http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 02.12.2017
Published: 17.12.2017

Year: **2017**, *Number*: **19**, *Pages*: **27-47**
Original Article

TRAPEZOIDAL LINGUISTIC CUBIC HESITANT FUZZY TOPSIS METHOD AND APPLICATION TO GROUP DECISION MAKING PROGRAM

Aliya Fahmi[1,*] <aliyafahmi@gmail.com>
Saleem Abdullah[2] <saleemabdullah81@yahoo.com>
Fazli Amin[1] <fazliamin@hu.edu.pk>

[1]*Hazara University, Mathematics Department, Mansehra, Pakistan.*
[2]*Abdul Wali Khan University, Mathematics Department, Madan, Pakistan.*

Abstract – In this paper, we define a new idea of trapezoidal linguistic cubic hesitant fuzzy number. We discuss some basic operational laws of trapezoidal linguistic cubic hesitant fuzzy number and hamming distance of trapezoidal linguistic cubic hesitant fuzzy number. Furthermore, we develop Trapezoidal linguistic cubic hesitant fuzzy TOPSIS method to solve the MCDM method based on trapezoidal linguistic cubic hesitant fuzzy TOPSIS method. Finally, an illustrative example is given to verify and demonstrate the practicality and effectiveness of the proposed method.

Keywords *– Trapezoidal linguistic cubic hesitant fuzzy number, trapezoidal linguistic cubic hesitant fuzzy TOPSIS method, MCDM, Numerical Application*

1 Introduction

Selecting a proper supplier amongst different suppliers is a grave substance for highest organization. In skills that are worried with huge gage production the raw resources and unit parts can equal up to 70% creation cost. In such circumstances the procurement department can presentation a vital role in cost reduction, and supplier selection is one of the most energetic functions of getting management [7]. So, by means of an fitting method for this purpose is a vital issue, supplier selection has been revealed to be a multiple criteria decision making (MCDM) problem [13]. In a typical MCDM problem, multiple and usually incompatible criteria are instantaneously booked into description for making a decision. A wide-ranging review and organization of the MCDM approaches for vendor

[*]*Corresponding Author.*

selection has been carried out in [5]. Torra [25] defined the Hesitant fuzzy sets. Xia et al. [26] defined hesitant fuzzy information aggregation in decision making.

(MCDM) is apprehensive with arranging and explaining decision and development problems relating multiple criteria. Typically, there does not exist an inimitable optimal solution for such problems and it is necessary to use decision maker's performance to differentiate between solutions. MCDM has been an active area of research since the 1970's. Different methods have been offered by many researchers, including the Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and MCDM. Some of the most broadly used multi-criteria decision exploration methods is the TOPSIS method, which was proposed by Hwang and Yoon in 1981 [8], and extended by Yoon in 1987 [15], as well as by Hwang et al. in 1993 [16]. In the TOPSIS method, the optimal alternative is nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). When control inaccurate and imprecise date, mostly modelling social Judgments, it is more representative and instinctive to use linguistic assessments instead of numerical evaluations. Thus, in many former studies, the TOPSIS method was used in concurrence with fuzzy logic. Numerous fuzzy TOPSIS methods and applications have been established since the 1990s, e.g., for supplier selection [17] [18], finance [19] [20], power industry [21] [22], and negotiation problems [23]. In our study, we employ a fuzzy extension of the TOPSIS method presented by Chen [24].

Cubic sets introduced by Jun et al. [9], are the generalizations of fuzzy sets and intuitionistic fuzzy sets, in which there are two representations, one is used for the degree of membership and other is used for the degree of non-membership. The membership function is hold in the form of interval while non-membership is thought over the normal fuzzy set. Aliya et al. [2] defined the cubic TOPSIS method and cubic grey ananlysis set. Aliya et al. [1] defined the idea of triangular cubic fuzzy set and hamming distance. Due to the motivation and inspiration of the above discussion in this paper we generalized the concept of trapezoidal linguistic hesitant fuzzy sets, trapezoidal linguistic intutionistic hesitant fuzzy sets, interval-valued trapezoidal linguistic intuitionstic hesitant fuzzy number, trapezoidal linguistic hesitant fuzzy TOPSIS method, interval-valued trapezoidal linguistic hesitant fuzzy TOPSIS method, interval-valued trapezoidal linguistic intuitionstic hesitant fuzzy TOPSIS method and introudce the concept of trapezoidal linguistic cubic hesitant fuzzy sets. If we take only one element in the membership degree of the trapezoidal linguistic cubic hesitant fuzzy number, i.e.instead of interval we take a fuzzy number, than we get trapezoidal linguistic intuitionistic hesitant fuzzy numbers, similarly if we take memebrship degree as hesitant fuzzy number and nonmembership degree equal to zero, than we get trapezoidal linguistic hesitant fuzzy numbers.

In section 2, we firstly introduced some basic definitions of the fuzzy set and cubic set. In section 3, we develop trapezoidal linguistic cubic hesitant fuzzy number and hamming distance. In section 4, we develop trapezoidal linguistic cubic hesitant fuzzy TOPSIS method different steps in the proposed trapezoidal linguistic cubic hesitant fuzzy TOPSIS method are presented. A numerical example of the proposed model is presented in section 5. In section 6, we discuss comparison to different method. The paper is concluded in section 7.

2 **Preliminaries**

Definition [14] 2.1. *Let* H *be a universe of discourse. The idea of fuzzy set was presented by Zadeh and defined as following:* $J=\{h,\Gamma_J(h)\,|\,h\in H\}$. *A fuzzy set in a set* H *is defined* $\Gamma_J : H \to I$, *is a membership function,* $\Gamma_J(h)$ *denoted the degree of membership of the element* h *to the set* H *, where* $I=[0,1]$ *. The collection of all fuzzy subsets of* H *is denoted by* I^H *. Define a relation on* I^H *as follows:*

$$(\forall \Gamma,\eta \in I^H)(\Gamma \le \eta \Leftrightarrow (\forall h\in H)(\Gamma(h)\le \eta(h))).$$

Definition [4] 2.2. *An Atanassov intuitionistic fuzzy set on* H *is a set*

$$J=\{h,\Gamma(h),\eta(h):\ h\in H\}$$

where Γ_J *and* η_j *are membership and non-membership function, respectively*

$$\Gamma_J(h):\ h\to[0,1], h\in H\to \Gamma_J(h)\in[0,1];\eta_J(h):\ h\to[0,1], h\in H\to \eta_J(h)\in[0,1]$$

and

$$0\le \Gamma_J(h)+\eta_J(h)\le 1 \ \text{ for all } \ h\in H\ .\ \pi_J(h)=1-\Gamma_J(h)-\eta_J(h).$$

Definition [9] 2.3. *Let* H *be a nonempty set. By a cubic set in* H *we mean a structure* $F=\{h,\alpha(h),\beta(h):\ h\in H\}$ *in which* α *is an IVF set in* H *and* β *is a fuzzy set in* H. *A cubic set* $F=\{h,\alpha(h),\beta(h):\ h\in H\}$ *is simply denoted by* $F=\langle\alpha,\beta\rangle$. *Denote by* C^H *the collection of all cubic sets in* H. *A cubic set* $F=\langle\alpha,\beta\rangle$ *in which* $\alpha(h)=0$ *and* $\beta(h)=1$ *(resp.* $\alpha(h)=1$ *and* $\beta(h)=0$ *for all* $h\in H$ *is denoted by 0 (resp. 1). A cubic set* $D=\langle\lambda,\xi\rangle$ *in which* $\lambda(h)=0$ *and* $\xi(h)=0$ *(resp.* $\lambda(h)=1$ *and* $\xi(h)=1$ *) for all* $h\in H$ *is denoted by 0 (resp. 1).*

Definition [9] 2.4. *Let* H *be a non-empty set. A cubic set* $F=(C,\lambda)$ *in* H *is said to be an internal cubic set if* $C^-(h)\le\lambda(h)\le C^+(h)$ *for all* $h\in H$.

