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A STUDY ON PRE-m_X CONTINUOUS FUNCTION

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Abstract – The aim of this paper is to introduce the concept of pre m_X continuous function and to show some of its application. Also the concept of pre m_X open mapping and pre m_X homeomorphism is studied. The concept of pre m_X open set has already been introduced by the authors in 2011.In this paper a topology is considered which is generated from m_X structure and it is denoted as T_{m_X}. The concept of pre m_X continuous function is discussed in the topological space (X, T_{m_X}) generated from (X, m_X).

***Keywords* –** Pre m_X continuous function, Pre m_X open mapping, Topology generated by m_X structure.

1. Introduction and Preliminaries

The concept of m_X-open set has been introduced by H. Maki in 1996.[8] and the concept of preopen set has been introduced by Mashour et al [9] .Lots of applications of preopen set and m_X structure in ordinary topological space has been introduced by various researchers.[1][2][3]. The concept of m_X pre-open set has been introduced by Ennis Rosas, Neelamegarajan Rajesh, Carlos Carpintero[17]. And the concept of Pre m_X open set has been introduced by the authors in 2011[4]. In this paper the concept of Pre m_X continuous function, Pre m_X irresolute continuous function, Pre m_X open mapping, Introduction. Pre m_X irresolute mapping, Pre m_X homeomorphism etc are introduced and some properties are discussed.

In the second section the concept of pre m_X-continuous function, pre m_X irresolute continuous function is discussed.

In the third section, the concept of pre m_X open mapping etc is introduced and their connection are shown. Lastly the concept of pre m_X homeomorphism is introduced and some of its utility is studied.

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Let us rememorize some of the basic concepts used by various researchers.

Defintion 1.1. [8] A structure is said to be a m_X structure iff $\phi \in m_X$, $X \in m_X$. From this structure the following operators may be defined as below:

For any subset A of X

$$m_X \text{ IntA} = \cup\{G: G \subseteq A \text{ , G is a } m_X \text{ open set in X}\}$$
$$m_X \text{ ClA} = \cap\{G: G \supseteq A \text{ , G is a } m_X \text{ closed set in X}\}$$

The subset A of X is said to be a

1.[8] open m_{X-} set in a m_X structure if m_X intA=A

2. [9] Preopen set in ordinary topological space if $A \subset int(cl(A))$

3. [14] m_X-regular open set in m_X structure if A= m_{X-}int m_{X-}clA.

4. [8] m_X-generalized closed set in m_X structure if there exist a m_X-open set containing A such that $m_X ClA \subset U$ whenever $A \subset U$.

5. [17] m_X--preopen set in X if $A \subseteq m_X Int(m_X Cl(A))$

6. [4] Pre-m_X open set on an m_X structure if $A \subseteq Int(m_{X-}Cl(A)))$.

From the above definitions a connection between the sets are shown in the following figüre

m_X- dense
↓
m_X- open → m_X-pre open → pre-m_X open → b-$m_X$open
↑
regular m_X-open

Definition 1.2. A mapping f : X→Y is said to be a

1. [9] pre continuous function in an ordinary topological space if $f^{-1}(A) \subset PO(X)$ for every open set A in Y.

2. [14] m_X-regular continuous function in a m_X structure if $f^{-1}(A)$ is a m_X regular open set in X for every m_X-regular open set A in Y.

3. [13] m_X-generalized continuous function in a m_X structure if $f^{-1}(A)$ is a m_X closed set in X for every m_X-closed set A in Y.

4. [8] m_X-continuous function in a m_X structure if $f^{-1}(A)$ is a m_X open set in X whenever A is an m_X open set in Y.

5. [9] Preopen mapping in an ordinary topological space if the image of each open set in X is a preopen set in Y.

6. [8] m_X-open mapping in a m_X structure if image of each m_X -open set in X is a m_X -open set in Y.

7. [14] m_X- regular-open mapping in a m_X structure if the image of each m_X -open set in X is a m_X – regular open set in Y.

8.[9] pre irresolute continous function in an ordinary topological space if $f^{-1}(U)\subset PO(X)$ for every $U\subset PO(Y)$,

9. [17] m_X pre irresolute continuous function in a m_X structure if the inverse image of every m_X pre open set in Y is a m_X pre open set in X.

Definition 1.3 [9] A bijective mapping $f:(X,\tau) \to (Y,\sigma)$ from X to Y is called a pre homeomorphism if both f and f^{-1} are pre irresolute mappings.

Throughout this paper we are considering the topological space as the structure formed by introducing the missing elements in m_X structure i.e. along with the elements of m_X structure we are also introducing the elements which are essentially needed for a topological space .Let us name this type of topological space as a topological space generated by an m_X structure and denote it as T_{m_X} .

Let X = {a,b,c} and the corresponding m_X structure be {ϕ, X, {a,b},{b,c}}. It is not a topology since finite intersection of the elements in m_X is not in m_X. Now T_{m_X} ={ϕ, X, {a,b},{b,c}, {b}}.This is a topology generated by an m_X structure.

