cup}$
$[int(F_A\widetilde{\cup}G_A)^*)\widetilde{\cap}int(F_A)]$
$\widetilde{\subset}[(int(F_A)\widetilde{\cap}G_A)\widetilde{\cup}(int(F_A)\widetilde{\cap}G_A)^*]\widetilde{\cup}$
$int[(F_A\widetilde{\cup}F_A^*)\widetilde{\cap}G_A]$
$= [(int(F_A)\widetilde{\cap}int(G_A))\widetilde{\cup}(int(F_A)\widetilde{\cap}int(G_A))^*]\widetilde{\cup}$
$int[(F_A\widetilde{\cap}G_A)\widetilde{\cup}(F_A^*\widetilde{\cap}G_A)]$
$\widetilde{\subset}[int(F_A\widetilde{\cap}G_A)\widetilde{\cup}(int(F_A\widetilde{\cap}G_A))^*]\widetilde{\cup}$
$int[(F_A\widetilde{\cap}G_A)\widetilde{\cup}(F_A\widetilde{\cap}G_A)^*]$
$= cl^*(int(F_A\widetilde{\cap}G_A))\widetilde{\cup}int(cl^*(F_A\widetilde{\cap}G_A))$.
This shows that $F_A\widetilde{\cap}G_A$ soft $b-I-$open set.
(b) Straightforward. □

Definition 21. *Let F_A be a soft subset in a soft ideal topological space (X,τ,A,I). F_A is said to be soft $b-I$-closed set if F_A^c is soft $b-I$-open set.*

The collection of all soft $b-I-$closed sets subsets in (X,τ,A,I) will be denoted by $SbIC(X)$.

Theorem 3. *Let F_A be to a subset of a soft ideal topological space (X,τ,A,I). If F_A is soft $b-I$-closed set, then $cl^*(int(F_A))\widetilde{\cap}int(cl^*(F_A))\widetilde{\subset}F_A$.*

Proof. Since F_A is soft $b-I$-closed set, then $\widetilde{X}-F_A$ is soft $b-I$-open set in X. Thus,

$$\begin{aligned}\widetilde{X}-F_A&\widetilde{\subset}cl^*(int(\widetilde{X}-F_A))\widetilde{\cup}int(cl^*(\widetilde{X}-F_A))\\&\widetilde{\subset}cl(int(\widetilde{X}-F_A))\widetilde{\cup}int(cl(\widetilde{X}-F_A))\\&=(\widetilde{X}-(int(cl(F_A))))\widetilde{\cup}(\widetilde{X}-(cl(int(F_A))))\\&\widetilde{\subset}(\widetilde{X}-int(cl^*(F_A)))\widetilde{\cup}(\widetilde{X}-(cl^*(int(F_A)))).\end{aligned}$$

Hence we obtain $cl^*(int(F_A))\widetilde{\cap}int(cl^*(F_A))\widetilde{\subset}F_A$. □

Remark 7. *For soft subset F_A of a soft ideal topological space (X,τ,A,I) we have $\widetilde{X}-int(cl^*(F_A))\neq cl^*(int(\widetilde{X}-F_A))$ as seen in the following example.*

Example 7. *Let $X=\{h_1,h_2\}$, $A=\{e_1,e_2,e_3\}$,*
$\tau=\left\{\widetilde{\emptyset},\widetilde{X},\{(e_1,\{h_1\}),(e_2,\{h_2\})\},\{(e_1,\{h_2\}),(e_2,\{h_1\})\}\right\}$ *and $I=S(X)$.*
For $F_A=\{(e_1,\{h_1\}),(e_2,\{h_1\}),(e_3,X)\}$ we have
$\widetilde{X}-int(cl^*(F_A))=\{(e_1,\{h_2\}),(e_2,\{h_1\}),(e_3,X)\}$
but $cl^*(int(\widetilde{X}-F_A))=\widetilde{\emptyset}$.

Corollary 1. *Let F_A be a soft subset of a soft ideal topological space (X,τ,A,I) such that $\widetilde{X}-int(cl^*(F_A))=cl^*(int(\widetilde{X}-F_A))$. Then F_A is soft $b-I$-closed set if and only if $cl^*(int(F_A))\widetilde{\cup}int(cl^*(F_A)\widetilde{\subset}F_A$.*

Corollary 2. *In a soft ideal topological space (X,τ,A,I) the following statements are hold:*

(a) If F_A soft $b-I$-closed set and G_A soft open set, then $F_A\widetilde{\cup}G_A$ soft $b-I$-closed set.

