Karadeniz Fen Bilimleri Dergisi, 15(1), 171-181, 2025. DOI: 10.31466/kfbd.1517746

KFBD

Karadeniz Fen Bilimleri Dergisi
The Black Sea Journal of Sciences
ISSN (Online): 2564-7377 https://dergipark.org.tr/tr/pub/kfbd

Araştırma Makalesi / Research Article

Predicting $P_c(4440)$ and $P_c(4457)$ Spins with an Different Alternative Method Instead of Using the Breit-Wigner Approach

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Abstract

The heavy quark symmetries play a necessary role in understanding heavy hadron states. If the $P_c(4440)$ and $P_c(4457)$ states are indeed $\bar{D}^*\Sigma_c$ molecules, they can be classified as heavy quark spin symmetry partners. These acceptances pave the way to clarify spin states that are not determined experimentally. Despite detailed studies on the $P_c(4440)$ and $P_c(4457)$ states, their spin states remain undetermined. While the Breit-Wigner parameterization is the conventional method for obtaining resonance parameters, it is unsuitable for near-threshold resonances such as the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ due to its failure to account for the threshold effect. To rectify this limitation, the recently proposed alternative distribution called the Sill is employed to predict their spin states. The use of the Sill values for the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ may assist in determining the spin states of the $P_c(4440)$ and $P_c(4457)$.

Keywords: $P_c(4440)$, $P_c(4457)$, HQSS, Hadronic molecule.

Breit-Wigner Yaklaşımından Farklı Bir Yöntemle $P_c(4440)$ ve $P_c(4457)$ Spinlerinin Tahmin Edilmesi

Öz

Ağır kuark simetrileri ağır hadronların anlaşılmasında önemli bir rol oynar. Eğer $P_c(4440)$ ve $P_c(4457)$ durumları gerçekten $\bar{D}^*\Sigma_c$ molekülü ise, ağır kuark spin simetri eşleri olarak sınıflandırılabilirler. Bu kabuller, deneysel olarak belirlenemeyen spin durumlarını açıklığa kavuşturulmasının yolunu açmaktadır. $P_c(4440)$ ve $P_c(4457)$ durumları detaylı çalışılmalarına rağmen spin durumları henüz belirlenememiştir. Breit-Wigner parametrizasyonu rezonansları elde etmekte geleneksel yöntem olmasına rağmen, eşik etkisini dikkate almadığından dolayı $P_c(4312)$, $P_c(4440)$ ve $P_c(4457)$ gibi eşiğe yakın rezonanslar için uygun değildir. Bu eksikliği bertaraf etmek için yakın zamanda önerilen ve Sill adı verilen alternatif dağılım, spin durumlarını tahmin etmekte kullanılacaktır. $P_c(4312)$, $P_c(4440)$ ve $P_c(4457)$ durumları için Sill değerlerinin kullanılması, $P_c(4440)$ ve $P_c(4457)$' nin spin durumlarının belirlenmesinde yardımcı olabilir.

Anahtar Kelimeler: $P_c(4440)$, $P_c(4457)$, AKSS, Hadronik molekül.

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Geliş/Received: 17.07.2024 **Kabul/Accepted:** 24.02.2025 **Yayın/Published:** 15.03.2025

Yldırım, D. (2025). Predicting Pc(4440) and Pc(4457) Spins with an Different Alternative Method Instead of Using the Breit-Wigner Approach. *Karadeniz Fen Bilimleri Dergisi*, 15(1), 171-181.

1. Introduction

In the last twenty years, scientists have observed many hadron states, some of which have been difficult to categorize. After the discovery of the $\chi_{c1}(3872)$ (Choi et al., 2020), the interest in hadrons has increased. During this time interval, significant effort has been focused mainly on understanding the $\chi_{c1}(3872)$, $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ in the charm sector and the $Z_b(10610)$ and $Z_b(10650)$ in the bottom sector. These resonances, which do not fit the conventional quark model, called exotics, are primarily explained as a hadronic molecule due to its proximity to the $D^{*0}\bar{D}$, $\bar{D}\Sigma_c$, $\overline{D^*}\Sigma_c$, and $B^{(*)}\bar{B}^{(*)}$ thresholds, respectively. In addition to molecular interpretation, various alternative approaches have been documented in the literature, including tetraquark, pentaquark, and hadrocharmonium models, among others.

