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State Feedback Switching Control of an Underactuated Planar Manipulator with Partial Feedback Linearization

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Underactuated manipulator,
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Abstract: In this paper, position control problem of a two-degree-of-freedom underactuated manipulator is considered and a state feedback control structure with energy-based switching is proposed. The mechanism has two revolute joints that move the two links on horizontal plane. Difficulties in system control arise with the fact that the manipulator has less control input signals than system degrees of freedom, and has complex nonholonomic structure. Furthermore, in horizontal operational conditions, position control of the system is a challenging work since free joint is not affected by the gravity. The system has only one actuator at the shoulder joint and the elbow joint is completely free. Therefore, the system cannot be stabilized around any equilibrium point by a linear state feedback control method. In this study, a control system that utilizes two stabilizing state feedback controllers and an energy-based supervisory switching mechanism is proposed. Stabilizing controllers are obtained utilizing the partial feedback linearization method. Proposed control is tested by computer simulations. First the open-loop plant dynamic model is obtained by Euler-Lagrange formulation and state-space modeling approach. Then, a simulation model of the system in closed loop with proposed control scheme is developed using the dynamic model and control law. Simulation tests are performed with respect to three different initial conditions. Performance of the control system is observed and revealed via simulations.

Eksik Tahrikli Düzlemsel Bir Manipülatörün Kısmi Geri Beslemeli Doğrusallaştırmayla Durum Geri Beslemeli Anahtarlamalı Denetimi

Anahtar Kelimeler
Durum geri beslemeli denetim,
Anahtarlamalı denetim,
Eksik tahrikli manipülatör,
Kısmi geri beslemeli doğrusallaştırma

Özet: Bu makalede iki serbestlik dereceli eksik tahrikli bir manipülatörün pozisyon denetimi ele alınmış ve enerji tabanlı anahtarlamayla durum geri beslemeli denetim yapısı önerilmiştir. Mekanizmada iki mafsalı düzlemsel alanda hareket ettiren iki döner eklem mevcuttur. Sistemdeki zorluklar, sistem serbestlik derecesi sayısından daha az denetim sinyali bulunmasından ve karmaşık nonholonomik yapısından kaynaklanır. Dahası, düzlemsel çalışma şartlarında, serbest eklem yer çekiminden etkilenmediği için pozisyon denetimi zordur. Sistemde sadece omuz ekleminde eyleyici bulunur ve dirsek eklemi tamamen serbesttir. Bu nedenle sistem hiçbir denge noktası etrafında doğrusal durum geri beslemeli denetim yöntemiyle kararlı hale getirilemez. Bu çalışmada, iki kararlılaştıran durum geri beslemeli denetimci ve bir enerji tabanlı anahtarlama mekanizmasından yararlanan denetim sistemi önerilmektedir. Önerilen denetim bilgisayarlı benzetim ile test edilmiştir. Önce Euler-Lagrange formülasyonu ve durum-uzay modelleme yaklaşımıyla açık çevrim dinamik model elde edilmiştir. Daha sonra açık çevrim dinamik model ve denetim kuralıyla önerilen denetim şemasını içeren kapalı çevrim sistemin benzetim modeli oluşturulmuştur. Üç farklı başlangıç değeri için benzetim testleri uygulanmıştır. Denetim sisteminin başarımı benzetim yoluyla gözlemlenmiş ve ortaya konmuştur.

1. Introduction

In recent years, there has been a rising interest in the analysis and control of underactuated manipulators in which the control inputs are fewer than the manipulator degrees-of-freedom. This corresponds to the case where all joints of the system are not equipped with actuators, or are not directly

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controllable. Underactuated character of manipulators may be intentional or result of actuator failures. Manipulators that are designed deliberately to be underactuated have several advantages such as reduced weight and power consumption, better security, and so on. In space research and industry for instance, underactuated manipulator application has promising prospect, as the payload transport to outer space is very expensive, and manipulators of space robots cannot be further reduced in weight because of driving motors despite employing extremely lightweight and high strength materials. Using underactuated joints not only reduces the total weight of space robots but also can improve the flexibility of systems. In addition, fully actuated manipulators may be desired to have the capability to run stably and with a sufficient amount of accuracy under underactuated conditions, which may result from an actuator failure. Underactuated systems introduce difficulties in motion control compared with fully actuated systems [1]. In order to control the underactuated manipulator, the passive joints that are not actuated should be driven via the acceleration coupling they have with the actuated or active joints.