For a topology generated by m_X structure let us denote the interior as Int_{Tm_X} and the closure as Cl_{Tm_X} .Now since $m_X \subseteq T_{m_X}$, $m_X\ Int \le Int_{Tm_X} \le Cl_{Tm_X} \le m_X\ Cl$.

2. Pre m_X Continuous Function

In this section the concept of pre m_X continuous function, pre m_X irresolute continuous mapping, pre m_X open mapping, pre m_X homeomorphism are introduced and their properties are studied.

Definition 2.1.A function $f:(X, T_{m_X}) \to (Y, T_{m_Y})$ is said to be a pre m_X-continuous function if the inverse image of each m_X-open set in Y is a pre m_X -open set in X.

Example 2.2. Let X = {a, b, c, d} and the m_X structure be m_X = {ϕ, X, {a,b}, {c}}, Tm_X={ϕ, X, {a,b}, {c}, {a,b,c}}.

Let Y = {x,y,z, t} then m_X structure is $m_X(y)$={ϕ, Y, {x}, {y}} and Tm_X={ϕ,Y,{x},{y},{x,y}}

Let us consider a mapping f : $(X,T_{m_X}) \to (Y,T_{m_Y})$ such that f(a) = x, f(b)=y, f(c)=z, f(d)=t. Now the inverse image of each m_X open set in Y are respectively ϕ, X, {a}, {b}. Now a subset A of X is said to be a Pre-m_X open set on an m_X structure if $A \subseteq Int_{Tm_X}(m_X\text{-}Cl(A)))$. Here ϕ, X, {a},{b} are all pre m_X open set. Hence f is a pre m_X continuous function.

Theorem 2.3. Let f :$(X,T_{m_X}) \to (Y,T_{m_Y})$ be a mapping from X to Y. Every m_X continuous function f is also a pre m_X –continuous function.

Proof: Let $x \in X$ and V be any m_X open set containing f(x).Since f is a m_X – continuous function there exist $U \in m_X(X)$ containing x such that $f^{-1}(V)$ is m_X- open in X. By the figure indicating the connection of the set ,it is shown that every m_X open set is a pre m_X open set, thus $f^{-1}(V)$ is a pre m_X –open set. Hence the proof.

Remark 2.4. The converse of the theorem is not true, which follows from the example 2.2. Here the function is a pre m_X continuous function but not a m_X continuous function since the inverse image of {x}, {y} are respectively {a}, {b} which are not a m_X open set in X.

Theorem 2.5. Let f :$(X,T_{m_X}) \to (Y,T_{m_Y})$ be a mapping from X to Y. Every m_X -preirresolute continuity is pre m_X-continuous.

Proof: Let V be a m_X-open set in Y. Since every m_X open set in Y is also a m_X pre open set in Y thus V is a m_X pre open set in Y and f being m_X pre irresolute continuous function from definition 1.1(9), $f^{-1}(V)$ is a m_X- preopen set in X i.e. inverse image of a m_X open set in Y is a m_X–preopen set in X. Again since m_X–preopen set is a pre m_X–open set in X. Hence f is a pre m_X-continuous

Remark 2.6. The converse of the theorem is not true which follows from the following example: Let

X={a,b,c,d},
m_X= {ϕ, X, {a},{b},{a,c},{b,c}},
Tm_X ={ϕ,X,{a},{b},{c},{a,b,c}},
Y= {m,n,l} and m_Y = {ϕ,Y,{m}, {l},{n,l},{m,n}},
Tm_Y={ϕ,Y,{m}, {l}, {n}, {m,l}, {n,l}, {m,n}}.

Let f: X→Y be a mapping defined by f(a)= m, f(b)=l, f(c) = f(d)= n. Then clearly f is pre m_X- continuous but it is not a m_X-preirresolute continuity. Since

$f^{-1}(\{m,n\}) = \{a,d\} \not\subset m_X\text{-}PO(X)$.

Theorem 2.7. Let f :$(X, T_{m_X}) \to (Y, T_{m_Y})$. Every m_X - regular continuity is pre m_X-continuity.

Proof: Let $x \in X$ and V be any m_X open set of Y containing f(x). Since f is m_X – regular continuous there exist $U \in m_X$ containing x such that $f^{-1}(V)$ is m_X- regular open in X. By figure indicating connections between various set**,** $f^{-1}(V)$ is pre m_X- open in X. Hence the proof.

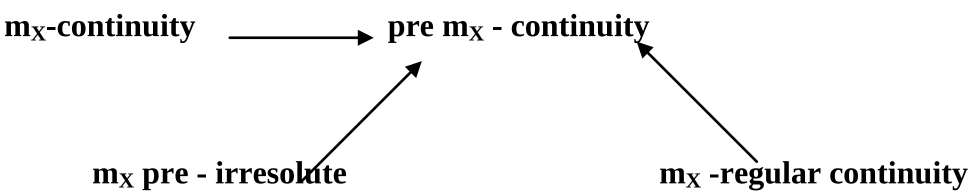
Remark 2.8. The converse of the theorem is not true, which follows from the following example : Let

$X=\{a,b,c,d\}$,
$m_X=\{\phi,X,\{d\},\{b\},\{c\},\{a,b\},\{a,c\}\}$,
$Tm_X=\{\phi,X,\{d\},\{b\},\{c\},\{a\}, \{a,b\}, \{a,c\} \{b,d\}, \{d,c\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$and
$Y=\{m,n,l\}$and $m_Y=\{\phi,Y,\{l\},\{m,n\},\{n,l\}\}$ and $Tm_Y =\{\phi,Y,\{l\},\{n\},\{m,n\},\{n,l\}\}$.