Definition [9] 2.5. *Let* H *be a non-empty set. A cubic set* $F=(C,\lambda)$ *in* H *is said to be an external cubic set if* $\lambda(h)\notin(C^-(h),C^+(h))$ *for all* $h\in H$.

3. **Trapezoidal Linguistic Cubic Hesitant Fuzzy Number**

Definition 3.1. *Let* $\tilde{b}$ *be the trapezoidal linguistic cubic hesitant fuzzy number on the set of real numbers, its interval value trapezoidal linguistic hesitant fuzzy set is defined as:*

$$\lambda_{\tilde{b}}(h)=\begin{cases} s_{\theta},\frac{(h-r)}{(s-r)}[\omega_{b,}^{-}\omega_{b}^{+}] & r\le h<s \\ [\omega_{b}^{-},\omega_{b}^{+}] & s\le h<t \\ s_{\theta},\frac{(t-h)}{(d^{-}-s^{-})}[\omega_{b}^{-},\omega_{b}^{+}] & t\le h<u \\ 0 & otherwise \end{cases}$$

and its trapezoidal linguistic hesitant fuzzy set

$$\Gamma_{\tilde{b}}(h)=\begin{cases} s_{\theta},\frac{s-h+(h-r)\eta_{\tilde{b}}}{(r-s)}, & r\le h<s \\ \eta_{\tilde{b}} & s\le h\le t \\ s_{\theta},\frac{h-t+(u-h)\eta_{\tilde{b}}}{(u-t)} & t<h\le u \\ 0 & \text{otherwise} \end{cases}$$

where $0\le\lambda_{\tilde{b}}(h)\le 1, 0\le\Gamma_{\tilde{b}}(h)\le 1$ and $r,s,t,u,$ are real numbers. The values of $\omega_{b^-},\omega_{b^+}$ consequently the maximum values of interval value hesitant fuzzy set and $\eta_{\tilde{b}}$ minimum value of hesitant fuzzy set. Then the TrLCHFN $\tilde{b}$ basically denoted by

$$\tilde{b}=s_{\theta},[(r,s,t,u)];\langle[\omega_{b}^{-},\omega_{b}^{+}],\eta_{\tilde{b}}\rangle.$$

Further, the TrLCHFN reduced to a TLCHFN. Moreover, if $\omega_{b}^{-}=1,\omega_{b}^{+}=1$ and $\eta_{\tilde{b}}=0$, if the TrLCHFN $\tilde{b}$ is called a normal TLCHFN denoted as

$$\tilde{b}=s_{\theta},[(r,s,t,u)];[\langle(1,1)],(0)]\rangle.$$

Therefore, the TrLCHFN considered now can be regarded as generalized TrLCHFN. Such numbers remand the doubt information in a more flexible approach than normal fuzzy numbers as the values $\omega_{b}^{-},\omega_{b}^{+},\eta_{\tilde{b}}\in[0,1]$ can be interpreted as the degree of confidence in the quantity characterized by $r,s,t,u.$ Then $\tilde{b}$ is called trapezoidal linguistic cubic hesitant fuzzy number (TrLCHFN).

Definition 3.2. Let

$$h=\begin{Bmatrix} s_{\theta}, \\ [r, \\ s, \\ t, \\ u], \\ \langle[\omega_{b}^{-}, \\ \omega_{b}^{+}], \\ \eta_{\tilde{b}}\rangle \end{Bmatrix}, h_1=\begin{Bmatrix} s_{\theta_1}, \\ [r_1, \\ s_1, \\ t_1, \\ u_1], \\ \langle[\omega_{b_1}^{-}, \\ \omega_{b_1}^{+}], \\ \eta_{\tilde{b}_1}\rangle \end{Bmatrix} \text{ and } h_2=\begin{Bmatrix} s_{\theta_2}, \\ [r_2, \\ s_2, \\ t_2, \\ u_2], \\ \langle[\omega_{b_2}^{-}, \\ \omega_{b_2}^{+}], \\ \eta_{\tilde{b}_2}\rangle \end{Bmatrix}$$

are three TrCLHFNs, which can be described as follows:

$$\ddot{h}^{c} = \{\alpha^{c} \mid \alpha \in \ddot{h}\} = \left\{ \begin{matrix} s_{\theta}, [r, s, t, u], \\ \langle [\omega_{b}^{-}, \omega_{b}^{+}], \eta_{\tilde{b}} \rangle \end{matrix} \right\};$$

$$\ddot{h}_1 \cup \ddot{h}_2 = \left\{ \begin{matrix} s_{\theta(h_1) \cup \theta(h_2)}, [r_1 \cup r_2, s_1 \cup s_2, t_1 \cup t_2, u_1 \cup u_2], \\ \langle [\omega_{b_1}^{-} \cup \omega_{b_2}^{-}, \omega_{b_1}^{+} \cup \omega_{b_2}^{+}], \eta_{\tilde{b}_1} \cap \eta_{\tilde{b}_2} \rangle \end{matrix} \right\};$$

$$\ddot{h}_1 \cap \ddot{h}_2 = \left\{ \begin{matrix} s_{\theta(h_1) \cap \theta(h_2)}, [r_1 \cap r_2, s_1 \cap s_2, t_1 \cap t_2, u_1 \cap u_2], \\ \langle [\omega_{b_1}^{-} \cap \omega_{b_2}^{-}, \omega_{b_1}^{+} \cap \omega_{b_2}^{+}], \eta_{\tilde{b}_1} \cup \eta_{\tilde{b}_2} \rangle \end{matrix} \right\};$$

$$\ddot{h}_1 \oplus \ddot{h}_2 = \left\{ \begin{matrix} s_{\theta(h_1)+\theta(h_2)}, [r_1 + r_2 - r_1 r_2, s_1 + s_2 - s_1 s_2, \\ t_1 + t_2 - t_1 t_2, u_1 + u_2 - u_1 u_2], \\ \langle [\omega_{b_1}^{-} + \omega_{b_2}^{-} - \omega_{b_1}^{-}\omega_{b_2}^{-}, \omega_{b_1}^{+} + \omega_{b_2}^{+} - \omega_{b_1}^{+}\omega_{b_2}^{+}], \\ \eta_{\tilde{b}_1}\eta_{\tilde{b}_2} \rangle \end{matrix} \right\};$$

$$\lambda\ddot{h} = \left\{ \begin{matrix} \{\lambda h \mid a \in \ddot{h}\} = s_{\theta^{\lambda}(h)}, [1-(1-r)^{\lambda}], [1-(1-s)^{\lambda}], \\ [1-(1-t)^{\lambda}], [1-(1-u)^{\lambda}]; \\ \langle [1-(1-\omega_{b}^{-})^{\lambda}], [1-(1-\omega_{b}^{+})^{\lambda}], \\ (\eta_{\tilde{b}})^{\lambda} \rangle \end{matrix} \right\};$$

$$\ddot{h}^{\lambda} = \left\{ \begin{matrix} \{\alpha^{\lambda} \mid a \in \ddot{h}\} = [s_{\lambda \times \theta(h)}], [(r)^{\lambda}, (s)^{\lambda}, (t)^{\lambda}, (u)^{\lambda}], \\ \langle [(\omega_{b}^{-})^{\lambda}, (\omega_{b}^{+})^{\lambda}], 1-(1-\eta_{\tilde{b}})^{\lambda} \rangle \end{matrix} \right\}.$$