Fully linearized models of underactuated manipulators can be controlled in the presence of gravity [2]. Stated otherwise, the system can be stabilized around equilibrium points by continuous time-invariant controllers. However, in this case, the number of equilibrium points of the system is few. The system considered in this study operates in horizontal plane and the system is not controllable since the gravity does not affect the manipulator joints [3]. In this case, a discontinuous or time-varying controller is needed. Arai and Tachi [4] have proposed a method to control the position of an underactuated manipulator that has driving motors and angular position sensors on active joints. In this study, passive joints are equipped with holding brakes and angular position sensors. For exciting passive joints to their desired positions, method of dynamic decoupling is utilized and holding brakes are engaged to maintain desired positions. In the same study, the authors have derived the controllability conditions of underactuated manipulators of concern [4]. Arai [5] has proved the controllability of a 3-DOF underactuated manipulator without holding brakes. Bergerman [6] presented results on controllability of planar manipulators and robust optimal sequential control methods for the underactuated manipulators. In the same study, collision-free motion of the manipulator using a graphical method has also been proposed [6]. On the other side, a discrete time approach have proposed based on sensitivity functions for the same system in year 2000 [7]. Izumi et al. [8] have presented a controller which uses a switching method based on fuzzy-energy regions. Zhixiang et al. [9] have presented a discontinuous control method, which divides the system into active and passive subsystem and designed adaptive laws with backstepping algorithm to control overall system. Knoll [10] has proposed a control method based on sliding mode control that uses similarities to the double integrator. Seifried [11] has presented a controller for minimum phase underactuated multibody systems, where an optimization-based design procedure has been used. Xia et al. [12] have proposed a solution to control problem of the manipulator by fuzzy control method such that the joint errors are fed back to the fuzzy system as inputs. The MIMO fuzzy controller has been divided into subsystems and control performance has been tested by simulation of three-link underactuated manipulator [12].

State feedback control method used in this study is based on the proposed combination of partial feedback linearization and energy-based switching control. Stabilizing controllers are obtained utilizing the partial feedback linearization method. Proposed control is tested by computer simulations.

2. Material and Method

2.1. Manipulator dynamics

Dynamic equation of a two-link manipulator can be obtained by Lagrange formulation. Lagrange function is written in terms of kinetic and potential energy of the system:

$$L = K - P \tag{1}$$

where K and P represent the total kinetic and potential energy terms, respectively. Then Lagrange function found by using (1) is written in Lagrange equation as follows:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial \dot{q}_j} = \tau_j \tag{2}$$

where τ_j, q_j and $\dot{q}_j$ represent input torque value, joint angles and joint velocities respectively. By using (2), dynamic equation of a multi-link manipulator can be obtained as follows:

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + f_v\dot{q} + f_s sgn(\dot{q}) = \tau \tag{3}$$

In (3) q represents generalized coordinates vector of manipulator while $M(q)$, $c(q,\dot{q})$ and $g(q)$ represent mass inertia terms, coriolis and centripetal terms and gravity terms respectively. General equation of motion for a two-degree-of-freedom manipulator moving in horizontal plane, which has only one actuator at shoulder joint, can be given as follows:

$$M(q)\ddot{q} + h(q,\dot{q}) = \tau \tag{4}$$

where the matrix $h(q,\dot{q})$ includes carioles, centripetal and gravity terms. The terms included in the general model given in (4) are as follows:

$$q = [q_1 \quad q_1]^T \tag{5}$$

$$\tau = [\tau_1 \quad 0]^T \tag{6}$$

$$M(q) = \begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{12}(q) & m_{22}(q) \end{bmatrix} \tag{7}$$

