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Research Article

Bayesian estimation of inverse weibull distribution scale parameter under the different loss functions

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ABSTRACT

In this paper, the Bayesian estimators for the Inverse Weibull Distribution (IWD) scale parameter are derived when the shape parameter of distribution is known. The Bayesian estimators for the parameter are obtained by using the Gamma prior under the different types of loss functions such as square error loss function (Self), Entropy loss function (Elf), Precautionary loss function (Plf), Linear exponential loss function (Linexlf) and nonlinear exponential loss function (Nlinexlf). A classical maximum likelihood estimator (mle) for the parameter is also derived. To compare the efficiency of the parameter estimation methods, a simulation study is carried out. The comparison is based on mean square error.

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INTRODUCTION

The Weibull distribution is frequently used in reliability engineering, especially for analyzing lifetime data. The probability density function (pdf) of the Weibull is a uni-modal or decreasing function. Also, the Weibull hazard function depends on its shape parameter. Depending on the parameter's value, the hazard function decreases or increases. If the data have a non-monotone hazard function, the Weibull distribution is not considered the appropriate model, for example, lung and breast cancer patients' mortalities [1-3]. In these circumstances, the problem is to find an appropriate distribution for the analysis of such data sets. Kundu and Howlader (2010) remarked that the IWD is an appropriate model for these data sets [4]. The IWD might be considered a suitable model when the study concludes that the pdf of the data can be unimodal [3].

Keller and Kamath (1982) used the IWD to investigate the failures of mechanical components subject to deterioration [5]. Calabria and Pulcini (1994) studied parameter estimations of IWD based on classical and Bayesian methods [6]. In the Bayesian aspect, they used informative priors. Some crucial theoretical properties of the IWD are given in [5]. Kundu and Howlader (2010) considered Bayesian inference for IWD type II censored data in their study [4]. Helu and Samawi (2015) studied progressively the first failure censoring data in their work and used Lindey's methods to derive a Bayesian estimator for IWD parameters [7]. Bi and Gui (2017) studied the stress-strength reliability of IWD [8]. Nasar and Kaser (2017) described frequentist and Bayesian estimation for the parameters of the IWD based

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on an adaptive type-II progressive hybrid censoring scheme [9]. To obtain Bayesian estimation, they used the Lindley approximation. Singh and Tripathi considered the parameter estimation of an IWD when it is known that samples are progressive type-I interval censored [10]. They proposed an EM algorithm to obtain maximum likelihood estimates and mid-point estimates. They obtained Bayes estimates under the square error loss function. Under the entropy loss function, Jana and Bera (2022) developed Bayes estimators of the IWD parameters [11]. They, also investigated the reliability of multi-component stress-strength model using classical and Bayesian approaches.

Suppose that Y is a random variable from the Weibull distribution. The pdf of Y is given as follows:

$$f\,(y;\lambda,\alpha) = \alpha\lambda y^{\alpha-1}e^{-\lambda y^{\alpha}}, y > 0;\ \lambda,\alpha > 0$$

where α is a shape and λ is a scale parameter of the distribution. If we take the transformation of Y with X = 1/Y, then X has the IWD, and the pdf of X is derived as follows:

$$f(x) = \alpha\lambda\left(x^{-(\alpha+1)}\right)\left(e^{-\lambda x^{-\alpha}}\right), x > 0;\ \alpha,\lambda > 0 \quad (1)$$

The cumulative distribution function (cdf) of X is written as

$$F(x) = e^{-\lambda x^{-\alpha}}, x > 0. \quad (2)$$

The expected value and the variance of the IWD are

$$E(X) = \lambda^{\frac{1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) \quad (3)$$

and

$$Var(X) = \lambda^{\frac{2}{\alpha}}\left(\Gamma\left(1-\frac{2}{\alpha}\right) - \left(\Gamma\left(1-\frac{1}{\alpha}\right)\right)^2\right). \quad (4)$$

respectively, where Γ is the Gamma function.

