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ECG Noise Cancellation with Recursive Gauss-Seidel Algorithm

Metin HATUN[1*]

Abstract

Electrocardiogram (ECG) signals provide information about heart functions and some cardiac diseases. However, various interferences distort the ECG waveforms during its measurement and transmission can cause inaccurate analysis and diagnosis. So, this unwanted disturbance signals must be eliminated and an acceptable ECG signal must be extracted the noisy ECG recordings. Researchers developed several methods to overcome the undesired noises and interferences contaminated to the ECG recordings. The adaptive filtering techniques have attracted the attention of scientists due to their adaptation mechanism to time-varying nature of undesired signals. Most of the presented adaptive filtering algorithms are gradient-based and have the advantage of simple implementation, but are affected negatively by disturbance signals; for example, they can have slow convergence rates and poor steady-state properties. Least squares-based algorithms are advantageous due to their faster convergence rates and better steady-state properties. In this paper, Recursive Gauss-Seidel (RGS) algorithm, which is an alternative least squares-based method to Recursive Least Squares (RLS) algorithm with less computational complexity, is presented to obtain an acceptable waveform from noisy ECG recordings. The denoising performance of the RGS algorithm is studied and compared to the widely used gradient-based algorithms and the popular RLS algorithm.

Keywords: ECG denoising, Adaptive filtering, Gauss-Seidel, Recursive algorithm.

Tekrarlamalı Gauss-Seidel Algoritması ile EKG Gürültüsünün Temizlenmesi

Öz

Elektrokardiyogram (EKG) sinyalleri kalp fonksiyonları ve bazı kalp hastalıkları hakkında bilgi sağlar. Ancak ölçüm ve iletim sırasında EKG dalga formlarını bozan çeşitli girişimler, hatalı analiz ve tanıya neden olabilir. Bu nedenle, bu istenmeyen bozucu sinyallerin ortadan kaldırılması ve gürültülü EKG kayıtlarından kabul edilebilir bir EKG sinyalinin çıkarılması gerekmektedir. Araştırmacılar, EKG kayıtlarına bulaşan istenmeyen gürültü ve girişimlerin üstesinden gelmek için çeşitli yöntemler geliştirdiler. Uyarlanabilir filtreleme teknikleri, istenmeyen sinyallerin zamanla değişen doğasına uyum sağlama mekanizmaları nedeniyle bilim adamlarının dikkatini çekmiştir. Sunulan uyarlanabilir filtreleme algoritmalarının çoğu eğim tabanlıdır ve basit gerçekleme avantajına sahiptir, ancak bozucu sinyallerden olumsuz etkilenirler; örneğin, yavaş yakınsama hızlarına ve zayıf kalıcı-durum özelliklerine sahip olabilirler. En küçük kareler tabanlı algoritmalar, daha hızlı yakınsama ve daha iyi kalıcı-durum yanıtları nedeniyle avantajlıdır. Bu makalede, gürültülü EKG kayıtlarından kabul edilebilir bir dalga şekli elde etmek için, Tekrarlamalı En Küçük Kareler (RLS) algoritmasına göre daha az hesaplama karmaşıklığına sahip, en küçük kareler tabanlı alternatif bir yöntem olan Tekrarlamalı Gauss-Seidel (RGS) algoritması sunulmaktadır. RGS algoritmasının gürültü temizleme performansı araştırılmış ve yaygın olarak kullanılan eğim tabanlı algoritmalar ve popüler RLS algoritması ile karşılaştırılmıştır.

Anahtar Kelimeler: EKG gürültü temizleme, Uyarlamalı filtreleme, Gauss-Seidel, Tekrarlamalı algoritma.