$$m_{11}(q) = (m_1 r_2^2 + m_2 l_1^2 + I_1) + (m_2 r_2^2 + I_2) + 2 m_1 l_1 r_2 \cos q_2 \tag{8}$$

$$m_{12}(q) = (m_2 r_2^2 + I_2) + m_2 l_1 r_2 \cos q_2 \tag{9}$$

$$m_{22}(q) = (m_2 r_2^2 + I_2) \tag{10}$$

$$h(q,\dot{q}) = [h_1(q,\dot{q}) \quad h_2(q,\dot{q})]^T \tag{11}$$

$$h_1(q,\dot{q}) = -m_2 l_1 r_2 \left(2\dot{q}_1\dot{q}_2 + \dot{q}_2{}^2\right)\sin q_2 + f_{v1}\dot{q}_1 + f_{s1} sgn(\dot{q}_1) \tag{12}$$

$$h_2(q,\dot{q}) = m_2 l_1 r_2 \dot{q}_1{}^2 \sin q_2 + f_{v2}\dot{q}_2 + f_{s2} sgn(\dot{q}_2) \tag{13}$$

Input torque and physical parameters of the manipulator are given in Table 1.

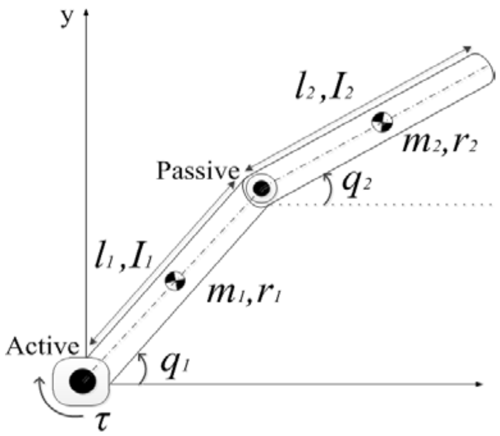

Figure 1. Two-link planar underactuated manipulator

Table 1. Physical parameters and variables

Symbol	Physical meaning
τ_1	Input torque (Nm)
m_1	First link mass of manipulator (kg)
m_2	Second link mass of manipulator (kg)
l_1	First link length (m)
l_2	Second link length (m)
r_1	Distance from first joint to first link's center of mass (m)
r_2	Distance from second joint to second link's center of mass (m)
I_1	First link moment of inertia about center of mass (kg m^2)
I_2	Second link moment of inertia about center of mass (kg m^2)

2.2. Control design

Since there is no actuator at the second link of manipulator, system dynamics have second-order nonholonomic constraints which are not integrable. This condition prevents the manipulator from being fully linearizable. The manipulator operates in horizontal plane and the system is not controllable since the gravity does not affect the manipulator joints. In this case the terms related with the gravity are excluded in dynamic equations of the system. In the end, unlike vertical operation condition the system becomes uncontrollable [1]. As a solution to this problem, a system of partly stable controllers with a switching mechanism is proposed.

2.2.1. Partly stable controllers

Partly stable controllers can be obtained using partial feedback linearization technique. This method can be applied to all underactuated systems because of the positive definiteness property of inertia matrix [13]. It is not possible to control both joint positions at the same time with a controller based on partial feedback linearization. Therefore only one joint can be stabilized at a time by this technique. If this method is applied for the actuated joint of the system, it is referred to as collocated partial feedback linearization and if the linearization is made for unactuated joint, it is referred as non-collocated linearization. However, in this case the system must have strong inertial coupled [14]:

$$m_2 r_2^2 + I_2 > m_2 l_1 r_2 \tag{14}$$

In other words, the number of active joints is greater than or equal to the number of passive joints.