The reliability function of X,

$$R(x) = 1-\left(e^{-\lambda x^{-\alpha}}\right), x \geq 0, \alpha,\lambda > 0 \quad (5)$$

and the hazard function

$$H(x) = \frac{\lambda\left(\alpha^{-\alpha-1}\right)\left(e^{-\lambda x^{-\alpha}}\right)}{\left(1-e^{-\lambda x^{-\alpha}}\right)}, x \geq 0, \alpha,\lambda > 0 \quad (6)$$

is given. IWD has a scale parameter (α) and a shape parameter (λ). The λ is equal to the slope of the regression model, which is obtained via the graphical method. So it's known as a slope. When the slope takes a value between zero and one, failure rates increase. If α = 1, this distribution is called the Inverse Exponential distribution, and if α = 2, then the Inverse Rayleigh distribution. If the λ is a constant and the α increases, the kurtosis of the distribution increases, and parallel to this, the height decreases. If the value of the scale parameter decreases, the height of the curve increases, and this distribution is sharper. Density curves for different values of λ and α are shown in Figure 1 and Figure 2, respectively.

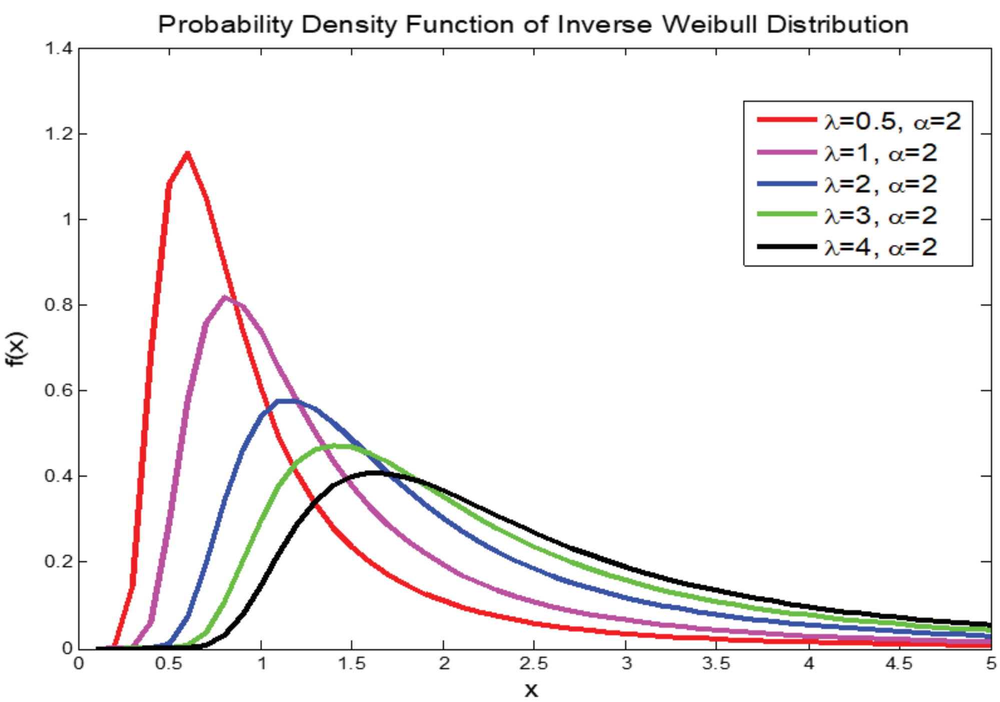

Figure 1. IWD density curves with various values of λ when α = 2.