[1]Bursa Uludağ Üniversity, Department of Electrical-Electronics Engineering, Bursa, Turkey, metinh@uludag.edu.tr

[*]Sorumlu Yazar/Corresponding Author

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1. Introduction

Noise reduction or denoising by filtering is an important preprocessing step in engineering applications (Vaseghi, 2008; Clifford et al., 2006; Berkaya et al., 2018). The electrocardiogram (ECG) signal is deformed during its measurement by some dominant types of noise; these are Power Line Interference (PLI), Baseline Wander (BW) noise, Muscle Artifact (MA) and Electrode Movement (EM) artifact, or a mixture of these in varying amounts. The PLI noise is a sinusoidal signal about 50 Hz (or 60 Hz) frequency arise from the electromagnetic field of the power lines. The BW noise is a low frequency signal occurs by the patient's breathing or body movements and shifts the baseline of the ECG signal. The MA noise, or electromyogram (EMG) wave is caused by the electrical activity of muscle movements near the electrodes. The EM noise appears by very slow changes in the impedance of the skin electrode with electrode movements and therefore a temporary baseline shift occurs in the ECG signal at a very low frequency. These noise types can lead to its incorrect observation and therefore incorrect analysis of cardiac functions or misdiagnosis of heart diseases (Chatterjee et al., 2020).

Even if the ECG waveform is distorted by various noise sources during its measurement, the cleaned ECG signal can be obtained from its noisy recordings with the lowest possible error by ECG denoising process. Several signal processing methods have been proposed in the literature for ECG denoising and to obtain an acceptable ECG waveform (Chatterjee et al., 2020; Malghan and Hota, 2020; Mir and Singh, 2021). Adaptive noise cancellation process is an effective noise removal technique among these and is implemented using adaptive filters. Adaptive filtering algorithms can be classified roughly as gradient-based and least squares-based. The Least Mean Squares (LMS) algorithm and the Normalized LMS (NLMS) algorithm are the most popular gradient-based algorithms, and the Recursive Least Squares (RLS) algorithm is the most used least squares-based algorithm in adaptive filtering applications (Haykin, 2002). The gradient-based algorithms have the advantage of simple implementation because of their lower computational complexity compared with least squares-based algorithms, and preferred due to their lower processing load. The major limitations of the gradient-based algorithms are their relatively slow convergence rates. The RLS-based algorithms are preferred due to their faster convergence speed despite the high computational load.

Some of the gradient-based algorithms used for adaptive cancellation of PLI, BW, MA and EM noises from noisy ECG recordings can be summarized as follows: The LMS algorithm and its computationally simplified versions, Sign-Regressor LMS (SRLMS), Sign-Error LMS (SELMS) and Sign-Sign LMS (SSLMS) algorithms (Rahman et al., 2009). The authors report that the SRLMS algorithm produces better results than LMS and the other counterparts in SNR improvement.

The NLMS algorithm uses the normalized step-size parameter that account the variation in signal level at filter output, and thus, it is obtained the more stable and fast converging LMS version (Haykin, 2002). The sign-based counterparts of the NLMS algorithm are Normalized SRLMS (NSRLMS), Normalized SELMS (NSELMS) and Normalized SSLMS (NSSLMS) algorithms (Rahman et al., 2011). It is reported that these normalized versions of LMS-based algorithms are also remove non-stationary noise efficiently, and the NSRLMS algorithm performs better than the other versions.

The Error Normalized LMS (ENLMS) algorithm uses the error vector instead of the data vector to obtain the normalized step-size parameter. Consequently, a large value of the excess mean square error reduces the normalized step-size parameter and thus reduces the signal distortion in the denoised signal. The sign-based equivalents of the ENLMS algorithm are Error Normalized SRLMS (ENSRLMS), Error Normalized SELMS (ENSELMS) and Error Normalized SSLMS (ENSSLMS) algorithms (Rahman et al., 2012). Among the presented equivalents, the ENSRLMS algorithm is reported to produce better results in SNR improvement than other counterparts.

A Variable Step-Size LMS (VSSLMS) algorithm, which is obtained by adding a variable step-size parameter to the normalized step-size parameter of the NLMS algorithm, is also proposed for ECG denoising (Gowri et al., 2014). The Sign-Regressor VSSLMS (SRVSSLMS) and Sign-Error VSSLMS (SEVSSLMS) algorithms are the complexity reduced versions of the original VSSLMS algorithm. Error Normalized VSSLMS (ENVSSLMS) and Sign-Regressor ENVSSLMS (ENSRVSSLMS) algorithms are also proposed. It is reported that the original VSSLMS algorithm and its Sign-Regressor version (SRVSSLMS) give the best results among the presented algorithms.