Example 3.3. Let

$$\alpha = \left\{ \begin{matrix} s_3, [0.4, 0.6, \\ 0.8, 0.10], \\ \langle [0.2, 0.4], \\ 0.3 \rangle \end{matrix} \right\}, \quad \alpha_1 = \left\{ \begin{matrix} s_4, [0.2, 0.4, \\ 0.6, 0.8], \\ \langle [0.4, 0.6], \\ 0.5 \rangle \end{matrix} \right\} \ \textit{and} \ \alpha_2 = \left\{ \begin{matrix} s_5, [0.9, 0.11, \\ 0.13, 0.15], \\ \langle [0.11, 0.13], \\ 0.10 \rangle \end{matrix} \right\}$$

be three TrCLHFNs. Then,

$$\alpha^{c} = \left\{ \begin{matrix} s_3, [0.4, 0.6, 0.8, 0.10], \\ \langle [0.2, 0.4], 0.3 \rangle \end{matrix} \right\},$$

$$\alpha_1 \cup \alpha_2 = \left\{ \begin{matrix} s_{4\cup 5}, [0.2 \cup 0.9, \\ 0.4 \cup 0.11, 0.6 \cup 0.13, \\ 0.8 \cup 0.15], \\ \langle [0.4 \cup 0.11, 0.6 \cup 0.13], \\ 0.5 \cap 0.10 \rangle \end{matrix} \right\};$$

$$a_1 \cap a_2 = \left\{ \begin{matrix} s_{4\cap 5}, [0.2 \cap 0.9, \\ 0.4 \cap 0.11, 0.6 \cap 0.13, \\ 0.8 \cap 0.15], \\ \langle [0.4 \cap 0.11, 0.6 \cap 0.13], \\ 0.5 \cup 0.10 \rangle \end{matrix} \right\};$$

$$\alpha_1 \oplus \alpha_2 = \left\{ \begin{matrix} s_{4+5}, [0.2 + 0.9 - (0.2)(0.9), \\ 0.4 + 0.11 - (0.4)(0.11), \\ 0.6 + 0.13 - (0.6)(0.13), \\ 0.8 + 0.15 - (0.8)(0.15)], \\ \langle [0.4 + 0.11 - (0.4)(0.11), \\ 0.6 + 0.13 - (0.6)(0.13)], \\ (0.5)(0.10) \rangle \end{matrix} \right\};$$

$$\alpha_1 \otimes \alpha_2 = \left\{ \begin{matrix} s_{4\times 5}, [(0.2)(0.9), (0.4)(0.11), (0.6)(0.13), \\ (0.8)(0.15)], [(0.4)(0.11), (0.6)(0.13)], \\ 0.5 + 0.10 - (0.5)(0.10) \rangle \end{matrix} \right\};$$

$$\lambda = 0.25, 0.25, 0.25, 0.25$$

$$\lambda\alpha = \left\{ \begin{matrix} s_{3\times 0.25}, [1 - (1 - 0.4)^{0.25}, 1 - (1 - 0.6)^{0.25}, \\ 1 - (1 - 0.8)^{0.25}, 1 - (1 - 0.10)^{0.25}], \\ \langle [1 - (1 - 0.2)^{0.25}, 1 - (1 - 0.4)^{0.25}], \\ (0.3)^{0.25} \rangle \end{matrix} \right\};$$

$$\alpha^{\lambda} = \left\{ \begin{matrix} s_{3^{0.25}}, [(0.4)^{0.25}, (0.6)^{0.25}, \\ (0.8)^{0.25}, (0.10)^{0.25}], \\ \langle [(0.2)^{0.25}, (0.4)^{0.25}], \\ 1 - (1 - 0.3)^{0.25} \rangle \end{matrix} \right\}.$$

Definition 3.4. Let

$$b_1 = \left\{ \begin{matrix} s_{\theta_1}, [r_1, s_1, t_1, u_1]; \\ \langle [\omega_{b_1}^-, \omega_{b_1}^+], \eta_{b_1} \rangle \end{matrix} \right\} \quad and \quad b_2 = \left\{ \begin{matrix} s_{\theta_2}, [r_2, s_2, t_2, u_2]; \\ \langle [\omega_{b_2}^-, \omega_{b_2}^+], \eta_{b_2} \rangle \end{matrix} \right\}$$

be two TrLCHFNs. The hammimg distance between b_1 and b_2 is defined as follows:

$$\underset{H}{d}(\tilde{b}_1, \tilde{b}_2) = \left\{ \begin{matrix} \frac{1}{12}[|s_{\theta_1 - \theta_2}|, \langle [|r_1 - r_2| + |s_1 - s_2| + |t_1 - t_2| + \\ |u_1 - u_2|] + \max[\{|\omega_{b_1}^- - \omega_{b_2}^-|, |\omega_{b_1}^+ - \omega_{b_2}^+|\}, |\eta_{b_1} - \eta_{b_2}|]] \end{matrix} \right\}$$

the TrLCHFN

$$b_1 = \left\{ \begin{matrix} s_{\theta_1}, [r_1, s_1, \\ t_1, u_1]; \\ \langle [\omega_{b_1^-}, \omega_{b_1^+}], \\ \eta_{b_1} \rangle \end{matrix} \right\} \quad and \quad b_2 = \left\{ \begin{matrix} s_{\theta_2}, [r_2, s_2, \\ t_2, u_2]; \\ \langle [\omega_{b_2^-}, \omega_{b_2^+}], \\ \eta_{b_2}] \rangle \end{matrix} \right\}$$

reduces to a TrLCHF

$$b_1 = \left\{ \begin{matrix} s_{\theta_1}, [r_1, s_1, \\ t_1, u_1]; \\ \langle [1,1], \\ 0 \rangle \end{matrix} \right\} \quad and \quad b_2 = \left\{ \begin{matrix} s_{\theta_2}, [r_2, s_2, \\ t_2, u_2]; \\ \langle [1,1], \\ 0 \rangle \end{matrix} \right\}.$$

If $\omega_{b_1^-} = 1, \omega_{b_2^-} = 1$ and $\omega_{b_1^+} = 1, \omega_{b_2^+} = 1,$ if $\eta_{b_1} = 0$ and $\eta_{b_2} = 0$.

Example 3.5. Let

$$b_1 = \left\{ \begin{matrix} s_4, [0.6, 0.8, \\ 0.10, 0.12]; \\ \langle [0.22, 0.24], \\ 0.23 \rangle \end{matrix} \right\} \quad and \quad b_2 = \left\{ \begin{matrix} s_2, [0.2, 0.4, \\ 0.6, 0.8]; \\ \langle [0.11, 0.13], \\ 0.12 \rangle \end{matrix} \right\}$$

be two TrLCHFNs. The hammimg distance between $\tilde{b}_1$ and $\tilde{b}_2$ is defined as follows:

$$\underset{H}{d}(\tilde{b}_1,\tilde{b}_2)=\left\{\begin{array}{c}\frac{1}{12}[|s_{4-2}|,\langle[|0.6-0.2|+|0.8-0.4|\\ +|0.10-0.6|+\\ |0.12-0.8|]+\\ \max[\left\{\begin{array}{c}|0.22-0.11|,\\ |0.24-0.13|\end{array}\right\},\\ |0.23-0.12|]]\\ \frac{1}{12}[|s_2|,\langle[|0.4|+\\ |0.4|+|-0.5|+\\ |-0.68|]+\max[\left\{\begin{array}{c}|0.11|,\\ |0.11|\end{array}\right\},\\ |0.11|]]=s_{0.3483}\end{array}\right\}.$$

4 Trapezoidal Linguistic Cubic Hesitant Fuzzy TOPSIS Method

In this section we apply trapezoidal cubic fuzzy set to linguistic hesitant TOPSIS method. We define a new extension of trapezoidal linguistic cubic hesitant fuzzy TOPSIS method by using trapezoidal cubic fuzzy set.