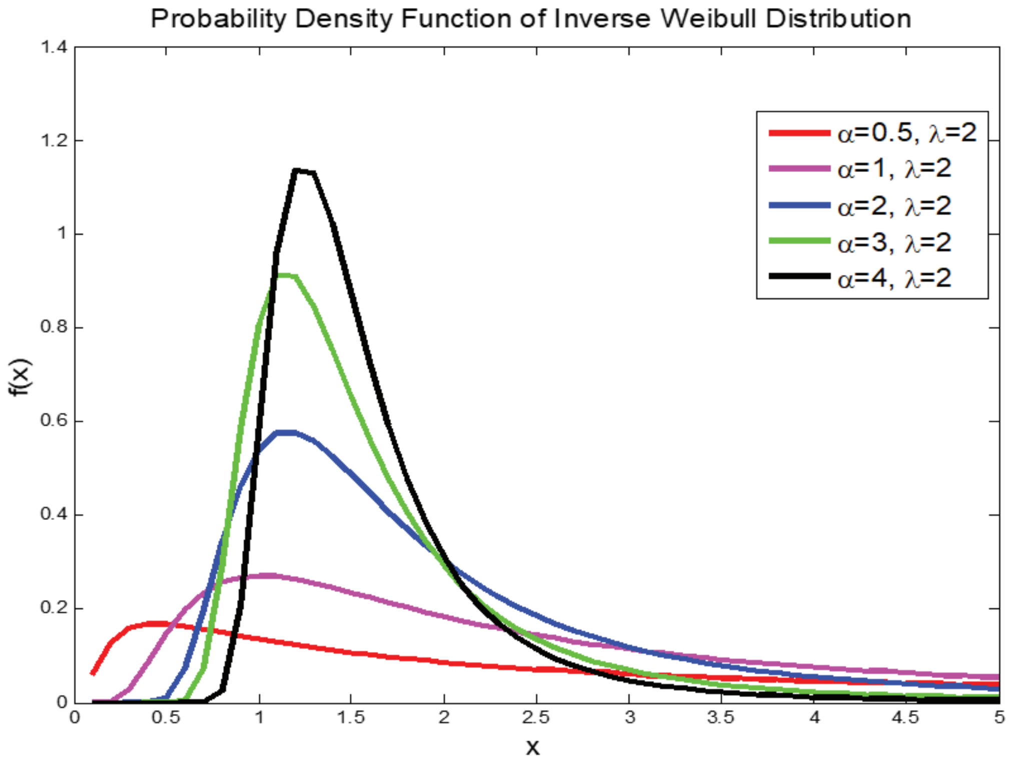

Figure 2. IWD density curves with various values α when $\lambda = 2$.

The IWD might be considered an alternative to the Log-Normal and the Gamma distributions, especially in life testing and reliability engineering. This study deals with the estimation of the scale parameter of the IWD. The estimation is considered in both classical and Bayesian aspects.

The rest of the study is as follows: In Section 2, the mle of λ is derived when the α is known. In Section 3, the Bayesian estimators with the Gamma prior under the Self, Elf, Plf, Linexlf, and Nlinexlf are discussed for the λ in IWD. In Section 4, the mle and the Bayes estimators for λ are compared based on the mean squared error criteria. In Section 5, the main observations of the simulation results are given. Concluding remarks are presented at the end of the paper.

MLE for the Scale Parameter of IWD

Mle is one of the most widely used techniques of estimation in statistics. In this section, for IWD, the mle for λ is derived when α is known. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the IWD when α is known. The likelihood is given as

$$L(\lambda|\alpha, x) = \alpha^n \lambda^n \prod_{i=1}^{n} x_i^{-(\alpha+1)} e^{-\lambda(\sum_{i=1}^{n} x_i^{-\alpha})} \quad (7)$$

then, the log-likelihood function (l) corresponding to (7) can be obtained as

$$l(\lambda|\alpha, x) = n(\log \alpha) + n(\log \lambda) - (\alpha+1)(\textstyle\sum_{i=1}^{n} \log(x_i)) - \lambda(\sum_{i=1}^{n}(x_i^{-\alpha})) \quad (8)$$

By taking the first derivative of l relating to the parameter λ and setting the equation to zero, the following estimating equation can be derived:

$$\frac{dl(\lambda|\alpha, x)}{d\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i^{-\alpha} = 0. \quad (9)$$

Then mle for the λ

$$\hat{\lambda}_{mle} = \frac{n}{\sum_{i=1}^{n} x_i^{-\alpha}} \quad (10)$$

is obtained.

Bayesian Estimator for the Scale Parameter of IWD Under The Different Loss Functions

The selection of appropriate priors and appropriate loss functions are the most important issues in Bayesian inference. The symmetric square loss function is frequently used in Bayesian theory. In this loss function, positive and negative prediction errors are given equal weight. It may not be appropriate to use a symmetric loss function for a different valuation of estimation errors [6], [12], [13], [14]. This paper deals with comparing estimation results for different loss functions in IWD. Parsian and Kinmari (2002) and Misra and van der Meulen (2003) studied asymmetric Linexlf, and they estimated the Normal distribution location parameter θ [15], [16]. Nematollahi and Shariati (2009) estimated the scale parameter of the Gamma distribution under the Elf [17]. Azimi et al. (2012) studied the comparison of Bayesian estimation methods under different loss functions for a progressive censored Rayleigh distribution [18].

Suppose that X_i, $i = 1, \ldots, n$ is an iid random sample from the IWD, and the shape parameter α is known. The likelihood is given as

$$L(\lambda|x,\alpha) = \prod_{i=1}^{n} f\,(x;\ \alpha,\lambda) \tag{11}$$

$$L= \prod_{i=1}^{n}(\alpha\lambda)\left(x_i^{\ -(\alpha+1)}\right) e^{-\lambda x_i^{-\alpha}} = \alpha^n\lambda^n e^{-\lambda T}\prod_{i=1}^{n}\left(x_i^{\ -(\alpha+1)}\right) \tag{12}$$

where $T= \sum_{i=1}^{n}(x_i^{\ -\alpha})$.