The Data Error Normalized VSS-LMS (DENVSS-LMS) algorithm uses both the error vector and the data vector together with a certain ratio to obtain the normalized step-size parameter (Gowri et al., 2015; Gowri et al., 2017). Its sign-based derivations are also proposed to obtain reduced complexity versions of the original algorithm.

The Normalized Variable step-size LMS (NVLMS) algorithm and its sign-based versions use a different variable step-size strategy in addition to the normalized step-size parameter in the NLMS algorithm (Salman et al., 2017).

The Least Mean Fourth (LMF), Normalized LMF (NLMF), and Error Normalized LMF (ENLMF) algorithms minimize the cost function in the least mean-fourth sense, and their sign-based versions are also used to reduce the computational complexities of the original versions in ECG denoising (Karthik and Sugumar, 2013).

Least Mean Mixed-Norm (LMMN) algorithm and Sign-Regressor LMMN (SRLMMN) algorithm minimize the cost function both in the least mean-square sense and in the least mean-fourth sense, and thus, work as a hybrid form of the LMS and LMF algorithms (Faiz and Kale, 2022). Sign-based equivalents of the mentioned algorithms also reduce the computational burden of the original algorithms.

In this paper, the Recursive Gauss-Seidel (RGS) algorithm is presented as an alternative least squares-based algorithm for ECG denoising. The RGS algorithm is performed using one step Gauss-Seidel (GS) iteration over a sampling interval and therefore has less implementation load than the RLS algorithm. It also converges faster than gradient-based algorithms. The RGS algorithm has been proposed to adjust parameters of a self-tuning controller (Hatun and Koçal, 2012). The one-step GS iteration has been used previously for auto-regressive modeling (Koçal, 1998). The Euclidean Direction Search algorithm, which was introduced from the perspective of an optimization algorithm, is also implemented using a one-step GS iteration in a sampling interval and has also been used in some adaptive filtering applications such as system identification, channel equalization, noise cancellation, blind source separation, and image restoration (Xu et al., 1998, 1999a, 1999b; Mabey et al., 2004; Bose, 2004). However, single-step GS iteration has not been used for ECG denoising purpose. The aim of this paper is to use the RGS algorithm for ECG noise cancellation and compare the noise reduction performance with commonly used algorithms.

Organization of the paper: The RGS algorithm for ECG denoising is presented in Section 2. In Section 3, some comparative simulations are presented for ECG denoising. Conclusions are explained in Section 4.

2. Recursive Gauss-Seidel Algorithm

Adaptive noise cancelation process shown in Figure 1 is an important application area of adaptive filters (Haykin, 2002).

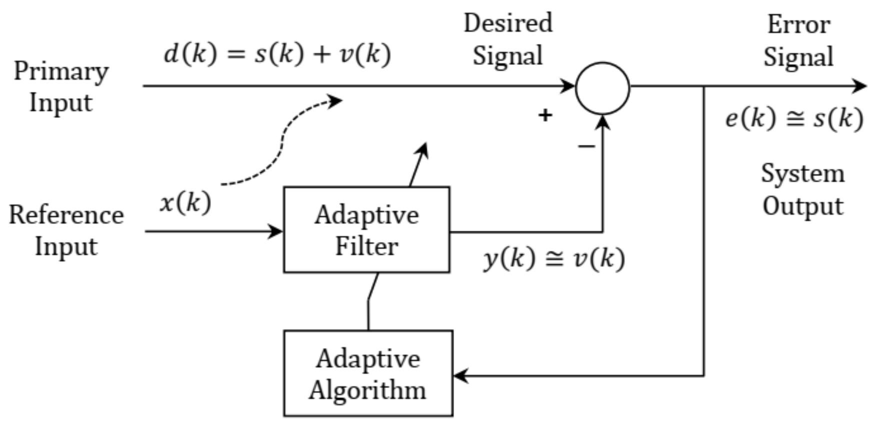

Figure 1. Adaptive noise cancellation process.