Let $f:(X, Tm_X)\rightarrow(Y,Tm_Y)$ be a function defined by f(a) = m, f(b)=l, f(c) = f(d)= n. Then clearly f is pre m_X-continuous but it is not a m_X - regular continuous. Since

$f^{-1}(\{m,n\}) = \{a,d\}\not\subset Tm_X$

We denote the relation discussed above by a figure below.



Definition 2.9. Let (X, Tm_X) be a space with a m_X –structure. For $A\subseteq X$, the pre-m_X-closure and the pre- m_X-interior of A, denoted by $Pm_XCl(A)$ and $Pm_XInt(A)$ respectively are defined as the following:

$Pm_XCl(A)=\cap\{F\subseteq X:A\subseteq F$, F is Pre m_X-closed in X$\}$and
$Pm_XInt(A)=\cup\{U\subseteq X:U\subseteq A$,U is Pre-$m_X$ open in X$\}$.

Theorem 2.10.

(1) A is a pre-m_X-open set iff $Pm_XInt(A)= A$
(2) A is a pre-m_X-closed set iff $Pm_XCl(A)= A$

Proof : (1) Let if possible A be a pre-m_X-open set then obviously $Pm_XInt(A)= A$
Conversely let $Pm_XInt(A)= A$, then

$Pm_XInt(A)= A= \cup\{U\subseteq X:U\subseteq A$,U is Pre-$m_X$ open in X$\}$.

Since arbitrary union of pre-m_X-open set is a pre-m_X-open set[From theorem 3.3 of [17], and A being the arbitrary union of pre-m_X-open set, A is a pre-m_X-open set.This proves the theorem.

(2) can be proved similarly.

Lemma 2.11. For any subset A, B of X the following properties hold.

(i) $Pm_XInt(\phi) = \phi$, $Pm_XInt(X) = X$, $Pm_XCl(\phi) = \phi$, $Pm_XCl(X) = X$
(ii) $Pm_XInt\, Pm_XInt(A) = Pm_XInt(A)$, $Pm_XClPm_XCl(A) = Pm_XCl(A)$
(iii) $Pm_XInt(A) \subseteq A \subseteq Pm_XCl(A)$
(iv) $Pm_XInt(A) \subseteq Pm_XInt(B)$, $Pm_XCl(A) \subseteq Pm_XCl(B)$ whenever $A \subseteq B$
(v) $Pm_XInt(\cup A_i: i \in I) \supseteq \cup \{Pm_XInt(A_i): i \in I\}$,
$Pm_XCl(\cap A_i: i \in I) \subseteq \cap \{Pm_XCl(A_i): i \in I\}$
(vi) $Pm_XCl(\cup A_i: i \in I) \supseteq \cup \{Pm_XCl(A_i): i \in I\}$,
$Pm_XInt(\cap A_i: i \in I) \subseteq \cap \{Pm_XInt(A_i): i \in I\}$
(vii) $Pm_XInt(X-A)= X- Pm_XCl(A)$.

Proof : (i), (iii), (iv), (v), (vi) and (vii) are obvious.

To prove (ii)

From (iii) , $Pm_XInt(A) \subseteq A$ and from (iv), $Pm_XIntPm_XInt(A) \subseteq Pm_XInt(A)$

Now we have to prove that

$$Pm_XIntPm_XInt(A) \supseteq Pm_XInt(A)$$

From definition it follows that,

$$Pm_XInt(A)=\cup\{U\subseteq X:U\subseteq A,U \text{ is Pre-}m_X \text{ open in } X\} \supseteq U$$

So $Pm_XInt\, Pm_XInt(A) \supseteq Pm_XInt\,(U) = U$, U is a Pre-$m_X$ open set in X

Thus $Pm_XInt\, Pm_XInt(A) \supseteq \cup\{U\subseteq X:U\subseteq A,U \text{ is Pre-}m_X \text{ open in } X\}= Pm_XInt(A)$

Thus $Pm_XInt\, Pm_XInt(A) = Pm_XInt\,(A)$

Remark 2.12: From Lemma 2.11(ii) and theorem 2.10, it is obvious that $Pm_XInt(A)$ is a Pre m_X open set and $Pm_XCl(A)$ is a Pre m_X Closed set

Theorem 2.13: Let $f:(X,Tm_X)\to(Y,Tm_Y)$ be a function from X to Y . Then the followings are equivalent.

i) f is a pre m_X-continuous function.
ii) for each m_X open set V in Y,$f^{-1}(V)$ is pre m_X open.
iii) for each m_X closed set B in Y,$f^{-1}(B)$ is pre m_X closed.
iv) $f(\, p\, m_X\, Cl(A)) \subseteq m_XCl(f(A))$ for $A\subseteq X$.
v) $p\, m_X\, Cl(f^{-1}(B))\subseteq f^{-1}(m_XCl(B))$ for $B\subseteq Y$.
vi) $f^{-1}(m_XInt(B))\subseteq p\, m_X\, Int(f^{-1}(B))$ for $B\subseteq Y$.

