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INTERSECTİONAL (α,A)-SOFT NEW-İDEALS İN PU-ALGEBRAS

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***Abstract* –** The new notions, intersectional A-soft *new*-ideals and intersectional (α, A)-soft *new*-ideals in PU-algebras are introduced and their properties are investigated. The relations between an intersectional A-soft *new*-ideals and an intersectional (α, A)-soft *new*-ideals are provided. The homomorphic image of an intersectional (α, A)-soft *new*-ideals is studied.

***Keywords* –** PU-algebra, Soft PU-algebra, fuzzy soft PU-algebra, homomorphic image of an intersectional (α, A)-soft *new*-ideals

1 Introduction

Imai and Is´eki [6] in 1966 introduced the notion of a BCK-algebra. Is´eki [7] introduced BCI-algebras as a super class of the class of BCK-algebras. In [4,5], Hu and Li introduced a wide class of abstract algebras, BCH-algebras. They are shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. Megalai and Tamilarasi[15] introduced the notion of a TM-algebra, which is a generalization of BCK/BCI/BCH-algebras and several results are presented. Mostafa et al, in [17] introduced a new algebraic structure called PU-algebra, which is a dual for TM-algebra and they investigated severed basic properties. Moreover, they derived new view of several ideals on PU-algebra. The concept of fuzzy sets was introduced by Zadeh [22]. In 1991, Xi [20] applied the concept of fuzzy sets to BCI, BCK, MV -algebras. Since its inception, the theory of fuzzy sets, ideal theory and its fuzzification has been developed in many directions and applied to a

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wide variety of fields. Mostafa et al [18,19] introduced the notion of α-fuzzy and $(\tilde{\alpha},\alpha)$-cubic new-ideal of P U -algebra. They discussed the homomorphic image (pre image) of α-fuzzy and $(\tilde{\alpha},\alpha)$-cubic new-ideal of P U -algebra under homomorprhism of P U -algebras. Molodtsov [16] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free form the difficulties that have troubled the usual theoretical approaches. Maji et al [12,13,14] described the application of soft theory and studied several operations on the soft sets. Many Mathematicians have studied the concept of soft set of some algebraic structures. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. Çagman et al. [1, 2, 3] introduced fuzzy parameterized (FP) soft sets and their related properties. They proposed a decision making method based on FP-soft set theory, and provided an example which shows that the method can be successfully applied to the problems that contain uncertainties. Jun [8] applied Molodtsovs notion of soft sets to the theory of BCK/BCI-algebras and introduced the notion of soft BCK/BCI-algebras and soft subalgebras and then investigated their basic properties. Jun and Park [9] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. They introduced the notion of soft ideals and idealistic soft BCK/BCI-algebras and gave several examples. Jun et al. [10] introduced the notion of soft p-ideals and p-idealistic soft BCI-algebras and investigated their basic properties. Using soft sets, they gave characterization of (fuzzy) p-ideals in BCI-algebras. Moreover, Jun et al. [11] applied a fuzzy soft set introduced by Maji et al. [12] as a generalization of the standard soft sets for dealing with several kinds of theories in BCK/BCI-algebras. They defined the notions of fuzzy soft BCK/BCI-algebras, (closed) fuzzy soft ideals, and fuzzy soft p-ideals, and investigated related properties. Yang et al. [21] introduced the concept of the interval-valued fuzzy soft set; they studied the algebraic properties of the concept and they analyzed a decision problem by using an interval-valued fuzzy soft set.

In this paper, we introduce the notions of soft PU-algebras , A-soft *new*-ideals , (α, A)-soft *new*-ideals and discuss various operations introduced in on these concepts. Using soft sets, we give characterizations of (α, A)-soft *new*-ideals in PU-algebras. The relations between A-soft *new*-ideals and (α, A)-soft *new*-ideals in PU-algebras is provided. The homomorphic image of an intersectional α-soft *new*-ideals are studied.

2 Preliminaries

Now, we will recall some known concepts related to PU-algebra from the literature, which will be helpful in further study of this article.