The primary noise $v(k)$ that contaminated to the ECG measurements is estimated as $y(k)$ at the output of the adaptive filter from the reference noise input $x(k)$. Consequently, the cleaned ECG signal $e(k)$ is obtained by subtracting $y(k)$ from $d(k)$ thereby ensuring that the system output $e(k)$ and defined as

$$e(k) = d(k) - y(k) = d(k) - \boldsymbol{x}^T(k)\widehat{\boldsymbol{w}}(k). \tag{1}$$

This is the best estimate of clean ECG signal $s(k)$ in the minimum mean square error sense (Haykin, 2002). The reference noise vector and estimated parameter vector are given as follows, respectively:

$$\boldsymbol{x}(k) = [x(k) \quad x(k-1) \quad \cdots \quad x(k-M+1)]^T \tag{2}$$

$$\widehat{\boldsymbol{w}}(k) = [\widehat{w}_0(k) \quad \widehat{w}_1(k) \quad \cdots \quad \widehat{w}_{M-1}(k)]^T \tag{3}$$

where M is the filter length. The RGS algorithm can be obtained by minimizing the following error function.

$$\mathrm{J}(\boldsymbol{w}, k) = \sum_{i=1}^{k} \lambda^{k-i}[d(i) - \boldsymbol{x}^T(i)\widehat{\boldsymbol{w}}(i)]^2 \tag{4}$$

The optimal parameter estimations are obtained as follows from (4) for k-step data.

$$\widehat{\boldsymbol{w}}(k) = \mathbf{R}(k)^{-1}\boldsymbol{p}(k) \tag{5}$$

These parameters are also the solution of the following normal equation.

$$\mathbf{R}(k)\widehat{\boldsymbol{w}}(k) = \boldsymbol{p}(k) \tag{6}$$

The estimation of $\mathbf{R}(k)$ and $\boldsymbol{p}(k)$ are updated as follows, recursively:

$$\mathbf{R}(k) = \lambda\, \mathbf{R}(k-1) + \boldsymbol{x}(k)\boldsymbol{x}^T(k) \tag{7}$$

$$\boldsymbol{p}(k) = \lambda\, \boldsymbol{p}(k-1) + \boldsymbol{x}(k)d(k) \tag{8}$$

where $\lambda \leq 1$ is the forgetting factor and taken as close to 1, and then, the following one-step Gauss-Seidel iteration can be applied to solve the normal equation (6) for $i = 1, 2, \ldots, M$ as follows:

$$\hat{w}_i(k+1) = \left[p_i(k) - \sum_{j=1}^{i-1} R_{ij}(k)\hat{w}_j(k+1) - \sum_{j=i+1}^{M} R_{ij}(k)\hat{w}_j(k) \right] \Big/ R_{ii}(k), \quad (9)$$

where $\hat{w}_j(k)$, $p_i(k)$, and $R_{ij}(k)$ are the elements of $\hat{\boldsymbol{w}}(k)$, $\boldsymbol{p}(k)$, and $\mathbf{R}(k)$, respectively. Note that the iteration index in the RGS algorithm is the time index k, that is, equations (7), (8) and (9) are implemented one-step at each sampling interval. This implementation allows to the computational complexity of the RGS algorithm becomes less than RLS (Hatun and Koçal, 2012).

The implementation steps of the adaptive noise cancellation process using the RGS algorithm can be summarized as follows:

1. To initialize: take $k = 1$ and set initial values as : $\hat{\boldsymbol{w}}(0) = \mathbf{0}_{M\times 1}$, $\boldsymbol{p}(0) = \mathbf{0}_{M\times 1}$ $\mathbf{R}(0) = \beta \mathbf{I}_{M\times M}$, where $\mathbf{0}$ is zero vector, $\mathbf{I}$ is unit matrix with suitable dimensions and β is a scalar and can be taken typically as $1, 0.1, 0.01, 0.001, \ldots$ etc.
2. Take the measurement data $d(k)$ and form the data vector $\boldsymbol{x}(k)$ given by (2).
3. Update the estimations of $\mathbf{R}(k)$ and $\boldsymbol{p}(k)$ as given by (7) and (8), respectively.
4. Update the vector of filter parameter estimates $\hat{\boldsymbol{w}}(k)$ using single-step GS iteration given by (9).
5. Estimate the noise signal $v(k)$ contaminating the ECG signal as $y(k) = \boldsymbol{x}^T(k)\hat{\boldsymbol{w}}(k)$.
6. Obtain the clean ECG signal $s(k)$ as $e(k) = d(k) - y(k)$ using (1).
7. Go to step 2 and repeat the same steps of adaptive noise cancellation process from step 2 to 7.