Proof: (i) $\Leftrightarrow$ (ii). Obvious.

(ii) $\Rightarrow$ (iii). Obvious.

(iii) ⇒ (iv). For $A \subseteq X$.

$f^{-1}(m_XCl(f(A)))=f^{-1}(\cap\{F\subseteq Y:f(A)\subseteq F$ and F is m_X closed in Y$\})$
$\supseteq \cap\{f^{-1}(F)\subseteq X: A \subseteq f^{-1}(F)$ and $f^{-1}(F)$ is pre m_X closed in X$\}$
[since every m_X closed in X is a pre m_X closed set in X, so arbitrary intersection of m_X closed set in X containing f(A) is a superset of intersection of Pre m_X closed set in X containing f(A). And f being pre m_X-continuous function, $f^{-1}(F)$ is pre m_X closed in X whenever F is a m_X closed in Y]
$= p\ m_X\ Cl(A)$

implies $f^{-1}(m_XCl(f(A))) \supseteq p\ m_X\ Cl(A)$

i.e. $f(f^{-1}(m_XCl(f(A)))) \supseteq f(p\ m_X\ Cl(A))$

i.e. $m_XCl(f(A)) \supseteq f(f^{-1}(m_XCl(f(A)))) \supseteq f(p\ m_X\ Cl(A))$

i.e. $m_XCl(f(A)) \supseteq f(p\ m_X\ Cl(A))$

(iv) ⇒ (v). Let $A=f^{-1}(B)$ then $f(A) =ff^{-1}(B) \subseteq B$. From (iv)

$f(p\ m_X\ Cl(A)) = f(p\ m_X\ Cl(f^{-1}(B))) \subseteq m_XCl(f(A)) \subseteq m_XCl((B))$
$\Rightarrow f^{-1}f (p\ m_X\ Cl(f^{-1}(B))) \subseteq f^{-1}m_XCl((B))$
$\Rightarrow pm_XCl(f^{-1}(B))) \subseteq f^{-1}f (p\ m_X\ Cl(f^{-1}(B))) \subseteq f^{-1}m_XCl((B))$.

(v) ⇒ (vi). from (v) $X - Pm_X\ Cl(f^{-1}(B)))\supseteq X - f^{-1} (Cl((B))\Rightarrow Pm_XInt(f^{-1}(B))\supseteq f^{-1}(Int(B))$.

(vi) ⇒ (i). For $x\in X$ and for each m_X open set V containing f(x),from (vi),it follows

$x\in f^{-1}(V)=f^{-1}(m_X\ Int(V)) \subseteq pm_XInt(f^{-1}(V))$

From lemma 2.11(iii), $pm_XInt(f^{-1}(V)) \subseteq f^{-1}(V)$. So $pm_XInt(f^{-1}(V)) = f^{-1}(V)$. Thus $f^{-1}(V)$ is a m_X open set in X. This implies that f is a pre m_X continuous function.

Theorem 2.14. Let $f : (X, Tm_X)\to(Y, Tm_Y)$ be a pre m_X-continuous function. Then the following statements holds:

(i) $f^{-1}(V)\subseteq Pm_XInt(m_XCl(f^{-1}(V)))$ for each m_X-open set V in Y.
(ii) $Pm_XCl(m_XInt(f^{-1}(G))) \subseteq f^{-1}(G)$ for each m_X-closed set G in Y.
(iii) $f(Pm_XCl(m_XInt(A)))\subseteq m_XCl(f(A))$ for $A\subseteq X$.
(iv) $Pm_XCl(m_XInt(f^{-1}(B)))\subseteq f^{-1}(m_XCl(B))$ for $B\subseteq Y$.
(v) $f^{-1}(m_XInt(C))\subseteq Pm_XInt(m_XCl(f^{-1}(C)))$ for $C\subseteq Y$.

Proof: To Prove (i) Let V be a m_X open set in Y. Since f is a pre m_X-continuous function, $f^{-1}(V)$ is pre m_X-open in X. Therefore $f^{-1}(V) =Pm_XInt(f^{-1}(V) \subseteq Pm_XInt(m_XCl(f^{-1} (V)))$.

(i) ⇒ (ii). Let $G =Y –V$ be a m_X-closed set in Y.From (ii)

$X – f^{-1}(V) \supseteq X - Pm_XInt(m_XCl(f^{-1} (V)))$

$\Rightarrow f^{-1}(G) \supseteq Pm_XCl(m_XInt(X - f^{-1}(V)))$
$\Rightarrow f^{-1}(G) \supseteq Pm_XCl(m_XInt(f^{-1}(G)))$.

