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ANOTHER QUASI μs -OPEN AND QUASI μs-CLOSED FUNCTIONS

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Abstaract − The purpose of this paper is to give a new type of open functions called quasi μs-open function. Also, we obtain its characterizations and its basic properties.

Keywords − *Topological spaces, μs-open set, μs-closed set, μs-interior, μs-closure, quasi μs-open function, quasi μs-closed function.*

1 Introduction

Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of open functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences.

Recently, as a generalization of closed sets, the notion of μs-closed sets were introduced and studied by Veera Kumar [7]. In this paper, we will continue the study of related functions by involving μs-open sets. We introduce and characterize the concept of quasi μs-open functions.

2 Preliminaries

Throughout this paper (X, τ),(Y, σ)and (Z, η) (or X, Y and Z)represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), cl(A), int(A) and A^C denote the closure of A, the interior of A and complement of A respectively.

We recall the following definitions which are useful in the sequel.

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Definition 2.1. *A subset A of a space (X, τ) is called:*

1. *α-open set [4] if A$\subseteq$ int(cl(int(A))).*

2. *semi-open set [2] if A$\subseteq$ cl(int(A)).*

The complements of the above mentioned open sets are called their respective closed sets.

The α-closure [4](resp.semi-closure [1]) of a subset A of X, denoted by αcl(A) (resp.scl(A)) is defined to be the intersection of all α-closed (resp. semi-closed) sets of (X, τ) containing A.

Definition 2.2. *A subset A of a space (X, τ) is called:*

1. *a gα*-closed set [3, 5] if αcl(A)$\subseteq$int(U) whenever A$\subseteq$U and U is α-open in (X, τ). The complement of gα*-closed set is called gα*-open set.*

2. *a μ-closed set [6] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is gα*-open in (X, τ). The complement of μ-closed set is called μ-open set.*

3. *a μs-closed set [7] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is gα*-open in (X, τ). The complement of μs-closed set is called μs-open set.*

The union (resp. intersection) of all μs-open (resp. μs-closed) sets, each contained in (resp. containing) a set A in a space X is called the μs-interior(resp. μs-closure) of A and is denoted by μs-int(A)(resp. μs-cl(A)).

Definition 2.3. *[7] A function f : (X, τ)$\rightarrow$(Y, σ) is called a μs-irresolute (resp. μs-continuous)if f^{-1}(V) is is μs-closed in X for every μs-closed (resp. closed) subset V of Y.*

Definition 2.4. *A function f : (X, τ)$\rightarrow$(Y, σ) is called a μs-open (resp. μs-closed) if f(V) is μs-open (resp. μs-closed) in Y for every open (resp. closed) subset V of X.*

3 Quasi μs-open Functions

We introduce a new definitions as follows.

Definition 3.1. *A function f : X $\rightarrow$ Y is said to be quasi μs-open if the image of every μs-open set in X is open in Y.*
It is evident that, the concepts quasi μs-openness and μs-continuity coincide if the function is a bijection.

Theorem 3.2. *A function f : X $\rightarrow$ Y is quasi μs-open if and only if for every subset U of X, f(μs-int(U))$\subset$ int(f(U)).*

Proof: Let f be a quasi μs-open function. Now, we have int(U)$\subset$ U and μs-int(U) is a μs-open set. Hence, we obtain that f(μs-int(U))$\subset$ f(U). As f (μs-int(U)) is open, f (μs-int(U))$\subset$ int(f(U)).

Conversely, assume that U is a μs-open set in X. Then, f(U) = f(μs-int(U)) $\subset$ int(f(U)) but int(f(U))$\subset$ f(U). Consequently, f(U) = int(f(U)) and hence f is quasi μs-open.

Lemma 3.3. *If a function $f : X \to Y$ is quasi μs-open, then μs-int($f^{-1}(G)$) $\subset$ f^{-1}(int(G)) for every subset G of Y.*

Proof: Let G be any arbitrary subset of Y. Then, μs-int(f^{-1}(G)) is a μs-open set in X and f is quasi μs-open, then f(μs-int(f^{-1}(G)))$\subset$ int(f(f^{-1}(G))) $\subset$ int(G). Thus, μs-int(f^{-1}(G)) $\subset$ f^{-1}(int(G)).

Definition 3.4. *A subset S of a space (X, τ) is called a μs-neighbourhood of a point x of X if there exists a μs-open set U such that $x \in U \subset S$.*

Theorem 3.5. *For a function $f : X \to Y$, the following are equivalent: (i) f is quasi μs-open; (ii) For each subset U of X, f(μs-int(U))$\subset$ int(f(U)); (iii) For each $x \in X$ and each μs-neighbourhood U of x in X, there exists a neighbourhood f(U) of f(x) in Y such that f(V) $\subset$ f(U).*

(i) $\Rightarrow$ (ii): It follows from Theorem 3.2.

