



Intuitionistic Fuzzy Soft Expert Sets and its Application in Decision Making

Said Broumi^{1,*} (broumisaid78@gmail.com)
Florentin Smarandache² (fsmarandache@gmail.com)

¹Faculty of letters and Humanities, Hay El Baraka Ben M'sik Casablanca B.P. 7951, University of Hassan II -Casablanca, Morocco.

²Department of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA.

Abstract - In this paper, we first introduced the concept of intuitionistic fuzzy soft expert sets (IFSESs for short) which combines intuitionistic fuzzy sets and soft expert sets. We also define its basic operations, namely complement, union, intersection, AND and OR, and study some of their properties. This concept is a generalization of fuzzy soft expert sets (FSESs). Finally, an approach for solving MCDM problems is explored by applying intuitionistic fuzzy soft expert sets, and an example is provided to illustrate the application of the proposed method.

Keywords - Intuitionistic fuzzy sets, soft expert sets, intuitionistic fuzzy soft expert sets, decision making.

1. Introduction

Intuitionistic fuzzy set (IFS in short) on a universe was introduced by Atanassov [7] in 1983, as a generalization of fuzzy set [13]. The conception of IFS can be viewed as an appropriate /alternative approach in case where available information is not sufficient to define the impreciseness by the conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. A detailed theoretical study may be found in [7].

Soft set theory was originally introduced by Molodtsov [3] as a general mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory,

** Edited by Irfan Deli (Area Editor) and Naim Çağman (Editor-in-Chief).

*Corresponding Author.

rough set theory, and probability theory. A soft set is in fact a set-valued map which gives an approximation description of objects under consideration based on some parameters. After Molodtsov's work, Maji et al. [26] introduced the concept of fuzzy soft set, a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties and also discussed their properties. Also, Maji et al. [27] devoted the concept of intuitionistic fuzzy soft sets by combining intuitionistic fuzzy sets with soft sets. Then, many interesting results of soft set theory have been studied on fuzzy soft sets [19, 20, 24, 25], on intuitionistic fuzzy soft set theory [21, 22, 23, 27], on possibility fuzzy soft set [31], on generalized fuzzy soft sets [5,29], on generalized intuitionistic fuzzy soft [12, 28], on possibility intuitionistic fuzzy soft set [14], on possibility vague soft set [8] and so on. All these research aim to solve most of our real life problems in medical sciences, engineering, management, environment and social science which involve data that are not crisp and precise. Moreover all the models created will deal only with one expert. To redefine this one expert opinion, Alkhazaleh and Salleh in 2011 [29] defined the concept of soft expert set in which the user can know the opinion of all the experts in one model and give an application of this concept in decision making problem. Also, they introduced the concept of the fuzzy soft expert set [30] as a combination between the soft experts set and the fuzzy set. After Alkhazaleh's work, many researchers have worked with the concept of soft expert sets [1, 2, 4, 6, 9, 10, 11, 15, 16, 18, 33].

Until now, there is no study on soft experts in intuitionistic fuzzy environment, so there is a need to develop a new mathematical tool called "intuitionistic fuzzy soft expert sets.

The paper is organized as follows. In Section 2, we first recall the necessary background on intuitionistic fuzzy sets, soft set, intuitionistic fuzzy soft sets, soft expert sets, fuzzy soft expert sets. Section 3 reviews various proposals for the definition of intuitionistic fuzzy soft expert sets and derive their respective properties. Section 4 presents basic operations on intuitionistic fuzzy soft expert sets. Section 5 presents an application of this concept in solving a decision making problem. Finally, we conclude the paper.

2. Preliminaries

In this section, we will briefly recall the basic concepts of intuitionistic fuzzy sets, soft set, soft expert sets and fuzzy soft expert sets.

Let U be an initial universe set of objects and E the set of parameters in relation to objects in U . Parameters are often attributes, characteristics or properties of objects. Let $P(U)$ denote the power set of U and $A \subseteq E$.

2.1. Intuitionistic Fuzzy Set

Definition 2.1 [7]: Let U be an universe of discourse then the intuitionistic fuzzy set A is an object having the form $A = \{ \langle x, \mu_A(x), \omega_A(x) \rangle, x \in U \}$, where the functions $\mu_A(x)$, $\omega_A(x) : U \rightarrow [0,1]$ define respectively the degree of membership, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$0 \leq \mu_A(x) + \omega_A(x) \leq 1.$$

For two IFS,

$$A_{IFS} = \{ \langle x, \mu_A(x), \omega_A(x) \rangle \mid x \in X \}$$

and

$$B_{IFS} = \{ \langle x, \mu_B(x), \omega_B(x) \rangle \mid x \in X \}$$

Then,

1. $A_{IFS} \subseteq B_{IFS}$ if and only if

$$\mu_A(x) \leq \mu_B(x), \omega_A(x) \geq \omega_B(x)$$

2. $A_{IFS} = B_{IFS}$ if and only if ,

$$\mu_A(x) = \mu_B(x), \omega_A(x) = \omega_B(x) \text{ for any } x \in X.$$

3. The complement of A_{IFS} is denoted by A_{IFS}^o and is defined by

$$A_{IFS}^o = \{ \langle x, \omega_A(x), \mu_A(x) \rangle \mid x \in X \}$$

4. $A \cap B = \{ \langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\omega_A(x), \omega_B(x)\} \rangle \mid x \in X \}$

5. $A \cup B = \{ \langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\omega_A(x), \omega_B(x)\} \rangle \mid x \in X \}$

As an illustration, let us consider the following example.

Example 2.2. Assume that the universe of discourse $U = \{x_1, x_2, x_3, x_4\}$. It may be further assumed that the values of x_1, x_2, x_3 and x_4 are in $[0, 1]$ Then, A is an intuitionistic fuzzy set (IFS) of U , such that,

$$A = \{ \langle x_1, 0.4, 0.6 \rangle, \langle x_2, 0.3, 0.7 \rangle, \langle x_3, 0.2, 0.8 \rangle, \langle x_4, 0.2, 0.8 \rangle \}$$

2.2. Soft Set

Definition 2.3. [3] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U . Consider a nonempty set $A, A \subset E$. A pair (K, A) is called a soft set over U , where K is a mapping given by $K : A \rightarrow P(U)$.

As an illustration, let us consider the following example.