Step 1: Suppose that a trapezoidal linguistic cubic hesitant fuzzy TOPSIS method decision-making problem under multiple attributes has m students and n decision attributes. The framework of trapezoidal linguistic cubic hesitant decision matric can be exhibit as follows:

$$\beta=\left\{\begin{array}{c}s_{\theta_{11}}[r_{11},s_{11},t_{11},u_{11}]\langle[B_{11}^-,B_{11}^+],\eta_{11}\rangle\\ s_{\theta_{12}}[r_{12},s_{12},t_{12},u_{12}]\langle[B_{12}^-,B_{12}^+],\eta_{12}^S\rangle.......\\ s_{\theta_{n1}}[r_{n1},s_{n1},t_{n1},u_{n1}]\langle[B_{n1}^-,B_{n1}^+],\eta_{1n}\rangle\\ s_{\theta_{12}}[r_{21},s_{21},t_{21},u_{21}]\langle[B_{21}^-,B_{21}^+],\eta_{21}\rangle\\ s_{\theta_{22}}[r_{22},s_{22},t_{22},u_{22}]\langle[B_{22}^-,B_{22}^+],\eta_{22}\rangle.......\\ s_{\theta_{n2}}[r_{n2},s_{n2},t_{n2},u_{n2}]\langle[B_{n2}^-,B_{n2}^+],\eta_{2n}\rangle\\ ....\\ ....\\ s_{\theta_{m1}}[r_{m1},s_{m1},t_{m1},u_{m1}]\langle[B_{m1}^-,B_{m1}^+],\eta_{m1}\rangle\\ s_{\theta_{m2}}[r_{m2},s_{m2},t_{m2},u_{m2}]\langle[B_{m2}^-,B_{m2}^+],\eta_{m2}\rangle.......\\ s_{\theta_{mn}}[r_{mn},s_{mn},t_{mn},u_{mn}]\langle[B_{mn}^-,B_{mn}^+],\eta_{mn}\rangle\end{array}\right\}$$

Step 2: Construct normalized trapezoidal linguistic cubic hesitant fuzzy TOPSIS decision matrix $R=[\beta_{ij}]$. The normalized value r_{ij} is calculated as:

$$\beta=\left\{\begin{array}{c}\frac{s_\theta}{\sqrt{\sum_{i=1}^{n}(s_{\theta_{ij}})^2}},\left[\frac{r}{\sqrt{\sum_{i=1}^{n}(r_{ij})^2}},\frac{s}{\sqrt{\sum_{i=1}^{n}(s_{ij})^2}},\frac{t}{\sqrt{\sum_{i=1}^{n}(t_{ij})^2}},\frac{u}{\sqrt{\sum_{i=1}^{n}(u_{ij})^2}}\right];\\ \left\langle\left[\frac{B^-}{\sqrt{\sum_{i=1}^{n}(B_{ij}^-)^2}},\frac{B^+}{\sqrt{\sum_{i=1}^{n}(B_{ij}^+)^2}}\right],\frac{\eta}{\sqrt{\sum_{i=1}^{n}(\eta)^2}}\right\rangle\end{array}\right\}$$

Step 3: Make the weighted normalized trapezoidal linguistic cubic hesitant fuzzy TOPSIS decision matrix by multiplying the normalized trapezoidal linguistic cubic hesitant decision matrix by its associated weights. The weight vector $W=(w_1,w_2,...w_n)$ collected of the isolated weights $w_j(j=1,2,3,...n)$ for each attribute C_j satisfying $\sum_{j=1}^{n}W_j=1$. The weighted normalized value is deliberate by $B_j=v_{ij}w_j$ where $0\le B_j\le 1$ $i=1,2,3....,m$ and $j=1,2,3...,n.$

Step 4: Identify positive ideal solution (α^*) and negative ideal solution (α^-). The trapezoidal linguistic cubic hesitant fuzzy TOPSIS method positive-ideal solution (TrLCHFPIS, α^*) and the trapezoidal linguistic cubic hesitant fuzzy negative-ideal solution (TrLCHFNIS, α^-) is shown as

$$\alpha_i^+=\left\{\begin{array}{c}s_{\theta_1},[(r_1,s_1,t_1,u_1),\\(r_2,s_2,t_2,u_2).....\\s_{\theta_n}(r_n,s_n,t_n,u_n)]\\\{(B_1^+,\eta_1)(B_2^+,\eta_2)\\.......(B_n^+,\eta_n)\end{array}\right\}=\max_i s_{\theta_{ij}},\max_i(r_{ij},s_{ij},t_{ij},u_{ij})\{\max_i(B_{ij}^+)\}\{\min_i(\eta_i)\},$$

$$\alpha_i^-=\left\{\begin{array}{c}s_{\theta_1},[(r_1,s_1,t_1,u_1),\\(r_2,s_2,t_2,u_2).....\\s_{\theta_n}(r_n,s_n,t_n,u_n)],\\\{(B_1^-,\eta_1)(B_2^-,\eta_2)\\.......(B_n^-,\eta_n)\end{array}\right\}=\min_i s_{\theta_{ij}},\min_i(r_{ij},s_{ij},t_{ij},u_{ij}),\{\min_i(B_1)\}\{\max_i(\eta_i)\}.$$

Step 5: Estimated separation measures, using the n-dimensional euclidean distance. The separation of each candidate from the TrLCHPIS $q_i^*\langle[B^-,B^+],\eta\rangle$ is given as

$$q_i^+\langle[B^-,B^+],\eta\rangle=\langle\tfrac{1}{12n}\begin{pmatrix}\left|s_{\theta_{ij}-\theta_j}\right|,\left|r_{ij}-r_j\right|+\left|s_{ij}-s_j\right|+\\\left|t_{ij}-t_j\right|+\left|u_{ij}-u_j\right|,\\\max\{\left|B_{ij}-B_j^-\right|,\\\left|B_{ij}-B_j^+\right|,\left|\eta_{ij}-\eta_j\right|\}\end{pmatrix}\rangle.$$

The separation of each candidate from the TrLCHNIS $q_i^-\langle[B^-,B^+],\eta\rangle$ is given as

$$q_i^-\langle[B^-,B^+],\eta\rangle=\langle\tfrac{1}{12n}\begin{pmatrix}\left|s_{\theta_{ij}-\theta_j}\right|,\left|r_{ij}-r_j\right|+\\ \left|s_{ij}-s_j\right|+\left|t_{ij}-t_j\right|,\\ \max\{\left|B_{ij}-B_j^-\right|,\\ \left|B_{ij}-B_j^+\right|,\left|\eta_{ij}-\eta_j\right|\}\end{pmatrix}\rangle.$$

Step 6: Calculate similarities to ideal solution. This progression comprehends the similitudes to an ideal solution by Eqs. $Z_i=\frac{q_i^-\langle[B^-,B^+],\eta\rangle}{q_i^-\langle[B^-,B^+],\eta\rangle+q_i^+\langle[B^-,B^+],\eta\rangle}$.

Figure 1

Flow chart of the extended trapezoidal linguistic cubic hesitant fuzzy TOPSIS method

5 Numerical Example

Assume that an automotive company is in a decision-making situation for purchasing one of the main items for their newly introduced automobile. A committee of three decision-makers $\{D_1, D_2, D_3\}$ want to select the most promising vendor for supplying the item. After a preliminary screening, three alternatives $B_1, B_2, ..., B_n$ remain for further evaluations.

