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ON SOME NEW SUBSETS OF NANO TOPOLOGICAL SPACES

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Abstaract — In this paper, we introduce some kernels in nano topological spaces, nano $\wedge_r$-set and nano λ-closed sets investigate some of their properties.

Keywords — *nano $\wedge_r$-set, nano $\wedge_\pi$-set, nano λ-closed set and nano λ_π-closed set*

1 Introduction

Lellis Thivagar et al [4] introduced a nano kernel to nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

In this paper, we introduce some kernels, nano $\wedge_r$-set, nano $\wedge_\pi$-set, nano λ-closed set and nano λ_π-closed set in nano topological spaces and investigate some of their properties.

2 Preliminary

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

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Definition 2.1. *[5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Proposition 2.2. *[3] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. *[3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*
2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*
3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. *[3] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[3] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.*

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. *[3] A subset H of a nano topological space $(U, \tau_R(X))$ is called nano regular-open $H = Nint(Ncl(H))$.*

The complement of the above mentioned set are called their respective closed set.

Definition 2.7. *[1] Let H be a subset of a space $(U, \tau_R(X))$ is nano π-open if the finite union of nano regular-open sets.*

Definition 2.8. *[4] Let $(U, \tau_R(X))$ be a nano topological spaces and $H \subseteq U$. The nano $Ker(H) = \bigcap\{U : H \subseteq U, U \in \tau_R(X)\}$ is called the nano kernal of H and is denoted by $\mathcal{N}Ker(H)$.*

Definition 2.9. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

1. *nano g-closed [2] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*
2. *nano rg-closed set [6] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano regular-open.*

3 On Some New Subsets of Nano Topological Spaces

Definition 3.1. *A subset H of a space $(U, \tau_R(X))$ is called a nano $\wedge$-set if $H = \mathcal{N}Ker(H)$.*

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$. Then $\{a\}$ is nano $\wedge$-set.*

Definition 3.3. *A subset H of a space $(U, \tau_R(X))$ is called nano λ-closed if $H = L \cap F$ where L is a nano $\wedge$-set and F is nano closed.*

Example 3.4. *In Example 3.2, then $\{b, c, d\}$ is nano λ-closed.*

Lemma 3.5.
1. *Every nano $\wedge$-set is nano λ-closed.*
2. *Every nano open set is nano λ-closed.*

3. Every nano closed set is nano λ-closed.

Remark 3.6. *The converses of statements in Lemma 3.5 are not necessarily true as seen from the following Examples.*

Example 3.7. *In Example 3.2,*

1. *then $\{c\}$ is nano λ-closed but not nano $\wedge$-set.*
2. *then $\{a, c\}$ is nano λ-closed but not nano open.*
3. *then $\{b, d\}$ is nano λ-closed but not nano closed.*

Lemma 3.8. *For a subset H of a space $(U, \tau_R(X))$, the following conditions are equivalent.*

1. *H is nano λ-closed.*
2. *$H = L \cap Ncl(H)$ where L is a nano $\wedge$-set.*
3. *$H = \mathcal{N}Ker(H) \cap Ncl(H)$.*

Lemma 3.9. *A subset $H \subset (U, \tau_R(X))$ is nano g-closed if and only if $Ncl(H) \subset \mathcal{N}Ker(H)$.*

Definition 3.10. *Let H be a subset of a space $(U, \tau_R(X))$ is nano πg-closed if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.*

Example 3.11. *In Example 3.2, then $\{b, c, d\}$ is nano πg-closed.*

Remark 3.12. *For a subset of a space $(U, \tau_R(X))$, we have the following implications:*

$$\textbf{nano closed} \rightarrow \textbf{nano } g\textbf{-closed} \rightarrow \textbf{nano } \pi g\textbf{-closed} \rightarrow \textbf{nano } rg\textbf{-closed}$$

None of the above implications are reversible.

Theorem 3.13. *For a subset H of a space $(U, \tau_R(X))$, the following conditions are equivalent.*

1. *H is nano closed.*
2. *H is nano g-closed and nano λ-closed.*

Proof. (1) $\Rightarrow$ (2): Obvious by Remark 3.12 and (3) of Lemma 3.5.

(2) $\Rightarrow$ (1): Since H is nano g-closed, by Lemma 3.9, $Ncl(H) \subset \mathcal{N}Ker(H)$. Since H is nano λ-closed, by Lemma 3.8, $H = \mathcal{N}Ker(H) \cap Ncl(H) = Ncl(H)$. Hence H is nano closed.

Remark 3.14. *In a nano topological space, the concepts of nano g-closed sets and nano λ-closed sets are independent as seen from the following Examples.*

Example 3.15. *In Example 3.2,*

1. *then* $\{b, c\}$ *is nano g-closed set but not nano* λ*-closed.*
2. *then* $\{a\}$ *is* λ*-closed set but not nano g-closed.*

Remark 3.16. *Theorem 3.13 together with Remark 3.14 and Example 3.15 gives a decomposition of nano closed set into a nano g-closed set and a* λ*-nano closed set.*

Definition 3.17. *Let* H *be a subset of a space* $(U, \tau_R(X))$*. Then*

1. *The nano r-Kernel of the set* H*, denoted by* $\mathcal{N}r$*-Ker(H), is the intersection of all nano regular-open supersets of* H*.*
2. *The nano* π*-Kernel of the set* H*, denoted by* $\mathcal{N}\pi$*-Ker(H), is the intersection of all nano* π*-open supersets of* H*.*