Example 2.4 . Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2, \dots, h_5\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, \dots, e_8\}$, where e_1, e_2, \dots, e_8 stand for the attributes “beautiful”, “costly”, “in the green surroundings”, “moderate”, respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the “attractiveness of the houses” in the opinion of a buyer, says Thomas, and may be defined like this:

$$A = \{e_1, e_2, e_3, e_4, e_5\};$$

$$K(e_1) = \{h_2, h_3, h_5\}, K(e_2) = \{h_2, h_4\}, K(e_3) = \{h_1\}, K(e_4) = U, K(e_5) = \{h_3, h_5\}.$$

2.3. Intuitionistic Fuzzy Soft Sets

Definition 2.5 [27] Let U be an initial universe set and $A \subset E$ be a set of parameters. Let $IFS(U)$ denotes the set of all intuitionistic fuzzy subsets of U . The collection (F, A) is termed to be the intuitionistic fuzzy soft set over U , where F is a mapping given by $F: A \rightarrow IFS(U)$.

Example 2.6 Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a word or sentence involving intuitionistic fuzzy words. Consider $E = \{\text{beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}\}$. In this case, to define a intuitionistic fuzzy soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by $U = \{h_1, h_2, \dots, h_5\}$ and the set of parameters

$A = \{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter ‘beautiful’, e_2 stands for the parameter ‘wooden’, e_3 stands for the parameter ‘costly’ and the parameter e_4 stands for ‘moderate’. Then the intuitionistic fuzzy set (F, A) is defined as follows:

$$(F, A) = \left\{ \begin{array}{l} \left(e_1 \left\{ \frac{h_1}{(0.1,0.6)}, \frac{h_2}{(0.2,0.7)}, \frac{h_3}{(0.6,0.2)}, \frac{h_4}{(0.7,0.3)}, \frac{h_5}{(0.2,0.3)} \right\} \right) \\ \left(e_2 \left\{ \frac{h_1}{(0.3,0.5)}, \frac{h_2}{(0.2,0.4)}, \frac{h_3}{(0.1,0.2)}, \frac{h_4}{(0.1,0.3)}, \frac{h_5}{(0.3,0.6)} \right\} \right) \\ \left(e_3 \left\{ \frac{h_1}{(0.4,0.3)}, \frac{h_2}{(0.6,0.3)}, \frac{h_3}{(0.2,0.5)}, \frac{h_4}{(0.2,0.6)}, \frac{h_5}{(0.7,0.3)} \right\} \right) \\ \left(e_4 \left\{ \frac{h_1}{(0.1,0.6)}, \frac{h_2}{(0.3,0.6)}, \frac{h_3}{(0.6,0.4)}, \frac{h_4}{(0.4,0.2)}, \frac{h_5}{(0.5,0.3)} \right\} \right) \end{array} \right\}$$

2.5. Soft Expert Sets

Definition 2.7 [29] Let U be a universe set, E be a set of parameters and X be a set of experts (agents). Let $O = \{1=\text{agree}, 0=\text{disagree}\}$ be a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$

A pair (F, E) is called a soft expert set over U , where F is a mapping given by $F : A \rightarrow P(U)$ and $P(U)$ denote the power set of U .

Definition 2.8 [29] An agree- soft expert set $(F, A)_1$ over U , is a soft expert subset of (F, A) defined as :

$$(F, A)_1 = \{F(\alpha) \mid \alpha \in E \times X \times \{1\}\}.$$

Definition 2.9[29] A disagree- soft expert set $(F, A)_0$ over U , is a soft expert subset of (F, A) defined as :

$$(F, A)_0 = \{F(\alpha) \mid \alpha \in E \times X \times \{0\}\}.$$

2.6. Fuzzy Soft Expert Sets

Definition 2.10 [30] A pair (F, A) is called a fuzzy soft expert set over U , where F is a mapping given by $F : A \rightarrow I^U$, and I^U denote the set of all fuzzy subsets of U .

3. Intuitionistic Fuzzy Soft Expert Sets

In this section, we generalize the fuzzy soft expert sets as introduced by Alkhazaleh and Salleh [30] to intuitionistic fuzzy soft expert sets and give the basic properties of this concept.

Let U be universal set of elements, E be a set of parameters, X be a set of experts (agents), $O = \{1=\text{agree}, 0=\text{disagree}\}$ be a set of opinions. Let $Z = E \times X \times O$ and

Definition 3.1 Let $U = \{u_1, u_2, u_3, \dots, u_n\}$ be a universal set of elements, $E = \{e_1, e_2, e_3, \dots, e_m\}$ be a universal set of parameters, $X = \{x_1, x_2, x_3, \dots, x_i\}$ be a set of experts (agents) and $O = \{1=\text{agree}, 0=\text{disagree}\}$ be a set of opinions. Let $Z = \{E \times X \times Q\}$ and $A \subseteq Z$. Then the pair (U, Z) is called a soft universe. Let $F : Z \rightarrow IF^U$ where IF^U denotes the collection of all intuitionistic fuzzy subsets of U . Suppose $F : Z \rightarrow IF^U$ be a function defined as:

$$F(z) = F(z)(u_i), \text{ for all } u_i \in U.$$

Then $F(z)$ is called an intuitionistic fuzzy soft expert set (IFSES in short) over the soft universe (U, Z) .

For each $z_i \in Z$. $F(z) = F(z_i)(u_i)$ where $F(z_i)$ represents the degree of belongingness and non-belongingness of the elements of U in $F(z_i)$. Hence $F(z_i)$ can be written as:

$$F(z_i) = \left\{ \left(\frac{u_i}{F(z_i)(u_i)} \right), \dots, \left(\frac{u_i}{F(z_i)(u_i)} \right) \right\}, \text{ for } i=1,2,3,\dots,n$$

where $F(z_i)(u_i) = \langle \mu_{F(z_i)}(u_i), \omega_{F(z_i)}(u_i) \rangle$ with $\mu_{F(z_i)}(u_i)$ and $\omega_{F(z_i)}(u_i)$ representing the membership function and non-membership function of each of the elements $u_i \in U$ respectively.

Sometimes we write F as (F, Z) . If $A \subseteq Z$. we can also have $IFSES(F, A)$.