Remark: Increasing the filter length M can improve the noise cancellation performance of the RGS algorithm, but if M is further increased, the denoising performance may not be obvious and the computational burden of the RGS algorithm increases too large.

3. ECG Denoising with RGS Algorithm

In this section, the success of the RGS algorithm is evaluated together with some commonly used adaptive algorithms by computer simulations. Some performance parameters given in Table 1 were used to evaluate the noise cancellation results obtained using the compared algorithms.

In this study, two simulation examples were performed to examine the noise cancellation performance of the RGS algorithm. The real ECG records were taken from the MIT-BIH arrhythmia database (Moody and Mark, 2005). Simulation example 1 was performed to cancel PLI noise from ECG recordings. It is the most dominant noise source and was generated synthetically. The ECG measurements can be corrupted also by a mixture of all noise sources such as PLI, BW, MA and EM noise. Simulation example 2 was performed to cancel such a mixed noise source from the ECG records. The BW, MA and EM noises were taken from the MIT-BIH Noise Stress Test database (Moody and Mark, 1999).

Table 1. The parameters for performance evaluations.

Name of the Performance Parameter	**Equation of the Performance Parameter**
Mean Square Error (*MSE*)	$MSE = \frac{1}{N}\sum_{k=1}^{N}\lvert s(k) - e(k)\rvert^2$
Percentage RMS Difference [%]	$PRD = \sqrt{\frac{\sum_{k=1}^{N}[s(k) - e(k)]^2}{\sum_{k=1}^{N}[s(k)]^2}} \times 100$
SNR improvement [dB]	$SNR_{imp} = SNR_{out} - SNR_{in}$
Output SNR [dB]	$SNR_{out} = 10\log_{10}\left\{\frac{\sum_{k=1}^{N}[e(k)]^2}{\sum_{k=1}^{N}[s(k) - e(k)]^2}\right\}$
Input SNR [dB]	$SNR_{in} = 10\log_{10}\left\{\frac{\sum_{k=1}^{N}[s(k)]^2}{\sum_{k=1}^{N}[v(k)]^2}\right\}$
Cross-correlation coefficient between clean signal and filtered signal	$\rho = \frac{\sum_{k=1}^{N}[s(k) - \bar{s}][e(k) - \bar{e}]}{\sqrt{\sum_{k=1}^{N}[s(k) - \bar{s}]^2 \sum_{k=1}^{N}[e(k) - \bar{e}]^2}}$

Example 1: In the first simulation example, the ECG record number 105 in the file "105m.mat" taken from the MIT-BIH arrhythmia database (Moody and Mark, 2005). The reference PLI noise with an amplitude of 1 V and a frequency of 50 Hz was synthetically generated and added to the ECG signal by multiplying by 0,5. The filter parameters of the algorithms compared were chosen as given in Table 2 to produce acceptable denoised ECG waveforms. In all algorithms used, the filter lengths were taken as M=16 and the initial values of the estimated parameter vectors were taken as zero vectors. The denoised ECG waveforms obtained by using the compared algorithms are given in Figure 2. The numerical values of the performance parameters calculated according to Table 1 are given in Table 3.

Table 2. Filter parameters of the compared algorithms for PLI noise cancellation.