(ii) ⇒ (iii). Let $A= f^{-1}(G)$ then from (iii)

$$Pm_XCl(m_XInt(A)) \subseteq A \Rightarrow f(Pm_XCl(m_XInt(A))) \subseteq f(A) \subseteq m_XCl(f(A)).$$

(iii) ⇒ (iv). Let $f(A)=B \Rightarrow A \subseteq f^{-1}(B)$ then from (iv)

$$f(Pm_XCl(m_XInt(A))) \subseteq f(Pm_XCl(m_XInt(f^{-1}(B)))) \subseteq m_XCl(B)$$
$$\Rightarrow Pm_XCl(m_XInt(f^{-1}(B))) \subseteq f^{-1}f(Pm_XCl(m_XInt(A))) \subseteq f^{-1}(m_XCl(B)).$$

(iv) ⇒ (v). it is obvious.

Definition 2.15.A function $f: (X, Tm_X) \to (Y, Tm_Y)$ is said to be a pre m_X irresolute continuous function iff the inverse image of each pre-m_X-open set in Y is a pre m_X open set in X.

Theorem 2.16. Consider a function $f: (X, Tm_X) \to (Y, Tm_Y)$ .Every pre m_X -irresolute continuous function is a pre m_X –continuous function.

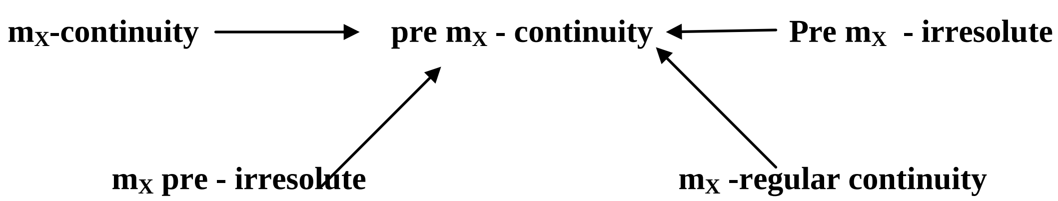
Proof: Let $x \in X$ and V be any m_X open in Y. Then we have V is a pre m_X-open in Y containing f(x). Since f is pre m_X irresolute map then $f^{-1}(V)$ is pre m_X -open in X. Hence the theorem.

Remark 2.17. The converse of the theorem is not true, which follows from the following example: Let

$X=\{a,b,c,d\}$,
$m_X=\{\phi,X,\{a,b\},\{b,c\}, \{a,c,d\}\}$,
$Tm_X=\{\phi,X, \{a,b\}, \{b,c\}, \{a,b,c\}, \{a,c,d\},\{b\}\}$,
$Y=\{x,y,z,t\}$
$m_Y= \{\phi,Y,\{x,y\},\{y,z\}\}$
$Tm_Y=\{\phi,Y, \{x,y\},\{y,z\},\{x,y,z\},\{y\}\}$.

Let $f: X \to Y$ be a mapping defined by f(a)=x, f(b)=y, f(c)=z, f(d)=t. Then clearly f is pre m_X - continuous, but it is not a pre m_X -irresolute map. since $f^{-1}(\{y\}) = \{b\}$ is not a pre m_X open set in X.

We denote the relation discussed above by a figure below.



Theorem 2.18. The following statements are equivalent for a function

$$f :(X, Tm_X) \rightarrow (Y, Tm_Y)$$

(i) f is pre m_X irresolute.
(ii) For each point x of X and each pre m_X neighborhood V of f(x), there exists a pre m_X - neighborhood U of x such that $f(U) \subseteq V$.
(iii) For each $x \in X$ and each $V \subset Pm_XO(Y)$, there exists $U \subset P\, m_XO(X)$ such that $f(U) \subset V$.

Proof. **(i) ⇒ (ii).** Assume that $x \in X$ and V is a pre m_X- open set in Y containing f(x).Since f is a pre m_X - irresolute and let $U = f^{-1}(V)$ be a pre m_X - open set in X containing x and hence $f(U) = f f^{-1} (V) \subseteq V$.

(ii) ⇒(iii). Assume that $V \subseteq Y$ is a pre m_X open set containing f(x). Then by (ii),there exists a pre m_X open set G such that $x \in G \subseteq f^{-1}(V)$. Therefore, $x \in f^{-1}(V)$. This shows that $f^{-1}(V)$ is a pre m_X neighborhood of x.

(iii)⇒ (i). Let V be a pre m_X-open set in Y, then $f^{-1}(V)$ is pre m_X neighborhood each x of X. Thus, for each x is a pre m_X interior point of $f^{-1}(V)$ which implies that $f^{-1}(V) \subset Int(m_X\text{-}Cl(f^{-1}(V))$. Therefore $f^{-1}(V)$ is a pre m_X open set in X and hence f is a pre m_X - reirresolute.

Theorem 2.19. The following are equivalent for a function $f : (X, Tm_X) \rightarrow (Y, Tm_Y)$

(i) f is pre m_X-irresolute continuous.
(ii) $f(Pm_XCl(v)) \subseteq Pm_X\text{-}Clf(v)$.
(iii) $Pm_XCl(f^{-1}(B)) \subseteq f^{-1}(Pm_X\text{-}Cl(B))$.
(iv) $Pm_X\text{-}Int(f^{-1}(A)) \supseteq f^{-1}(Pm_XInt(A))$.
(v) $f(Pm_X\text{-}Int(B)) \supseteq Pm_X\text{-}Intf(B)$ if f is bijective.