Definition 2.1. [17] A PU-algebra is a non-empty set X with a constant $0 \in X$ and a binary operation $*$ satisfying the following conditions:
(I) $0 * x = x$,
(II) $(x * z) * (y * z) = y * x$ for any $x, y, z \in X$.

On X we can define a binary relation "$\leq$" by: $x \leq y$ if and only if $y * x = 0$.

Example 2.2. [17] Let X = {0, 1, 2, 3, 4} be a set and ∗ is defined by

∗	0	1	2	3	4
0	0	1	2	3	4
1	4	0	1	2	3
2	3	4	0	1	2
3	2	3	4	0	1
4	1	2	3	4	0

Then (X, ∗, 0) is a PU-algebra.

Proposition 2.3. [17] In a **PU**-algebra (X,∗, 0) the following hold, for all x, y, z $\in X$
(a) x ∗ x = 0.
(b) (x ∗ z) ∗ z = x.
(c) x ∗ (y ∗ z) = y ∗ (x ∗ z).
(d) x ∗ (y ∗ x) = y ∗ 0.
(e) (x ∗ y) ∗ 0 = y ∗ x.
(f) If x ≤ y, then x ∗ 0 = y ∗ 0.
(g) (x ∗ y) ∗ 0 = (x ∗ z) ∗ (y ∗ z).
(h) x ∗ y ≤ z if and only if z ∗ y ≤ x.
(i) x ≤ y if and only if y ∗ z ≤ x ∗ z.
(j) In a **PU**-algebra (X, ∗, 0) , the following are equivalent:

(1) x = y, (2) x ∗ z = y ∗ z, (3) z ∗ x = z ∗ y.

(k) The right and the left cancellation laws hold in X .
(l) (z ∗ x) ∗ (z ∗ y) = x ∗ y,
(m) (x ∗ y) ∗ z = (z ∗ y) ∗ x.
(n) (x ∗ y) ∗ (z ∗ u) = (x ∗ z) ∗ (y ∗ u) for all x, y, z and u $\in X$.

Lemma 2.4.[17] If (X, ∗, 0) is a PU-algebra, then (X, ≤) is a partially ordered set.

Definition 2.5. [17] A non-empty subset S of a **PU**-algebra (X, ∗, 0) is called a sub-algebra of X if x ∗ y ∈ S whenever x, y ∈ S.

Definition 2.6. [17] A non-empty subset I of a **PU**-algebra (X, ∗, 0) is called a **new**-ideal of X if,
(i) 0 ∈ I,
(ii) (a ∗ (b ∗ x)) ∗ x ∈ I, for all a, b ∈ I and $x \in X$.

Theorem 2.7[17] Any sub-algebra S of a **PU**-algebra X is a **new**-ideal of X.

Example 2.8[17] Let X = {0, a, b, c} be a set with ∗ is defined by the following table:

∗	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then (X , ∗, 0) is a **PU**-algebra. It is easy to show that I_1 = {0, a}, I_2 = {0, b}, I_3 = {0, c} are **new-**ideals of X.

3 Basic Results on Soft Sets

Molodtsov [16] defined the notion of a soft sets as follows. Let U be an initial universe and E be the set of parameters. The parameters are usually "attributes, characteristics or properties of an object". Let P(U) denote the power set of U and A is a subset of E.

Definition 3.1 [16]**.** A pair (F , A) is called a soft set over U, where F is a mapping given by F : A → P(U).In other words, a soft set over a universe is a U parameterized family of subsets of the universe U . For $e \in A$, $F(e)$ may be considered as the set of e-elements or e- approximate elements of the soft set (F, A).Thus $(F, A) = \{F(e) \in P(U) : e \in A \subseteq E\}$.

Definition 3.2 [12]**.** Let (F , C) and (G, D) be two soft sets over a common universe U. The soft set (F , C) is called a soft subset of (G, D), if C ⊆ D and for all ε ∈ C, $F(\varepsilon) \subseteq G(\varepsilon)$. This relationship is denoted by (F , C)$\subseteq$ (G, D). Similarly (F , C) is called a soft superset of (G, D), if (G, D) is soft subset of (F , C). This relationship is denoted by (F , C)$\supseteq$ (G, D). Two soft sets (F , C) and (G, D) over U are said to be equal, if (F , C) is a soft subset of (G, D) and (G, D) is a soft subset of (F , C).