Example 3.18. *In Example 3.2,*

1. *then* $\{a\}$ *is nano r-Kernel.*
2. *then* $\{b, d\}$ *is nano* π*-Kernel.*

Definition 3.19. *A subset* H *of a space* $(U, \tau_R(X))$ *is called*

1. *nano* $\wedge_r$*-set if* $H = \mathcal{N}r$*-Ker(H).*
2. *nano* $\wedge_\pi$*-set if* $H = \mathcal{N}\pi$*-Ker(H).*

Example 3.20. *In Example 3.2,*

1. *then* $\{b, d\}$ *is nano* $\wedge_r$*-set.*
2. *then* $\{a\}$ *is nano* $\wedge_\pi$*-set.*

Definition 3.21. *A subset* H *of a space* $(U, \tau_R(X))$ *is called*

1. *nano* λ_r*-closed if* $H = L \cap F$ *where* L *is a nano* $\wedge_r$*-set and* F *is nano closed.*
2. *nano* λ_π*-closed if* $H = L \cap F$ *where* L *is a nano* $\wedge_\pi$*-set and* F *is nano closed.*

Example 3.22. *In Example 3.2,*

1. *then* $\{b, c, d\}$ *is nano* λ_r*-closed.*
2. *then* $\{a, c\}$ *is nano* λ_π*-closed.*

Lemma 3.23.

1. *Every nano closed set is nano* λ_r*-closed.*
2. *Every nano* $\wedge_r$*-set is nano* λ_r*-closed.*
3. *Every nano closed set is nano* λ_π*-closed.*
4. *Every nano* $\wedge_\pi$*-set is nano nano* λ_π*-closed.*

Remark 3.24. *The converses of the statements in Lemma 3.23 are not necessarily true as seen from the following Examples.*

Example 3.25. *In Example 3.2,*

1. *then* $\{b, d\}$ *is nano* λ_r*-closed set but not nano closed.*
2. *then* $\{a, c\}$ *is nano* λ_r*-closed set but not nano* $\wedge_r$*-set.*
3. *then* $\{b, d\}$ *is nano* λ_π*-closed set but not nano closed.*
4. *then* $\{b, c, d\}$ *is nano* λ_π*-closed but not nano* $\wedge_\pi$*-set.*

Lemma 3.26. *For a subset* H *of a space* $(U, \tau_R(X))$*, the following are equivalent.*

1. (a) *H is nano* λ_r*-closed.*
   (b) $H = L \cap Ncl(H)$ *where L is a nano* $\wedge_r$*-set.*
   (c) $H = \mathcal{N}r$*-Ker(H)* $\cap$ *Ncl(H).*
2. (a) *H is nano* λ_π*-closed.*
   (b) $H = L \cap Ncl(H)$ *where L is a nano* $\wedge_\pi$*-set.*
   (c) $H = \mathcal{N}\pi$*-Ker(H)* $\cap Ncl(H)$.

Lemma 3.27. 1. *A subset* $H \subset (U, \tau_R(X))$ *is nano* πg*-closed if and only if* $Ncl(H) \subset \mathcal{N}\pi$*-Ker(H).*

2. *A subset* $H \subset (U, \tau_R(X))$ *is nano rg-closed if and only if* $Ncl(H) \subset \mathcal{N}r$*-Ker(H).*

Theorem 3.28. *For a subset* H *of a space* $(U, \tau_R(X))$*, the following are equivalent.*

1. *H is nano closed.*
2. *H is nano* πg*-closed and nano* λ_π*-closed.*

Proof. (1)⇒(2) Proof follows by Remark 3.12 and (6) of Lemma 3.23.

(2)⇒(1) By Lemma 3.27 and Lemma 3.26(2), proof follows similar to the proof of Theorem 3.13.

Remark 3.29. *In a nano topological space, the concepts of nano* λ_π*-closed sets and nano* πg*-closed sets are independent as seen from the following Examples.*

Example 3.30. *Let* $U = \{a, b, c, d\}$ *with* $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ *and* $X = \{b, d\}$*. Then the nano topology* $\tau_R(X) = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}, U\}$*.*

1. *then* $\{a, c\}$ *is nano* πg*-closed set but not nano* λ_π*-closed.*
2. *then* $\{c, d\}$ *is nano* λ_π*-closed set but not nano* πg*-closed.*

Remark 3.31. *Theorem 3.28 together with Remark 3.29 and Example 3.30 gives a decomposition of nano closed set into a nano* λ_π*-closed set and a nano* πg*-closed set.*

Theorem 3.32. *For a subset* H *of a space* $(U, \tau_R(X))$*, the following are equivalent.*

1. *H is nano closed.*

2. *H is nano rg-closed and nano λ_r-closed.*

Proof. (1) $\Rightarrow$ (2) Proof follows by Remark 3.12 and (3) of Lemma 3.23.
(2) $\Rightarrow$ (1) By Lemma 3.27 and Lemma 3.26(1), proof follows similar to the proof of Theorem 3.13.

Remark 3.33. *In a nano topological space, the concepts of nano λ_r-closed sets and nano rg-closed sets are independent as seen from the following Examples.*

Example 3.34. *In Example 3.2,*

1. *then $\{a\}$ is nano λ_r-closed set but not nano rg-closed.*

2. *then $\{a, d\}$ is nano rg-closed set but not nano λ_r-closed.*

Remark 3.35. *Theorem 3.32 together with Remark 3.33 and Example 3.34 gives a decomposition of nano closed set into a nano λ_r-closed set and a nano rg-closed set.*

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