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Connectedness in Ditopological Texture Spaces Via Ideal

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Abstract − The main purpose of this paper, is to introduce the notion of ditopological texture spaces with ideal. We study the notions of $\star$-connected ditopological texture spaces with an ideal and $\star$-connected sets in ditopological texture spaces with ideal. Some new types of connectedness in $\star$-ditopological texture spaces namely, locally $\star$-connectedness, totally $\star$-disconnectedness, hyperconnectedness and $\star$-hyperconnectedness have investigated.

Keywords − *Texturing, Texture space, Bitopology, Ditopology, Connectedness, $\star$-connected sets, Ditopological texture spaces with ideal, $\star$-ditopological texture spaces, $\star$-separated sets, $\star$-connectedness, $\star$-component, Locally $\star$-connectedness, Totally $\star$-disconnectedness, $\star$-hyperconnected.*

1 Introduction

The notion of a texture space, under the name of fuzzy structure, was introduced by Brown in [2]. The motivation for the study of texture spaces is that they allow us to represent, for instance, classical fuzzy sets, L-fuzzy sets [14], intuitionistic fuzzy sets [1] and intuitionistic sets [9], as lattices of crisp subsets of some base set S. A detailed analysis of this relation between texture spaces and lattices of fuzzy sets of various kinds may be found in [5, 6, 9]. The concept of a ditopology on a texture space is introduced in [3] and corresponds in a natural way to a fuzzy topology. In general ditopological texture spaces may be regarded as natural generalizations of both topological spaces and bitopological spaces [16]. The notion of connectedness in ditopological texture space was introduced in [10], which is being extended by Tantawy et al. in [12]. In this paper, we introduce the notion of ditopological texture spaces with ideal. Also connectedness in ditopological texture spaces with an ideal is studied. We study the notions of $\star$-connectedness in ditopological texture spaces with ideal. Some new types of connectedness in $\star$-ditopological texture spaces namely, locally $\star$-connectedness, totally $\star$-disconnectedness, hyperconnectedness and $\star$-hyperconnectedness have investigated.

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2 Preliminary

The aim of this section is to collect the relevant definitions and results from texture space and ditopology which will be needed in the sequel.

Definition 2.1. [2]. Let X be a set. Then $L \subseteq P(X)$ is called texturing of X and X is said to be textured by L if L is separates the points of X, complete, completely distributive lattice with respect to inclusion, which contains X, ϕ, and for which arbitrary meet coincides with intersection and finite joins coincide with unions. The pair (X,L) is then known as a texture space.
In any texture space, the p-sets and q-sets for each $x \in X$ are the sets $p_x = \bigcap\{A \in L : x \in A\}$ and $q_x = \bigvee\{A \in L : x \notin A\}$.
A surjection $\sigma : L \to L$ is called a complementation if $\sigma^2(A) = A\ \forall A \in L$ and $A \subseteq B$ in L implies $\sigma(B) \subseteq \sigma(A)$. A texture with a complementation is said to be complemented.
We now recall the definition of a dichotomous topology (or ditopology for short) on a texture given in [2].

Definition 2.2. [3]. (L, τ, K) is called a ditopological texture space on X if

(1) $\tau \subseteq L$ satisfies

- **(a)** $X,\ \phi \in \tau$,
- **(b)** $G_1,\ G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$, and
- **(c)** $G_i \in \tau, i \in I \Rightarrow \bigvee_{i\in I} G_i \in \tau$, and

(2) $K \subseteq L$ satisfies

- **(a)** $X,\ \phi \in K$,
- **(b)** $F_1,\ F_2 \in K \Rightarrow F_1 \cup F_2 \in K$, and
- **(c)** $F_i \in K, i \in I \Rightarrow \bigwedge_{i\in I} F_i \in K$.

The elements of τ are called open and those of K are called closed. We refer to τ as the topology and to K as the cotopology of (τ,K).
In general there is no a priori relation between τ and K , but if σ is a complementation on (X, L), and τ, k are related by the relation $K = \sigma(\tau)$, then we call (τ, K) a complemented ditopology on (X, L, σ).
Finally, let $Z \subseteq X$. Then the closure of Z is the set $[Z] = \bigcap\{F \in K : Z \subseteq F\}$, the interior is $]Z[= \bigvee\{G \in \tau : G \subseteq Z\}\}$, the exterior is $ext(Z) = \bigvee\{G \in \tau : G \cap Z = \phi\}$ and Z is called dense in X if $[Z] = X$. Also, if $A \nsubseteq F\ \forall F \in K - \{X\}$, we say A is co-dense.

Example 2.1. [8].

(1) For any texture (X, L), a ditopology (τ, K) with $\tau = L$ is called discrete, and one with $K = L$ is called co-discrete.

(2) For any texture (X, L), a ditopology (τ, K) with $\tau = \{X, \phi\}$ is called indiscrete, and one with $K = \{X, \phi\}$ is called co-indiscrete.

(3) For any topology τ on X, (τ, τ'), $\tau' = \{X - G : G \in \tau\}$, is a complemented ditopology on the usual(crisp) set structure $(X, P(X), \sigma_X)$ of X, where $\sigma_X : P(X) \to P(X)$ defined by $\sigma_X(A) = A'$ where $A' = X - A\ \forall A \in P(X)$.

(4) For any bitopology (τ_1, τ_2) on X, (τ_1, τ_2') is a ditopology on$(X, P(X))$.

Definition 2.3. [10]. Let (X,L) be a texture space, $A \subseteq X$.Then, We define $\lambda(A)$ by $\lambda(A) = \bigvee_{x\in A} P_x$. Hence, $\lambda(A)$ is the smallest element of L containing A.

Definition 2.4. [10]. Let (X_i,L_i) be a texture spaces on X_i,$i = 1, 2$ and$f : X_1 \to X_2$ a mapping. We define the mapping $\tilde{f}^{-1} : L_2 \to L_1$ by $\tilde{f}^{-1}(l) = \lambda_1(f^{-1}(l)\ \forall l \in L_2$, where λ_1 is defined for L_1 as in definition 2.3.

Theorem 2.1. [10].The following are equivalent for a function $f : X_1 \to X_2$.

(1) $f^{-1}(l) \in L_1 \; \forall l \in L_2$.

(2) $\tilde{f}^{-1}(l) = f^{-1}(l) \; \forall l \in L_2$.