In order to design a partly stable state feedback controller, PD control accompanied with partial feedback linearization can be used. Since there is only one actuator at the shoulder joint and second link is completely free, it is needed to have at least two controllers in order to control both joint positions [15]. For designing a partly stable controller, dynamic equation of the two-link manipulator can be written as follows:

$$\ddot{q} = M^{-1}[-h(q,\dot{q}) + \tau] \tag{15}$$

By expanding (15):

$$\ddot{q}_1 = -\frac{m_{22}}{D}h_1 + \frac{m_{12}}{D}h_2 + \frac{m_{22}}{D}\tau_1 \tag{16}$$

$$\ddot{q}_2 = \frac{m_{12}}{D}h_1 - \frac{m_{11}}{D}h_2 - \frac{m_{12}}{D}\tau_1 \tag{17}$$

where D is the determinant of inertia matrix $M(q)$ and is expressed as follows:

$$D = m_{11}m_{22} - m_{12}^2 \tag{18}$$

Now one can give the system input u which is, in this case, τ_1 for (6) as follows:

$$\tau_1 = u = \frac{D}{m_{22}}\left(v_1 + \frac{m_{22}}{D}h_1 - \frac{m_{12}}{D}h_2\right) \quad (19)$$

where v_1 is the auxiliary input to the system. This results in the following system:

$$\ddot{q}_1 = v_1 \quad (20)$$

$$m_{22}\ddot{q}_2 + h_2(q,\dot{q}) = -m_{21}v_1 \quad (21)$$

Since (20) is now linear, a computed torque controller with feedforward acceleration could be applied for the purpose of the first link's position control:

$$v_1 = \ddot{q}_{d1} + K_{v1}(\dot{q}_{d1} - \dot{q}_1) + K_{p1}(q_{d1} - q_1) \quad (22)$$

where q_{d1} is the desired value of q_1. This is the outer loop control which is employed to move to a desired angle value q_1 for Link #1. The response of Link #2 is then given by the resulting nonlinear equation in (21), which represents internal dynamics with respect to an output $y = q_1$. The function of the outer loop control therefore is to move a given angle value for Link #1 and the same time excite the internal dynamics to any angle value for Link #2. First, the state variables, q_1, q_2, $\dot{q}_1$, $\dot{q}_2$ are fed back to the outer loop control law to calculate the variable v_1 using (22). Then, v_1 is used with the state variables to calculate the control input for the manipulator based on the inner loop control law presented in (19). One can see that the so called outer loop and inner loop are actually two steps for calculating the control signal u. They are divided in order to be expressed more clearly.

Now, substituting (22) into (20), one can get the error equation:

$$\alpha = \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = \begin{bmatrix}q_{d1} - q_1\\ \dot{q}_{d1} - \dot{q}_1\end{bmatrix} \quad (23)$$

$$\dot{\alpha} = \begin{bmatrix}0 & 1\\ -K_{p1} & -K_{v1}\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} \quad (24)$$

Similarly, controller that stabilizes only second joint position q_2 and torque input can be found as follows:

$$v_2 = \ddot{q}_{d2} + K_{v2}(\dot{q}_{d2} - \dot{q}_2) + K_{p2}(q_{d2} - q_2) \quad (25)$$

$$u = -\frac{D}{m_{12}}\left(v_2 - \frac{m_{12}}{D}h_1 + \frac{m_{11}}{D}h_2\right) \quad (26)$$

In the same manner one can get the error equation for second joint as follows:

$$\beta = \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} = \begin{bmatrix}q_{d2} - q_2\\ \dot{q}_{d2} - \dot{q}_2\end{bmatrix} \quad (27)$$

$$\dot{\beta} = \begin{bmatrix}0 & 1\\ -K_{p2} & -K_{v2}\end{bmatrix}\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} \quad (28)$$

with properly chosen K_{p2} and K_{v2} one can adjust the convergence rate of the errors to zero.

Now, it has been shown that any degree of freedom of the underactuated manipulator can be fully controlled. In order to control each link of the underactuted manipulator, partly stable controllers are designed. In this case a supervisory controller is needed to control the position of end point of second link. Therefore, a logic based switching controller that is based on error energy function of the joints is proposed which uses position and velocity errors of the joints.