Definition 3.3 [12]**.** Let (F , C) and (G, D) be any two soft sets over U.

(1) The intersection (H, E) of two soft sets (F , C) and (G, D) is defined as the soft set (H, E) = (F , C) $\tilde{\bigcap}$ (G, D), where E = C ∩ D and for all ε ∈ C□

$$H(C) = \begin{cases} F(\varepsilon) & if \ \varepsilon \in C \setminus D \\ G(\varepsilon) & if \ \varepsilon \in D \setminus C \\ F(\varepsilon) \bigcap G(\varepsilon) & if \ \varepsilon \in C \bigcap D \end{cases}$$

(2) The union (H, E) of two soft sets (F , C) and (G, D) is defined as the soft set (H, E) = (F , C) $\tilde{\bigcup}$ (G, D), where E = C ∪ D and for all ε ∈ C

$$H(C)=\begin{cases} F(\varepsilon) & if\ \varepsilon\in C\setminus D \\ G(\varepsilon) & if\ \varepsilon\in D\setminus C \\ F(\varepsilon)\bigcup G(\varepsilon) & if\ \varepsilon\in C\bigcap D \end{cases}$$

(3) The AND operation (F , C) AND (G, D) of two soft sets (F , C) and (G, D) is defined as the soft set (H, E) = (F , C) $\tilde{\wedge}$ (G, D), where H[α, β] = F [α] ∩ G[β] for all

(α, β) ∈ C × D.

(4) The OR operation (F , C) OR (G, D) of two soft sets (F , C) and (G, D) is defined as the soft set (H, E) = (F , C) $\tilde{\vee}$ (G, D), where H[α, β] = F [α] ∪ G[β] for all

(α, β) ∈ C × D.

Definition 3.4 [12]. Let (F , C) and (G, D) be two soft sets over U. Then,

(1) The $\wedge$-intersection of two soft sets (F , C) and (G, D) is defined as the soft set (H, E) = (F , C) $\wedge$ (G, D) over U, where E = C × D, where H[α, β] = F [α] ∩ G[β] for all (α, β) ∈ C × D.

(2) The $\vee$-union of two soft sets (F , C) and (G, D) is defined as the soft set (H, E) = (F , C) $\vee$ (G, D) over U, where E = C × D, where H[α, β] = F [α] ∪ G[β] for all (α, β) ∈ C × D.

(3) Let (F , C) and (G, D) be two soft sets over G and K, respectively.The Cartesian product of the soft sets (F , C) and (G, D), denoted by (F , C) × (G, D), is defined as (F , C) × (G, D) = (U, A × B), where U[α, β] = F [α] × G[β] for all (α, β) ∈ C × D.

4 Soft PU-algebras

In this section, we introduce the notion of soft PU-algebras. Let X and A be a PU-algebra and a nonempty set, respectively. A pair (F,A) is called a soft set over X if and only if F is a mapping from a set of A into the power set of X . That is, $F:A\rightarrow P(X)$ such that $F(x)=\phi$ if $x\notin A$. A soft set over X can be represented by the set of ordered pairs

$$\{(x,F(x)):x\in A,F(x)\in P(X)\}.$$

It is clear to see that a soft set is a parameterized family of subsets of the set X .

Definition 4.1. Let (F,A) be a soft set over X . Then (F,A) is called a soft PU-algebra over X , if $F(x)$ is a **new**-ideal of X , for all $x\in A$.

Example 4.2. Let X = {0, a, b, c} be a set in Example 2.8. Define a mapping $F:X\rightarrow P(X)$ by: F (0) = {0}, F (a) = {0, a}, F (b) = {0, b} and F (c) = {0, c}. It is clear that (F,X) is a soft PU-algebra over X .