Definition 2.5. [10]. Let (X_i,L_i,K_i) be a ditopological texture space on X_i, $i = 1, 2$ and $f : X_1 \to X_2$ a mapping. We say that f is continuous if
(1) $\tilde{f}^{-1}(G) \in \tau_1 \; \forall G \in \tau_2$, and
(2) $\tilde{f}^{-1}(F) \in K_1 \; \forall F \in K_2$.

Definition 2.6. [10]. Let(X,L) be a texture space and $\phi \neq Z \subseteq X$. $\{A, B\} \subseteq P(X)$ is said to be a partition of Z if $A \cap Z \neq \phi$, $Z \nsubseteq B$ and $A \cap Z = B \cap Z$. Here we may interchange the roles of A and B. Indeed if $\{A, B\}$ is a partition of Z, then we also have $B \cap Z \neq \phi$ and $Z \nsubseteq A$.

Definition 2.7. [10]. Let (L, τ, K) be a ditopological texture space on X and $Z \subseteq X$. Z is said to be connected if there exists no partition $\{G, F\}$ with $G \in \tau$ and $F \in K$.

Theorem 2.2. [10]. Let (X, L, τ, K) be a ditopological texture space, then X is connected if and only if $\tau \cap K = \{X, \phi\}$.

Theorem 2.3. [10]. Let Z be a connected set in a ditopological texture space (X_1, L_1, τ_1, K_1) and f be a continuous function of X_1 in to a ditopological texture space (X_2, L_2, τ_2, K_2) satisfying one of the equivalent conditions of Theorem 2.1. Then $f(Z)$ is connected in X_2.

Definition 2.8. [12]. A set which is τ-open as well as K-closed is said to be clopen.

Theorem 2.4. [12]. Let (X, L, τ, K) be a ditopological texture space, then the following are equivalent:

(1) X is connected.

(2) X has no a partition $\{A, B\} \subseteq P(X)$ with $A \in \tau$ and $B \in K$.

(3) There is no proper subset A of X which is clopen.

(4) X can not be expressed as an union of two nonempty disjoint subsets A, B of X with $A \in \tau$ and $B \in K'$.

(5) X can not be expressed as an union of two nonempty disjoint subsets A, B of X with $A \in \tau'$ and $B \in K$.

(6) X can not be expressed as an union of two separated subsets A, B of X.

Definition 2.9. [12]. Let (X, L, τ, K) be a ditopological texture space and let $Z \subseteq X$ with $x \in Z$. Then the component of Z w.r.t x is the maximal of all connected subsets of Z containing the point x and denoted by $C(Z, x)$, i.e

$$C(Z, x) = \bigvee \{Y \subseteq Z : x \; \in Y, \; Y \textit{ is connected}\}.$$

Theorem 2.5. [12]. Every clopen connected subset of a ditopological texture space (X, L, τ, K) is a component of X.

Theorem 2.6. [12]. Let (Y, L_Y, τ_Y, K_Y) be a subspace of a ditopological texture space (X, L, τ, K) and $A \subseteq Y$. Then

(1) $Cl_Y(A) = [A] \cap Y$.

(2) $]A[\subseteq Int_Y(A)$.

(3) $ext_Y(A) = Y \cap ext(A)$.

Definition 2.10. [12]. A ditopological texture space (X, L, τ, K) is said to be locally connected at a point $x \in X$ if and only if every open subset of X containing x contains a connected open set containing x. X is said to be locally connected if and only if it is locally connected at each of its points.

Theorem 2.7. [12]. Every connected ditopological texture space is a locally connected.

Definition 2.11. [12]. A ditopological texture space (X, L, τ, K) is said to be totally disconnected if and only if $\forall x,\ y \in X$ s.t $x \neq y$ $\exists$ non empty disjoint clopen proper subsets $A,\ B$ of X s.t $x \in A$ and $y \in B$.

Definition 2.12. [12]. A ditopological texture space (X, L, τ, K) is said to be extremely disconnected if for every open set $G \subseteq X$ we have $[G]$ is open in X.

Theorem 2.8. [12]. Let (Y, L_Y, τ_Y, K_Y) be a subspace of a ditopological texture space (X, L, τ, K). Then

(1) Every τ_Y open set is τ open set if and only if $Y \in \tau$.

(2) Every K_Y closed set is K closed set if and only if $Y \in K$.

Definition 2.13. [17]. A topological space (X, τ) is said to be hyperconnected if for every pair of nonempty open sets of X has a nonempty intersection.

Definition 2.14. [15]. A nonempty collection I of subsets of a nonempty set X is said to be an ideal on X, if it satisfies the following two conditions:

(1) $A \in I$ and $B \subseteq A \Rightarrow B \in I$,

(2) $A \in I$ and $B \in I \Rightarrow A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $()^* : P(X) \to P(X)$, called a local function of A with respect to τ and I, is denoted by $A^*(I, \tau)$ or $A^*(I)$ and defined as follows, for $A \subseteq X$, $A^*(I, \tau) = \{x \in X : O_x \cap A \notin I\ \forall\ O_x \in \tau\}$. A Kuratowski closure operator for the topology $\tau^*(I, \tau)$, called the $\star$-topology, finer than τ, is defined by $Cl^*(A) = A \cup A^*$ and $\tau^*(I, \tau)$ or $\tau^*(I)$ is defined by $\tau^*(I) = \{A \subseteq X : Cl^*(A') = A'\}$. Also, (X, τ, I) is called an ideal topological space or simply an ideal space.
For any ideal space (X, τ, I), the collection$\{G - V : G \in \tau, V \in I\}$ is a basis for τ^*.

Definition 2.15. [13]. Nonempty subsets $A,\ B$ of a topological space with an ideal I on X (X, τ, I) are said to be $\star$-separated sets if $Cl^*(A) \cap B = A \cap Cl(B) = \phi$.

Definition 2.16. [13]. A subset A of a topological space (X, τ, I) with an ideal I on X is said to be $\star_s$-connected if A is not the union of two $\star$-separated sets in (X, τ, I).

Definition 2.17. [13]. Let (X, τ, I) be a topological space with an ideal I on X and $x \in X$. The union of all $\star_s$-connected subsets of X containing x is called the $\star_s$-component of X containing x.

Definition 2.18. [11]. A subset A of an ideal topological space (X, τ, I) is said to be $\star$-dense if $Cl^*(A) = X$. An ideal topological space (X, τ, I) is said to be $\star$-hyperconnected if A is $\star$-dense for every nonempty open subset A of X has a nonempty intersection.