Algorithm:	**Filter Parameters:**	**Algorithm :**	**Filter Parameters:**
LMS	$\mu = 0.01$	SRLMS	$\mu = 0.01$
NLMS	$\mu = 0.1$, $\alpha = 0.01$	NSRLMS	$\mu = 0.1$, $\alpha = 0.01$
ENLMS	$\mu = 0.1$, $\alpha = 0.01$	ENSRLMS	$\mu = 0.1$, $\alpha = 0.01$
DENVSS-LMS	$\mu = 0.1$, $\alpha = 0.5$	DENVSS-SRLMS	$\mu = 0.1$, $\alpha = 0.5$
VSSLMS	$\mu = 0.99$	SRVSSLMS	$\mu = 0.99$
LMMN	$\mu = 0.02$, $\lambda = 0.5$	SRLMMN	$\mu = 0.02$, $\lambda = 0.5$
LMF	$\mu = 0.01$	SRLMF	$\mu = 0.01$
NLMF	$\mu = 0.1$, $\alpha = 0.01$	ENLMF	$\mu = 0.1$, $\alpha = 0.01$
RGS	$\lambda = 0.9995$, $\mathbf{R}(0) = \mathbf{I}_{M\times M}$	RLS	$\lambda = 0.9995$, $\mathbf{R}^{-1}(0) = \mathbf{I}_{M\times M}$