In this study, to compute the Bayesian estimate of the IWD scale parameter λ, a Gamma prior and five different loss functions are used. Choosing a prior distribution for the unknown model parameter is essential in Bayesian theory. Being a natural conjugate prior, in this study, the Gamma distribution is considered a prior for the scale parameter λ in IWD with hyperparameters a and b. Then it has the following density function:

$$\pi(\lambda) = \frac{b^a\lambda^{a-1}}{\Gamma(a)}e^{-b\lambda}, \lambda > 0, a, b > 0. \tag{13}$$

For the λ, the posterior distribution is computed as follows:

$$\pi\,(\lambda\,|x) = \frac{L(\lambda|x,\alpha)\pi(\lambda)}{\int_0^{\infty} L(\lambda|x,\alpha)\pi(\lambda)d\lambda} \tag{14}$$

$$\pi(\lambda|x) = \frac{\lambda^{n+a-1}e^{-\lambda(T+b)}}{\int_0^{\infty}\alpha^{n+a-1}e^{-\alpha(T+b)}d\lambda} = \frac{(T+b)^{n+a}}{\Gamma(n+a)}\lambda^{n+a-1}e^{-\lambda(T+b)}. \tag{15}$$

This distribution is appropriate for the Gamma distribution with parameters $(n + a)$ and $(T + b)$.

Bayesian Estimator under the Self

In Bayesian estimation, a frequently used loss function is the symmetric Self. The Self is given as

$$L(\lambda,\hat{\lambda}) = \left(\lambda - \hat{\lambda}\right)^2. \tag{16}$$

The Self gives equal weight to both overestimation and underestimation. If the Self is used as the loss function in Bayesian inference, the estimator is the expected value of the posterior distribution. Then, for the λ, it is given as

$$\hat{\lambda}_{Self} = E(\lambda|x) \tag{17}$$

then the Bayes estimator

$$\hat{\lambda}_{Self} = \frac{(n+a)}{(T+b)} \tag{18}$$

is obtained.

Bayesian Estimator under the Elf

The Elf is a useful asymmetric loss function that was pointed out by [6]. Let $f(x, \lambda)$represent the pdf of the random variable X, and λ is the parameter. If $\hat{\lambda}$ is an estimator for λ, then general Elf is defined by

$$L(\lambda,\hat{\lambda}) = \left(\left(\frac{\hat{\lambda}}{\lambda}\right)^{c_1} - c_1 ln\frac{\hat{\lambda}}{\lambda} - 1\right) \tag{19}$$

[3]. In Elf, if $c_1 > 0$, then an overestimation error is more important than an underestimation error. If $c_1 < 0$, then an underestimation error is more important than an overestimation error. Under the general Elf, the Bayesian estimator of λ is given as

$$\hat{\lambda}_{Gelf}\,(X) = \left(E(\,\lambda^{-c_1}\,|X)\right)^{-c_1} \tag{20}$$

when $E_\lambda(.)$ exists and is finite. If $c_1 = -1$ then the Bayesian estimator under the Elf is the same as under the Self. If in the Elf, c_1 is taken as 1 then for the λ, the Bayesian estimator becomes

$$\hat{\lambda}_{Elf}\,(X) = \left(E\left(\frac{1}{\lambda}\middle|X\right)\right)^{-1}. \tag{21}$$

The expectation is computed as follows:

$$E\left(\frac{1}{\lambda}\middle|X = x\right) = \int_0^{\infty}\frac{1}{\lambda}\frac{(T+b)^{n+a}}{\Gamma(n+a)}\lambda^{n+a-1}e^{-\lambda(T+b)}d\lambda = \frac{T+b}{n+a-1} \tag{22}$$

Then the estimator for the λ under the Elf

$$\hat{\lambda}_{Elf}\,(X = x) = \left(\frac{T+b}{n+a-1}\right)^{-1} = \frac{n+a-1}{T+b} \tag{23}$$

is obtained.