(b) If F_A soft $b-I$-closed set and G_A soft $\alpha-I$-closed set, then $F_A\widetilde{\cup}G_A$ soft $b-I$-closed set.

Proof. It is obvious from Theorem 2. □

4 Soft $b-I-$Continuous Functions

Definition 22. *Let $f:(X,\tau,E,I)\rightarrow(Y,\sigma,K)$ be a soft mapping. If $f^{-1}(G_B)$ is soft $b-I-$open set in (X,τ,E,I) for each soft open set G_B of (Y,σ,K), then f is called soft $b-I-$continuous function.*

Definition 23. *Let $f:(X,\tau,E,I)\rightarrow(Y,\sigma,K)$ be a soft mapping.If $f^{-1}(G_B)$ is soft $\beta-I-$open set in (X,τ,E) for each soft open set (G_B) of (Y,σ,K), then f is called soft $\beta-I-$continuous function.*

Definition 24. [15] *A soft mapping $f:(X,\tau,E,I)\rightarrow(Y,\sigma,K)$ is called soft semi–$I-$continuous if $f^{-1}(G_B)$ is soft semi–$I-$open set in (X,τ,E) for each soft open set G_B of (Y,σ,K).*

Definition 25. *Let $f:(X,\tau,E,I)\rightarrow(Y,\sigma,K)$ be a soft mapping. If $f^{-1}(G_B)$ is soft pre–$I-$open set in (X,τ,E) for each soft open set G_B of (Y,σ,K), then f is called soft pre–$I-$continuous function.*

We can write the following results from the above descriptions.

Corollary 3. *Let $f:(X,\tau,E,I)\rightarrow(Y,\sigma,K)$ be a soft function. Then:*
(a) If f is soft $b-I$-continuous, then f is soft $\beta-I-$continuous.
(b) If f is soft semi–I-continuous, then f is soft $b-I-$continuous.
(c) If f is soft pre–I-continuous, then f is soft $b-I-$continuous.

Not that the converses is not true in general. As the following examples shown.

Example 8. *Let $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$,*
$\tau=\left\{\widetilde{\emptyset},\widetilde{X},\{(e_1,\{h_1\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\right\}$, $I=\left\{\widetilde{\emptyset}\right\}$ *and*
$F_A=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$. *Moreover, let $Y=\{y_1,y_2\}$, $K=\{k_1,k_2\}$,*
$\sigma=\left\{\widetilde{\emptyset},\widetilde{Y},H_K\right\}$, *where $H_K=\{(k_1,\{y_1\}),(k_2,\{y_1\})\}$.*
Then $f:(X,\tau,A,I)\rightarrow(Y,\sigma,K)$ denoted by
$u(h_1)=y_1$, $u(h_2)=y_2$, $p(e_1)=k_1$, $p(e_2)=k_2$ *is soft $\beta-I-$continuous function but is not soft $b-I$-continuous function Because, for soft open set H_K in Y, $f^{-1}(H_K)=F_A$ is soft $\beta-I-$open set but is not soft $b-I$-open set.*

Example 9. *Let $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$,*
$\tau=\left\{\widetilde{\emptyset},\widetilde{X},\{(e_1,\{h_1\}),(e_2,\{h_2\})\},\{(e_1,\{h_2\}),(e_2,\{h_1\})\}\right\}$, $I=\left\{\widetilde{\emptyset}\right\}$
and $F_A=\{(e_1,\{h_1\})\}$. In addition to, let $Y=\{y_1,y_2\}$, $K=\{k_1,k_2\}$, $\sigma=\left\{\widetilde{\emptyset},\widetilde{Y},H_K\right\}$,
where $H_K=\{(k_1,\{y_1\})\}$ .Then $f:(X,\tau,A,I)\rightarrow(Y,\sigma,K)$
denoted by $u(h_1)=y_1$, $u(h_2)=y_2$, $p(e_1)=k_1$, $p(e_2)=k_2$ is soft $b-I-$continuous function
but is not soft semi–I-continuous function. Because, for soft open set H_K in Y, $f^{-1}(H_K)=F_A$ is soft $b-I-$open set but is not soft semi–I-open set.