3 Ideal Ditopological Texture Spaces

In this section we introduce a ditopological texture space finer than the given ditopological texture space (X, L, τ, K) on the same set X by using the ideal notion. We extend the notion of connectedness to such spaces and study some of its basic properties. We denote by (X, L, τ, K, I) as a ditopological texture space with an ideal I on X.

Definition 3.1. Let (τ, K) be a ditopological space on any texture space with an ideal (X, L, I). Then

(1) define the local function $()^*_\tau : P(X) \to P(X)$ by $A^*(I, \tau) = \{x \in X : O_x \cap A \notin I\ \forall\ O_x \in \tau\}\ \forall\ A \in P(X)$. A Kuratowski closure operator $Cl^*_\tau(.)$ for the topology $\tau^*(I, \tau)$, called the $*$-topology, finer than τ, induced by $Cl^*_\tau(A) = A \cup A^*(I, \tau)$, where $\tau^* = \{G \subseteq X : Cl^*_\tau(G') = G'\}$.

(2) let $K' = \{X - F : F \in K\}$, which is a topology on X, so we again define a local function $()^*_{K'} : P(X) \to P(X)$, where $A^*(I, K')$ is the local function of A w.r.t $I,\ K'$. Also a Kuratowski closure operator $Cl^*_{K'}(.)$ for the topology $K'^*(I, K')$, called the $*$-topology, finer than K'. Hence, $K'^{*'} = \mathcal{K}^*$ is a family of closed subsets of X finer than K.

(3) let (X, L^*) be the smallest texture structure space containing L, τ^* and $\mathcal{K}^*$. Hence, $(\tau^*, \mathcal{K}^*)$ is called the $\star$-ditopology on (X, L^*), finer than (τ, K) on (X, L).

Finally, let $Z \subseteq X$. Then the $\star$-closure of Z is the set $[Z]^{\mathcal{K}^*} = \bigcap\{F \in \mathcal{K}^* : Z \subseteq F\}$, the $\star$-interior is $]Z[^{\tau^*} = \bigvee\{G \in \tau^* : G \subseteq Z\}\}$, the $\star$-exterior is $ext^{\tau^*}(Z) = \bigvee\{G \in \tau^* : G \cap Z = \phi\}$ and Z is called $\star$-dense in X if $[Z]^{\mathcal{K}^*} = X$. Also if $A \nsubseteq F \ \ \forall F \in \mathcal{K}^* - \{X\}$, we say A is $\star$-co-dense.

Examples 3.1. **(1)** If $I = \phi$, then $A^*(I, \tau) = [A]^\tau$ and $A^*(I, K') = [A]^K \ \forall A \in P(X)$. Hence, $Cl^*_\tau(A) = [A]^\tau$, $Cl^*_{K'}(A) = [A]^K$, $\tau^* = \tau$ and $\mathcal{K}^* = K$.

(2) If $I = P(X)$, then $A^*(I, \tau) = \phi$ and $A^*(I, K') = \phi \ \forall A \in P(X)$. Hence, $Cl^*_\tau(A) = A$, $Cl^*_{K'}(A) = A$, $\tau^* = P(X)$ and $\mathcal{K}^* = P(X)$.

(3) If $I \subseteq J$, then $A^*(I, \tau) \subseteq A^*(J, \tau)$ and $A^*(I, K') \subseteq A^*(J, K')$. Hence, the $\star$- ditopological texture space $(X, L^*, \tau^*(J), \mathcal{K}^*(J))$ is finer than the $\star$- ditopological texture space $(X, L^*, \tau^*(I), \mathcal{K}^*(I))$.

Remark 3.1. In the case of $\star$- ditopological texture space, we choose $L^* \supseteq L$. Indeed sometimes $\tau^*, \mathcal{K}^* \nsubseteq L$, as in the following examples.

Examples 3.2. **(1)** Let $X = \{a, b, c\}$, $L = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$, $\tau = \{X, \phi, \{b\}\}$, $K = \{X, \phi, \{c\}\}$ and $I = \{\phi, \{b\}\}$ be an ideal on X. Then $\tau^* = \{X, \phi, \{b\}, \{a, c\}\}$, $\mathcal{K}^* = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\tau^* \nsubseteq L$. Hence, $L^* = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$.

(2) Let $X = \{a, b, c\}$, $L = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}$, $K = L$ and $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ be an ideal on X. Then $\tau^* = P(X)$, $\mathcal{K}^* = P(X)$ and τ^*, $\mathcal{K}^* \nsubseteq L$. Hence, $L^* = P(X)$.

(3) Let $X = [0, 1]$, $L = \{[0, r] : r \in X\} \cup \{\phi\}$, $\tau = K = \{X, \phi\}$, $I = \{A \subseteq X : A \ is \ finite\}$. Then $\tau^* = \tau_\infty$ [15], where τ_∞ is the cofinite topology, $\tau_\infty \nsubseteq L$, also $\mathcal{K}^* = \tau'_\infty$ and $\mathcal{K}^* \nsubseteq L$. Hence, $L^* = P(X)$.

Theorem 3.1. Let (L, τ, K) be a ditopological texture space on X, I be an ideal on X and $(X, L^*, \tau^*, \mathcal{K}^*$ be the $\star$-ditopological texture space w.r.t I. Then

(1) $\beta(I, \tau) = \{G - V : G \in \tau, V \in I\}$ is a basis of τ^*.

(2) $\beta(K', I) = \{F - V : F \in K', V \in I\}$ is a basis of $\mathcal{K}^{*'}$.

Proof. **(1)** Since $X \in \tau, \phi \in I$, then $X - \phi \in \beta$, hence $X \in \beta$ and $\bigcup_{i \in I}(G_i - V_i) = X$. Also, let $G_1 - V_1, G_2 - V_2 \in \beta$, s.t $x \in (G_1 - V_1) \cap (G_2 - V_2)$, then $x \in (G_1 \cap G_2) - (V_1 \cup V_2) \in \beta$. Therefore, $x \in (G_1 \cap G_2) - (V_1 \cup V_2) \subseteq (G_1 - V_1) \cap (G_2 - V_2)$. Hence, β is a basis of τ^*.

(2) By a similar way.