Example 3.2 Let $U = \{u_1, u_2, u_3\}$ be a set of elements, $E = \{e_1, e_2\}$ be a set of decision parameters, where $e_i (i = 1, 2, 3)$ denotes the parameters $E = \{e_1 = \text{beautiful}, e_2 = \text{cheap}\}$ and $X = \{x_1, x_2\}$ be a set of experts. Suppose that $F : Z \rightarrow IF^U$ is function defined as follows:

$$\begin{aligned}
 F(e_1, x_1, 1) &= \left\{ \left(\frac{u_1}{\langle 0.1, 0.8 \rangle}, \frac{u_2}{\langle 0.1, 0.6 \rangle}, \frac{u_3}{\langle 0.4, 0.5 \rangle} \right) \right\}, \\
 F(e_2, x_1, 1) &= \left\{ \left(\frac{u_1}{\langle 0.5, 0.25 \rangle}, \frac{u_2}{\langle 0.25, 0.6 \rangle}, \frac{u_3}{\langle 0.4, 0.4 \rangle} \right) \right\}, \\
 F(e_1, x_2, 1) &= \left\{ \left(\frac{u_1}{\langle 0.2, 0.7 \rangle}, \frac{u_2}{\langle 0.4, 0.3 \rangle}, \frac{u_3}{\langle 0.6, 0.2 \rangle} \right) \right\}, \\
 F(e_2, x_2, 1) &= \left\{ \left(\frac{u_1}{\langle 0.2, 0.6 \rangle}, \frac{u_2}{\langle 0.3, 0.2 \rangle}, \frac{u_3}{\langle 0.3, 0.5 \rangle} \right) \right\}, \\
 F(e_1, x_1, 0) &= \left\{ \left(\frac{u_1}{\langle 0.2, 0.4 \rangle}, \frac{u_2}{\langle 0.1, 0.9 \rangle}, \frac{u_3}{\langle 0.2, 0.5 \rangle} \right) \right\}, \\
 F(e_2, x_1, 0) &= \left\{ \left(\frac{u_1}{\langle 0.3, 0.4 \rangle}, \frac{u_2}{\langle 0.2, 0.7 \rangle}, \frac{u_3}{\langle 0.5, 0.2 \rangle} \right) \right\}, \\
 F(e_1, x_2, 0) &= \left\{ \left(\frac{u_1}{\langle 0.3, 0.4 \rangle}, \frac{u_2}{\langle 0.1, 0.6 \rangle}, \frac{u_3}{\langle 0.6, 0.3 \rangle} \right) \right\} \\
 F(e_2, x_2, 0) &= \left\{ \left(\frac{u_1}{\langle 0.4, 0.4 \rangle}, \frac{u_2}{\langle 0.8, 0.2 \rangle}, \frac{u_3}{\langle 0.2, 0.4 \rangle} \right) \right\}
 \end{aligned}$$

Then we can view the intuitionistic fuzzy soft expert set (F, Z) as consisting of the following collection of approximations:

$$\begin{aligned}
 (F, Z) &= \{ (e_1, x_1, 1) = \left\{ \left(\frac{u_1}{\langle 0.1, 0.8 \rangle}, \frac{u_2}{\langle 0.1, 0.6 \rangle}, \frac{u_3}{\langle 0.4, 0.5 \rangle} \right) \right\}, \\
 &\{ (e_2, x_1, 1) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.25 \rangle}, \frac{u_2}{\langle 0.25, 0.6 \rangle}, \frac{u_3}{\langle 0.4, 0.4 \rangle} \right) \right\}, \\
 &\{ (e_1, x_2, 1) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.7 \rangle}, \frac{u_2}{\langle 0.4, 0.3 \rangle}, \frac{u_3}{\langle 0.6, 0.2 \rangle} \right) \right\}, \\
 &\{ (e_2, x_2, 1) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.6 \rangle}, \frac{u_2}{\langle 0.3, 0.2 \rangle}, \frac{u_3}{\langle 0.3, 0.5 \rangle} \right) \right\}, \\
 &\{ (e_1, x_1, 0) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.4 \rangle}, \frac{u_2}{\langle 0.1, 0.9 \rangle}, \frac{u_3}{\langle 0.2, 0.5 \rangle} \right) \right\}, \\
 &\{ (e_2, x_1, 0) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.4 \rangle}, \frac{u_2}{\langle 0.2, 0.7 \rangle}, \frac{u_3}{\langle 0.5, 0.2 \rangle} \right) \right\}, \\
 &\{ (e_1, x_2, 0) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.4 \rangle}, \frac{u_2}{\langle 0.1, 0.6 \rangle}, \frac{u_3}{\langle 0.6, 0.3 \rangle} \right) \right\}, \\
 &\{ (e_2, x_2, 0) = \left\{ \left(\frac{u_1}{\langle 0.4, 0.4 \rangle}, \frac{u_2}{\langle 0.8, 0.2 \rangle}, \frac{u_3}{\langle 0.2, 0.4 \rangle} \right) \right\}.
 \end{aligned}$$

Then (F, Z) is an intuitionistic fuzzy soft expert set over the soft universe (U, Z) .

Definition 3.3. For two intuitionistic fuzzy soft expert sets (F, A) and (G, B) over a soft universe (U, Z) . Then (F, A) is said to be an intuitionistic fuzzy soft expert subset of (G, B) if

- i. $B \subseteq A$
- ii. $F(\varepsilon)$ is an intuitionistic fuzzy subset of $G(\varepsilon)$, for all $\varepsilon \in A$

This relationship is denoted as $(F, A) \widetilde{\subseteq} (G, B)$. In this case, (G, B) is called an intuitionistic fuzzy soft expert superset (IFSES superset) of (F, A) .

Definition 3.4. Two intuitionistic fuzzy soft expert sets (F, A) and (G, B) over soft universe (U, Z) are said to be equal if (F, A) is a intuitionistic fuzzy soft expert subset of (G, B) and (G, B) is an intuitionistic fuzzy soft expert subset of (F, A) .

Definition 3.5. An IFSES (F, A) is said to be a null intuitionistic fuzzy soft expert sets denoted $(\tilde{\emptyset}, A)$ and defined as:

$$(\tilde{\emptyset}, A) = F(\alpha) \text{ where } \alpha \in Z.$$

Where $F(\alpha) = \langle 0, 1 \rangle$, that is $\mu_{F(\alpha)} = 0$ and $\omega_{F(\alpha)} = 1$ for all $\alpha \in Z$.

Definition 3.6. An IFSES (F, A) is said to be an absolute intuitionistic fuzzy soft expert sets denoted $(F, A)_{abs}$ and defined as:

$$(F, A)_{abs} = F(\alpha), \text{ where } \alpha \in Z.$$

Where $F(\alpha) = \langle 1, 0 \rangle$, that is $\mu_{F(\alpha)} = 1$ and $\omega_{F(\alpha)} = 0$, for all $\alpha \in Z$.