Definition 4.3. Let (F,A) and (G,B) be two soft PU-algebras over X . Then (F,A) is called a soft PU-subalgebra of (G,B), denoted by $(F,A)\prec(G,B)$, if it satisfies:
(i) $A\subset B$,
(ii) $F(x)$ is subalgebra of $G(x)$, for all $x\in A$.

Proposition 4.4. A soft set (F,A) over X is a soft PU-algebra, if and only if each $\Phi\neq F\ [\varepsilon]$ is a **new-**ideal of X , for all $\varepsilon\in A$.

Proof. Let (F,A) be a soft PU-algebra over X . Then by above definition, $F\ [\varepsilon]$ is a **new-**ideal of X , for all $\varepsilon\in A$. It follows that for all $\varepsilon\in A$, $F\ [\varepsilon]\neq\Phi$ is a **new-**ideal of X .
Conversely, let us consider that (F,A) is a soft set over X such that for all $\varepsilon\in A$, $F\ [\varepsilon]\neq\Phi$ is a **new-**ideal of X , whenever $F\ [\varepsilon]\neq\Phi$. Since $F\ [\varepsilon]$ is a **new-**ideal of X .
Hence (F,A) is a soft PU-algebra over X .

Theorem 4.5. Let (F,A) and (G,B) be two soft PU-algebras over X . If $A\cap B\neq\phi$,then the intersection $(F,A)\tilde{\cap}(G,B)$ is a soft PU-algebra over X .

Proof. We can write $(F,A)\tilde{\cap}(G,B)=(H,E)$, where $E=A\cap B$ and for all $\varepsilon\in E$, it is defined as

$$H(\varepsilon)=\begin{cases} F(\varepsilon) & if\ \ \varepsilon\in A\setminus B\\ G(\varepsilon) & if\ \ \varepsilon\in B\setminus A\\ F(\varepsilon)\cup G(\varepsilon) & if\ \ \varepsilon\in A\cap B\end{cases}$$

Since for all $\varepsilon\in E$ either $\varepsilon\in A\setminus B$ or $\varepsilon\in B\setminus A$, $\varepsilon\in A\cap B$. If $\varepsilon\in A\setminus B$, then H[ε]=$F$ [ε]. As F [ε] is a new-ideal over X , then H[ε] is a new-ideal over X . If $\varepsilon\in B\setminus A$, then H[ε] = G[ε]. As G[ε] is a new-ideal over X , then H[ε] is a new-ideal over X . If, then H[ε] = F [ε] ∩ G[ε]. As F [ε] and G[ε] are both new-ideals over X , then H[ε] is a new-ideal over X . In all cases, H[ε] is a new-ideal over X . Hence (H, E) = (F , A) $\tilde{\cap}$ (G, B) is a soft PU-algebra over X . □

5 Intersectional (α,A)-soft New PU- ideals

In what follow let X and A be a PU -algebra and a non empty set, respectively.

Definition 5.1. Let $E=X$ be a PU-algebra. Given a subalgebra A of E ,let F_A be an A-soft set over U . Then F_A is called an intersectional A-soft PU-subalgebra over U if it satisfies the following condition:

$$F_A(x*y)\supseteq F_A(x)\cap F_A(y),\ \text{for all}\ x,y\in X.$$

Definition 5.2 Let (X, $*$, 0) be a **PU**-algebra. F_A is called intersectional A -soft **new**-ideal over U if it satisfies the following conditions:

(F_1) $F_A(0) \supseteq F_A(x)$,
(F_2) $F_A((x*(y*z))*z) \supseteq F_A(x) \bigcap F_A(y)$, for all $x, y, z \in X$.

Example 5.3 . Consider the PU-algebra (Z;$*$,0) as the initial universeset U , where $a*b = b-a \ \forall a,b \in Z$. Let $E = X = \{0,a,b,c\}$ be a PU-algebra with the following Cayley table:

Define a soft set (F_A, X) over U by

$$F_X : X \to P(U) \quad x \mapsto \begin{cases} Z & if \ x \in \{0,a\} \\ 2Z & if \ x \in \{b,c\} \end{cases}$$

Then F_A is an intersectional A-soft (subalgebra) new ideal over U .