□

Definition 3.2. Let (X, L, τ, K, I) be a ditopological texture space with an ideal, $Y \subseteq X$ s.t $Y \in L$ and let L_Y, τ_Y, K_Y, I_Y are the restriction of L, τ, K, I on I, then $(Y, L_Y, \tau_Y, K_Y, I_Y)$ is a ditopological texture subspace with an ideal I_Y on Y.

Theorem 3.2. Let (X, L, τ, K, I) be a ditopological texture space with an ideal I on X and $A \subseteq Y \subseteq X$. Then

(1) $A^*(\tau_Y, I_Y) = Y \cap A^*(\tau, I)$.

(2) $A^*(K'_Y, I_Y) = Y \cap A^*(K', I)$.

Proof. **(1)** $A^*(\tau_Y, I_Y) = \{y \in Y : O_y \cap A \notin I_Y \ \forall \ O_y \in \tau_Y\} = \{y \in Y : (Y \cap G) \cap A \notin I_Y \ \forall \ Y \cap G \in \tau_Y\} = \{y \in Y : G \cap A \notin I \ \forall \ G \in \tau\} = Y \cap \{y \in X : G \cap A \notin I \ \forall \ G \in \tau\} = Y \cap A^*(\tau, I)$.

(2) By a similar way.

□

Theorem 3.3. Let $(Y, L_Y, \tau_Y, K_Y, I_Y)$ be a ditopological texture subspace with an ideal I_Y on Y of a ditopological texture space (X, L, τ, K, I) and $A \subseteq Y \subseteq X$. Then

(1) $[A]^{\mathcal{K}^*_Y} = Y \cap [A]^{\mathcal{K}^*}$, where $\mathcal{K}^*{}_Y$ is the family of $\mathcal{K}^*$-closed subsets of Y.

(2) $]A[^{\tau^*} \subseteq Int^{\tau^*_Y}(A)$, where τ^*_Y is the family of τ^*-open subsets of Y.

(3) $ext^{\tau^*_Y}(A) = Y \cap ext^{\tau^*}(A)$.

Proof. Immediate from Theorem 2.6, Definition 3.1 and Theorem 3.2. □

Remark 3.2. Note that, the equality in Theorem 3.3(2) holds for all subsets of Y if and only if Y is τ^*-open. Indeed, if $x \in Int^{\tau^*_Y}(A)$. Then $x \in \bigvee\{G \cap Y \in \tau^*_Y : G \cap Y \subseteq A\}$. Since $Y \in \tau^*$, then $x \in]A[^{\tau^*}$.

Theorem 3.4. Let $(Y, L_Y, \tau_Y, K_Y, I_Y)$ be a ditopological texture subspace with an ideal I_Y on Y of a ditopological texture space (X, L, τ, K, I) and consider the $\star$-ditopological texture space $(X, L^*, \tau^*, \mathcal{K}^*)$. Then

(1) Every τ^*_Y-open set is τ^*-open set if and only if $Y \in \tau^*$.

(2) Every $\mathcal{K}^*{}_Y$-closed set is $\mathcal{K}^*$-closed set if and only if $Y \in \mathcal{K}^*$.

Proof. **(i)** Suppose that every τ^*_Y-open set is τ^*-open set, then $Y \in \tau^*_Y \subseteq \tau^*$. Conversely, if $Y \in \tau^*$ and $A \subseteq Y$ is τ^*_Y-open, then $A = Y \cap G$ for some $G \in \tau^*$, but $Y \in \tau^*$, hence $A \in \tau^*$.

(2) By a similar way.

□

Corollary 3.1. Let $(Y, L_Y, \tau_Y, K_Y, I_Y)$ be a τ-open subspace of (X, L, τ, K, I), consider the $\star$-ditopological texture space $(X, L^*, \tau^*, \mathcal{K}^*)$ and $A \subseteq Y$. Then A is τ^*_Y-open set if and only if it is τ^*-open set.

Proof. Immediate from Theorem 3.4. □

Corollary 3.2. Let $(Y, L_Y, \tau_Y, K_Y, I_Y)$ be a K-closed subspace of (X, L, τ, K, I), consider the $\star$-ditopological texture space $(X, L^*, \tau^*, \mathcal{K}^*)$ and $A \subseteq Y$. Then A is $\mathcal{K}^*_Y$-closed set if and only if it is $\mathcal{K}^*$-closed set .

Proof. Immediate from Theorem 3.4. □

Definition 3.3. Let (X, L, τ, K, I) be a ditopological texture space with an ideal on X and $Z \subseteq X$. $\{A, B\} \subseteq P(X)$, where $(A, B) \in \tau^* \times K$ or $(A, B) \in \mathcal{K}^* \times \tau$, is said to be a $\star$-partition of Z if $A \cap Z \neq \phi$, $Z \nsubseteq B$ and $A \cap Z = B \cap Z$.

Definition 3.4. Let (X, L, τ, K, I) be a ditopological texture space with an ideal on X and $Z \subseteq X$. Z is said to be $\star$-connected if there exists no $\star$-partition of Z.

Theorem 3.5. Let (X, L, τ, K, I) be a ditopological texture space with an ideal on X and $(X, L^*, \tau^*, \mathcal{K}^*)$ be a $\star$-ditopological texture space, then the following are equivalent:

(1) X is $\star$-connected.

(2) There is no proper subset A of X with $A \in \tau^* \cap K$ and $A \in \mathcal{K}^* \cap \tau$.

(3) X can not be expressed as an union of two nonempty disjoint subsets A, B of X with $A \in \tau$ and $B \in \mathcal{K}^{*'}$ (resp. $A \in \tau^*$ and $B \in K'$).

(4) X can not be expressed as an union of two nonempty disjoint subsets A, B of X with $A \in \tau'$ and $B \in \mathcal{K}^*$ (resp. $A \in \tau^{*'}$ and $B \in K$).

Proof. Immediate by Theorem 2.4 and Definition 3.4. □

Theorem 3.6. Let (X, L, τ, K, I) be a $\star$-connected ditopological texture space with an ideal. Then (X, L, τ, K) is connected.

Proof. Immediate. □

Remark 3.3. The converse of Theorem 3.6 is not true in general, as in the following example. Let $X = \{a,b,c\}$, $L = \{X, \phi, \{b\}, \{c\}, \{b,c\}, \{a,b\}\}$, $\tau = \{X, \phi, \{b\}, \{a,b\}\}$, $K = \{X, \phi, \{c\}\}$, and $I = \{\phi, \{a\}, \{b\}, \{a,b\}\}$ be an ideal on X, then $\tau^* = P(X)$, $\mathcal{K}^* = P(X)$ and $L^* = P(X)$. Then (X, L, τ, K) is connected but (X, L, τ, K, I) is $\star$-disconnected.