Definition 3.7. Let (F, A) be an IFSES over a soft universe (U, Z) . An agree- intuitionistic fuzzy soft expert set (agree- IFSES) over U , denoted as $(F, A)_1$ is an intuitionistic fuzzy soft expert subset of (F, A) which is defined as :

$$(F, A)_1 = \{F(\alpha) \mid \alpha \in E \times X \times \{1\}\}.$$

Definition 3.8. Let (F, A) be a IFSES over a soft universe (U, Z) . A disagree- intuitionistic fuzzy soft expert set (disagree- IFSES) over U , denoted as $(F, A)_0$ is a intuitionistic fuzzy soft expert subset of (F, A) which is defined as :

$$(F, A)_0 = \{F(\alpha) \mid \alpha \in E \times X \times \{0\}\}.$$

Example 3.9 consider Example 3.2 .Then the agree- intuitionistic fuzzy soft soft expert set

$$\begin{aligned} (F, A)_1 = & \{((e_1, x_1, 1), \{(\frac{u_1}{\langle 0.1, 0.8 \rangle}), (\frac{u_2}{\langle 0.1, 0.6 \rangle}), (\frac{u_3}{\langle 0.4, 0.5 \rangle})\}), \\ & ((e_2, x_1, 1), \{(\frac{u_1}{\langle 0.5, 0.25 \rangle}), (\frac{u_2}{\langle 0.25, 0.6 \rangle}), (\frac{u_3}{\langle 0.4, 0.4 \rangle})\}), \\ & ((e_1, x_2, 1), \{(\frac{u_1}{\langle 0.2, 0.7 \rangle}), (\frac{u_2}{\langle 0.4, 0.3 \rangle}), (\frac{u_3}{\langle 0.6, 0.2 \rangle})\}), \\ & ((e_2, x_2, 1), \{(\frac{u_1}{\langle 0.2, 0.6 \rangle}), (\frac{u_2}{\langle 0.3, 0.2 \rangle}), (\frac{u_3}{\langle 0.3, 0.5 \rangle})\})\} \end{aligned}$$

And the disagree-intuitionistic fuzzy soft expert set over U

$$\begin{aligned} (F, A)_0 = & \{((e_1, x_1, 0), \{(\frac{u_1}{\langle 0.2, 0.4 \rangle}), (\frac{u_2}{\langle 0.1, 0.9 \rangle}), (\frac{u_3}{\langle 0.2, 0.5 \rangle})\}), \\ & ((e_2, x_1, 0), \{(\frac{u_1}{\langle 0.3, 0.4, 0.6 \rangle}), (\frac{u_2}{\langle 0.2, 0.7 \rangle}), (\frac{u_3}{\langle 0.5, 0.2 \rangle})\}), \end{aligned}$$

$$((e_1, x_2, 0), \{ (\frac{u_1}{\langle 0.3, 0.4 \rangle}, (\frac{u_2}{\langle 0.1, 0.6 \rangle}, (\frac{u_3}{\langle 0.6, 0.3 \rangle}))$$

$$((e_2, x_2, 0), \{ (\frac{u_1}{\langle 0.4, 0.4 \rangle}, (\frac{u_2}{\langle 0.8, 0.2 \rangle}, (\frac{u_3}{\langle 0.2, 0.4 \rangle})) \})$$

4. Basic Operations on Intuitionistic Fuzzy Soft Expert Sets

In this section, we introduce some basic operations on IFSES, namely the complement, AND, OR, union and intersection of IFSES, derive their properties, and give some examples.

Definition 4.1 Let (F, A) be an IFSES over a soft universe (U, Z) . Then the complement of (F, A) denoted by $(F, A)^c$ is defined as:

$$(F, A)^c = \tilde{c}(F(\alpha)) \text{ for all } \alpha \in U.$$

where \tilde{c} is an intuitionistic fuzzy complement .

Example 4.2 Consider the IFSES (F, Z) over a soft universe (U, Z) as given in Example 3.2. By using the intuitionistic fuzzy complement for $F(\alpha)$, we obtain $(F, Z)^c$ which is defined as:

$$(F, Z)^c = \{ (e_1, x_1, 1) = \{ (\frac{u_1}{\langle 0.8, 0.1 \rangle}, (\frac{u_2}{\langle 0.6, 0.1 \rangle}, (\frac{u_3}{\langle 0.5, 0.4 \rangle}) \} \},$$

$$\{ (e_2, x_1, 1) = \{ (\frac{u_1}{\langle 0.25, 0.5 \rangle}, (\frac{u_2}{\langle 0.6, 0.25 \rangle}, (\frac{u_3}{\langle 0.4, 0.4 \rangle}) \} \},$$

$$\{ (e_1, x_2, 1) = \{ (\frac{u_1}{\langle 0.7, 0.2 \rangle}, (\frac{u_2}{\langle 0.3, 0.4 \rangle}, (\frac{u_3}{\langle 0.2, 0.6 \rangle}) \} \},$$

$$\{ (e_2, x_2, 1) = \{ (\frac{u_1}{\langle 0.6, 0.2 \rangle}, (\frac{u_2}{\langle 0.2, 0.3 \rangle}, (\frac{u_3}{\langle 0.5, 0.3 \rangle}) \} \},$$

$$\{ (e_1, x_1, 0) = \{ (\frac{u_1}{\langle 0.4, 0.2 \rangle}, (\frac{u_2}{\langle 0.9, 0.1 \rangle}, (\frac{u_3}{\langle 0.5, 0.2 \rangle}) \} \},$$

$$\{ (e_2, x_1, 0) = \{ (\frac{u_1}{\langle 0.4, 0.3 \rangle}, (\frac{u_2}{\langle 0.7, 0.2 \rangle}, (\frac{u_3}{\langle 0.2, 0.5 \rangle}) \} \},$$

$$\{ (e_1, x_2, 0) = \{ (\frac{u_1}{\langle 0.4, 0.3 \rangle}, (\frac{u_2}{\langle 0.6, 0.1 \rangle}, (\frac{u_3}{\langle 0.3, 0.6 \rangle}) \} \},$$

$$\{ (e_2, x_2, 0) = \{ (\frac{u_1}{\langle 0.4, 0.4 \rangle}, (\frac{u_2}{\langle 0.2, 0.8 \rangle}, (\frac{u_3}{\langle 0.4, 0.2 \rangle}) \} \}.$$

Proposition 4.3 If (F, A) is an IFSES over a soft universe (U, Z) , then,

$$((F, A)^c)^c = (F, A).$$

Proof. Suppose that is (F, A) an IFSES over a soft universe (U, Z) defined as $(F, A) = F(e)$. Now let $IFSES(F, A)^c = (G, B)$. Then by Definition 4.1, $(G, B) = G(e)$ such that $G(e) = \tilde{c}(F(e))$, Thus it follows that:

$$(G, B)^c = \tilde{c}(G(e)) = (\tilde{c}(\tilde{c}(F(e)))) = F(e) = (F, A).$$

Therefore

$((F, A)^c)^c = (G, B)^c = (F, A)$. Hence it is proven that $((F, A)^c)^c = (F, A)$.