2.2.2. Switching control

In order to make the whole system stable by using partly stable controllers together, a switching control algorithm is needed. To constitute switching control system, the error between desired joint position value and actual joint angle position value can be defined as follows:

$$e_i \triangleq q_{di} - q_i \quad (29)$$

Then each joint energy-like error function is defined as given below [15]:

$$E_i \triangleq e_i^2 + \dot{e}_i^2 \quad (30)$$

Let us name the controller that stabilizes only ith link as C_i. If C_1 is active while first joint error energy E_1 decreases, then second joint error energy E_2 starts to increase because q_1 link angle can only be stabilized by its associated controller. Similarly, when C_2 is chosen as the active controller, then E_2 decreases while E_1 increases.

Considering these behaviors, in order to bring the error of both joints to zero and move the links to their predefined positions with zero velocity, logic based switching rule is defined as follows:

$$\hat{I} = \begin{cases}1 \; if \; E_1 \geq E_2\\ 2 \; if \; E_1 < E_2\end{cases} \quad (30)$$

where $\hat{I}$ is the switching index that decides which controller is to be chosen as active.

2.2.3. Simulation tests

In practical applications joint frictions cannot be ignored. Especially, at the free rotating joint, frictions must definitely be taken into account. This is a necessity resulting from the fact that friction at the actuated joint can be compensated for by partial feedback linearization whereas the same is not true for the unactuated joint [13]. Therefore, in simulations, viscous friction of the free joint is considered and integrated in the model [2]. In simulations, sampling interval is chosen as 0.01 s and total simulation time is adjusted as 20 s.

Considering joint velocity ranges, gain parameters K_{vi} and K_{pi} of partly stable controllers are adjusted as:

$$\begin{cases} K_{v1} = 14 \\ K_{p1} = 49 \\ K_{v2} = 22 \\ K_{p2} = 121 \end{cases} \quad (31)$$

In the simulation, physical parameters given in Table 2 are used. The controller that stabilizes only the first joint is applied to simulated system and performance of controller is examined. Similarly, by using dynamical equations, second controller that stabilizes only second joint is made up in the same manner.

System state variables are chosen as joint angles and velocities. For simulations, we define desired state vector as:

$$x_d = [0 \quad 0 \quad 0 \quad 0]^T \quad (32)$$

and initial state vectors:

$$x_i(0) = [q^T(0) \quad \dot{q}^T(0)]^T \quad (33)$$

are as given below:

$$\begin{cases} x_1(0) = [0 \quad \pi/2 \quad 0 \quad 0]^T \\ x_2(0) = [3\pi/4 \quad \pi/2 \quad 0 \quad 0]^T \\ x_3(0) = [\pi \quad \pi \quad 0 \quad 0]^T \end{cases} \quad (34)$$

Sampling interval for simulations is set to 10 ms and elapsed time to carry out each simulation under proposed control is 1.2 s, which implies little computational effort and gives opportunity to apply the algorithm to a real system.

3. Results

Simulations are carried out using Simulink models of the two controllers designed for stabilizing the links of the underactuated manipulator. Figure 2 depicts the closed-loop system with state feedback controllers using partial feedback linearization for link#1 and link#2. The partly stable controller that combines and switches between the two controllers has the block diagram representation in Figure 3.

Simulation outputs given in Figures 4 to 6 give controlled manipulator system angles, velocities and applied input torques. From the given responses in figures, it can be seen that simulated angle and velocity values converge to zero and system is stabilized. If initially the error between desired state vector and initial state vector increases, accordingly amplitude of input torque and velocity values also increase and the system goes to desired angle positions in about 10 s and only a slight increase occurs in this duration of settling time when the initial error values are high.

Table 2. Manipulator parameters used in simulations

Parameter	Value
m_1	0.12 kg
m_2	0.05 kg
l_1	0.1 m
l_2	0.15 m
r_1	0.0976 m
r_2	0.09 m
I_1	0.019 kgm^2
I_2	0.0004 kgm^2
f_{v1}	0 Nms
f_{v2}	0.01 Nms

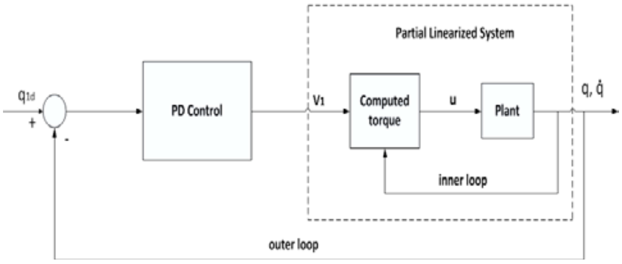

Figure 2. Block diagram of the partial feedback linearization control.