Theorem 3.7. Let (X, L, τ, K, I) be a $\star$-connected ditopological texture space with an ideal. Then (X, L, τ, K) is locally connected.

Proof. Immediate by Theorem 2.7 and Theorem 3.6. □

Theorem 3.8. X is $\star$-connected if for all pair of point $x,\ y \in X$ with $x \neq y$ there exists a $\star$-connected set $Z \subseteq X$ with $x,\ y \in Z$.

Proof. Suppose that X is $\star$-disconnected. Then by Theorem 3.5 there exists a proper subset A of X s.t $A \in \tau^* \cap K$ or $A \in \tau \cap \mathcal{K}^*$. If $A \in \tau^* \cap K$. Then we choose $x,\ y \in X$ with $x \in A$ and $y \notin A$. If there exists a $\star$-connected set Z with $x,\ y \in Z$, then $Z \nsubseteq A$ and $A \cap Z \neq \phi$. Hence, $\{A, A\}$ is a partition of Z, which is a contradiction with the $\star$-connectedness of Z. By a similar way if $A \in \tau \cap \mathcal{K}^*$. Hence, we get the proof. □

Corollary 3.3. Let Z be a $\star$-connected set in a ditopological texture space with an ideal $(X_1, L_1, \tau_1, K_1, I_1)$ and f be a continuous function of X_1 into a ditopological texture space with an ideal $(X_2, L_2, \tau_2, K_2, I_2)$ satisfying one of the equivalent conditions of Theorem 2.1. Then $f(Z)$ is $\star$-connected in X_2.

Proof. Suppose that $f(Z)$ is not $\star$-connected in X_2. Let $\{G, F\} \subseteq P(X)$ be a partition of $f(Z)$ with $G \in \tau_2^*$ and $F \in K_2$ or $G \in \tau_2$ and $F \in \mathcal{K}_2^*$. If $G \in \tau_2^*$ and $F \in K_2$. Then $f(Z) \cap G \neq \phi$, $f(Z) \nsubseteq F$ and $f(Z) \cap G = f(Z) \cap F$. Since $\tilde{f}^{-1}(G) = f^{-1}(G)$ and $\tilde{f}^{-1}(F) = f^{-1}(F)$, then $\{f^{-1}(G), f^{-1}(F)\}$ is a partition of Z where $f^{-1}(G) \in \tau_1^*$ and $f^{-1}(F) \in K_1$, which is a contradiction with the $\star$-connectedness of Z. By a similar way if $G \in \tau_2$ and $F \in \mathcal{K}_2^*$. Hence, we get the proof.

□

Definition 3.5. Let (X, L, τ, K, I) be a ditopological texture space with an ideal and let $Z \subseteq X$ with $x \in Z$.Then the $\star$-component of Z w.r.t x is the maximal of all $\star$-connected subsets of Z containing the point x and denoted by $C(Z, x)$, i.e

$$C(Z,x) = \bigvee\{Y \subseteq Z : \text{x} \in Y, Y is \star -connected\}.$$

Theorem 3.9. If Z is a $\star$-component and $\text{ext}^{\tau^*}(Z) = \phi$, then $Z = [Z]^*$.

Proof. We want to prove that $[Z]^* \subseteq Z$. So Let (X, L, τ, K, I) be a ditopological texture space with an ideal, Z be a $\star$-component subset of $(X, L^*, \tau^*, \mathcal{K}^*)$ and $x \notin Z$, but Z is a maximal $\star$-connected set, then $Z \cup \{x\}$ can not be $\star$-connected set. Take a partition $\{A, B\}$ of $Z \cup \{x\}$ s.t $(A, B) \in \tau^* \times K$ with $A \cap (Z \cup \{x\}) \neq \phi$, $Z \cup \{x\} \nsubseteq B$ and $(Z \cup \{x\}) \cap A = (Z \cup \{x\}) \cap B$. Since $Z \subseteq Z \cup \{x\}$, then $Z \cap A = Z \cap B$ and Z is $\star$-connected. Hence, either $Z \cap A = \phi$ or $Z \subseteq B$. Suppose $Z \cap A = \phi$. Since $x \notin Z$ and $A \cap (Z \cup \{x\}) \neq \phi$, then $x \in A$. Hence, $x \in ext^{\tau^*}(Z)$, which is a contradiction. If $Z \subseteq B$ and $Z \cup \{x\} \nsubseteq B$, then $x \notin B$ and $[Z]^* \subseteq B$. Hence, $x \notin [Z]^*$. By a similar way if we take a partition $\{A, B\}$ of $Z \cup \{x\}$ s.t $(A, B) \in \tau \times \mathcal{K}^*$. This completes the proof. □

Theorem 3.10. Let $\{Z_i : i \in I\}$ be a family of $\star$-connected subsets in L^* with $\bigcap_{i \in I} Z_i \neq \phi$, then $\bigvee_{i \in I} Z_i$ is also $\star$-connected.

Proof. Suppose that $Z = \bigvee_{i \in I} Z_i$ is $\star$-disconnected. Then we may choose a partition $\{A, B\}$ of Z s.t $(A, B) \in \tau^* \times K$ or $(A, B) \in \tau \times \mathcal{K}^*$ with $Z \cap A \neq \phi$, $Z \nsubseteq B$ and $A \cap Z = B \cap Z$. Since $Z_i \subseteq Z\ \forall i \in I$, then $A \cap Z_i = B \cap Z_i\ \forall i \in I$. But Z_i is $\star$-connected, then either $Z_i \cap B = \phi$ or $Z_i \subseteq A$. Now we choose $x \in \bigcap_{i \in I} Z_i$, then $x \in Z_i\ \forall i \in I$, so either $x \in A$ or $x \notin B$. Suppose $x \in A$, then $A \cap Z_i \neq \phi\ \forall i \in I$, $A \cap Z_i = B \cap Z_i$ and Z_i is $\star$-connected, then $Z_i \subseteq B\ \forall i \in I$. Hence, $Z = \bigvee_{i \in I} Z_i \subseteq B$, which is a contradiction. Now suppose $x \notin B$, since $x \in Z_i\ \forall i \in I$, then $Z_i \nsubseteq B$, $A \cap Z_i = B \cap Z_i\ \forall i \in I$ and Z_i is $\star$-connected. Hence, $A \cap Z_i = \phi$ and $A \cap Z = A \cap (\bigvee_{i \in I} Z_i) = \bigvee_{i \in I}(A \cap Z_i) = \bigvee_{i \in I}(\phi) = \phi$, which is a contradiction. □

Corollary 3.4. Let $\{Z_i : i \in I\}$ be a family of $\star$-connected subsets of a ditopological texture space with an ideal (X, L, τ, K, I) s.t one of the members of the family intersects every other members, then $Z = \bigvee_{i \in I} Z_i$ is $\star$-connected.