Definition 4.4 Let (F, A) and (G, B) be any two IFSES s over a soft universe (U, Z) . Then the union of (F, A) and (G, B) , denoted by $(F, A) \tilde{\cup} (G, B)$ is an IFSES defined as $(F, A) \tilde{\cup} (G, B) = (H, C)$, where $C = A \cup B$ and

$$H(\alpha) = F(\alpha) \tilde{\cup} G(\alpha), \text{ for all } \alpha \in C$$

where

$$H(\alpha) = \begin{cases} F(\alpha) & \alpha \in A - B \\ G(\alpha) & \alpha \in A - B \\ s(F(\alpha), G(\alpha)) & \alpha \in A \cap B \end{cases}$$

Where s is a s- norm.

Proposition 4.5 Let (F, A) , (G, B) and (H, C) be any three IFSES over a soft universe (U, Z) . Then the following properties hold true.

- (i) $(F, A) \tilde{\cup} (G, B) = (G, B) \tilde{\cup} (F, A)$
- (ii) $(F, A) \tilde{\cup} ((G, B) \tilde{\cup} (H, C)) = ((F, A) \tilde{\cup} (G, B)) \tilde{\cup} (H, C)$
- (iii) $(F, A) \tilde{\cup} (F, A) \subseteq (F, A)$
- (iv) $(F, A) \tilde{\cup} (\Phi, A) = (\Phi, A)$

Proof

(i) Let $(F, A) \tilde{\cup} (G, B) = (H, C)$. Then by definition 4.4, for all $\alpha \in C$, we have $(H, C) = H(\alpha)$

Where

$H(\alpha) = F(\alpha) \tilde{\cup} G(\alpha)$ However $H(\alpha) = F(\alpha) \tilde{\cup} G(\alpha) = G(\alpha) \tilde{\cup} F(\alpha)$ since the union of these sets are commutative by definition 4.4. Therefore $(H, C) = (G, B) \tilde{\cup} (F, A)$. Thus the union of two IFSES are commutative i.e $(F, A) \tilde{\cup} (G, B) = (G, B) \tilde{\cup} (F, A)$.

- (ii) The proof is similar to proof of part(i) and is therefore omitted
- (iii) The proof is straightforward and is therefore omitted.
- (iv) The proof is straightforward and is therefore omitted.

Definition 4.6 Let (F, A) and (G, B) be any two IFSES over a soft universe (U, Z) . Then the intersection of (F, A) and (G, B) , denoted by $(F, A) \tilde{\cap} (G, B)$ is an IFSES defined as $(F, A) \tilde{\cap} (G, B) = (H, C)$ where $C = A \cup B$ and

$$H(\alpha) = F(\alpha) \tilde{\cap} G(\alpha), \text{ for all } \alpha \in C$$

where

$$H(\alpha) = \begin{cases} F(\alpha) & \alpha \in A - B \\ G(\alpha) & \alpha \in A - B \\ t(F(\alpha), G(\alpha)) & \alpha \in A \cap B \end{cases}$$

Where t is a t-norm

Proposition 4.7 If (F, A) , (G, B) and (H, C) are three IFSES over a soft universe (U, Z) , then,

- (i) $(F, A) \tilde{\cap} (G, B) = (G, B) \tilde{\cap} (F, A)$
- (ii) $(F, A) \tilde{\cap} ((G, B) \tilde{\cap} (H, C)) = ((F, A) \tilde{\cap} (G, B)) \tilde{\cap} (H, C)$
- (iii) $(F, A) \tilde{\cap} (F, A) \subseteq (F, A)$
- (iv) $(F, A) \tilde{\cap} (\Phi, A) = (\Phi, A)$

Proof

- (i) The proof is similar to that of Proposition 4.5 (i) and is therefore omitted
- (ii) The proof is similar to the proof of part (i) and is therefore omitted
- (iii) The proof is straightforward and is therefore omitted.
- (iv) The proof is straightforward and is therefore omitted.

Proposition 4.8. If (F, A) , (G, B) and (H, C) are three IFSES over a soft universe (U, Z) , then,

- (i) $(F, A) \tilde{\cup} ((G, B) \cap (H, C)) = ((F, A) \tilde{\cup} (G, B)) \tilde{\cap} ((F, A) \tilde{\cup} (H, C))$
- (ii) $(F, A) \tilde{\cap} ((G, B) \tilde{\cup} (H, C)) = ((F, A) \tilde{\cap} (G, B)) \tilde{\cup} ((F, A) \tilde{\cap} (H, C))$

Proof. The proof is straightforward by definitions 4.4 and 4.6 and is therefore omitted.

Proposition 4.9 If (F, A) , (G, B) are two IFSES over a soft universe (U, Z) , then,

- i. $((F, A) \tilde{\cup} (G, B))^c = (F, A)^c \tilde{\cap} (G, B)^c$.
- ii. $((F, A) \tilde{\cap} (G, B))^c = (F, A)^c \tilde{\cup} (G, B)^c$.

Proof.

- (i) suppose that (F, A) and (G, B) be IFSES over a soft universe (U, Z) defined as:

$(F, A) = F(\alpha)$ for all $\alpha \in A \subseteq Z$ and $(G, B) = G(\alpha)$ for all $\alpha \in B \subseteq Z$. Now, due to the commutative and associative properties of IFSES, it follows that: by Definition 4.10 and 4.11, it follows that:

$$\begin{aligned} (F, A)^c \tilde{\cap} (G, B)^c &= (F(\alpha))^c \tilde{\cap} (G(\alpha))^c \\ &= (\tilde{c}(F(\alpha))) \tilde{\cap} (\tilde{c}(G(\alpha))) \\ &= (\tilde{c}(F(\alpha)) \tilde{\cap} G(\alpha)) \\ &= ((F, A) \tilde{\cup} (G, B))^c. \end{aligned}$$

- (ii) The proof is similar to the proof of part (i) and is therefore omitted.

Definition 4.10 Let (F, A) and (G, B) be any two IFSES over a soft universe (U, Z) . Then “ (F, A) AND (G, B) ” denoted $(F, A) \tilde{\wedge} (G, B)$ is a defined by:

$$(F, A) \tilde{\wedge} (G, B) = (H, A \times B)$$

Where $(H, A \times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$. and \cap represent the basic intersection.

Definition 4.11 Let (F, A) and (G, B) be any two IFSES over a soft universe (U, Z) . Then “ (F, A) OR (G, B) ” denoted $(F, A) \tilde{\vee} (G, B)$ is a defined by:

$$(F, A) \tilde{\vee} (G, B) = (H, A \times B)$$

Where $(H, A \times B) = H(\alpha, \beta)$ such that $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$. and \cup represent the basic union.