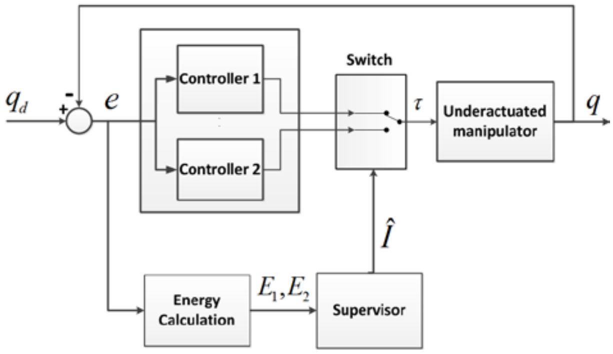

Figure 3. Block diagram of proposed control system.

There are several recent studies in literature dealing with the problem of controlling the same configuration of the system considered in this paper. One significant study in the paper by He et al. uses nilpotent approximation and iterative steering methods to solve the problem [16]. It also uses numerical simulations to show effectiveness of the method. A comparative simulation study of the method in aforementioned paper with the proposed method is shown in Figures 7 and 8. Firstly, to start with the same conditions, initial conditions in the mentioned paper are chosen also for simulation of proposed method. In Figure 7, the graph on the left is the response of the method by He et al. Here initial conditions for both systems is $x_{n1}(0)$=[-0.18, 0.26, 0, 0]. In Figure 8, joint position responses for initial condition of $x_{n2}(0)$=[-0.16, -0.24, 0, 0] are given.

It can be seen from Figures 7 and 8 that proposed switching partial linearization method gives better results than nilpotent iterative steering method (NISM) that is proposed in the reference paper by

Hen et al. [16]. Joint positions are settling their final positions in a shorter time and oscillation around the final value is very small if proposed method is used. It is also evident from the mean square error (MSE) analysis given in Table 3 and Figures 9 and 10 that the manipulator position control performance is better for proposed method over NISM in terms of cumulative error as well.

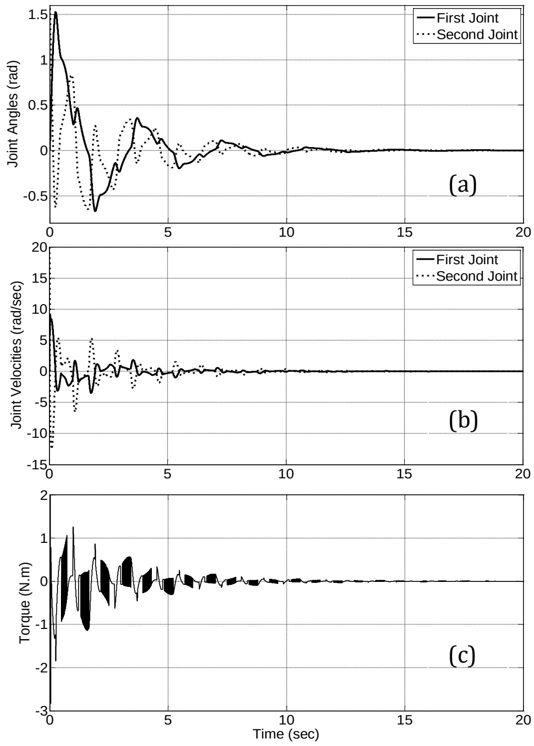

Figure 4. Joint angles (a), joint velocities (b), and applied input torque (c) for initial condition $x_1(0)$