Linguistic variables of ratings of alternatives by decision-maker 1

	C_1	C_2	C_3
B_1	$\left\{ s_2, \begin{bmatrix} 0.2, 0.3, \\ 0.4, 0.5 \end{bmatrix}; \langle [0.20, 0.24], 0.22 \rangle \right\}$	$\left\{ s_4, \begin{bmatrix} 0.4, 0.5, \\ 0.6, 0.7 \end{bmatrix}; \langle [0.10, 0.12], 0.11 \rangle \right\}$	$\left\{ s_2, \begin{bmatrix} 0.1, 0.2, \\ 0.3, 0.4 \end{bmatrix}; \langle [0.5, 0.7], 0.6 \rangle \right\}$
B_2	$\left\{ s_6, \begin{bmatrix} 0.14, 0.15, \\ 0.16, 0.17 \end{bmatrix}; \langle [0.1, 0.3], 0.2 \rangle \right\}$	$\left\{ s_6, \begin{bmatrix} 0.11, 0.12, \\ 0.13, 0.14 \end{bmatrix}; \langle [0.4, 0.6], 0.5 \rangle \right\}$	$\left\{ s_4, \begin{bmatrix} 0.4, 0.5, \\ 0.6, 0.7 \end{bmatrix}; \langle [0.10, 0.12], 0.11 \rangle \right\}$
B_3	$\left\{ s_2, \begin{bmatrix} 0.2, 0.3, \\ 0.4, 0.5 \end{bmatrix}; \langle [0.20, 0.24], 0.22 \rangle \right\}$	$\left\{ s_2, \begin{bmatrix} 0.1, 0.2, \\ 0.3, 0.4 \end{bmatrix}; \langle [0.5, 0.7], 0.6 \rangle \right\}$	$\left\{ s_6, \begin{bmatrix} 0.11, 0.12, \\ 0.13, 0.14 \end{bmatrix}; \langle [0.4, 0.6], 0.5 \rangle \right\}$

Linguistic variables of ratings of alternatives by decision-maker 2

	C_1	C_2	C_3
B_1	$\left\langle s_4, \begin{bmatrix} 0.4, 0.5, \\ 0.6, 0.7 \end{bmatrix}; [0.10, 0.12], 0.11 \right\rangle$	$\left\langle s_2, \begin{bmatrix} 0.2, 0.3, \\ 0.4, 0.5 \end{bmatrix}; [0.20, 0.24], 0.22 \right\rangle$	$\left\langle s_6, \begin{bmatrix} 0.14, 0.15, \\ 0.16, 0.17 \end{bmatrix}; [0.1, 0.3], 0.2 \right\rangle$
B_2	$\left\langle s_6, \begin{bmatrix} 0.11, 0.12, \\ 0.13, 0.14 \end{bmatrix}; [0.4, 0.6], 0.5 \right\rangle$	$\left\langle s_2, \begin{bmatrix} 0.1, 0.2, \\ 0.3, 0.4 \end{bmatrix}; [0.5, 0.7], 0.6 \right\rangle$	$\left\langle s_6, \begin{bmatrix} 0.11, 0.12, \\ 0.13, 0.14 \end{bmatrix}; [0.4, 0.6], 0.5 \right\rangle$
B_3	$\left\langle s_2, \begin{bmatrix} 0.2, 0.3, \\ 0.4, 0.5 \end{bmatrix}; [0.20, 0.24], 0.22 \right\rangle$	$\left\langle s_6, \begin{bmatrix} 0.14, 0.15, \\ 0.16, 0.17 \end{bmatrix}; [0.1, 0.3], 0.2 \right\rangle$	$\left\langle s_2, \begin{bmatrix} 0.1, 0.2, \\ 0.3, 0.4 \end{bmatrix}; [0.5, 0.7], 0.6 \right\rangle$

$v_1 = 0.2, v_2 = 0.3, v_3 = 0.5$

Step 1: In this step we aggregated TrLCHF-decision matrix D_1, D_2, D_3 based on opinions of the experts after weights values for the experts are obtained, the evaluating values provided by different experts can be aggregated based on the TrLCHFWG operator as below: The aggregated TrLCHF-decision matrix can be defined as follows:

Table 3. The aggregated TrLCHF-decision matrix

	C_1	C_2	C_3
B_1	$\left\langle s_{1.5157}, \begin{bmatrix} 0.1365, \\ 0.1893, \\ 0.2483, \\ 0.3157 \end{bmatrix}; \begin{bmatrix} 0.0635, \\ 0.0773 \end{bmatrix}, 0.4751 \right\rangle$	$\left\langle s_{1.8661}, \begin{bmatrix} 0.1976, \\ 0.2701, \\ 0.3482, \\ 0.4339 \end{bmatrix}; \begin{bmatrix} 0.0938, \\ 0.1136 \end{bmatrix}, 0.3274 \right\rangle$	$\left\langle s_{3.4641}, \begin{bmatrix} 0.1202, \\ 0.1753, \\ 0.2331, \\ 0.2943 \end{bmatrix}; \begin{bmatrix} 0.3291, \\ 0.5417 \end{bmatrix}, 0.3464 \right\rangle$
B_2	$\left\langle s_{2.0476}, \begin{bmatrix} 0.0521, \\ 0.0564, \\ 0.0607, \\ 0.0652 \end{bmatrix}; \begin{bmatrix} 0.1159, \\ 0.2247 \end{bmatrix}, 0.6309 \right\rangle$	$\left\langle s_{2.1074}, \begin{bmatrix} 0.0644, \\ 0.0999, \\ 0.1382, \\ 0.1800 \end{bmatrix}; \begin{bmatrix} 0.3031, \\ 0.4706 \end{bmatrix}, 0.6968 \right\rangle$	$\left\langle s_{4.8989}, \begin{bmatrix} 0.2692, \\ 0.3366, \\ 0.4100, \\ 0.4921 \end{bmatrix}; \begin{bmatrix} 0.2651, \\ 0.4067 \end{bmatrix}, 0.2345 \right\rangle$
B_3	$\left\langle s_{1.3195}, \begin{bmatrix} 0.0853, \\ 0.1329, \\ 0.1848, \\ 0.2421 \end{bmatrix}; \begin{bmatrix} 0.0854, \\ 0.1039 \end{bmatrix}, 0.5457 \right\rangle$	$\left\langle s_{2.1074}, \begin{bmatrix} 0.0739, \\ 0.1092, \\ 0.1472, \\ 0.1887 \end{bmatrix}; \begin{bmatrix} 0.2131, \\ 0.3738 \end{bmatrix}, 0.5293 \right\rangle$	$\left\langle s_{3.4641}, \begin{bmatrix} 0.1051, \\ 0.1609, \\ 0.2196, \\ 0.2816 \end{bmatrix}; \begin{bmatrix} 0.4522, \\ 0.6535 \end{bmatrix}, 0.5477 \right\rangle$

Step 2: Construct normalized trapezoidal linguistic cubic hesitant fuzzy TOPSIS decision matrix $R = [\beta_{ij}]$. The normalized value r_{ij} is calculated as:

Table 4. The normalized trapezoidal linguistic cubic hesitant fuzzy TOPSIS decision matrix