Example 10. *Let F_A be a soft set of a soft ideal topological space (X,τ,E,I) as in Example 2.*
Moreover, let $Y=\{y_1,y_2,y_3,y_4\}$, $K=\{k_1,k_2,k_3\}$, $\sigma=\left\{\widetilde{\emptyset},\widetilde{Y},H_K\right\}$, where
$H_K=\{(k_1,\{y_2,y_4\}),(k_2,\{y_1,y_3\}),(k_3,\{y_1,y_3,y_4\})\}$.

Then $f:(X,\tau,A,I)\rightarrow(Y,\sigma,K)$ *denoted by* $u(h_i)=y_i$, $p(e_j)=k_j$ *(for* $1\leq i\leq 3$, $1\leq j\leq 4$.*) is soft* $b-I-$*continuous function but is not soft semi−I-continuous function.*

Because, for soft open set H_K *in* Y, $f^{-1}(H_K)=F_A$ *is soft* $b-I-$*open set but is not soft semi−I-open set.*

Definition 26. [12] *Let* $f:(X,\tau,E,I)\rightarrow(Y,\sigma,K)$ *be a soft function. Then* f *is said to be soft b-continuous if* $f^{-1}(G_B)$ *is soft b-open set in* (X,τ,E) *for each soft open set* G_B *of* (Y,σ,K).

Remark 8. *It is clear that soft* $b-I$*-continuity implies soft b-continuity. But the converse is not true in general as shown by the following example:*

Example 11. *Let* $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$,

$\tau=\left\{\widetilde{\emptyset},\widetilde{X},\{(e_1,\{h_1\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\right\}$, $I=S(X)$ *and*

$F_A=\{(e_1,X),(e_2,\{h_1\})\}$. *Moreover, let* $Y=\{y_1,y_2\}$, $K=\{k_1,k_2\}$, $\sigma=\left\{\widetilde{\emptyset},\widetilde{Y},H_K\right\}$,

where $H_K=\{(k_1,Y),(k_2,\{y_1\})\}$ *.Then* $f:(X,\tau,A,I)\rightarrow(Y,\sigma,K)$ *denoted by* $u(h_1)=y_1$, $u(h_2)=y_2$, $p(e_1)=k_1$, $p(e_2)=k_2$ *is soft* $b-I-$*continuous function*

but is not soft b-continuous function. Because, for soft open set H_K *in* Y, $f^{-1}(H_K)=F_A$

is soft $b-I-$*open set but not soft* $b-$*open set.*

Theorem 4. *Let* $f:(X,\tau,A,I)\longrightarrow(Y,\sigma,K)$ *be a soft function.* f *is soft* $b-I$*-continuous function if and only if then for each soft point* F_A^x *in* X *and each soft open set* V_K *in* Y *containing* $f(F_A^x)$ *there exists soft* $b-I$ *open set* G_A *containing* F_A^x *such that* $f(G_A)\widetilde{\subset}V_K$.

Proof. $\Rightarrow$: Let F_A^x be a soft point in X and V_K be soft open set in Y containing $f(F_A^x)$. Set $G_A=f^{-1}(V_K)$, then since f is soft $b-I$-continuous function, then G_A is soft $b-I$-open set containing F_A^x and $f(G_A)\widetilde{\subset}V_K$.

$\Leftarrow$: Let V_K be any soft open set in Y containing $f(F_A^x)$. Then by hypothesis there exists G_A soft $b-I$-open set such that $f(G_A)\widetilde{\subset}V_K$ and hence $G_A\widetilde{\subset}f^{-1}(V_K)$. Let $G_A=f^{-1}(V_K)$. Therefore $f^{-1}(V_K)$ is soft $b-I$-open set. This shows that f is soft $b-I$-continuous function. □

Theorem 5. *Let* $f:(X,\tau,E,I)\rightarrow(Y,\sigma,K)$ *be a soft function. If* f *is soft* $b-I$*-continuous function, then for each* $F_A^x\in X$ *the graph function* $g:X\rightarrow X\times Y$, *defined by* $g(F_A^x)=(F_A^x,f(F_A^x))$ *is soft* $b-I$*-continuous function.*