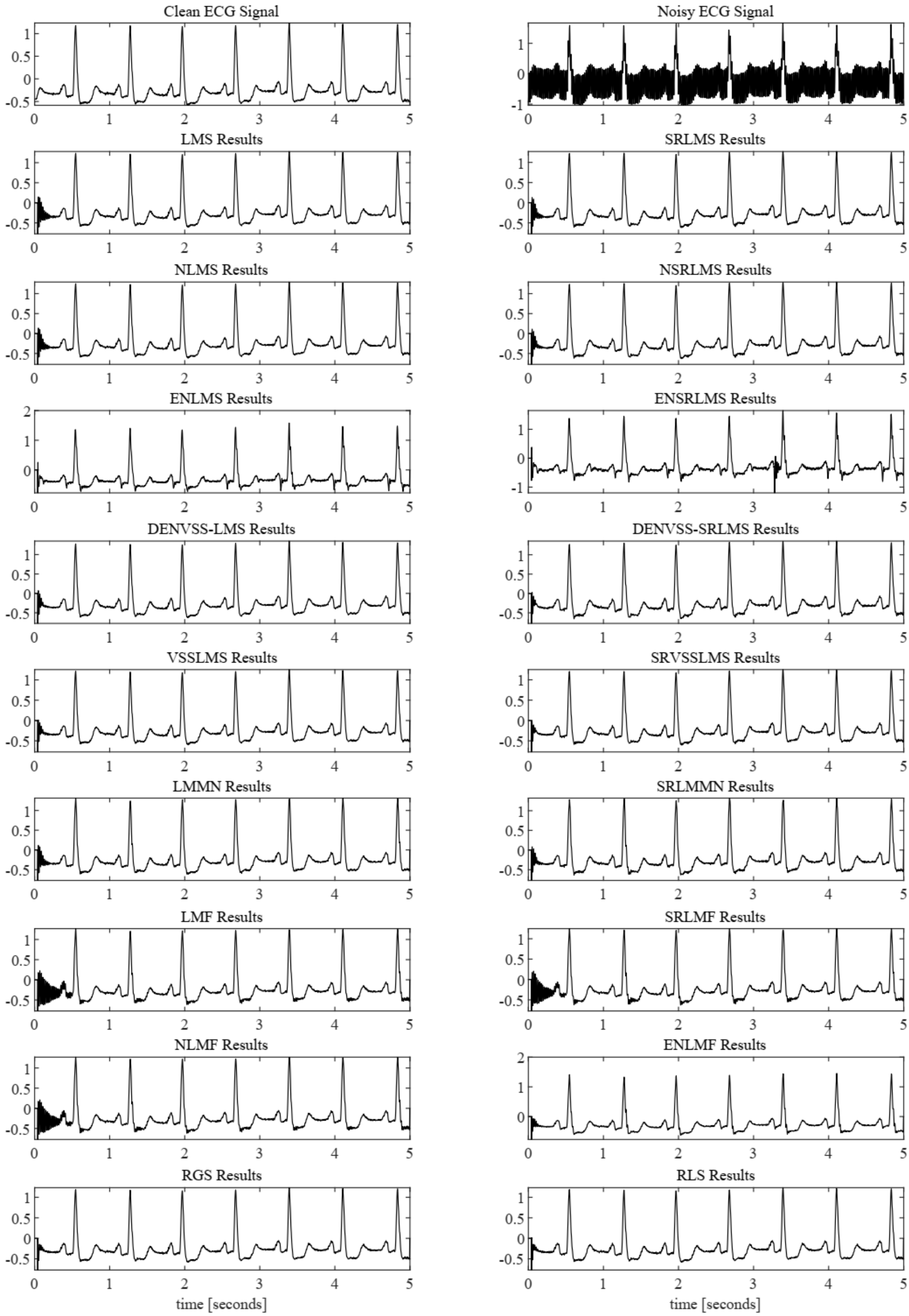

Figure 2. Comparison of the denoised ECG waveforms obtained by cancellation of PLI noise.

Table 3. Performance parameters of the compared algorithms for PLI noise cancellation.

Algorithm:	MSE %	SNR_{out} [dB]	SNR_{imp} $[dB]$	PRD %	ρ
LMS	0.2833	17.7070	16.7749	13.5221	0.98617
SRLMS	0.2851	17.7429	16.8109	13.5659	0.98668
NLMS	0.2866	17.7414	16.8093	13.6020	0.98686
NSRLMS	0.3023	17.5942	16.6621	13.9685	0.98701
ENLMS	0.8963	13.6179	12.6858	24.0531	0.97469
ENSRLMS	1.3033	12.2200	11.2880	29.0043	0.96341
DENVSS-LMS	0.3068	17.6224	16.6904	14.0714	0.98794
DENVSS-SRLMS	0.3569	17.0858	16.1538	15.1776	0.98736
VSSLMS	0.2118	18.7906	17.8585	11.6931	0.98874
SRVSSLMS	0.2116	18.8316	17.8995	11.6857	0.98891
LMMN	0.3360	17.1277	16.1956	14.7262	0.98586
SRLMMN	0.3822	16.6654	15.7333	15.7065	0.98494
LMF	0.4589	15.4589	14.5269	17.2110	0.97648
SRLMF	0.4310	15.7443	14.8123	16.6787	0.97811
NLMF	0.4392	15.6806	14.7485	16.8378	0.97797
ENLMF	0.3533	16.8140	15.8819	15.1010	0.98433
RGS	0.1968	18.9522	18.0202	11.2697	0.98909
RLS	0.1972	18.9422	18.0101	11.2813	0.98907

According to the results in Figure 2 and the obtained performance parameters in Table 3, the RGS and RLS algorithms produced similar results, and also better results than the gradient-based algorithms.

Example 2: In the second computer simulation, the ECG record number 103 taken from the file "103m.mat" in the MIT-BIH arrhythmia database (Moody and Mark, 2005). The BW, MA and EM noise recordings were taken from the “bwm.mat”, “mam.mat” and “emm.mat” files in MIT-BIH Noise Stress Test database (Moody and Mark, 1999). The original BW, MA, and EM noise recordings were used as reference noise signals and also added to the ECG signal by multiplying by 0,5. The PLI noise, which was generated synthetically and used in Example 1, was also used in Example 2. The filter parameters of the compared algorithms in Table 4 were chosen so that acceptable ECG waveforms were obtained. For all algorithms used, the filter lengths were chosen as M=16 and the initial values of the parameter estimates were taken as zero vectors. The compared algorithms for mixed noise cancellation produced the denoised ECG signals shown in Figure 3. The numerical values of the performance parameters calculated according to Table 1 are given in Table 5. According to the results in Figure 3 and Table 5, it was seen that the RGS algorithm has similar performance to the RLS algorithm and also has better results than the gradient-based algorithms.

Table 4. Filter parameters of the compared algorithms for mixed noise cancellation.