The $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ are observed in the $J/\Psi p$ invariant mass distribution of the decay $\Lambda_b \rightarrow J/\Psi p K^-$ (Aaij et al., 2015; Aaij et al., 2019). All these states consist of at least five quarks, specifically $uud\bar{c}c$. This composition makes them suitable to be considered as pentaquarks (Cheng and Liu, 2019; Pimikov et al., 2020; Weng et al., 2019). However, the proximity of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ states (referred to as P_c states to avoid repetition) to the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ states (referred to as P_c states to avoid repetition) to $\bar{D}\Sigma_c$ and $\bar{D}^*\Sigma_c$ thresholds, along with their small decay widths, as given in Table 1, are also notable. Therefore, it is commonly suggested that they are hadronic molecules (Du et al., 2020; Xiao et al., 2019). However, other proposals exist, such as hadrocharmonium (Eides et al., 2020) and virtual states (Fernandez-Ramirez et al., 2019).

The P_c states are currently favored to have a molecular interpretation, but their spin states are not entirely clear. Determining the spin state is essential for distinguishing between the different P_c states. So far, we cannot determine the spin state of the $P_c(4440)$ and $P_c(4457)$ with the data at hand, but it is predicted that either the $P_c(4440)$ and $P_c(4457)$ are $J = \frac{1}{2}^-$ and $J = \frac{3}{2}^-$, or vice versa. In hadron physics, it is generally expected that a higher mass state will have a higher spin. Many papers have attempted to determine their spin states using different approaches. Some studies suggest that the spin of the $P_c(4440)$ is $J^P = \frac{1}{2}^-$ and $P_c(4457)$ is $J^P = \frac{3}{2}^-$ (Chen et al., 2019; Liu et al., 2023; Zhang et al., 2023), while others have the opposite identification (Karliner and Rosner, 2015; Liu et al., 2021; Yamaguchi et al.,2020). Additionally, Ref. (Chen et al., 2019) suggests that the $P_c(4440)$ and $P_c(4457)$ are $J^P = \frac{3}{2}^-$ or $P_c(4440)$ is $\Sigma\bar{D}$ with $J^P = \frac{1}{2}^-$ and $P_c(4457)$ is $\Sigma^*\overline{D^*}$ with $J^P = \frac{5}{2}^-$. These results might change when other contributions, such as one pion exchange, are considered. For instance, it is pointed out that the selection of quantum numbers changes when pion contribution is included

(Valderrama, 2019). With contact interactions only, the $P_c(4440)$ is $J^P = \frac{1}{2}^-$ and $P_c(4457)$ is $J^P = \frac{3}{2}^-$ but one pion exchange contribution reverses the situation. The preferred spin states of the $P_c(4440)$ become $\frac{3}{2}^-$ and $P_c(4457)$ becomes $\frac{1}{2}^-$ with the contribution of one pion exchange due to attractive/repulsive consequences for the channel. Some literature even suggests positive parity designation for the $P_c(4440)$ and $P_c(4457)$ states (Xiang et al., 2019). As can be seen, there are different proposals regarding their spin states. Since the scope of this work is the leading order, only contact interactions are taken into consideration.