Proof. $\Rightarrow$: Let f is soft $b-I$-continuous function and $F_A^x\in X$ and $W_{A\times B}$ be any open set of $X\times\ Y$ containing $g(F_A^x)$. Then there exists a funtametal open set $U_A\times V_B$ such that $g(F_A^x)=(F_A^x,f(F_A^x))\widetilde{\in}U_A\times V_B\widetilde{\subset}W_{A\times B}$. In the cause of f is soft $b-I$-continuous function, there exists a soft $b-I$-open set U_{A_0} of X containing F_A^x such that $f(U_{A_0})\subset V_B$. By Theorem 2, $U_{A_0}\cap U_A$ is soft $b-I-$open set in (X,τ) and $g(U_{A_0}\widetilde{\cap}U_A)\widetilde{\subset}U_A\times V_B\widetilde{\subset}W_{A\times B}$. Hence g is soft $b-I$-continuous function.

$\Leftarrow$: Let g is soft $b-I$-continuous function and $F_A^x\in X$ and G_B be any soft open set of Y containing $f(F_A^x)$. Then $\widetilde{X}\times V_B$ is soft open in $X\times Y$ and since g is soft $b-I$-continuity, we have a soft $b-I$-open set U_A in (X,τ) containing F_A^x such that $g(U_A)\subset X\times V_B$. Therefore, we obtain $f(U_A)\widetilde{\subset}V_B$. Hence f is soft $b-I$-continuous function. □

Definition 27. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K)$ be a soft function. If $f^{-1}(G_B)$ is soft $b - I$-open set for every soft b-open set (G_B) of (Y, σ, K), then f is said to be soft $b - I$-irresolute function.*

Theorem 6. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K)$ be a soft funciton. If $f : (X, \tau, E, I) \to (Y, \sigma, K)$ is soft $b-I$-irresolute, then for each soft point F_A^x in X and each soft b-open set V_K in Y containing F_A^x, there exists a soft $b - I$-open set U_A containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$*

Proof. Let $F_A^x \widetilde{\in} X$ and V_K be any soft b-open set in Y containing $f(F_A^x)$.
By supposition, $f^{-1}(V_K)$ is soft $b - I$-open set in X.
Set $U_A = f^{-1}(V_K)$, then U_A is a soft $b - I$-open set in X containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$. □

Theorem 7. *If is $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$ for every soft b-open set V_K in Y, then $f^{-1}(H_K)$ is soft $b - I$-closed set in X for every soft b-closed set H_K in Y.*

Proof. Let H_K be any soft b-closed subset of Y and $V_K = \widetilde{Y} - H_K$.
Then V_K is soft b-open set in Y.
By hypothesis, $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$.
Therefore $f^{-1}(H_K) = \widetilde{X} - f^{-1}(V_K)$ is soft $b - I$-closed set in X. □

Theorem 8. *If f is soft $b - I$-irresolute, then $f^{-1}(H_K)$ is soft $b - I$-closed set in X for every soft b-closed set H_K in Y.*

Proof. Let V_K be any soft b-open set in Y and $H_K = \widetilde{Y} - V_K$.
Then by hypothesis, $f^{-1}(H_K) = \widetilde{X} - f^{-1}(V_K)$ is soft $b - I$-closed in X.
This shows that $f^{-1}(V_K)$ is soft $b - I$-open set in X and f is soft $b - I$-irresolute function. □

Theorem 9. *For each soft point F_A^x in X and each soft b-open set V_K in Y containing F_A^x, if there exists a soft $b - I$-open set U_A containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$, then for every soft b-open set V_K in Y, $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$.*

Proof. Let V_K be any soft b-open set in Y and $F_A^x \widetilde{\in} f^{-1}(V_K)$.
By hypothesis, there exists a soft $b - I$-open set U_A of X containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$.
Thus we attain $F_A^x \widetilde{\in} U_A \widetilde{\subset} cl^*(int(U_A)) \widetilde{\cup} int(cl^*(U_A))$
$\widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$ and hence
$F_A^x \widetilde{\in} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$.
Hence $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$. □

Theorem 10. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K, J)$ and $g : (Y, \sigma, K, J) \to (Z, \eta, M)$ be two soft functions, where I and J are ideals on X and Y respectively. Then the following are hold:*

(a) if f is soft $b - I$-continuous and g is soft continuous then gof is soft continuous,

(b) if f^{-1} is soft $b - I$-irresolute and g is soft b-continuous then gof is soft $b - I$-continuous

Proof. (a) Let H_C be a soft open set of (Z, η, M). Since g is soft continuous then $g^{-1}(H_C)$ is soft open in (Y, σ, K, J). Since f is soft $b-I$-continuous then $f^{-1}(g^{-1}(H_C)) = (gof)^{-1}(H_C)$ is soft $b-I$-open set in (X, τ, E, I). Therefore we obtain gof is soft $b-I$-continuous.