Proof. Let $Z_{i0} \in \{Z_i : i \in I\}$ s.t $Z_{i0} \cap Z_i \neq \phi \quad \forall i \in I$. Then $Z_{i0} \bigvee Z_i$ is $\star$-connected $\quad \forall i \in I$ by Theorem 3.10., hence the collection $\{Z_{i0} \vee Z_i : i \in I\}$ is a collection of a $\star$-connected subsets of X, which having a non-empty intersection. So $Z = \bigvee_{i \in I} Z_i$ is $\star$-connected by Theorem 3.10. □

Theorem 3.11. Let $Z \subseteq X$ be a $\star$-connected set, $Z \subseteq Y \subseteq [Z]^*$ and $ext^{\tau^*}(Z) \cap Y = \phi$, then Y is $\star$-connected.

Proof. Suppose that Y is $\star$-disconnected. Take a partition $\{A, B\}$ of Y with $(A, B) \in \tau^* \times K$. Then $Y \cap A \neq \phi$, $Y \nsubseteq B$ and $Y \cap A = Y \cap B$. Since $Z \subseteq Y$, then $Z \cap A = Z \cap B$ but Z is $\star$-connected, so either $Z \cap A = \phi$ or $Z \subseteq B$. Suppose $Z \cap A = \phi$, then $A \subseteq ext^*(Z)$ and $Y \cap A \subseteq Y \cap ext^*(Z)$. Hence, $A \cap Y = \phi$ is a contradiction. Now suppose $Z \subseteq B$, then $[Z]^* \subseteq B$, hence $Y \subseteq B$, which is a contradiction. □

Corollary 3.5. The $\mathcal{K}^*$-closure of $\star$-connected subset of a ditopological texture space (X, L, τ, K, I) with an ideal is $\star$-connected.

Proof. Immediate by Theorem 3.11. □

Corollary 3.6. Every $\star$-component of a ditopological texture space (X, L, τ, K, I) with an ideal is $\mathcal{K}^*$-closed set.

Proof. Immediate from Definition 3.5 and Corollary 3.5. □

4 $\star_s$-Connectedness in Ditopological Texture Spaces Modulo Ideal

Definition 4.1. Nonempty subsets A, B of a ditopological texture space with (X, L, τ, K, I) an ideal are said to be $\star$-separated sets if either $A \cap [B]^{\tau^*} = B \cap [A]^K = \phi$ or $A \cap [B]^{\mathcal{K}^*} = B \cap [A]^\tau = \phi$.

Theorem 4.1. Let (X, L, τ, K, I) be a ditopological texture space with an ideal and $(X, L^*, \tau^*, \mathcal{K}^*)$ be a $\star$-ditopological texture space, then the following are equivalent:

(1) X is $\star$-connected.

(2) There is no proper subset A of X with $A \in \tau^* \cap K$ and $A \in \mathcal{K}^* \cap \tau$.

(3) X can not be expressed as an union of two nonempty disjoint subsets A, B of X with $A \in \tau$ and $B \in \mathcal{K}^{*'}$ (resp. $A \in \tau^*$ and $B \in K'$).

(4) X can not be expressed as an union of two nonempty disjoint subsets A, B of X with $A \in \tau'$ and $B \in \mathcal{K}^*$ (resp. $A \in \tau^{*'}$ and $B \in K$).

(5) X can not be expressed as an union of two $\star$-separated sets.

Proof. Immediate from Theorem2.4 and Definition 4.1.

□

Theorem 4.2. Let (X, L, τ, K, I) be a ditopological texture space with an ideal I. If A and B are $\star$-separated sets of X s.t $A \cup B \in \tau \cap K$, then either A $(resp.B) \in \tau^* \cap K$ or A $(resp.B) \in \mathcal{K}^* \cap \tau$.

Proof. Suppose that A, B be a $\star$-separated sets s.t $A \cup B \in \tau \cap K$, then $[A \cup B]^{\tau^*} \in \tau^{*'}$. Since $[B]^{\tau^*} \in \tau^{*'}$, then $([B]^{\tau^*})' \in \tau^*$, it follows that $(A \cup B) \cap ([B]^{\tau^*})' \in \tau^*$. Then $A = (A \cap ([B]^{\tau^*})') \cup (B \cap ([B]^{\tau^*})' \in \tau^*$, hence $A \in \tau^*$. Since $A \cup B \in K$ and $[A]^K \in K$, then $(A \cup B) \cap [A]^K \in K$. Then, $A = (A \cap [A]^K) \cup (B \cap [A]^K) \in K$, it follows that $A \in K$. This means that, $A \in \tau^* \cap K$. The rest of the proof by s similar way.

□

Definition 4.2. A subset Z of a ditopological texture space with an ideal (X, L, τ, K, I) is called $\star_s$-connected if Z is not the union of two $\star$-separated sets in (X, L, τ, K, I).

Theorem 4.3. Let Y be a clopen subset of a ditopological texture space with an ideal (X, L, τ, K, I). Then Y is $\star_s$-connected if and only if it is $\star$-connected.

Proof. $\Rightarrow$**:** Suppose that Y is $\star$-disconnected, then either $\exists$ nonempty disjoint τ^*_Y-open and K_Y-open or $\mathcal{K}^*$-closed and τ-open subsets $A,\ B$ of Y s.t $Y = A \cup B$. Since $Y \in \tau \cap K$, by Theorem 2.8 and Theorem 3.4, A and B are τ^*-open and K-open or $\mathcal{K}^*$-closed and τ-closed subsets of X. Since A and B are disjoint, then either $B \cap [A]^{\tau^*} = A \cap [B]^K = \phi$ or $A \cap [B]^{\mathcal{K}^*} = B \cap [A]^\tau = \phi$. This implies that, $A,\ B$ are $\star$-separated sets in X s.t $Y = A \cup B$. Hence, Y is not $\star_s$-connected, which is a contradiction.