Definition 5.4 [3] . The complement of a soft set (F, A) is denoted by (F^C, A) and is defined by $(F, A)^C$, where $F^C : A \to P(U)$ is a mapping given by

$$F^C(x) = U - F(x) \ \forall \ x \in X \ .$$

Definition 5.5 Let F_A be an intersectional A-soft PU- subalgebra over U and $\alpha \in \bigcap_{x\in X} F_A^C \ (x)$.Then F_A^{α} is called intersectional (α, A)-soft PU- subalgebra over U (w.r.t. F_A) and is defined by $F_A^{\alpha}(x) = F_A(x) \bigcup \alpha$, for all $x \in X$.

Lemma 5.6 If F_A is intersectional A-soft PU- subalgebra over U and $\alpha \in \bigcap_{x\in X} F_A^C \ (x)$, then $F_A^{\alpha}(x*y) \supseteq F_A^{\alpha}(x) \bigcap F_A^{\alpha}(y)$, for all $x, y \in X$.

Proof: Let X be a **PU**-algebra and $\alpha \in \bigcap_{x\in X} F_A^C \ (x)$ .Then by Definitions (5.1, 5.5), we have

$$\begin{aligned} F_A^{\alpha}(x*y) = F_A(x*y) \bigcup \alpha\} &\supseteq \{F_A(x) \bigcap F_A(y)\} \bigcup \alpha \\ &= \{F_A(x) \bigcup \alpha\} \bigcap \{F_A(y) \bigcup \alpha\} \\ &= F_A^{\alpha}(x) \bigcap F_A^{\alpha}(y), \text{for all } x, y \in X. \end{aligned}$$

Definition 5.7 Let X be a **PU**-algebra, F_A^{α} is called intersectional (α, A)-soft PU-subalgebra of X if $F_A^{\alpha}(x*y) \supseteq F_A^{\alpha}(x) \bigcap F_A^{\alpha}(y)\}$, for all $x, y \in X$.

It is clear that intersectional (α, A)-soft PU- subalgebra over U is a generalization of intersectional A-soft PU- subalgebra over U.

Example 5.9 Let X = {0, 1, 2, 3} in which ∗ is defined by the following table:

∗	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then (X, ∗, 0) is a **PU**-algebra. Define an $F_A{}^{\alpha}$ by

$$F_X{}^{\alpha}(x) = \begin{cases} Z_4 \cup \{3\} & \text{if} \quad x \in \{0,1\} \\ Z_8 \cup \{3\} & \text{otherwise} \end{cases}$$

Routine calculations give that $F_X{}^{\alpha}$ is an intersectional (α, A)-soft subalgebra over U.

Lemma 5.10 Let $F_A{}^{\alpha}$ be an intersectional (α, A)-soft new ideal over U. If the inequality $x * y \le z$ holds in X, then $F_A{}^{\alpha}(y) \supseteq F_A{}^{\alpha}(x) \cap F_A{}^{\alpha}(z).$

Proof: Assume that the inequality $x * y \le z$ holds in X, then $z * (x * y) = 0$ and by (F_2^{α}) $F_A{}^{\alpha}((\overbrace{z * (x * y)}^{0}) * y) \supseteq F_A{}^{\alpha}(x) \cap F_A{}^{\alpha}(z)$. Since $F_A{}^{\alpha}(y) = F_A{}^{\alpha}(0 * y)$, then we have that $F_A{}^{\alpha}(y) \supseteq F_A{}^{\alpha}(x) \cap F_A{}^{\alpha}(z).$

Corollary 5.11 Let F_A be intersectional A-soft new ideal over U. If the inequality $x * y \le z$ holds in X, then $F_A(y) \supseteq F_A(x) \cap F_A(z).$

Lemma 5.12 If $F_A{}^{\alpha}$ is intersectional (α, A)-soft new ideal over U and if $x \le y,$ then $F_A{}^{\alpha}(x) = F_A{}^{\alpha}(y).$