Bayesian Estimator under the Linexlf

The Linexlf for the parameter λ is expressed as the following:

$$L(\lambda,\hat{\lambda}) = (e^{c(\hat{\lambda}-\lambda)} - 1 - c(\hat{\lambda} - \lambda)) \tag{24}$$

where $\hat{\lambda}$ is an estimate of λ and $c \neq 0$ [19]. This loss function is asymmetric. Many authors have discussed the Linex loss function [15],[20], [21], [22]. Under the Linexlf, the Bayesian estimator for λ is obtained following

$$\hat{\lambda}_{Linex} = \frac{-1}{c}\log\left(E\left[e^{-c\lambda}\,|X\right]\right) \tag{25}$$

[23]. For IWD, the $\hat{\lambda}_{Linexlf}$ is

$$E\left(e^{-c\lambda}\middle|x\right) = \int_0^{\infty}\frac{(T+b)^{n+a}}{\Gamma(n+a)}\,e^{-c\lambda}\lambda^{n+a-1}e^{-\lambda(T+b)}d\lambda = \int_0^{\infty}e^{-\lambda(c+T+b)}\frac{(T+b)^{n+a}}{\Gamma(n+a)}\lambda^{n+a-1}\,d\lambda \tag{26}$$

$$E\left(e^{-c\lambda}\middle|x\right)=\frac{(T+b)^{n+a}}{\Gamma(n+a)}\int_0^{\infty}e^{-t}\left(\frac{t}{c+T+b}\right)^{n+a-1}d\lambda$$
$$=\left(\frac{(T+b)^{n+a}}{\Gamma(n+a)}\right)\left(\frac{1}{(c+T+b)^{n+a-1}}\right)\int_0^{\infty}(e^{-t})(t^{n+a-1})d\lambda \quad (27)$$

$$\int_0^{\infty}(e^{-t})(t^{n+a-1})d\lambda=\Gamma(n+a)$$
$$E\left(e^{-c\lambda}\middle|x\right)=\frac{(T+b)^{n+a}}{(c+T+b)^{n+a-1}} \quad (28)$$

$$\hat{\lambda}_{Linexlf}=-\frac{1}{c}\log\left(E\left(e^{-c\lambda}\middle|x\right)\right)$$
$$\hat{\lambda}_{Linexlf}=-\frac{1}{c}\log\left(\frac{(T+b)^{n+a}}{(c+T+b)^{n+a-1}}\right) \quad (29)$$

computed.

Bayesian Estimator under the Plf

The Plf is introduced in [14] as follows

$$L(\lambda,\hat{\lambda})=\frac{(\hat{\lambda}-\lambda)^2}{\hat{\lambda}}. \quad (30)$$

This function is asymmetric. By solving the equation that follows, the Bayesian estimator under the Plf is obtained as

$$\hat{\lambda}^2_{plf}=E(\lambda^2|X). \quad (31)$$

Then

$$E(\lambda^2|x)=\int_0^{\infty}\lambda^2\frac{(T+b)^{n+a}}{\Gamma(n+a)}\lambda^{n+a-1}e^{-\lambda(T+b)}d\lambda$$
$$=\int_0^{\infty}\lambda^{n+a+1}\frac{(T+b)^{n+a}}{\Gamma(n+a)}e^{-\lambda(T+b)}d\lambda \quad (32)$$

$$E(\lambda^2|x)=\frac{\Gamma(n+a+2)}{\Gamma(n+a)(T+b)^2} \quad (33)$$

is computed. Finally, under the Plf, the Bayesian estimator of λ can be obtained as

$$\hat{\lambda}_{plf}=\frac{\sqrt{(n+a+1)(n+a)}}{(T+b)}. \quad (34)$$

Bayesian Estimator under the Nlinexlf

The Nlinex loss function is given as

$$L_{Nlinexlf}(\hat{\lambda},\lambda)=k\left[e^{cD}-cD^2-cD-1\right],k>0,c>0. \quad (35)$$

In Equation (35) D= $\hat{\lambda}$ -λ represents estimation error. For lack of generality, one can assume k = 1. The estimator under Nlinexlf $\hat{\lambda}_{Nlinexlf}$ is obtained as follows:

$$\hat{\lambda}_{Nlinexlf}=-\frac{[\ln E\left(e^{-c\lambda}\right)-2E(\lambda)]}{(c+2)} \quad (36)$$

[24]. Then at first,

$$E\left(e^{-c\lambda}\right)=\int_0^{\infty}e^{-c\lambda}\,\pi(\lambda\,|x)d\lambda \quad (37)$$

is computed. From (15), it is known that the posterior distribution is the Gamma, then

$$E((\lambda|x))=\frac{(n+a)}{(T+b)} \quad (38)$$

and

$$E\left(e^{-c\lambda}\middle|x\right)=\frac{(T+b)^{n+a}}{(c+T+b)^{n+a-1}}. \quad (39)$$