Several methods are used to study resonances, such as the Breit-Wigner, Flatte parameterization, and effective-range expansion. The Breit-Wigner parametrization is the most accepted method, and experimental data is primarily analyzed using this method (Aubert et al., 2008; Bonder et al., 2012; Zyla et al., 2008). These methods differ from each other in their handling of threshold effects. The Breit-Wigner and the Flatte parameterization are affected by threshold effects, while the effective-range expansion is not affected by nearby thresholds. The effective range expansion parameterization is commonly used in compared with the Flatte parametrization because it can produce negative effective ranges (Kang et al., 2017). Regarding the effects of nearby thresholds, Ref. (Kang et al., 2017) emphasized this point and studied the $\chi_{c1}(3872)$, with near-threshold resonances, using the effective-range expansion approximation up to and including the effective-range contribution. It was noted that more data are required for a more precise conclusion about the $\chi_{c1}(3872)$. For instance, taking into account the threshold effect, the authors of Ref. (Kang et al., 2016) examined the $Z_b^{(,)}$ states using effective-range expansion. They stated that the Breit-Wigner function aligns with the data when the resonance is above the threshold, approximately 3 MeV. Refs. (Cleven et al., 2011; Cleven, 2013) suggested that if the $Z_b^{(,)}$ are considered a molecular state, the data should not be interpreted using the Breit-Wigner parametrization. Therefore, the Breit-Wigner parameterization has some complexities. Recently, an alternative method called the Sill has been proposed for near-threshold resonances instead of the Breit-Wigner (Giacosa et al., 2021). This approach has been shown to be better in some circumstances, including exotic states. With this new parameterization, our study aims to examine the pentaquark states and see whether they will contribute to determining their spin states.

2. Formalism

The extracted properties of a resonance, such as mass and decay width, depend on how a resonance is defined. It means that if the definition of resonance changes, these properties will also change. The Breit-Wigner distribution formula is the most used and accepted description of resonance is given as

$$d^{BW}(E) = \frac{\Gamma}{2\pi}\frac{1}{(E-M)^2+\frac{\Gamma^2}{4}} \quad , \tag{1}$$

where M is the mass and Γ the decay width of the resonance. This formula is mainly used to describe unstable states. While the formula is effective for most resonances, especially fundamental particles like Z^0 and $W^{\pm}$, it has limitations regarding near-threshold resonances and complex structures. For example, the threshold effect masks the resonance region, so using the Breit-Wigner formalism deviates from unitarity and analyticity. Using the real and imaginary parts of the resonance, the Sill resonance values were calculated utilizing Eq. 2. Additionally, the branching ratios calculated using the Breit-Wigner parameterization for near-threshold states may not accurately represent the decay probabilities (Chen et al., 2016). According to Ref. (Cleven et al., 2011), the Breit-Wigner formula should not be used for near-threshold states. It is important to examine which approach is more suitable and applicable across a wider range in order to understand near-threshold exotic resonances.

A new formalism was proposed to address the threshold issue inherent in the Breit-Wigner distribution. The distribution function was modified as described in Ref. (Giacosa et al., 2021), known as the Sill:

$$d^{Sill}(E) = \frac{2E}{\pi}\frac{\sqrt{E^2-E_{th}^2}\tilde{\Gamma}}{(E^2-M^2)^2+\left(\sqrt{E^2-E_{th}^2}\tilde{\Gamma}\right)^2}\theta(E-E_{th}) \ , \tag{2}$$

where $\tilde{\Gamma}$ is defined as $\tilde{\Gamma} = \Gamma\, M/\sqrt{M^2 - E_{th}^2}$. The Sill has threshold effects and can be applied to mesons and exotics because this new distribution is not sensitive to the inner content of the hadron. According to the paper, when the Sill is applied to various ranges of resonances, it provides better results with experiments compared to the results obtained with the Breit-Wigner formalism. Using the real and imaginary parts of the resonance, the Sill resonance values were calculated utilizing Eq. 2. Additionally, a method was proposed for the applicability of the Sill approach. The authors stated that, compared with the Breit-Wigner, the Sill is preferred when the ratio $\Gamma/\Delta m$ is greater than 1/3. Here, Γ represents the decay width of the state, and Δm represents the state's distance to the threshold. This criterion also suits P_c resonances.