Proof: If $x \le y,$ then $y * x = 0.$ Hence by the definition of **PU**-algebra and its properties we have $F_A{}^{\alpha}(x) = F_A(x) \cup \alpha = F_A(0 * x) \cup \alpha = F_A((y * x) * x) \cup \alpha = F_A(y) \cup \alpha\} = F_A{}^{\alpha}(y).$

Corollary 5.13 If F_A is intersectional A-soft PU-new ideal over U and if $x \le y,$ then $F_A(x) = F_A(y).$

Lemma 5.14 If F_A is intersectional A-soft new ideal over U and $\alpha \in \bigcap_{x \in X} F_A^C(x)$,then

(F_1^{α}) $F^{\alpha}(0) \supseteq F^{\alpha}(x),$
(F_2^{α}) $F^{\alpha}((x * (y * z)) * z) \supseteq F^{\alpha}(x) \cap F^{\alpha}(y),$ for all $x, y, z \in X.$

Proof: Let X be a **PU**-algebra and $\alpha \in \bigcap_{x \in X} F_A^C(x)$ .Then by Definitions (5.2, 5.5), we have:

$$F_A^{\alpha}(0) = F_A(0) \cup \alpha \supseteq F_A(x) \cup \alpha = F_A^{\alpha}(x), \text{ for all } x \in X.$$

$$\begin{aligned} F_A^{\alpha}((x*(y*z))*z) &= F_A((x*(y*z))*z) \cup \alpha \\ &\supseteq \{F_A(x) \cap F_A(y)\} \cup \alpha\} \\ &= \{F_A(x) \cup \alpha\} \cap \{F_A(y) \cup \alpha\}\} \\ &= \{F_A^{\alpha}(x) \cap F_A^{\alpha}(y)\}, \text{ for all } x, y, z \in X. \end{aligned}$$

Definition 5.8 Let X be a **PU**-algebra, F_A^{α} is called intersectional (α, A)-soft new ideal of X if

$(F_1 I)$ $F_A^{\alpha}(0) \supseteq F_A^{\alpha}(x)$,

$(F_2 I)$ $F_A^{\alpha}((x*(y*z))*z) \supseteq F_A^{\alpha}(x) \cap F_A^{\alpha}(y)$, for all $x, y, z \in X$.

Remark. In what follows, denote by S(U) the set of all soft sets over U by C¸ aˇgman et al. [2,3].

Definition 5.15 Let f be a mapping from X to Y, F_X , $F_Y \in S(U)$

(1) The soft set $f^{-1}(F_Y) = \{ (x, f^{-1}(F_Y)(x) : x \in X, f^{-1}(F_Y)(x) \in P(U) \}$, where $f^{-1}(F_Y)(x) = F_Y(f(x))$,is called the soft pre-image of F_Y under f.

(2) The soft se $f\ (F_X) = \{ (y, f(F_X)(y) : y \in Y, f(F_X)(y) \in P(U) \}$
where

$$f(F_X)(y) = \begin{cases} \bigcup_{x \in f^{-1}(y)} F_X(x) & \text{if } f^{-1}(y) \neq \Phi \\ \Phi & \text{otherwise} \end{cases}$$

is called the soft image of F_X under f.

Proposition 5.16. For any PU-algebras X and Y, let $f : X \to Y$ be a function. Then

$$(\forall F_X \in S(U))\ (F_X^{\alpha} \tilde{\subseteq} f^{-1}(f(F_X)) \qquad \textbf{(*)}$$

Proof: Since $f^{-1}(f(x)) \neq \Phi \quad \forall\ x \in X$. Hence

$$F_X^{\alpha}(x) \subseteq \bigcup_{\beta \in f^{-1}(f(x))} F_X^{\alpha}(\beta) = f(F_X^{\alpha})(f(x)) = f^{-1}(f(F_X^{\alpha}))(x) \quad \forall x \in X ,$$

and therefore (*) is valid.

Theorem 5.17 Let $(X,*,0)$ and $(Y,*^{\backslash},0^{\backslash})$ be **PU**-algebras and $f:X\to Y$ be a homomorphism. If $F_Y^{\alpha}\in S(U)$ is α-intersectional A-soft PU-new ideal over Y, then the soft pre-imag $(f^{-1})(F_Y^{\alpha})$ of F_Y^{α} is intersectional (α,A)-soft PU-new ideal over Y, of X.