$\Leftarrow$**:** Suppose that Y is not $\star_s$-connected in X, then $\exists$ $\star$-separated sets $A,\ B$ s.t $Y = A \cup B$. By Theorem 4.2 $A \in \tau^* \cap K$. By Theorem 2.8 and Theorem 3.4, $A \in \tau^*_Y \cap K_Y$. Hence, Y is $\star$-disconnected by Theorem 4.1, which is a contradiction.

□

Theorem 4.4. Let Z be a $\star_s$-connected subset of a ditopological texture space with an ideal I on X (X, L, τ, K, I) and $A,\ B$ are $\star$-separated subsets of X with $Z \subseteq A \cup B$, then either $Z \subseteq A$ or $Z \subseteq B$.

Proof. Let $Z \subseteq A \cup B$ for some $\star$-separated subsets $A,\ B$ of X. Since $Z = (Z \cap A) \cup (Z \cap B)$, then $(Z \cap A) \cap ([Z \cap B]^{\tau^*}) \subseteq A \cap [B]^{\tau^*} = \phi$. By a similar way, we have $(Z \cap B) \cap ([Z \cap A]^K) = \phi$, $(Z \cap A) \cap ([Z \cap B]^{\mathcal{K}^*}) = \phi$ and $(Z \cap B) \cap ([Z \cap A]^\tau) = \phi$. Suppose that $Z \cap A$ and $Z \cap B$ are nonempty. Then Z is not $\star_s$-connected, which is a contradiction. Thus, either $Z \cap A = \phi$ or $Z \cap B = \phi$. This implies that, $Z \subseteq A$ or $Z \subseteq B$. □

Theorem 4.5. Let $\{Z_i : i \in J\}$ be a nonempty family of $\star_s$-connected subsets of a ditopological texture space with an ideal (X, L, τ, K, I) with $\bigcap_{i \in J} Z_i \neq \phi$, then $\bigvee_{i \in J} Z_i$ is also $\star_s$-connected.

Proof. Suppose that $Z = \bigvee_{i \in J} Z_i$ is not $\star_s$-connected. Then $Z = A \cup B$ for some two $\star$-separated subsets $A,\ B$ of X. Since $\bigcap_{i \in J} Z_i \neq \phi$, then $\exists\ x \in \bigcap_{i \in J} Z_i\ \forall i \in J$, so $x \in Z_i\ \forall i \in J$ and $x \in A$ or $x \in B$. Suppose that $x \in A$. Since $Z_i \subseteq A \cup B\ \forall i \in J$, then $Z_i \subseteq A$ or $Z_i \subseteq B\ \forall i \in J$ by Theorem 4.4. Since $A \cap B = \phi$, $Z_i \subseteq A; \forall i \in J$, then $Z \subseteq A$. This implies that, $B = \phi$, which is a contradiction. The rest of the proof is similar. □

Corollary 4.1. Let $\{Z_i : i \in J\}$ be a nonempty family of $\star_s$-connected subsets of a ditopological texture space with an ideal (X, L, τ, K, I) s.t one of the members of the family intersects every other members, then $Z = \bigvee_{i \in J} Z_i$ is $\star_s$-connected.

Proof. The proof is similar to Corollary 3.4. □

Definition 4.3. Let (X, L, τ, K, I) be a ditopological texture space with an ideal and let $Z \subseteq X$ with $x \in Z$. Then the $\star_s$-component of Z w.r.t x is the maximal of all $\star_s$-connected subsets of Z containing the point x.

Theorem 4.6. Every $\star_s$-component of a ditopological texture space with an ideal (X, L, τ, K, I) is a maximal $\star_s$-connected subset of X.

Proof. Immediate from Definition 4.3. □

Theorem 4.7. Let (X, L, τ, K, I) be a ditopological texture space with an ideal. Then:

(1) Each point in X is contained in exactly one component of X.

(2) Any two components w.r.t two different points of X are either disjoint or identical.

(3) Every τ^*-open and K-closed $\star_s$-connected subset of X is a $\star_s$-component of X.

Proof. Immediate from Theorem 2.5 and Theorem4.6. □

Corollary 4.2. The set of all distinct $\star_s$-components of a ditopological texture space with an ideal (X, L, τ, K, I) partition the set X.

Proof. Immediate from Theorem 4.7. □

5 Relation Between the $\star$-Connectedness

In this section we introduce some new types of a $\star$-connectedness in a ditopological texture spaces with an ideal (X, L, τ, K, I).

Definition 5.1. A ditopological texture space with an ideal (X, L, τ, K, I) is said to be locally $\star$-connected at a point $x \in X$ if and only if every τ^*-open and K-closed set and for every $\mathcal{K}^*$-closed and τ-open set containing x contains a $\star$-connected open set containing x and is said to be locally $\star$-connected if and only if it is locally $\star$-connected at each of its points.

Theorem 5.1. Every $\star$-connected space is a locally $\star$-connected space.

Proof. Suppose that (X, L, τ, K, I) be a $\star$-connected ditopological texture space with an ideal and $(X, L^*, \tau^*, \mathcal{K}^*)$ be a $\star$-ditopological texture space. Then $\tau^* \cap K = \{X, \phi\}$ and $\mathcal{K}^* \cap \tau = \phi$, hence $\forall x \in X \ \exists \ X \in \tau^*$ which is $\star$-connected set and $x \in X \subseteq X$. Then X is locally $\star$-connected. □

Theorem 5.2. Every $\star$-component of a locally $\star$-connected ditopological texture space with an ideal (X, L, τ, K, I) is a $\star$-open set.

Proof. Let (X, L, τ, K, I) be a locally $\star$-connected ditopological texture space with an ideal, $x \in X$ and C be a $\star$-component of X w.r.t x. Since (X, L, τ, K, I) is a locally $\star$-connected space. Therefore, every τ^*-open and K-closed set and every $\mathcal{K}^*$-closed and τ-open set containing x contains a $\star$-connected open set G containing x, but C is the largest $\star$-connected set containing x, then $x \in G \subseteq C$, i.e C is a τ^*-nbd of x. Then C is a τ^*-nbd of each of its points. This implies that, C is a $\star$-open set. □

Definition 5.2. A ditopological texture space with an ideal (X, L, τ, K, I) is said to be totally $\star$-disconnected if and only if $\forall x,\ y \in X$ s.t $x \neq y\ \exists$ a non empty disjoint τ^*-open and K-closed or $\mathcal{K}^*$-closed and τ-open subsets $A,\ B$ of X s.t $x \in A$ and $y \in B$.