(b) Let H_C be a soft open set of (Z, η, M). Since g is soft b-continuous then $g^{-1}(H_C)$ is soft b-open set in (Y, σ, K, J). Since f^{-1} is soft $b-I$-irresolute then $f^{-1}(g^{-1}(H_C)) = (gof)^{-1}(H_C)$ is soft $b-I$-open set in (X, τ, E, I).Therefore we obtain gof is soft $b-I$-continuous. □

Lemma 3. [13] *If (X, τ, E, I) is an soft ideal topological space and F_A is soft subset of X, we denote by $\tau|_{F_A}$ the soft relative topology on F_A and $I|_{F_A} = \{F_A \cap I | I \in I\}$ is obviously an ideal on F_A.*

Lemma 4. *Let (X, τ, E, I) be a soft ideal topological space and V_A, F_A subsets of X such that $V_A \subset F_A$. Then $B^*(\tau|_{F_A}, E, I|_{F_A}) = B^*(\tau, E, I) \cap F_A$.*

Proof. Obvious. □

Theorem 11. *In a soft ideal topological space (X, τ, A, I) if U_A is soft open and F_A is soft $b-I-$open set, then $U_A \widetilde{\cap} F_A$ is soft $b-I-$open in $(U_A, \tau|_{U_A}, I|_{U_A})$*

Proof. We have $int_{U_A} V_A = int(V_A) \widetilde{\cap} U_A$ for any soft subset V_A of U_A, since U_A is soft open. Hence, by using this real and Lemma 5, proof is completed. □

Theorem 12. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K)$ be soft $b-I$-continuous function and U_A soft open set in X. Then the restriction $f|_{U_A} : (U_A, \tau|_{U_A}, E, I|_{U_A}) \to (Y, \sigma, K)$ is soft $b-I$-continuous.*

Proof. Let G_B be any soft open set of (Y, σ, K). Since f is soft $b-I$-continuous,
then $f^{-1}(G_B)$ is soft $b-I$-open set in X. For U_A soft open set,
by Theorem 8 $U_A \cap f^{-1}(G_B)$ is soft $b-I-$open set in $(U_A, \tau, E, I|_{U_A})$.
On the other hand, $(f|_{U_A})^{-1}(G_B) = U_A \cap f^{-1}(G_B)$ and
$(f|_{U_A})^{-1}(G_B)$ is soft $b-I$-open set in $(U_A, \tau|_{U_A}, E, I|_{U_A})$.
This shows that $f|_{U_A} : (U_A, \tau|_{U_A}, E, I|_{U_A}) \to (Y, \sigma, K)$ is soft $b-I$-continuous. □

5 Soft $b-I$-Open Functions and Soft $b-I$-Closed Functions

Definition 28. *A function $f : (X, \tau, E) \to (Y, \sigma, K, J)$ is said to be soft $b-I$-open (resp. soft $b-I$-closed) if the image of each soft open (resp. soft closed) set of X is soft $b-I$-open (resp. soft $b-I$-closed) set in (Y, σ, K, J).*

Definition 29. [12] *A function $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be soft b-open (resp. soft b-closed) if the image of each soft open (resp. soft closed) set of X is soft b-open (resp. soft b-closed) set in (Y, σ, K).*

We can give the following warning from the above two definitions

Remark 9. *(a) Every soft open function is soft $b-I$-open function.*
(b) Every soft $b-I$-open function is soft b-open function.

In the following examples as observed the converses are not true.