Proof: Since $((f^{-1}))(F_Y^{\alpha})$ is soft pre-image of F_Y under f. then $f^{-1}(F_Y^{\alpha})(x)=F_Y^{\alpha}(f(x))$ for all $x\in X$. Let $x\in X$, then $((f^{-1}))(F_Y^{\alpha})(0)\supseteq((f^{-1}))(F_Y^{\alpha})(x)$

Now let $x,y,z\in X$, then

$$\begin{aligned}((f^{-1}))(F_Y^{\alpha})((x*(y*z))*z) &= F_Y^{\alpha}(f((x*(y*z))*z))\\ &= F_Y^{\alpha}(f(x*(y*z))*^{\backslash} f(z))\\ &= F_Y^{\alpha}((f(x)*^{\backslash} f(y*z))*^{\backslash} f(z))\\ &= F_Y^{\alpha}((f(x)*^{\backslash}(f(y)*^{\backslash} f(z)))*^{\backslash} f(z))\\ &\supseteq F_Y^{\alpha}(f(x))\cap F_Y^{\alpha}(f(y))\}\\ &= ((f^{-1}))(F_Y^{\alpha})(x)\cap((f^{-1}))(F_Y^{\alpha})(y)\end{aligned}$$

and the proof is completed.

Theorem 5.18 Let $(X,*,0)$ and $(Y,*^{\backslash},0^{\backslash})$ be **PU**-algebras, $f:X\to Y$ be a injective homomorphism, F_X^{α} be an (α,X)-intersectional soft of X, $f(F_X^{\alpha})$ be the image of F_X^{α} under f and $F_X^{\alpha}(y)=F_X^{\alpha}(f(x))$ for all $x\in X$. If F_X^{α} is an (α,X)-intersectional soft new ideal over X, then $f(F_X^{\alpha})$ is an (α,Y)-intersectional soft new ideal over Y.

Proof: If at least one of $f^{-1}(x^{\backslash})$ and $f^{-1}(y^{\backslash})$, $f^{-1}(z^{\backslash})$ is empty, then

$$f(F_X{}^{\alpha}((x^{\backslash}*^{\backslash}(y^{\backslash}*^{\backslash}z^{\backslash}))*^{\backslash}z^{\backslash}))\supseteq f(F_X{}^{\alpha}(x^{\backslash}))\cap f(F_X{}^{\alpha}(y^{\backslash})\}.$$

is clear.

Assume that $f^{-1}(x^{\backslash})\neq\phi$, $f^{-1}(y^{\backslash})\neq\phi$ and $f^{-1}(z^{\backslash})\neq\phi$.Then

$$f(F_X{}^{\alpha}(x^{\backslash}))\cap f(F_X{}^{\alpha}(y^{\backslash}))=\Big(\bigcup_{x_1\in f^{-1}(x^{\backslash})}F_X{}^{\alpha}(x_1)\Big)\cap\Big(\bigcup_{y_1\in f^{-1}(y^{\backslash})}F_X{}^{\alpha}(y_1)\Big)=$$

$$\bigcup_{\substack{x_1\in f^{-1}(x^{\backslash}),\\ y_1\in f^{-1}(y^{\backslash})}}(F_X{}^{\alpha}(x_1))\cap(F_X{}^{\alpha}(y_1))\subseteq\bigcup_{\substack{x_1\in f^{-1}(x^{\backslash}),\\ y_1\in f^{-1}(y^{\backslash})\\ z_1\in f^{-1}(z^{\backslash})}}F_X{}^{\alpha}(x_1*(y_1*z_1))*z_1)=\bigcup_{x\in f^{-1}((x^{\backslash}*(y^{\backslash}*z^{\backslash}))*z^{\backslash})}F_X{}^{\alpha}(x)$$

Therefore $f(F_X{}^{\alpha})$ is a (α,Y)-intersectional soft new ideal.

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