Proposition 4.12 If (F, A) , (G, B) and (H, C) are three IFSES over a soft universe (U, Z) , then,

- i. $(F, A) \tilde{\wedge} ((G, B) \tilde{\wedge} (H, C)) = ((F, A) \tilde{\wedge} (G, B)) \tilde{\wedge} (H, C)$
- ii. $(F, A) \tilde{\vee} ((G, B) \tilde{\vee} (H, C)) = ((F, A) \tilde{\vee} (G, B)) \tilde{\vee} (H, C)$
- iii. $(F, A) \tilde{\vee} ((G, B) \tilde{\wedge} (H, C)) = ((F, A) \tilde{\vee} (G, B)) \tilde{\wedge} ((F, A) \tilde{\vee} (H, C))$
- iv. $(F, A) \tilde{\wedge} ((G, B) \tilde{\vee} (H, C)) = ((F, A) \tilde{\wedge} (G, B)) \tilde{\vee} ((F, A) \tilde{\wedge} (H, C))$

Proof. The proofs are straightforward by Definitions 4.10 and 4.11 and are therefore omitted.

Note: The “AND” and “OR” operations are not commutative since generally $A \times B \neq B \times A$.

Proposition 4.13. If (F, A) and (G, B) are two IFSES over a soft universe (U, Z) , then,

- i. $((F, A) \tilde{\wedge} (G, B))^c = (F, A)^c \tilde{\vee} (G, B)^c$.
- ii. $((F, A) \tilde{\vee} (G, B))^c = (F, A)^c \tilde{\wedge} (G, B)^c$.

Proof.

(i) suppose that (F, A) and (G, B) be IFSES over a soft universe (U, Z) defined as:

$(F, A) = (F(\alpha))$ for all $\alpha \in A \subseteq Z$ and $(G, B) = (G(\beta))$ for all $\beta \in B \subseteq Z$. Then by Definition 4.10 and 4.11, it follows that:

$$\begin{aligned} ((F, A) \tilde{\wedge} (G, B))^c &= ((F(\alpha) \tilde{\wedge} G(\beta))^c \\ &= (F(\alpha) \cap G(\beta))^c \\ &= (\tilde{c}(F(\alpha) \cap G(\beta))) \\ &= (\tilde{c}(F(\alpha)) \cup \tilde{c}(G(\beta))) \\ &= (F(\alpha))^c \tilde{\vee} (G(\beta))^c \\ &= (F, A)^c \tilde{\vee} (G, B)^c. \end{aligned}$$

(ii) the proof is similar to that of part (i) and is therefore omitted.

5. Application of Intuitionistic Fuzzy Soft Expert Sets in a Decision Making Problem.

In this section, we introduce a generalized algorithm which will be applied to the IFSES model introduced in Section 3 and used to solve a hypothetical decision making problem.

Suppose that company Y is looking to hire a person to fill in the vacancy for a position in their company. Out of all the people who applied for the position, three candidates were shortlisted and these three candidates form the universe of elements, $U = \{u_1, u_2, u_3\}$. The hiring committee consists of the hiring manager, head of department and the HR director of the company and this committee is represented by the set $\{p, q, r\}$ (a set of experts) while the set $Q = \{1 = \text{agree}, 0 = \text{disagree}\}$ represents the set of opinions of the hiring committee members. The hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3, e_4\}$ where the parameters e_i represent the characteristics or qualities that the candidates are assessed on, namely “relevant job experience”, “excellent academic qualifications in the relevant field”, “attitude and level of professionalism” and “technical knowledge” respectively. After interviewing all the three candidates and going through their certificates and other supporting documents, the hiring committee constructs the following IFSES.

$$\begin{aligned}
 (F, Z) = \{ & (e_1, p, 1) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.4 \rangle}, \frac{u_2}{\langle 0.1, 0.4 \rangle}, \frac{u_3}{\langle 0.1, 0.7 \rangle} \right) \right\}, \\
 & (e_2, p, 1) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.2 \rangle}, \frac{u_2}{\langle 0.25, 0.2 \rangle}, \frac{u_3}{\langle 0.2, 0.6 \rangle} \right) \right\}, \\
 & (e_3, p, 1) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.7 \rangle}, \frac{u_2}{\langle 0.4, 0.3 \rangle}, \frac{u_3}{\langle 0.1, 0.6 \rangle} \right) \right\}, \\
 & (e_4, p, 1) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.6 \rangle}, \frac{u_2}{\langle 0.3, 0.2 \rangle}, \frac{u_3}{\langle 0.3, 0.1 \rangle} \right) \right\}, \\
 & (e_1, q, 1) = \left\{ \left(\frac{u_1}{\langle 0.4, 0.6 \rangle}, \frac{u_2}{\langle 0.2, 0.3 \rangle}, \frac{u_3}{\langle 0.3, 0.2 \rangle} \right) \right\}, \\
 & (e_2, q, 1) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.3 \rangle}, \frac{u_2}{\langle 0.9, 0.1 \rangle}, \frac{u_3}{\langle 0.1, 0.2 \rangle} \right) \right\}, \\
 & (e_3, q, 1) = \left\{ \left(\frac{u_1}{\langle 0.1, 0.4 \rangle}, \frac{u_2}{\langle 0.6, 0.2 \rangle}, \frac{u_3}{\langle 0.2, 0.4 \rangle} \right) \right\}, \\
 & (e_4, q, 1) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.3 \rangle}, \frac{u_2}{\langle 0.8, 0.2 \rangle}, \frac{u_3}{\langle 0.3, 0.4 \rangle} \right) \right\}, \\
 & (e_1, r, 1) = \left\{ \left(\frac{u_1}{\langle 0.4, 0.5 \rangle}, \frac{u_2}{\langle 0.6, 0.4 \rangle}, \frac{u_3}{\langle 0.2, 0.4 \rangle} \right) \right\}, \\
 & (e_2, r, 1) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.7 \rangle}, \frac{u_2}{\langle 0.3, 0.2 \rangle}, \frac{u_3}{\langle 0.2, 0.2 \rangle} \right) \right\}, \\
 & (e_3, r, 1) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.2 \rangle}, \frac{u_2}{\langle 0.1, 0.6 \rangle}, \frac{u_3}{\langle 0.3, 0.2 \rangle} \right) \right\}, \\
 & (e_1, p, 0) = \left\{ \left(\frac{u_1}{\langle 0.1, 0.4 \rangle}, \frac{u_2}{\langle 0.3, 0.2 \rangle}, \frac{u_3}{\langle 0.2, 0.4 \rangle} \right) \right\}, \\
 & (e_3, p, 0) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.2 \rangle}, \frac{u_2}{\langle 0.2, 0.4 \rangle}, \frac{u_3}{\langle 0.3, 0.1 \rangle} \right) \right\}, \\
 & (e_4, p, 0) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.2 \rangle}, \frac{u_2}{\langle 0.6, 0.4 \rangle}, \frac{u_3}{\langle 0.4, 0.5 \rangle} \right) \right\}, \\
 & (e_1, q, 0) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.4 \rangle}, \frac{u_2}{\langle 0.1, 0.9 \rangle}, \frac{u_3}{\langle 0.1, 0.2 \rangle} \right) \right\}, \\
 & (e_2, q, 0) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.4 \rangle}, \frac{u_2}{\langle 0.2, 0.7 \rangle}, \frac{u_3}{\langle 0.3, 0.5 \rangle} \right) \right\}, \\
 & (e_3, q, 0) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.8 \rangle}, \frac{u_2}{\langle 0.1, 0.2 \rangle}, \frac{u_3}{\langle 0.6, 0.3 \rangle} \right) \right\}, \\
 & (e_4, q, 0) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.4 \rangle}, \frac{u_2}{\langle 0.6, 0.2 \rangle}, \frac{u_3}{\langle 0.3, 0.4 \rangle} \right) \right\}, \\
 & (e_1, r, 0) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.4 \rangle}, \frac{u_2}{\langle 0.3, 0.6 \rangle}, \frac{u_3}{\langle 0.25, 0.2 \rangle} \right) \right\}.
 \end{aligned}$$