Algorithm:	Filter Parameters:	Algorithm :	Filter Parameters:
LMS	$\mu = 0.001$	SRLMS	$\mu = 0.001$
NLMS	$\mu = 0.02$, $\alpha = 0.01$	NSRLMS	$\mu = 0.02$, $\alpha = 0.01$
ENLMS	$\mu = 0.004$, $\alpha = 0.01$	ENSRLMS	$\mu = 0.004$, $\alpha = 0.01$
DENVSS-LMS	$\mu = 0.01$, $\alpha = 0.5$	DENVSS-SRLMS	$\mu = 0.01$, $\alpha = 0.5$
VSSLMS	$\mu = 0.995$	SRVSSLMS	$\mu = 0.995$
LMMN	$\mu = 0.002$, $\lambda = 0.5$	SRLMMN	$\mu = 0.002$, $\lambda = 0.5$
LMF	$\mu = 0.002$	SRLMF	$\mu = 0.002$
NLMF	$\mu = 0.02$, $\alpha = 0.01$	ENLMF	$\mu = 0.005$, $\alpha = 0.01$
RGS	$\lambda = 0.9995$, $\mathbf{R}(0) = \mathbf{I}_{M \times M}$	RLS	$\lambda = 0.9995$, $\mathbf{R}^{-1}(0) = \mathbf{I}_{M \times M}$

Table 5. Performance parameters of the compared algorithms for mixed noise cancellation

Algorithm:	$\boldsymbol{MSE}$ %	$\boldsymbol{SNR_{out}}$ [dB]	$\boldsymbol{SNR_{imp}}$ $[\boldsymbol{dB}]$	$\boldsymbol{PRD}$ %	$\boldsymbol{\rho}$
LMS	4.3137	4.8621	8.2896	53.3195	0.86301
SRLMS	4.1621	5.2259	8.6534	52.3740	0.86248
NLMS	4.3229	4.7463	8.1738	53.3761	0.87476
NSRLMS	3.9969	5.2514	8.6789	51.3239	0.87671
ENLMS	8.1404	2.6929	6.1204	73.2460	0.72416
ENSRLMS	7.5983	3.2397	6.6673	70.7648	0.72421
DENVSS-LMS	4.5900	4.6899	8.1174	55.0007	0.85992
DENVSS-SRLMS	4.1602	5.2557	8.6832	52.3621	0.86534
VSSLMS	5.1643	3.3476	6.7751	58.3400	0.85048
SRVSSLMS	4.7753	3.9207	7.3483	56.0997	0.86029
LMMN	3.7790	5.5026	8.9301	49.9058	0.85197
SRLMMN	3.5584	6.0014	9.4289	48.4270	0.85914
LMF	2.7135	8.0256	11.4531	42.2887	0.87051
SRLMF	2.9604	7.8376	11.2651	44.1705	0.86384
NLMF	2.3427	9.2359	12.6634	39.2934	0.90298
ENLMF	2.3156	8.4775	11.9050	39.0651	0.89199
RGS	1.4187	10.1726	13.6001	30.5776	0.93672
RLS	1.4083	10.1667	13.5942	30.4651	0.93734

In both simulation examples, it was seen that the least squares based RGS and RLS algorithms have the best denoising performance. Because of the least squares based algorithms minimize a cumulative error function, their second-order statistical performance is better than the gradient-based algorithms (Haykin, 2002).

Figure 3. Comparison of the denoised ECG waveforms obtained by cancellation of mixed noise.

4. Conclusions and Recommendations

Recursive Gauss-Seidel (RGS) algorithm is presented in this paper as an alternative least squares-based algorithm for denoising of noisy ECG signals. Because of the RGS algorithm is performed by single-step Gauss-Seidel method during a sampling period, its computational burden is less than the popular RLS algorithm, and convergences faster than the commonly used gradient-based algorithms. According to the computer simulations performed, it was confirmed that the RGS algorithm gave better results than the mostly preferred gradient-based algorithms for both the transient phase and the steady-state phase of the algorithms and produced similar best results with the RLS algorithm. Simulation results shows that the ECG noise removal using the RGS algorithm enables more accurate observation of ECG signals and therefore more accurate analysis of heart functions and accurate diagnosis of heart diseases. Consequently, the RGS algorithm produces more suitable results than the gradient-based algorithms for wireless telemetry and healtcare applications. High noise levels contaminated to the ECG measurements negatively affect the performance of not only the RGS algorithm but also all the algorithms used. To overcome this insufficiency and achieve better denoising performance, using the RGS algorithm in a hybrid manner with the contribution of other denoising techniques such as Wavelet transform is considered as a future work.

Statement of Conflicts of Interest

The author declares that there is no conflict of interest.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

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