Proof: (i) ⇒ (ii). Let $x \in X$ and $V \subseteq X$ then

$$Pm_XCl(v) \subseteq Pm_XCl(f^{-1}(f(v)) \subseteq Pm_X\text{-}Cl(f^{-1}(Pm_X\text{-}Cl(f (v))) = f^{-1}(Pm_X\text{-}Clf(v))$$
$$\Rightarrow f(Pm_X\text{-}Cl(v)) \subseteq ff^{-1}(Pm_X\text{-}Cl(f (v)) \subseteq Pm_X\text{-}Cl(f(v)).$$

Therefore $f(Pm_XCl(v)) \subseteq Pm_X\text{-}Clf(v)$.

(ii) ⇒ (iii). Let $x \in X$ and $V \subseteq X$ and $B \subseteq Y$ such that $V = f^{-1}(B)$ then

$$f(Pm_X\text{-}Cl(f^{-1} (B))) \subseteq Pm_XCl\, ff^{-1}(B) \subseteq Pm_XCl\, (B)$$
$$\Rightarrow f^{-1}f(Pm_XCl(f^{-1}(B))) \subseteq f^{-1}(Pm_XCl\, (B)) \Rightarrow Pm_XCl\, f^{-1}((B)) \subseteq f^{-1}(Pm_XCl(B)).$$

(iii) ⇒(iv) Let A be any subset of Y such that $B^C=A$. By (iii)

$$X - Pm_X\text{-}Cl(f^{-1}(B)) \supseteq X - f^{-1}(Pm_X\text{-}Cl(B))$$
$$\Rightarrow Pm_XIntf^{-1}(B^C) \supseteq f^{-1}(Pm_XInt\, (B^C))$$
$$\Rightarrow Pm_XIntf^{-1}(A) \supseteq f^{-1}(Pm_XInt\, (A)).$$

(iv) ⇒ (i) Let C be any sub set of Y such that $A=Pm_XIntC$. By (iv)

$$Pm_XIntf^{-1}(Pm_XIntC)\supseteq f^{-1}(Pm_XInt(C))\supseteq Pm_XIntf^{-1}(Pm_XIntC)$$

Therefore $f^{-1}(Pm_XInt(C))=Pm_Xintf^{-1}(Pm_XIntC)$.

Therefore f is a pre m_X irresolute continuous.

(ii)⇔(v) Let A be a subset of X and f is a bijective then

$f(X-A)=X-f(A)$ and $X-A=A^C=B$ (say)

Now,

$$f(Pm_Xcl(A))\subseteq Pm_X\text{-}clf(A)$$
$$\Rightarrow X\text{-}f(Pm_Xcl(A))\supseteq X\text{-}Pm_X\text{-}clf(A)$$
$$\Rightarrow f(Pm_Xint(B))\supseteq Pm_XInt(f(B))$$

Converse part holds similarly
Hence the statements are equivalent is proved as follows

(i) ⇒(ii) ⇒(iii) ⇒(iv) ⇒(i) ,
↕
(v)

Theorem 2.20.
(1) If $f:(X, Tm_X)\to(Y, Tm_Y)$ is pre m_X irresolute and $g:(Y, Tm_Y)\to(Z, Tm_Z)$ is pre m_X continuous then gof is pre m_X continuous.
(2) If $f:(X, Tm_X)\to(Y, Tm_Y)$ is pre m_X irresolute and $g:(Y, Tm_Y)\to(Z, Tm_Z)$ is m_X continuous then gof is pre m_X continuous.
(3) If $f:(X, Tm_X)\to(Y, Tm_Y)$ is pre m_X continuous and $g:(Y, Tm_Y)\to(Z, Tm_Z)$ is m_X continuous then gof is pre m_X continuous.
(4) If $f:(X, Tm_X)\to(Y, Tm_Y)$ is pre m_X irresolute continuous and $g:(Y, Tm_Y)\to(Z, Tm_Z)$ is pre m_X irresolute continuous then gof is pre m_X irresolute continuous.

Proof: To Prove (1) Let W be any m_X-open set of Z. since f is pre m_X irresolute then

$$(gof)^{-1}(w)=f^{-1}(g^{-1}(w))$$

is pre m_X open in X and hence gof is a pre m_X continuous function.

The other can be proved similarly.

3. Pre m_X Open Mapping

In this section the concept of Pre m_X open mapping is introduced and also the concept of Pre m_X irresolute mapping is introduced and some of its properties were discussed.

Definition 3.1. A function f : (X, Tm_X) →(Y, Tm_Y) is said to be a pre m_X -open mapping if the image of each Pre m_X open set in X is a m_X -open set in Y.

Example 3.2. Let X = {a,b,c} and Y = {x,y,z}. Let m_X = {ϕ,X,{a,b},{c,b}}. Then Tm_X={ϕ,X,{a,b}, {b,c},{b}}. Here the pre m_X open sets are ϕ,X,{a,b},{c,b}, {b}. Let

m_Y= {ϕ,Y,{x,y},{y,z},{y}} and Tm_Y={ϕ,X,{ x,y},{y,z},{y}}.