	C_1	C_2	C_3
B_1	$\left\{ s_{0.9142}, \begin{bmatrix} 0.0823, \\ 0.1141, \\ 0.1497, \\ 0.1904 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0383, \\ 0.0466 \end{bmatrix}, 0.2865 \right\rangle \right\}$	$\left\{ s_{0.9292}, \begin{bmatrix} 0.0984, \\ 0.1345, \\ 0.1733, \\ 0.2161 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0467, \\ 0.0565 \end{bmatrix}, 0.1630 \right\rangle \right\}$	$\left\{ s_{0.9717}, \begin{bmatrix} 0.0337, \\ 0.0491, \\ 0.0653, \\ 0.0825 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0923, \\ 0.1519 \end{bmatrix}, 0.0971 \right\rangle \right\}$
B_2	$\left\{ s_{0.9476}, \begin{bmatrix} 0.0241, \\ 0.0261, \\ 0.0281, \\ 0.0301 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0536, \\ 0.1039 \end{bmatrix}, 0.2921 \right\rangle \right\}$	$\left\{ s_{0.9149}, \begin{bmatrix} 0.0279, \\ 0.0433, \\ 0.0600, \\ 0.0781 \end{bmatrix}; \left\langle \begin{bmatrix} 0.1315, \\ 0.2043 \end{bmatrix}, 0.3025 \right\rangle \right\}$	$\left\{ s_{0.9821}, \begin{bmatrix} 0.0539, \\ 0.0674, \\ 0.0821, \\ 0.0986 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0531, \\ 0.0815 \end{bmatrix}, 0.0471 \right\rangle \right\}$
B_3	$\left\{ s_{0.8948}, \begin{bmatrix} 0.0578, \\ 0.0901, \\ 0.1253, \\ 0.1641 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0579, \\ 0.0704 \end{bmatrix}, 0.3700 \right\rangle \right\}$	$\left\{ s_{0.9442}, \begin{bmatrix} 0.0331, \\ 0.0489, \\ 0.0659, \\ 0.0845 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0954, \\ 0.1674 \end{bmatrix}, 0.2371 \right\rangle \right\}$	$\left\{ s_{0.9572}, \begin{bmatrix} 0.0291, \\ 0.0444, \\ 0.0606, \\ 0.0778 \end{bmatrix}; \left\langle \begin{bmatrix} 0.1249, \\ 0.1805 \end{bmatrix}, 0.1513 \right\rangle \right\}$

Step 3: Make the weighted normalized trapezoidal linguistic cubic hesitant fuzzy TOPSIS decision matrix by multiplying the normalized trapezoidal linguistic cubic hesitant decision matrix by its associated weights. $w_1 = 0.3427, w_2 = 0.3492, w_3 = 0.3079.$

Table 5. The weighted normalized trapezoidal linguistic cubic hesitant fuzzy TOPSIS decision matrix

	C_1	C_2	C_3
B_1	$\left\{ s_{0.3132}, \begin{bmatrix} 0.0282, 0.0391, \\ 0.0513, 0.0652 \end{bmatrix}; \left\langle \begin{bmatrix} 0.131, \\ 0.0159 \end{bmatrix}, 0.0981 \right\rangle \right\}$	$\left\{ s_{0.3244}, \begin{bmatrix} 0.0343, 0.0469, \\ 0.0605, 0.0754 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0153, \\ 0.0197 \end{bmatrix}, 0.0569 \right\rangle \right\}$	$\left\{ s_{0.2991}, \begin{bmatrix} 0.0103, 0.0151, \\ 0.0201, 0.0254 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0284, \\ 0.0467 \end{bmatrix}, 0.0298 \right\rangle \right\}$
B_2	$\left\{ s_{0.3247}, \begin{bmatrix} 0.0082, 0.0089, \\ 0.0096, 0.0103 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0183, \\ 0.0356 \end{bmatrix}, 0.1001 \right\rangle \right\}$	$\left\{ s_{0.3194}, \begin{bmatrix} 0.0097, 0.0151, \\ 0.0209, 0.0272 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0459, \\ 0.0713 \end{bmatrix}, 0.1056 \right\rangle \right\}$	$\left\{ s_{0.3023}, \begin{bmatrix} 0.0165, 0.0207, \\ 0.0252, 0.0303 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0163, \\ 0.0251 \end{bmatrix}, 0.0145 \right\rangle \right\}$
B_3	$\left\{ s_{0.3066}, \begin{bmatrix} 0.0198, 0.0308, \\ 0.0429, 0.0562 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0198, \\ 0.0241 \end{bmatrix}, 0.1267 \right\rangle \right\}$	$\left\{ s_{0.3297}, \begin{bmatrix} 0.0115, 0.0171, \\ 0.0231, 0.0295 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0333, \\ 0.0584 \end{bmatrix}, 0.0827 \right\rangle \right\}$	$\left\{ s_{0.2947}, \begin{bmatrix} 0.0089, 0.0136, \\ 0.0186, 0.0239 \end{bmatrix}; \left\langle \begin{bmatrix} 0.0384, \\ 0.0555 \end{bmatrix}, 0.0465 \right\rangle \right\}$

Step 4: Identify positive ideal solution (α^*) and negative ideal solution (α^-). The trapezoidal linguistic cubic hesitant fuzzy TOPSIS method positive-ideal solution (TrLCHPIS, α^*) and the trapezoidal linguistic cubic hesitant fuzzy negative-ideal solution (TrLCHNIS, α^-) is shown as

$$\alpha_i^{+}=\left\{\left\{\begin{matrix} s_{0.3244}, \\ \begin{bmatrix}0.0343, 0.0469, \\ 0.0605, 0.0754\end{bmatrix}; \\ \left\langle \begin{bmatrix}0.0284, \\ 0.0467\end{bmatrix}, \\ 0.0298 \right\rangle \end{matrix}\right\}, \left\{\begin{matrix} s_{0.3247}, \\ \begin{bmatrix}0.0165, 0.0207, \\ 0.0252, 0.0303\end{bmatrix}; \\ \left\langle \begin{bmatrix}0.0459, \\ 0.0713\end{bmatrix}, \\ 0.0145 \right\rangle \end{matrix}\right\}, \left\{\begin{matrix} s_{0.3297}, \\ \begin{bmatrix}0.0198, 0.0308, \\ 0.0429, 0.0562\end{bmatrix}; \\ \left\langle \begin{bmatrix}0.0384, \\ 0.0584\end{bmatrix}, \\ 0.0465 \right\rangle \end{matrix}\right\}\right\}$$