$$\{(e_2, r, 0) = \left\{ \left(\frac{u_1}{\langle 0.4, 0.5 \rangle}, \left(\frac{u_2}{\langle 0.4, 0.2 \rangle}, \left(\frac{u_3}{\langle 0.4, 0.3 \rangle} \right) \right) \right\} \}.$$

$$\{(e_3, r, 0) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.2 \rangle}, \left(\frac{u_2}{\langle 0.3, 0.5 \rangle}, \left(\frac{u_3}{\langle 0.5, 0.1 \rangle} \right) \right) \right\} \}.$$

Next the IFSES (F , Z) is used together with a generalized algorithm to solve the decision making problem stated at the beginning of this section. The algorithm given below is employed by the hiring committee to determine the best or most suitable candidate to be hired for the position. This algorithm is a generalization of the algorithm introduced by Alkhazaleh and Salleh (see [30]) which is used in the context of the IFSES model that is introduced in this paper. The generalized algorithm is as follows:

Algorithm

1. Input the IFSES (F , Z).
2. Find the values of $\mu_{F(z_i)}(u_i) - \omega_{F(z_i)}(u_i)$ for each element $u_i \in U$ where $\mu_{F(z_i)}(u_i)$, and $\omega_{F(z_i)}(u_i)$ are the membership function and non-membership function of each of the elements $u_i \in U$ respectively.
3. Find the highest numerical grade for the agree- IFSES and disagree- IFSES.
4. Compute the score of each element $u_i \in U$ by taking the sum of the products of the numerical grade of each element for the agree- IFSES and disagree IFSES, denoted by A_i and D_i respectively.
5. Find the values of the score $r_i = A_i - D_i$ for each element $u_i \in U$.

Table I. Values of $\mu_{F(z_i)}(u_i) - \omega_{F(z_i)}(u_i)$ for all $u_i \in U$.

	u_1	u_2	u_3		u_1	u_2	u_3
$(e_1, p, 1)$	-0.2	-0.3	-0.6	$(e_3, p, 0)$	0.1	-0.2	0.2
$(e_2, p, 1)$	0.1	0.05	-0.4	$(e_4, p, 0)$	0.1	-0.2	- 0.1
$(e_3, p, 1)$	-0.5	0.1	-0.5	$(e_1, q, 0)$	-0.2	-0.8	-0.1
$(e_4, p, 1)$	-0.4	0.1	0.2	$(e_2, q, 0)$	-0.1	-0.5	-0.2
$(e_1, q, 1)$	-0.2	-0.1	0.1	$(e_3, q, 0)$	-0.6	-0.1	0.3
$(e_2, q, 1)$	0	0.8	-0.1	$(e_4, q, 0)$	-0.1	0.4	-0.1
$(e_3, q, 1)$	-0.3	0.4	-0.2	$(e_1, r, 0)$	-0.1	-0.3	0.05
$(e_4, q, 1)$	0.2	0.6	-0.1	$(e_2, r, 0)$	-0.1	0.2	0.1
$(e_1, r, 1)$	-0.1	0.2	-0.2	$(e_4, r, 0)$	0.1	-0.2	0.4
$(e_2, r, 1)$	-0.4	0.1	0				
$(e_3, r, 1)$	0.3	-0.5	0.1				
$(e_1, p, 0)$	-0.3	0.1	-0.2				

6. Determine the value of the highest score, $s = \max_{u_i} \{ r_i \}$. Then the decision is to choose element as the optimal or best solution to the problem. If there is more than one element with the highest r_i score, then any one of those elements can be chosen as the optimal solution.

Then we can conclude that the optimal choice for the hiring committee is to hire candidate u_i to fill the vacant position

Table I gives the values of $\mu_{F(z_i)}(u_i) - \omega_{F(z_i)}(u_i)$ for each element $u_i \in U$. The notation a, b gives the values of $\mu_{F(z_i)}(u_i) - \omega_{F(z_i)}(u_i)$.

In Table II and Table III, we give the highest numerical grade for the elements in the agree-IFSES and disagree-IFSES respectively.

Table II. Numerical Grade for Agree-IFSES

	u_i	Highest Numeric Grade
$(e_1, p, 1)$	u_1	-0.2
$(e_2, p, 1)$	u_1	0.1
$(e_3, p, 1)$	u_2	0.1
$(e_4, p, 1)$	u_3	0.2
$(e_1, q, 1)$	u_3	0.1
$(e_2, q, 1)$	u_2	0.8
$(e_3, q, 1)$	u_2	0.4
$(e_4, q, 1)$	u_2	0.6
$(e_1, r, 1)$	u_2	0.2
$(e_2, r, 1)$	u_2	0.1
$(e_3, r, 1)$	u_1	0.3

Score (u_1) = -0.1 + 0.3 = 0.2

Score (u_2) = 0.1 + 0.8 + 0.4 + 0.6 + 0.2 + 0.1 = 2.2

Score (u_3) = 0.2 + 0.1 = 0.3

Table III. Numerical Grade for Disagree-IFSES

	u_i	Highest Numeric Grade
$(e_1, p, 0)$	u_2	0.1
$(e_3, p, 0)$	u_3	0.2
$(e_4, p, 0)$	u_1	0.1
$(e_1, q, 0)$	u_3	-0.1
$(e_2, q, 0)$	u_1	-0.1
$(e_3, q, 0)$	u_3	0.3
$(e_4, q, 0)$	u_2	0.4
$(e_1, r, 0)$	u_3	0.05
$(e_2, r, 0)$	u_2	0.2
$(e_4, r, 0)$	u_3	0.4

$$\text{Score} (u_1) = 0.1 - 0.1 = 0$$

$$\text{Score} (u_2) = 0.1 + 0.4 + 0.2 = 0.7$$

$$\text{Score} (u_3) = 0.2 - 0.1 + 0.3 + 0.05 + 0.4 = 0.85$$

Let A_i and D_i represent the score of each numerical grade for the agree- IFSES and disagree- IFSES respectively. These values are given in Table IV.