In the field of QCD, effective field theories can be used to study exotic states by considering QCD symmetries. When dealing with heavy quarks, symmetries are crucial in describing the heavy spectra and predicting possible states. Heavy quark spin symmetry arises in the heavy quark limit (Neubert, 1994). Heavy quark's spin becomes decoupled in this limit. Therefore, it implies that the pseudoscalar meson D and the vector meson D^* are degenerate. Due to this degeneration, they can be expressed together as a linear combination of them in a nonrelativistic superfield matrix, which is described (Falk, 1992; Grinstein, 1992; Nieves, 2012),

$$H_c = \frac{1}{\sqrt{2}}\left[D + \overrightarrow{D^*} \cdot \vec{\sigma}\right] \tag{3}$$

where $\vec{\sigma}$ is the Pauli matrices. The H superfield respects heavy quark rotations and transforms as a doublet.

During the paper, we are interested in a spin-$\frac{1}{2}$ Σ and spin-$\frac{3}{2}$ Σ^* states; therefore, we are dealing with the baryons whose total light spin is 1. In the limit of heavy quark mass goes to infinity, these baryons are degenerate; therefore, we can put them in a superfield matrix (Cho, 1994; Liu et al., 2018; Lu, 2019):

$$\overrightarrow{S_c} = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_c + \overrightarrow{\Sigma_c^*} \quad . \tag{4}$$

In this representation, the spin-$\frac{3}{2}$ heavy baryon field is referred to the condition $\sigma \cdot \vec{\Sigma} *= 0$, which ensures that the Σ^* is a spin-$\frac{3}{2}$ field. With the H_c and $\vec{S}_c$ superfields, the Lagrangian containing the contact range interaction without derivatives can be written as in a covariant representation (Liu et al., 2018),

$$\mathcal{L} = C_a \overrightarrow{S^\dagger} \cdot \vec{S}\mathrm{Tr}[\bar{H}^\dagger \bar{H}] + C_b \sum_{i=1}^{3} \overrightarrow{S^\dagger} \cdot \left(J_i \vec{S}\right)\mathrm{Tr}[\bar{H}^\dagger \sigma_i \bar{H}] \tag{5}$$

where C_a and C_b are low energy coupling constants. The matrices J_i with i = 1, 2, 3 represent the spin-1 angular momentum. The Lagrangian is assumed to be independent of heavy flavor. For simplicity, we are ignoring isospin in our study. The Lagrangian associated with this leads to the formation of seven contact $\bar{D}^{(*)}\Sigma^{(*)}$ molecules. The resulting states and contact potentials can be found in Table 2.

To search for bound states, we need to solve the Lippmann-Schwinger equation. The solution of the Lippmann-Schwinger equation corresponds to the mass of the bound state. In momentum space, the Lippmann-Schwinger equation can be expressed as:

$$\phi(k) + \int \frac{d^3p}{(2\pi)^3} < k|V|p> \frac{\phi(p)}{B+\frac{p^2}{2\mu}}=0 \, , \tag{6}$$

where $\phi(k)$ is the vertex function, B the binding energy, and μ the reduced mass of the system. For the regularization of the contact range potential with the regulator function f(x), the potential is given as

$$\langle p|V\,|k\,\rangle = C\,(\Lambda)\, f\left(\frac{p}{\Lambda}\right) f\left(\frac{k}{\Lambda}\right), \tag{7}$$

where $f(x) = e^{-x^2}$ is the Gaussian regulator function, which is chosen to regularize the ultraviolet divergence. The low energy constants' dependence on the Λ cutoff is undesirable. However, this dependence is mild and can be reinterpreted as contact terms. The Λ cutoff is varied from 1.0 GeV to 1.5 GeV, but no significant changes in masses are observed for these two values. Therefore, the Λ cutoff is chosen for numerical calculations as 1.0 GeV. As a result, the dependence of cutoff is not a concern in this context.

The symmetry we are considering is exact when the mass of the heavy quark is approaching infinity. Since the mass of heavy quarks is finite, it is expected to have some level of deviation. For heavy quark spin symmetry, this deviation is approximately on the order of $\mathcal{O}(\Lambda_{QCD}/m_Q)$. Given that Λ_{QCD}~200 MeV, the estimated deviation is about 15% in the charm sector. The deviation is primarily due to the influence of corrections and subleading orders. Including this uncertainty in numeric calculations causes deviations at the states' locations. Although these deviations provide valuable information about the limits of HQSS, no results outside the uncertainty bounds are expected. In some cases, the parameterization of the Sill completely changes its definition (Yıldırım, 2023); in here, no significant change is seen.