Example 12. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,

$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, F_A\right\}$,*where* $F_A = \{(e_1, \{h_2\})\}$. *Also, let* $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$,

$\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}, \{(k_1, \{y_2\}), (k_2, \{y_1\})\}\right\}$ *and* $J = \left\{\widetilde{\emptyset}\right\}$

Then the soft function $f : (X, \tau, A, I) \to (Y, \sigma, K)$ *denoted by*
$u(h_1) = y_1$, $u(h_2) = y_2$, $p(e_1) = k_1$, $p(e_2) = k_2$ *is soft* $b - I-$*open set but is not soft open set. Because, for soft open set* F_A *in* X,
$f(F_A) = H_K$ *is soft* $b - I-$*open set but is not soft -open set.*

Example 13. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, F_A\right\}$,
where $F_A = \{(e_1, X), (e_2, \{h_1\})\}$. *Also, let* $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$,
$\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, \{(k_1, \{y_1\})\}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\right\}$ *and* $J = S(Y)$

Then the soft function $f : (X, \tau, A, I) \to (Y, \sigma, K)$ *denoted by*
$u(h_1) = y_1$, $u(h_2) = y_2$, $p(e_1) = k_1$, $p(e_2) = k_2$ *is soft* $b-$*open but is not soft* $b - I$*-open set. Because, for soft open set* F_A *in* X,
$f(F_A) = H_K$ *is soft* $b-$*open set but is not soft* $b - I$*-open set.*

Theorem 13. *A function* $f : (X, \tau, E) \longrightarrow (Y, \sigma, J, K)$ *is a soft* $b - I$*-open if and only if for each* $F_A^x \widetilde{\in} X$ *and each soft open set* U_A *containing* F_A^x*, there exists a soft* $b - I$*-open set* W_K *containing* $f(F_A^x)$ *such that* $W_K \widetilde{\subset} f(U_A)$.

Proof. $\Rightarrow$:Let's face it $F_A^x \widetilde{\in} X$ and U_A be any soft open set containing F_A^x. Since f is soft $b - I$-open function, $f(U_A)$ is soft $b - I$-open set in Y. Set $W_K = f(U_A)$, then $f(F_A^x) \widetilde{\in} W_K$ and W_K is soft $b - I$-open set such that $W_K \widetilde{\subset} f(U_A)$.
$\Leftarrow$: Obvious. □

Theorem 14. *Let* $f : (X, \tau, E) \longrightarrow (Y, \sigma, J, K)$ *be a soft* $b - I$*-open function. If* W_K *is soft set in* Y *and* U_A *is soft closed set in* X *containing* $f^{-1}(W_K)$*, then there exists a soft* $b - I$*-closed set* H_K *in* Y *containing* W_K *such that* $f^{-1}(H_K) \widetilde{\subset} U_A$.

Proof. Let U_A be a soft closed set in X. Since $G_A = \widetilde{X} - U_A$ is soft open set in X. Since f is soft $b - I$-open function, $f(G_A)$ is soft $b - I$-open set in Y. Therefore $H_K = \widetilde{Y} - f(G_A)$ is soft $b - I$-closed set in Y and $f^{-1}(H_K) = f^{-1}(\widetilde{Y} - f(G_A)) = \widetilde{X} - f^{-1}(f(G_A)) \widetilde{\subset} \widetilde{X} - G_A = U_A$. □

Theorem 15. *The following phrases are equivalent for any bijective soft function* $f : (X, \tau, E, I) \longrightarrow (Y, \sigma, K, J)$*;*

(a) $f^{-1} : (Y, \sigma, J) \longrightarrow (X, \tau, I)$ *is soft* $b - I$*-continuous function,*
(b) f *is soft* $b - I$*-open function,*
(c) f *is soft* $b - I$*-closed function.*

Proof. Obvious. □

Theorem 16. *Let* $f : (X, \tau, E, I) \longrightarrow (Y, \sigma, K, J)$ *and* $g : (Y, \sigma, K, J) \longrightarrow (Z, \eta, L, K)$ *be two soft functions. The followings hold:*

(a) gof *is soft* $b-I$*-open function if* f *is soft open function and* g *is soft* $b-I$*-open function.*

(b) f *is soft* $b - I$*-open function if* gof *is soft open function and* g *is soft* $b - I$*-continuous function.*

Proof. This is obvious. □

6 Conclusion

Our purpose in this paper is to define upper and lower soft $b-I-$continuous functions and study their various properties. Moreover, we obtain some characterizations and several properties concerning such functions. We expect that results in this paper will be basis for further applications of soft mappings in soft sets theory and corresponding information systems.

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