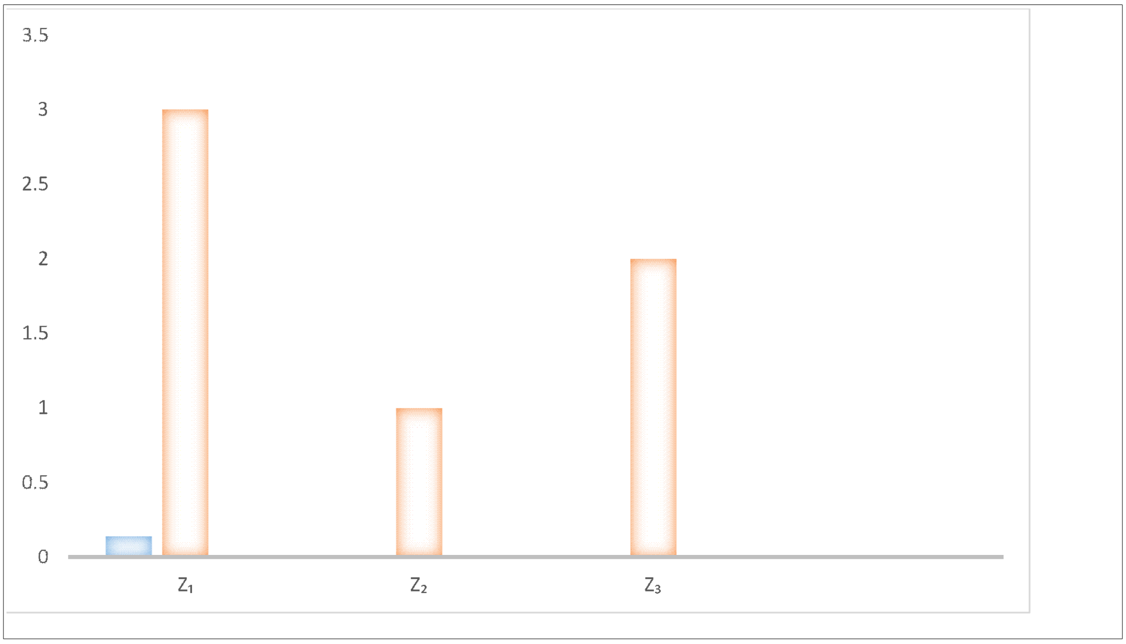
$$\alpha_i^{-}=\left\{\left\{\begin{matrix} s_{0.2991}, \\ \begin{bmatrix}0.0103, 0.0151, \\ 0.0201, 0.0254\end{bmatrix}; \\ \left\langle \begin{bmatrix}0.131, \\ 0.0159\end{bmatrix}, \\ 0.0981 \right\rangle \end{matrix}\right\}, \left\{\begin{matrix} s_{0.3023}, \\ \begin{bmatrix}0.0082, 0.0089, \\ 0.0096, 0.0103\end{bmatrix}; \\ \left\langle \begin{bmatrix}0.0163, \\ 0.0251\end{bmatrix}, \\ 0.0145 \right\rangle \end{matrix}\right\}, \left\{\begin{matrix} s_{0.2947}, \\ \begin{bmatrix}0.0089, 0.0136, \\ 0.0186, 0.0239\end{bmatrix}; \\ \left\langle \begin{bmatrix}0.0198, \\ 0.0241\end{bmatrix}, \\ 0.1267 \right\rangle \end{matrix}\right\}\right\}$$

Step 5: Estimated separation measures, using the n-dimensional euclidean distance. The separation of each candidate from the TrLCHPIS $q_i^{*}\langle[B^-,B^+],\eta\rangle$ is given as $q_1^{+}\langle[B^-,B^+],\eta\rangle=0.2104$, $q_2^{+}\langle[B^-,B^+],\eta\rangle=0.3288, q_3^{+}\langle[B^-,B^+],\eta\rangle=0.6732$.

The separation of each candidate from the TrLCHNIS $q_i^{-}\langle[B^-,B^+],\eta\rangle$ is given as the separation of each candidate from the TrLCNIS $q_i^{-}\langle[B^-,B^+],\eta\rangle$ is given as $q_1^{-}\langle[B^-,B^+],\eta\rangle=0.0345$. $q_2^{-}\langle[B^-,B^+],\eta\rangle=0.2344, q_3^{-}\langle[B^-,B^+],\eta\rangle=0.1234$.

Step 6: Calculate similarities to ideal solution. This progression comprehends the similitudes to an ideal solution by Eqs.

$$Z_1=\tfrac{0.0054}{0.2449}=0.1408, Z_2=\tfrac{0.0127}{0.5632}=0.4161, Z_3=\tfrac{0.1234}{0.7966}=0.1549.$$



6 Comparison Analyses

In direction to verify the rationality and efficiency of the proposed approach, a comparative study is steered consuming the methods of cubic TOPSIS method [2] , which is special case of TrLCHTFNs, to the similar expressive example.

6.1. A Comparison Analysis With The Existing MCDM Method Cubic TOPSIS Method

[2] TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method is a popular approach to multi-attribute decision making problems. Assuming that there are N alternatives and M attributes, the procedure of TOPSIS starts from the construction of the scores matrix $X=[x_{ij}]$ where x_{ij} denotes score of the ith alternative with respect to the jth attribute and can be summarized as follows: The proposed method which is applied to solve this problem and the computational procedure are summarized as follows:
Step 1: Set a number of alternatives and some attributes or criteria. There are 3 criteria used as a basis for decision making in academic scholarship. The criteria include:

Table 6 Cubic TOPSIS method

	C_1	C_2	C_3
B_1	$\langle[0.20,0.24],0.22\rangle$	$\langle[0.10,0.12],0.11\rangle$	$\langle[0.5,0.7],0.6\rangle$
B_2	$\langle[0.1,0.3],0.2\rangle$	$\langle[0.4,0.6],0.5\rangle$	$\langle[0.10,0.12],0.11\rangle$
B_3	$\langle[0.20,0.24],0.22\rangle$	$\langle[0.5,0.7],0.6\rangle$	$\langle[0.4,0.6],0.5\rangle$

Step 2: Calculation of normalized decision matrix

Table 7 Normalized decision matrix

	C_1	C_2	C_3
B_1	$\langle [0.5234, 0.6281], 0.5757 \rangle$	$\langle [0.5232, 0.6279], 0.5756 \rangle$	$\langle [0.4767, 0.6674], 0.5721 \rangle$
B_2	$\langle [0.2673, 0.8019], 0.5346 \rangle$	$\langle [0.4558, 0.6838], 0.5698 \rangle$	$\langle [0.5232, 0.6279], 0.5756 \rangle$
B_3	$\langle [0.5234, 0.6281], 0.5757 \rangle$	$\langle [0.4767, 0.6674], 0.5721 \rangle$	$\langle [0.4558, 0.6838], 0.5698 \rangle$

Step 3: Calculation of the weighted aggregated CF-decision matrix

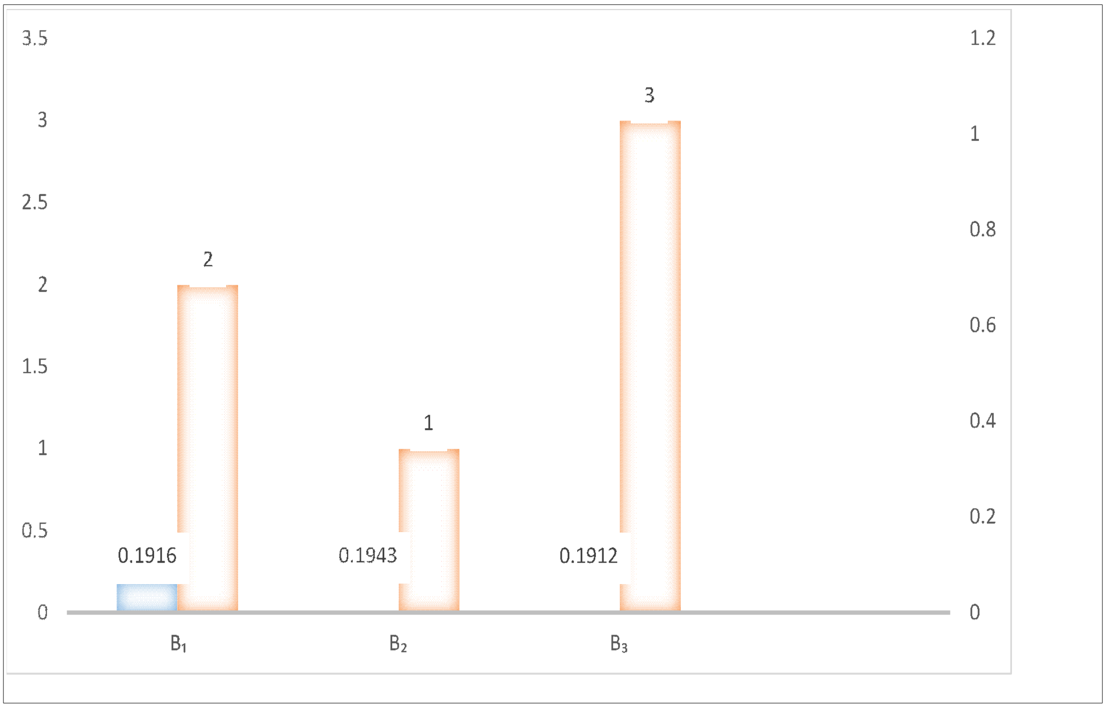
$$v_1 = 0.3, v_2 = 0.2, v_3 = 0.5,$$

Table 8 weighted aggregated CF-decision matrix

B_1	$\langle [0.5005, 0.6482], 0.5738 \rangle$
B_2	$\langle [0.4431, 0.7018], 0.5618 \rangle$
B_3	$\langle [0.4811, 0.6646], 0.5721 \rangle$