Theorem 5.3. The $\star$-components of a totally $\star$-disconnected ditopological texture space with an ideal (X, L, τ, K, I) are the singleton subsets of X.

Proof. Suppose that Y be a subset of a totally $\star$-disconnected ditopological texture space with an ideal (X, L, τ, K, I), which containing more than one point of X. Let $y_1,\ y_2 \in Y \subseteq X$ s.t $y_1 \neq y_2$, since X is totally $\star$-disconnected, then $\exists$ a non empty disjoint τ^*-open and K-closed or $\mathcal{K}^*$-closed and τ-open proper subsets $A,\ B$ of X s.t $y_1 \in A$ and $y_2 \in B$. Clearly, $\{A, A\}$ is a partition of Y in both cases, then Y is $\star$-disconnected set, but the $\star$-components are $\star$-connected set, hence no subsets of X containing more than one point can be a $\star$-component of X. □

Definition 5.3. Let (X, L, τ, K) be a ditopological texture space. Then X is said to be hyperconnected if every pair of nonempty τ-open and K-open proper subsets $A,\ B$ of X respectively, has a nonempty intersection, i.e
(X, L, τ, K) is said to be hyperconnected if $\forall\ \ A \in \tau$ and $B \in K'$ we have $A \cap B \neq \phi$.

Theorem 5.4. Every hyperconnected ditopological texture space is connected.

Proof. Suppose that (X, L, τ, K) be a disconnected ditopological texture space, then there exists a proper subset A of X with $A \in \tau \cap K$. Then $A \in \tau$ and $A' \in K$ s.t $A \cap A' = \phi$, hence X is not hyperconnected, which is a contradiction. □

Remark 5.1. The converse of Theorem 5.4 is not true in general, for the following example,
let $X = \{a, b, c\}$, $L = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$, $\tau = \{X, \phi, \{a\}\}$, $K = \{X, \phi, \{b\}\}$. Then (X, L, τ, K) is connected but not hyperconnected.

Theorem 5.5. Every hyperconnected space is a locally connected space.

Proof. Immediate by Theorem 2.7 and Theorem 5.4. □

Definition 5.4. Let (X, L, τ, K, I) be a ditopological texture space with an ideal and $(X, L^*, \tau^*, \mathcal{K}^*)$ be a $\star$-ditopological texture space. Then X is said to be a $\star$-hyperconnected if A is $\star$-dense for ever nonempty τ-open subset A of X.

Theorem 5.6. Every $\star$-hyperconnected ditopological texture space is $\star$-connected.

Proof. Suppose that (X, L, τ, K, I) be a $\star$-disconnected ditopological texture space with an ideal. Then either $\exists\ A \in \tau$ and $B \in \mathcal{K}^{*'}$ or $A \in \tau^*$ and $B \in K'$ s.t $A \cap B = \phi$ and $X = A \cup B$. Then $B = \phi$, which is a contradiction.Then (X, L, τ, K, I) be a $\star$-connected. □

Theorem 5.7. Every $\star$-hyperconnected ditopological texture space is locally $\star$-connected.

Proof. Immediate by Theorem 5.1 and Theorem 5.6. □

Theorem 5.8. Let (X, L, τ, K, I) be a $\star$-hyperconnected ditopological texture space with an ideal, then (X, L, τ, K) is hyperconnected.

Proof. Immediate. □

Remark 5.2. The converse of Theorem 5.8 not true in general, for the following example, let $X = \{a, b, c\}$, $L = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$, $\tau = \{X, \phi, \{b\}\}$, $K = \{X, \phi, \{c\}\}$ and $I = \{\phi, \{b\}\}$ be an ideal on$X$, then $\tau^* = \{X, \phi, \{b\}, \{a, c\}\}$ and $\mathcal{K}^* = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Hence, $L^* = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Then (X, L, τ, K) is hyperconnected and (X, L, τ, K, I) is not $\star$-hyperconnected.

Theorem 5.9. . Let (X, L, τ, K, I) be a $\star$-hyperconnected ditopological texture space with an ideal, then (X, L, τ, K) is connected.

Proof. Immediate by Theorem 3.6 and Theorem 5.6. □

Theorem 5.10. Let (X, L, τ, K, I) be a $\star$-hyperconnected ditopological texture space with an ideal, then (X, L, τ, K) is locally connected.

Proof. Immediate by Theorem 2.7, Theorem 3.6 and Theorem 5.6. □

Theorem 5.11. The following implications hold for a ditopological texture space (X, L, τ, K, I) with an ideal.

$$\begin{array}{lcc}
(X, L, \tau, K, I) \text{ is } \star\text{-hyperconnected} & \Rightarrow & (X, L, \tau, K) \text{ is hyperconnected} \\
\Downarrow & & \Downarrow \\
(X, L, \tau, K, I) \text{ is } \star\text{-connected} & \Rightarrow & (X, L, \tau, K) \text{ is connected} \\
\Downarrow & & \Downarrow \\
(X, L, \tau, K, I) \text{ is locally } \star\text{-connected} & & (X, L, \tau, K) \text{ is locally connected}
\end{array}$$

Proof. Immediate by Theorem 2.7, Theorem 3.6, Theorem 5.5, Theorem 5.6, Theorem 5.8 and Theorem 5.10. □

6 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. The notion of a texture space, under the name of fuzzy structure, was introduced by Brown in [2], as a means of representing a lattice of fuzzy sets as a lattice of crisp subsets of some base set. The notion of connectedness in ditopological texture spaces was initiated by Diker in [10], which is being extended in [12]. The main purpose of this paper, is to introduce the notion of ditopological texture spaces with ideal (X, L, τ, K, I), which is finer than the given ditopological texture space (X, L, τ, K) on the same set X. We study the notions of $\star$-connected ditopological texture spaces with an ideal and $\star$-connected sets in ditopological texture spaces with ideal. Moreover, we introduce new types of connectedness in $\star$-ditopological texture spaces namely, locally $\star$-connectedness, totally $\star$-disconnectedness, hyperconnectedness and $\star$-hyperconnectedness have investigated.

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