For the $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecular system, there are seven equations corresponding to seven possible s-wave states. Three P_c states can be identified as heavy quark spin symmetry partners: $\bar{D}\Sigma_c$ as $P_c(4312)$, and the $\bar{D}^*\Sigma_c$ molecules as $P_c(4440)$, and $P_c(4457)$. Therefore, three observed P_c states are appropriate as labels for hadronic molecules. $m_D = 1868$ MeV, $m_{D^*} = 2009$ MeV, $m_{\Sigma_c} = 2454$ MeV and $m_{\Sigma_c^*} = 2518$ MeV values are used in the numerical calculations (Workman et al., 2022). The determination of two unknown counterterms, C_a and C_b, are essential for accurate predictions of the complete spectrum of these molecular states. Initially, the states $P_c(4440)$ and $P_c(4457)$ are considered as the $\bar{D}^*\Sigma_c$ molecule, with two unknown coupling constants. By utilizing the masses of the $P_c(4440)$ and $P_c(4457)$ states, we can determine the values of C_a and C_b. The spin states of these molecular states have two potential options, labeled as option 1 and option 2. In option 1, the $P_c(4440)$ is assigned a spin of $J^P = \frac{1}{2}^-$ and the $P_c(4457)$ is assigned $J^P = \frac{3}{2}^-$, while the spin states are interchanged in option 2. The mass of the $P_c(4312)$ state has not been incorporated as input in the existing literature, so this information is still pending. As a follow-up step, the $P_c(4312)$ state is used in combination with the alternative spin states of $P_c(4440)$, labeled as option 3 and option 4. These options consider the $P_c(4440)$ with spin $J^P = \frac{1}{2}^- (\frac{3}{2}^-)$ in conjunction with the $P_c(4312)$ state.

3. Results and Discussions

The Sill mass and width values obtained by Eq. 2, which are a few MeV higher than the Breit-Wigner values, are shown in the third column of Table 1. While Table 2 represents the results obtained by taking Breit-Wigner mass values as input from Table 1, Table 3 shows the results obtained by taking the Sill values from Table 1. First, Table 2 shows that all states are classified as bound states for all options, even when considering the uncertainty of heavy quark spin symmetry, suggesting that all are likely to be detectable in experiments. But there is an exception. In the last option, $\frac{1}{2}^-$ $\bar{D}^*\Sigma_c^*$ state is seen at the threshold. However, detecting three $\bar{D}^*\Sigma_c^*$ states is also challenging due to the low production rates in Λ_b decays. This issue is also discussed in Ref. (Du et al., 2020). The $\bar{D}^*\Sigma_c^*$ states do not interact with $J/\Psi p$ in s-wave, as explained in Ref. (Xiao, 2019), indicating that the $\bar{D}^*\Sigma_c^*$ with $\frac{5}{2}^-$ state is unlikely to be observed in the LHCb experiment. Second, the $\bar{D}\,\Sigma_c^*$ state is predicted rather consistent with the experimental value, $m_{P_c(4380)} = 4380$ MeV, especially within the first option. Additionally, other options are also compatible with the experimental results. Third, as expected, higher spin states have higher mass values for $\bar{D}^*\Sigma_c^*$ for options 1 and 3. This status is the opposite of the second and fourth options.

In Table 3, the obtained Sill mass values are used as input and seen parallel to the Breit-Wigner mass values, as expected, because there are no significant changes in the Sill masses. All results are found below the thresholds. Like the Breit-Wigner results, the first option yields the best value for the $P_c(4380)$ mass. Additionally, the first and third options support the expectation that higher spin is associated with a higher mass.