Step 4: Determination of the score value

$$Z_1 = 0.1916, Z_2 = 0.1943, Z_3 = 0.1912.$$



Comparison analysis with existing methods Table 9

Method	Ranking
Trapezoidal linguistic cubic hesitant fuzzy TOPSIS method	$B_2 > B_3 > B_1$
Cubic TOPSIS method [2]	$B_2 > B_1 > B_3$

7 Conclusion

In this paper, we define a new idea of trapezoidal linguistic cubic hesitant fuzzy number. We discuss some basic operational laws of trapezoidal linguistic cubic hesitant fuzzy number and hamming distance of trapezoidal linguistic cubic hesitant fuzzy number. Furthermore, we develop Trapezoidal linguistic cubic hesitant fuzzy TOPSIS method to solve the MCDM method based on trapezoidal linguistic cubic hesitant fuzzy TOPSIS method. Finally, an illustrative example is given to verify and demonstrate the practicality and effectiveness of the proposed method. We compared the proposed method to the existing methods, which shows the trapezoidal linguistic Cubic hesitant TOPSIS method are more fexible to deal uncertainties and fuzziness. In fact, this method is very simple and flexible. Hence, it is expected that proposed in this study may have more potential management applications.

Graphical Abstract

References

[1] A. Fahmi, S. Abdullah, F. Amin ,N. Siddque and A Ali. AGGREGATION OPERATORS ON TRIANGULAR CUBIC FUZZY NUMBERS AND ITS APPLICATION TO MULTI-CRITERIA DECISION MAKING PROBLEMS, Journal of Intelligent and Fuzzy System, 33 (2017),3323-3337..

[2] A. Fahmi, S. Abdullah, F. Amin and A. Ali, Precursor Selection for Sol--Gel Synthesis of Titanium Carbide Nanopowders by a New Cubic Fuzzy Multi-Attribute Group Decision-Making Model. Journal of Intelligent Systems. 0(0): -. Retrieved 6 Oct. 2017, from doi:10.1515/jisys-2017-0083.

[3] A. Fahmi, S. Abdullah, F. Amin and A. Ali, Weighted Average Rating (War) Method for Solving Group Decision Making Problem Using Triangular Cubic Fuzzy Hybrid Aggregation (Tcfha), Punjab University Journal of Mathematics, Vol. 50(1)(2018) pp. 23-34.

[4] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1) (1986) 87--96.

[5] De Boer, L., E. Labro, and P. Morlacchi, A review of methods supporting supplier selection, European Journal of Purchasing and Supply Management, vol.7, pp.75-89, 200.

[6] B. Gülçin, and G. Çifçi. "A novel hybrid MCDM approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers." Expert Systems with Applications 39.3 (2012): 3000-3011.

[7] S. H. Ghodsypour., and C. O'Brien, A decision support system for supplier selection using an integrated analytic hierarchy process and linear programming, International Journal of Production Economics, vol.56-57, pp.199-212, 1998.

[8] C. L. Hwang and Yoon K.: Multiple Attribute Decision Making: Methods and Application, Springer, New York (1981).

[9] Y. B. Jun, C. S. Kim and Ki. O. Yang, Cubic sets ,Annals of Fuzzy Mathematics and Informatics, 4(1), 83- 98: (2011).

[10] Yang, T. & Chou P.: Solving a multi-response simulation--optimization problem with discrete variables using a multipleattribute decision making method. Math Comput Simulat 68, 9--21, (2005).

[11] K. Yoon,. and C. Hwang. Multiple attribute decision making. Thousand Oaks, CA, Sage Publication, (1995).

[12] T. Yang and C. Hung Multiple-attribute decision making methods for plant layout design problem. Robotics and ComputerIntegrated Manufacturing 23, 126--137, (2005).

[13] C. A. Weber., J.R. Current, and W.C. Benton, Vendor selection criteria and methods, European Journal of Operational Research, vol.50, no.1, pp.2-18, 1991.

[14] L.A. Zadeh, Fuzzy sets, Inf. Control 18 (1965) 338--353.

[15] K. Yoon, A Reconciliation Among Discrete Compromise Solutions, Journal of The Operational Research Society 38 (1987) 277--286.

[16] C. lai Hwang, Y. jou Lai, T. yun Liu, A new approach for multiple objective decision making, Computers and Operations Research 20 (1993) 889--899.

[17] X. lin Wang, Z. bo Qing, Y. Zhang, Decision making of selecting manufacturing partner based on the supply chain: study on topsis method application, in: Cybernetics and Intelligent Systems, 2004 IEEE Conference on, Vol. 2, 2004, pp. 1118--1122. doi:10.1109/ICCIS.2004.1460746.

[18] Y. Yang, X. Li, D. Chen, T. Yu, W. Wang, Application of gc-topsis method in the process of supplier evaluation, in: Management and Service Science, 2009. MASS '09. International Conference on, 2009, pp. 1--4.doi:10.1109/ICMSS.2009.5301059.

[19] A. Bilbao-Terol, M. Arenas-Parra, V. Cañal-Fernández, J. Antomil-Ibias, Using {TOPSIS} for assessing the sustainability of government bond funds, Omega 49 (2014) 1 -- 17. doi:http://dx.doi.org/10.1016/j.omega.2014.04.005.

[20] M. Tavana, M. Keramatpour, F. J. Santos-Arteaga, E. Ghorbaniane, A fuzzy hybrid project portfolio selection method using data envelopment analysis, {TOPSIS} and integer programming, Expert Systems with Applications 42 (22) (2015) 8432 -- 8444. doi:http://dx.doi.org/10.1016/j.eswa.2015.06.057.

[21] L. Zhao, X. Gao, Y. Zhang, Compare and contrast analysis to the development pattern of energy producing provinces with the aim of carbon emission reducing, in: Management and Service Science (MASS), 2010 International Conference on, 2010, pp. 1--6. doi:10.1109/ICMSS.2010.5577654.

[22] L. yun Wu, Y. zhong Yang, Topsis method for green vendor selection in coal industry group, in: Machine Learning and Cybernetics, 2008 International Conference on, Vol. 3, 2008, pp. 1721--1725. doi:10.1109/ICMLC.2008.4620683.

[23] E. Roszkowska, T. Wachowicz, Application of fuzzy {TOPSIS} to scoring the negotiation offers in ill-structured negotiation problems, European Journal of

Operational Research 242 (3) (2015) 920 -- 932. doi:http://dx.doi.org/10.1016/j.ejor.2014.10.050.

[24] C. T. Chen, Extensions of the {TOPSIS} for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114 (1) (2000) 1 -- 9.

[25] V. Torra, Hesitant fuzzy sets, International Journal of Intelligent Systems, vol. 25, no. 6, pp. 529--539, 2010.

[26] M. Xia and Z. Xu, Hesitant fuzzy information aggregation in decision making, International Journal of Approximate Reasoning, vol. 52, no. 3, pp. 395--407, 2011.