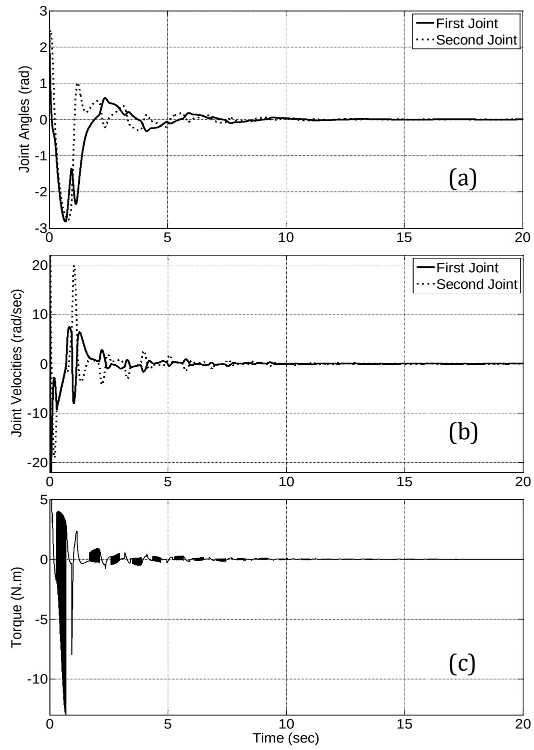

Figure 5. Joint angles (a), joint velocities (b), and applied input torque (c) for initial condition $x_2(0)$

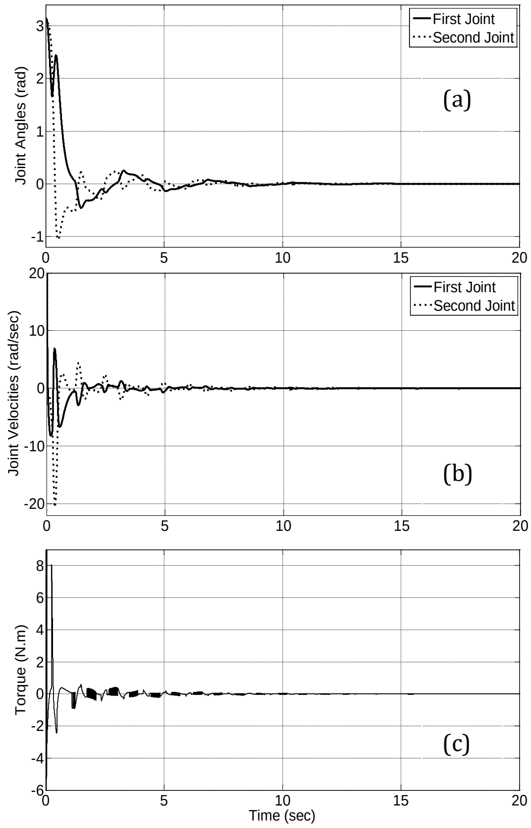

Figure 6. Joint angles (a), joint velocities (b), and applied input torque (c) for initial condition $x_3(0)$

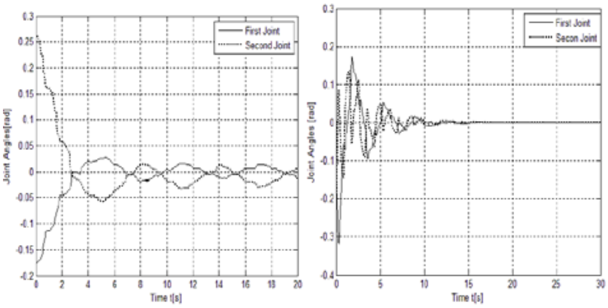

Figure 7. Comparison of joint position control results for $x_{n1}(0)$ using NISM [16] (left) and proposed method (right)