We used heavy quark symmetries to determine the spins of the $P_c(4440)$ and $P_c(4457)$. Using heavy quark spin symmetry, we easily identified the P_c states as $\bar{D}^*\Sigma_c$ molecular states. Initially, we considered the Breit-Wigner mass values as inputs of the $P_c(4440)$ and $P_c(4457)$ as hadronic $\bar{D}^*\Sigma_c$

molecules. Subsequently, we proposed two possible assignments to clarify their spin states and studied their corresponding uncertainties. We found that all $\bar{D}^{(*)}\Sigma_c^{(*)}$ states are bound for the first two

Table 1. Mass and decay widths obtained by the Breit-Wigner parametrization's (Zyla et al., 2020) and the Sill's approach for the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ resonances. The results are given in MeV.

	The Breit-Wigner $(E - i\,\Gamma/2)$	The Sill $(E - i\,\Gamma)$
$P_c(4312)$	4311.9-i9.8/2	4313.1-i5.0
$P_c(4440)$	4440.3-i20.6/2	4442.7-10.6
$P_c(4457)$	4457.2-i6.4/2	4458.3-i3.3

Table 2. The leading order contact range potential derived from heavy quark spin symmetry for the heavy meson and heavy baryon system depends on the linear combination of two, C_a and C_b, coupling constants. These coupling constants are determined from reproducing masses of the Breit-Wigner $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ with each possible options. Mass results are given in MeV.

Molecule	J^P	V	Option 1	Option 2	Option 3	Option 4	Threshold
$\bar{D}\,\Sigma_c$	$\frac{1}{2}^-$	C_a	4312	4308	$P_c(4312)$	$P_c(4312)$	4322
$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	C_a	4378	4373	4376	4376	4386
$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	$C_a - \frac{4}{3}C_b$	$P_c(4440)$	$P_c(4457)$	$P_c(4440)$	4462	4463
$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	$C_a + \frac{2}{3}C_b$	$P_c(4457)$	$P_c(4440)$	4455	$P_c(4440)$	
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$C_a - \frac{5}{3}C_b$	4501	4524	4501	4527	4527
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$C_a - \frac{2}{3}C_b$	4511	4517	4510	4523	
$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$C_a + C_b$	4524	4501	4521	4498	

options and could potentially be observed in experiments. We then included the $P_c(4312)$ as an input for the last two options with the possible spin options of the $P_c(4440)$ to determine the possible spin states of the $P_c(4440)$ and $P_c(4457)$. As a next step, the Sill parametrization is examined to see different parametrization contributions, and the obtained Sill masses of the pentaquarks are not much different from Breit-Wigner's predictions. Overall, the results appear to be highly consistent with one another, and if one takes both parametrizations with options 1 and 3 into consideration, the $P_c(4440)$ state of $\frac{1}{2}^-$-and the $P_c(4457)$ state of $\frac{3}{2}^-$ are favored. If one of any of the $\bar{D}^*\Sigma_c^*$ states is discovered, it will strengthen our conclusions. Further studies and the future run of the LHC will allow us to make a more definitive prediction.

Table 3. The leading order contact range potential derived from heavy quark spin symmetry for the heavy meson and heavy baryon system depends on the linear combination of two, C_a and C_b, coupling constants. These coupling constants are determined from reproducing masses of the Sill values of $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ with each possible options. Mass results are given in MeV.

Molecule	J^P	V	Option 1	Option 2	Option 3	Option 4
$\bar{D}\,\Sigma_c$	$\frac{1}{2}^-$	C_a	4315	4310	$P_c(4312)$	$P_c(4312)$
$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	C_a	4379	4374	4376	4378
$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	$C_a - \frac{4}{3}C_b$	$P_c(4440)$	$P_c(4457)$	$P_c(4440)$	4460
$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	$C_a + \frac{2}{3}C_b$	$P_c(4457)$	$P_c(4440)$	4455	$P_c(4440)$
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$C_a - \frac{5}{3}C_b$	4503	4524	4501	4524
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$C_a - \frac{2}{3}C_b$	4513	4518	4510	4524
$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$C_a + C_b$	4524	4503	4521	4500

Funding Support

There is no funding source.

Authors' Contributions

All the paper is designed, created and revised by one author.

Competing of Interest

The author declared no competing interests.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

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