Let f : (X, Tm_X) →(Y, Tm_Y) be a mapping such that f(a)=x, f(b)=y, f(c)=z. Then the mapping is a pre m_X open mapping .

Theorem 3.3. Consider a function f : (X, Tm_X) →(Y, Tm_Y) .Every pre m_X open map is a open map.

Proof: Let A be a open set in (X, Tm_X) then A is a pre m_X open set in (X,Tm_X). Since f is a pre m_X open map, f(A) is a m_X open set in (Y, Tm_Y). Since every m_X open set in (Y, Tm_Y) is also a open set . So f is a open map

Remark 3.4. The converse of the theorem is not true which follows from the following example : Let

X={x,y,z,t},
m_X={ϕ,X,{x,y},{y,z}}and
Tm_X={ϕ,X,{x,y},{y,z},{x,y,z},{y}}.

Let

Y={a,b,c,d},
m_Y={ϕ,Y, {a,b},{b,c}, {a,c,d}}
Tm_Y ={ϕ,Y,{a,b},{b,c},{a,c,d},{b},{a,b,c}}.

Let f : (X, Tm_X) →(Y, Tm_Y) is a map defined by f(x)= a, f(y)=b and f(z)=c, f(t)=d. Here f is a open map but not a pre m_X open mapping

Definition 3.5. A function f : (X, Tm_X) →(Y, Tm_Y) is said to be a pre m_X -irresolute mapping if the image of each Pre m_X open set in X is a pre m_X -open set in Y.

Example 3.6. The example 3.2 is also an example of Pre m_X -irresolute mapping

Theorem 3.7. Consider a function f : (X, Tm_X) →(Y, Tm_Y) . Every Pre m_X – open map is also a Pre m_X –irresolute map

Proof: Let A be a Pre m_X –open set in X . Since f is a Pre m_X –open map, f(A) is m_X –open set in Y. Every m_X –open set is also an open set and a Pre m_X –open set. Thus f(A) is a Pre m_X –open set. This proves that f is a Pre m_X –irresolute mapping.

Remark 3.8. The converse of the above theorem need not be true which follows from the following example : Let

$X = \{a,b,c,d\}$ and $Y=\{x,y,z,t\}$,
$m_X= \{\phi, X, \{a\}, \{b\}, \{c\}\}$ and
$Tm_X= \{\phi, X, \{a\}, \{b\}, \{c\},\{a,b\}, \{a,c\}, \{b,c\}\}$,
$m_Y= \{\phi, Y, \{x\}, \{y\}, \{z\}\}$ and
$Tm_Y= \{\phi, Y, \{x\}, \{y\}, \{z\},\{x,y\}, \{x,z\}, \{y,z\}\}$,

Let $f : (X, Tm_X) \rightarrow (Y, Tm_Y)$ is a map defined by $f(x)= a$, $f(y)=b$ and $f(z)=c$, $f(t)=d$. Then f is a pre m_X irresolute map but not a Pre m_X open map.

We denote the relation discussed above by a figure below.

Pre **m_X-Open map** ⟶ **m_X – Open map**

Pre m_X - irresolute map **Open Map**

Theorem 3.9. The following are equivalent for a function $f : (X, Tm_X) \rightarrow (Y, Tm_Y)$

(i) f is pre- m_X irresolute mapping.
(ii) $f^{-1}(Pm_XInt(v)) \supseteq Pm_X Int(f^{-1}(v))$
(iii) $f^{-1} (Pm_X Cl(v)) \subseteq Pm_XCl(f^{-1}(v))$
(iv) $Pm_XIntf(A) \supseteq f (Pm_XInt(A))$
(v) $f(Pm_XCl(B)) \supseteq Pm_XClf(B)$ if f is bijective.

Proof : (i) ⇒(ii). Let $x\in X$ and $V\subseteq X$ then

$$Pm_XInt(v) \supseteq Pm_XIntff^{-1}(v)) \supseteq Pm_XIntf(Pm_XIntf^{-1}(v)) =f(Pm_XIntf^{-1}(v))$$
$$\Rightarrow f^{-1}(Pm_XInt(v))\supseteq f^{-1}f(Pm_XIntf^{-1}(v))\supseteq Pm_XInt(f^{-1}(v)).$$

Therefore

$f^{-1} (Pm_XInt(v))\supseteq Pm_XInt(f^{-1}(v))$.

(ii) ⇔(iii). From (ii),

$$X - f^{-1}(Pm_Xint(v)) \subseteq X- Pm_Xint(f^{-1}(v)) \Rightarrow f^{-1}(Pm_Xclv) \subseteq Pm_Xcl(f^{-1}(v)).$$

The converse part may be proved similarly.

(ii) ⇒(iv). Let $x\in X$ and $V\subseteq X$ and let $f^{-1}(v)=A$. From (ii),

$$f^{-1} (Pm_Xintf(A)) \supseteq Pm_Xint(A)$$

Therefore $Pm_Xintf(A))\supseteq f(Pm_Xint(A))$.