(ii) $\Rightarrow$(iii): Let x $\in$ X and U be an arbitrary μs-neighbourhood of x in X. Then there exists a μs-open set V in X such that x $\in$ V $\subset$ U. Then by (ii), we have f (V) = f(μs-int(V)) $\subset$ int(f(V)) and hence f(V) = int(f(V)). Therefore, it follows that f(V) is open in Y such that f(x)$\in$ f (V)$\subset$ f (U).

(iii)$\Rightarrow$ (i): Let U be an arbitrary μs-open set in X. Then for each y $\in$ f (U), by (iii) there exists a neighbourhood Vy of y in Y such that Vy $\subset$ f (U). As Vy is a neighbourhood of y, there exists an open set Wy in Y such that y $\in$ Wy $\subset$ Vy . Thus f (U) = $\cup$ {Wy : y $\in$ f (U)} which is an open set in Y. This implies that f is quasi μs-open function.

Theorem 3.6. *A function $f : X \to Y$ is quasi μs-open if and only if for any subset B of Y and for any μs-closed set F of X containing f^{-1}(B), there exists a closed set G of Y containing B such that f^{-1}(G) $\subset$ F.*

Proof: Suppose f is quasi μs-open. Let B $\subset$ Y and F be a μs-closed set of X containing f^{-1}(B). Now, put G = Y - f (X - F). It is clear that f^{-1}(B) $\subset$ F implies B $\subset$ G. Since f is quasi μs-open, we obtain G as a closed set of Y. Moreover, we have f^{-1}(G)$\subset$ F.

Conversely, let U be a μs-open set of X and put B = Y \f (U). Then X \U is a μs-closed set in X containing f^{-1}(B). By hypothesis, there exists a closed set F of Y such that B $\subset$ F and f^{-1}(F) $\subset$ X \U. Hence, we obtain f(U) $\subset$ Y \F. On the other hand, it follows that B $\subset$ F, Y \F $\subset$ Y \B = f(U). Thus, we obtain f(U) = Y \F which is open and hence f is a quasi μs-open function.

Theorem 3.7. *A function $f : X \to Y$ is quasi μs-open if and only if f^{-1}(cl(B)) $\subset$ μs-cl(f^{-1}(B)) for every subset B of Y.*

Proof: Suppose that f is quasi μs-open. For any subset B of Y , f^{-1}(B)$\subset$ μs-cl(f^{-1}(B)). Therefore by Theorem 3.6, there exists a closed set F in Y such that B $\subset$ F and f^{-1}(F)$\subset$ μs-cl(f^{-1}(B)). Therefore, we obtain f^{-1}(cl(B)) $\subset$ f^{-1}(F)$\subset$ μs-cl(f^{-1}(B)).

Conversely, let B $\subset$ Y and F be a μs-closed set of X containing f^{-1}(B). Put W = cl_Y(B), then we have B $\subset$ W and W is closed and f^{-1}(W)$\subset$ μs-cl(f^{-1}(B))$\subset$ F. Then by Theorem 3.6, f is quasi μs-open.

Lemma 3.8. *Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Given that $g \ o \ f : X \to Z$ is quasi μs-open. If g is continuous injective, then f is quasi μs-open.*

Proof: Let U be a μs-open set in X. Then (g o f)(U) is open in Z since g o f is quasi μs-open. Again g is an injective continuous function, f (U) = g^{-1}(g o f (U)) is open in Y . This shows that f is quasi μs-open.

4 Quasi μs-Closed Functions

Definition 4.1. *A function $f : X \to Y$ is said to be quasi μs-closed if the image of each μs-closed set in X is closed in Y.*

Clearly, every quasi μs-closed function is closed as well as μs-closed.

Remark 4.2. *Every μs-closed (resp. closed) function need not be quasi μs-closed as shown by the following example.*

Example 4.3. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ by f(a)=b, f(b)=c and f(c)=a. Then clearly f is μs-closed as well as closed but not quasi μs-closed.*

Lemma 4.4. *If a function $f : X \to Y$ is quasi μs-closed, then $f^{-1}(int(B)) \subset \mu s\text{-}int(f^{-1}(B))$ for every subset B of Y.*

Proof: This proof is similar to the proof of Lemma 3.3.

Theorem 4.5. *A function $f : X \to Y$ is quasi μs-closed if and only if for any subset B of Y and for any μs-open set G of X containing $f^{-1}(B)$, there exists an open set U of Y containing B such that $f^{-1}(U) \subset G$.*

Proof: This proof is similar to that of Theorem 3.6.