Table IV. The score $r_i = A_i - D_i$

A_i	D_i	r_i
Score (u_1) = 0.2	Score (u_1) = 0	0.2
Score (u_2) = 2.2	Score (u_2) = 0.7	1.45
Score (u_3) = 0.3	Score (u_3) = 0.85	-0.55

Then $s = \max_{u_i} \{ r_i \} = r_2$, the hiring committee should hire candidate u_2 to fill in the vacant position

6. Conclusion

In this paper we have introduced the concept of intuitionistic fuzzy soft expert soft set and studied some of its properties. The complement, union, intersection, AND or OR operations have been defined on the intuitionistic fuzzy soft expert set. Finally, an application of this concept is given in solving a decision making problem. This new extension will provide a significant addition to existing theories for handling uncertainties, and lead to potential areas of further research and pertinent applications.

References

- [1] A. Arokia Lancy, C. Tamilarasi and I. Arockiarani, Fuzzy parameterization for decision making in risk management system via soft expert set, International Journal of Innovative Research and studies, Vol 2 issue 10, (2013) 339-344, from www.ijirs.com.
- [2] A. Arokia Lancy, I. Arockiarani, A Fusion of soft expert set and matrix models, International Journal of Research in Engineering and Technology, Vol 02, issue 12, (2013) 531-535, from <http://www.ijret.org>
- [3] D. Molodtsov, Soft set theory-first result, Computers and Mathematics with Applications, 37(1999) 19-31.
- [4] G. Selvachandran, Possibility Vague Soft Expert Set Theory.(2014) Submitted.
- [5] H. L. Yang, Notes On Generalized Fuzzy Soft Sets, Journal of Mathematical Research and Exposition, 31/ 3 (2011) 567-570.
- [6] I. Arockiarani and A. A. Arokia Lancy, Multi criteria decision making problem with soft expert set. International journal of Computer Applications, Vol 78- No.15,(2013) 1-4 , from www.ijcaonline.org.
- [7] K.T. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems 20(1),(1986) 87-96.

- [8] K. Alhazaymeh & N. Hassan, Possibility vague soft set and its application in decision making. *International Journal of Pure and Applied Mathematics* 77 (4), (2012) 549-563.
- [9] K. Alhazaymeh & N. Hassan, Application of generalized vague soft expert set in decision making, *International Journal of Pure and Applied Mathematics* 93(3), (2014) 361-367.
- [10] K. Alhazaymeh & N. Hassan, Generalized vague soft expert set, *International Journal of Pure and Applied Mathematics*, (in press).
- [11] K. Alhazaymeh & N. Hassan, Mapping on generalized vague soft expert set, *International Journal of Pure and Applied Mathematics*, Vol 93, No. 3 (2014) 369-376.
- [12] K.V. Babitha and J. J. Sunil, Generalized intuitionistic fuzzy soft sets and Its Applications, *Gen. Math. Notes*, 7/ 2 (2011) 1-14.
- [13] L.A. Zadeh, Fuzzy sets, *Information and Control*, Vol8 (1965) 338-356.
- [14] M. Bashir & A.R. Salleh & S. Alkhazaleh. 2012. Possibility intuitionistic fuzzy soft Sets. *Advances in Decision Sciences*, 2012, Article ID 404325, 24 pages.
- [15] M. Bashir & A.R. Salleh, Fuzzy parameterized soft expert set. *Abstract and Applied Analysis*, 2012, Article ID 25836, 15 pages.
- [16] M. Bashir & A.R. Salleh, Possibility fuzzy soft expert set. *Open Journal of Applied Sciences* 12,(2012) 208-211.
- [17] M. Borah, T. J. Neog and D. K. Sut, A study on some operations of fuzzy soft sets, *International Journal of Modern Engineering Research*, 2/ 2 (2012) 157-168.
- [18] N. Hassan & K. Alhazaymeh, Vague soft expert set theory. *AIP Conference Proceedings* 1522, 953 (2013) 953-958.
- [19] N. Çağman, S. Enginoğlu, F. Çıtak, Fuzzy soft set theory and Its Applications. *Iranian Journal of Fuzzy System* 8(3) (2011) 137-147.
- [20] N. Çağman, F Çıtak , S. Enginoğlu, Fuzzy parameterized fuzzy soft set theory and its applications, *Turkish Journal of Fuzzy System* 1/1 (2010) 21-35.
- [21] N. Çağman, S. Karataş, Intuitionistic fuzzy soft set theory and its decision making, *Journal of Intelligent and Fuzzy Systems* DOI:10.3233/IFS-2012-0601.
- [22] N. Çağman, I. Deli, Intuitionistic fuzzy parametrized soft set theory and its decision making, *Applied Soft Computing* 28 (2015) 109–113.
- [23] N. Çağman, F. Karaaslan, IFP –fuzzy soft set theory and its applications, Submitted.
- [24] N. Çağman, I. Deli, Product of FP-Soft Sets and its Applications, *Hacettepe Journal of Mathematics and Statistics*, 41/3 (2012) 365 - 374.
- [25] N. Çağman, I. Deli, Means of FP-aoft aets and its applications, *Hacettepe Journal of Mathematics and Statistics*, 41/5 (2012) 615–625.
- [26] P. K. Maji, A. R. Roy and R. Biswas, Fuzzy soft sets, *Journal of Fuzzy Mathematics*, 9/3 (2001) 589-602.
- [27] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, *The Journal of Fuzzy Mathematics*, 9/3 (2001) 677-692.
- [28] P. Majumdar, S. K. Samanta, Generalized fuzzy soft sets, *Computers and Mathematics with Applications*, 59 (2010) 1425-1432
- [29] S. Alkhazaleh & A.R. Salleh, Fuzzy soft expert set and its application. *Applied Mathematics* 5(2014) 1349-1368.
- [30] S. Alkhazaleh & A.R. Salleh, Soft expert sets. *Advances in Decision Sciences* 2011, Article ID 757868, 12 pages.
- [31] S. Alkhazaleh, A. R. Salleh & N. Hassan, Possibility fuzzy soft sets. *Advances in Decision Sciences* (2011) Article ID 479756, 18 pages.

- [32] T. A. Albinaa and I. Arockiarani, SOFT EXPERT *pg SET, Journal of Global Research in Mathematical Archives, Vol 2, No. 3,(2014) 29-35