(iv)⇒i) Let $A=Pm_Xint(C)$.From (iv),

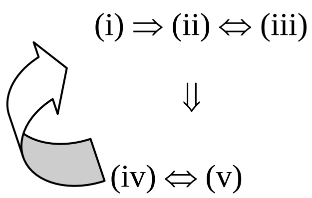
$$Pm_Xintf(Pm_Xint(C))\supseteq f(Pm_Xint(Pm_Xint(C))=f(Pm_Xint(C))\supseteq Pm_X intf(Pm_Xint(C))$$

Therefore $f(Pm_X int(C))$ is a pre-$m_X$open i.e. the image of a pre m_X open set is a pre m_X open set

(iv) ⇔(v) Let A be any subset of X and f is a bijective mapping then f(X – A)=X- f(A) and X – A=B(say).Therefore from (iv)

$$f(Pm_X intB) \subseteq Pm_X intf(B)$$
$$\Rightarrow Y\text{-}f(Pm_X intB) \supseteq Y\text{-} Pm_X int(f(B))$$
$$\Rightarrow f(Y - Pm_X int(B)) \supseteq Pm_X clf(B)$$
$$\Rightarrow f(Pm_X cl(B)) \supseteq Pm_X clf(B).$$

Converse part can be proved similarly. The equivalence relation is proved as below



4. Pre m_X Homeomorphism

In this section we introduce the concept of Pre m_X homeomorphism and study some of its properties.

Definition 4.1: A bijective mapping $f:(X,m_X) \to (Y,Tm_Y)$ from a space X into a space Y is called pre-m_X homeomorphism if f and f^{-1} are pre m_X-irresolute mapping.

Theorem 4.2:Let $f:(X,m_X) \to (Y,m_Y)$ be a bijective mapping from a m_X structure(X,m_X) to a topological space (Y, Tm_Y).The following statements are equivalent.

- **(i)** f is a pre m_X homeomorphism.
- **(ii)** f^{-1} is a pre m_X homeomorphism.
- **(iii)** f is a pre m_X irresolute mapping and a pre m_X irresolute continuous .
- **(iv)** The image of a pre m_X open set in X is a pre m_X open set in Y and a pre m_X continuous mapping.
- **(v)** $f^{-1}(P\, m_X Int(v)) = Pm_X Int(f^{-1}(v))$.
- **(vi)** $f^{-1}(Pm_X Cl(B)) = Pm_X\, cl(f^{-1}(B))$.
- **(vii)** $Pm_X Intf(A) = f(Pm_X Int(A))$.
- **(viii)** $f(Pm_X Cl(B)) = Pm_X Clf(B)$.

Proof: (i) ⇔(ii). it follows from the definition.

(i) ⇔(iii). Let f be a pre m_X homeomorphism implies that f and f^{-1} are pre m_X irresolute mapping .Now f^{-1} is a pre m_X irresolute mapping implies that $(f^{-1})^{-1}(A)$ i.e f(A) is a pre m_X open for each A being a pre m_X open set in X. Therefore f is a pre m_X irresolute mapping and a pre m_X irresolute continuous.

Converse: since f is a pre m_X irresolute mapping then f^{-1} is a pre m_X irresolute continuous. Hence f and f^{-1} are pre m_X irresolute continuous mapping. Then obviously f is a pre m_X homeomorphism.

(iii) ⇔(iv). Let f be a pre m_X irresolute mapping then for each pre m_X open set A of X , f(A) is a pre m_X open and f is also pre m_X irresolute continuous then by theorem **2.5** we say that image of a pre m_X open set in X is a pre m_X open set in Y and hence f is a pre m_X irresolute continuous mapping.

(iii) ⇒ (v). Let Let $x \in X$ and $V \subseteq X$, if f is pre m_X irresolute continuous then from theorem 3.7(iv)

$$Pm_X\, Intf^{-1}(A) \supseteq f^{-1}(\,Pm_X Int(A))\ldots\ldots(a)$$

and if f is pre m_X irresolute mapping then from theorem 3.8(ii)

$$f^{-1}(Pm_X Int(v)) \subseteq Pm_X Int(f^{-1}(v)) \ldots\ldots\ldots\ldots(b).$$

Combining (a) and (b) we get the result.

(v) ⇒(vi) since f is bijective and from (v)

$$X - f^{-1}(Pm_X int(v)) = X - Pm_X int(f^{-1}(v))$$
$$\Rightarrow f^{-1}(\,X - Pm_X int(v)) = Pm_X\, Cl(f^{-1}(v))$$
$$\Rightarrow f^{-1}(Pm_X Cl(v)) = Pm_X Cl(f^{-1}(v))$$

(vi) ⇒ (v). It is obvious.

(v) ⇒ (vii). Let $x \in X$ and $V \subseteq X$ and let $f^{-1}(v) = A$ then from(v),

$$Pm_X Int(v) = f(Pm_X Int(f^{-1}(v)) \Rightarrow Pm_X\, intf(A) = f(Pm_X int(A)).\text{proof}.$$

(vii) ⇒ (viii). It is obvious.

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