Definition 4.6. *A function $f : X \to Y$ is called μs^*-closed if the image of every μs-closed subset of X is μs-closed in Y.*

Theorem 4.7. *If $f : X \to Y$ and $g : Y \to Z$ are two quasi μs-closed functions, then $g \ o \ f: X \to Z$ is a quasi μs-closed function.*

Proof. Obvious.
Furthermore, we have the following theorem

Theorem 4.8. *Let $f : X \to Y$ and $g : Y \to Z$ be any two functions. Then: (i) if f is μs-closed and g is quasi μs-closed, then g o f is closed; (ii) if f is quasi μs-closed and g is μs-closed, then g o f is μs^* -closed; (iii) if f is μs^* -closed and g is quasi μs-closed, then g o f is quasi μs-closed.*

Proof. Obvious.

Theorem 4.9. *Let $f : X \to Y$ and $g : Y \to Z$ be two functions such that $g \ o \ f : X \to Z$ is quasi μs-closed. Then: (i) if f is μs-irresolute surjective, then g is closed. (ii) if g is μs-continuous injective, then f is μs^*-closed.*

Proof: (i) Suppose that F is an arbitrary closed set in Y. As f is μs-irresolute, $f^{-1}(F)$ is μs-closed in X. Since g o f is quasi μs-closed and f is surjective, (g o $f(f^{-1}(F))$) = g(F), which is closed in Z. This implies that g is a closed function. (ii) Suppose F is any μs-closed set in X. Since g o f is quasi μs-closed, (g o f)(F) is closed in Z. Again g is a μs-continuous injective function, g^{-1}(g o f(F)) = f (F), which is μs-closed in Y. This shows that f is μs*-closed.

Theorem 4.10. *Let X and Y be topological spaces. Then the function $g : X \to Y$ is a quasi μs-closed if and only if g(X) is closed in Y and $g(V) \setminus g(X \setminus V)$ is open in g(X) whenever V is μs-open in X.*

Proof: Necessity: Suppose g : X $\to$ Y is a quasi μs-closed function. Since X is μs-closed, g(X) is closed in Y and $g(V) \setminus g(X \setminus V) = g(V) \cap g(X) \setminus g(X \setminus V)$ is open in g(X) when V is μs-open in X.
Sufficiency: Suppose g(X) is closed in Y, $g(V) \setminus g(X \setminus V)$ is open in g(X) when V is μs-open in X, and let C be closed in X. Then $g(C) = g(X) \setminus (g(X \setminus C) \setminus g(C))$ is closed in g(X) and hence, closed in Y.

Corollary 4.11. *Let X and Y be topological spaces. Then a surjective function $g : X \to Y$ is quasi μs-closed if and only if $g(V) \setminus g(X \setminus V)$ is open in Y whenever V is μs-open in X.*

Proof: Obvious.

Corollary 4.12. *Let X and Y be topological spaces and let $g : X \to Y$ be a μs-continuous quasi μs-closed surjective function. Then the topology on Y is $\{g(V) \setminus g(X \setminus V) : V$ is μs-open in X$\}$.*

Proof: Let W be open in Y. Then $g^{-1}(W)$ is μs-open in X, and $g(g^{-1}(W)) \setminus g(X \setminus g^{-1}(W)) = W$. Hence, all open sets in Y are of the form $g(V) \setminus g(X \setminus V)$, V is μs-open in X. On the other hand, all sets of the form $g(V) \setminus g(X \setminus V)$, V is μs-open in X, are open in Y from Corollary 4.11.

Definition 4.13. *A topological space (X, τ) is said to be μs*-normal if for any pair of disjoint μs-closed subsets F_1 and F_2 of X, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.*

Theorem 4.14. *Let X and Y be topological spaces with X is μs*-normal. If $g : X \to Y$ is a μs-continuous quasi μs-closed surjective function, then Y is normal.*

Proof: Let K and M be disjoint closed subsets of Y . Then $g^{-1}(K)$, $g^{-1}(M)$ are disjoint μs-closed subsets of X. Since X is μs*-normal, there exist disjoint open sets V and W such that $g^{-1}(K) \subset V$ and $g^{-1}(M) \subset W$. Then $K \subset g(V) \setminus g(X \setminus V)$ and $M \subset g(W) \setminus g(X \setminus W)$. Further by Corollary 4.11, $g(V) \setminus g(X \setminus V)$ and $g(W) \setminus g(X \setminus W)$ are open sets in Y and clearly $(g(V) \setminus g(X \setminus V)) \cap (g(W) \setminus g(X \setminus W)) = \phi$. This shows that Y is normal.

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