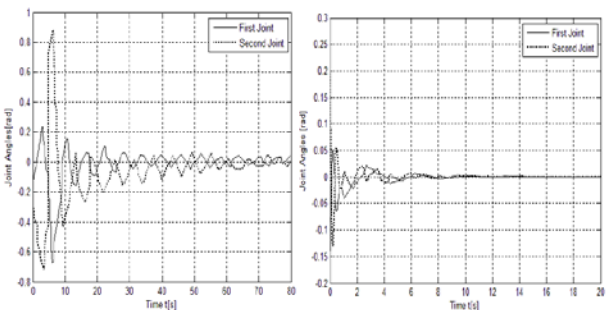

Figure 8. Comparison of joint position control results for $x_{n2}(0)$ using NISM [16] (left) and proposed method (right)

Table 3. Position MSE values of joints using two methods for performance comparison

Initial Condition	Method	MSE (Joint1)	MSE (Joint2)
$x_{n1}(0)$	Proposed	0.0001	0.0002
$x_{n1}(0)$	NISM [16]	0.0021	0.0079
$x_{n2}(0)$	Proposed	0.0210	0.0010
$x_{n2}(0)$	NISM [16]	0.0316	0.0968

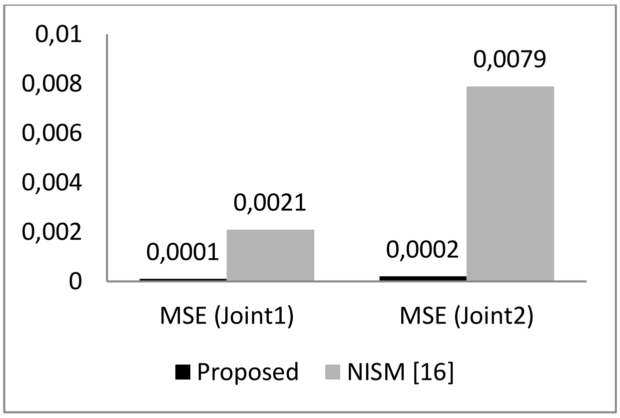

Figure 9. Comparison of position MSE values for joint 1 and joint 2 with initial condition of $x_{n1}(0)$

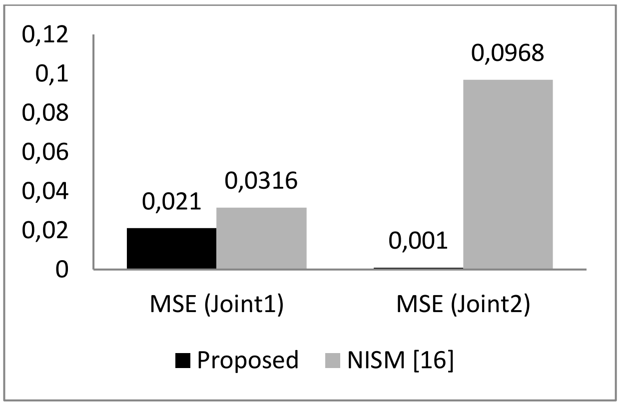

Figure 10. Comparison of position MSE values for joint 1 and joint 2 with initial condition of $x_{n2}(0)$

4. Discussion and Conclusion

Underactuated planar manipulator considered in this study has less control inputs than system degrees-of-freedom and has complex nonholonomic structure. Furthermore, in horizontal operation condition, linearized system is not controllable since free joint is not affected by the gravity. As a solution to the control problem, a control system that uses two partly stable feedback controllers and an energy-based supervisory switching mechanism is proposed. Each link of the manipulator is stabilized by a partial feedback linearizing controller. A mathematical model with a state-space structure convenient for state feedback control application is derived for the open-loop plant. A simulation model for the closed-loop system embracing the open-loop plant dynamics, partly stable controllers and the switching mechanism is developed. After successive simulations, it is observed that the proposed controller steers the system to predefined positions and stabilizes the system for various initial conditions. It is also shown that the errors are kept within a small range, and converge to zero quickly. Because of the switching control the error values always oscillate within very small peak amplitude around zero. Proposed method has shown superior transient and cumulative error performances in comparison with a recent study utilizing nilpotent approximation and iterative steering. Applied torque to the system is in a reasonable range and can be applied to real systems. In the light of the current simulation study, improving the time response performances and testing the controller on a real system are aimed for future studies.

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