Then, the Bayes estimator under Nlinexlf is as follows:

$$\hat{\lambda}_{Nlinexlf}=-\frac{[\ln\left(\frac{(T+b)^{n+a}}{(c+T+b)^{n+a-1}}\right)-2(n+a)(T+b)}{(c+2)} \quad (40)$$

SIMULATION STUDY

In a simulation study, different random samples from the IWD with the sample sizes n=10,30,50,70,90,110 are generated in Matlab. A simulation study is carried out 1000 times for the parameter values α = 2 and λ=2,3,4 where the prior hyperparameters are chosen as a = 2, b = 2. This is iterated 1000 times, and the λ is estimated using each of the methods given in the previous sections. The mean squared error (MSE) is used as a criterion to compare the efficiency of the methods. The MSE is calculated as follows:

$$MSE=\frac{\sum_{i=1}^{1000}\left(\boldsymbol{\lambda}_i-\hat{\boldsymbol{\lambda}}_i\right)^2}{\mathbf{1000}}$$

The results are given in Table 1.

RESULTS AND DISCUSSION

The main observations of the results from Table 1 are summarized below:

1) From Table 1, it can be said that the Bayes estimate using the Gamma prior under the Plf provides the smallest Mse values in most cases as compared to the other loss functions and the classical mle. Especially for small sample sizes, Bayes estimates give better results than classical mle, except for the linexlf and the Nonlinexlf.
2) The Linexlf and the Nlinexlf give the worst conclusions in all cases.
3) The Bayesian estimates under linexlf and nonlinexlf are sensitive to the values of the corresponding shape parameter c.

Table 1. Estimation results for the scale parameter λ

			λ=2		λ=3		λ=4	
		α = 2	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
Mle		***n* = 10**	2.20644	0.62068	3.32654	1.53461	4.45114	2.80048
Self			1.57090	0.30698	2.04200	1.07607	2.40437	2.71816
Elf			1.42129	0.43547	1.84753	1.45778	2.17539	3.47012
Plf			1.64401	0.26129	2.13703	0.91810	2.51626	2.39000
Linexlf	***c* = -10**		1.13154	1.58873	0.30863	7.43790	0.04522	16.44679
	***c* = 5**		0.66794	1.82347	0.95072	4.24738	1.14415	8.19917
	***c* = 10**		0.66837	1.79457	0.85270	4.63050	0.97563	9.16387
Nlinexlf	***c* = -10**		1.02170	2.43911	0.12471	10.18573	0.65762	21.91668
	***c* = 5**		0.92593	1.22029	1.26252	3.09159	1.50422	6.30024
	***c* = 10**		0.81880	1.427641	1.050918	3.832341	1.213758	7.794792
Mle		***n* = 30**	2.07868	0.15796	3.09597	0.35062	4.13027	0.61825
Self			1.84925	0.11318	2.59618	0.32256	3.27339	0.75132
Elf			1.78862	0.12930	2.51106	0.38826	3.16607	0.90439
Plf			1.87932	0.10799	2.63840	0.29548	3.32662	0.68412
Linexlf	***c* = -10**		3.16380	2.46902	6.31791	18.95462	7.49366	23.26038
	***c* = 5**		0.99492	1.07413	1.59093	2.07454	8.54872	38.44228
	***c* = 10**		1.11109	0.82798	1.56196	2.11680	1.91857	4.38625
Nlinexlf	***c* = -10**		3.49244	3.79986	7.24834	30.01407	2.07873	3.79572
	***c* = 5**		1.23902	0.65013	1.87814	1.36562	2.42006	2.63001
	***c* = 10**		1.23412	0.63158	1.73433	1.66474	2.14437	3.51741
Mle		***n* = 50**	2.04451	0.08798	3.06739	0.20508	4.08864	0.36833
Self			1.90612	0.07218	2.75382	0.18680	3.53995	0.40785
Elf			1.86838	0.078214	2.699294	0.211681	3.469854	0.469568
Plf			1.92490	0.07026	2.78095	0.17668	3.57482	0.38086
Linexlf	***c* = -10**		2.69097	0.63340	4.26068	2.30543	6.55848	11.14111
	***c* = 5**		1.05226	0.95238	1.79978	1.52877	2.43634	2.56340
	***c* = 10**		1.25340	0.59388	1.85760	1.36063	2.35641	2.77175
Nlinexlf	***c* = -10**		2.88718	0.97274	4.63739	3.62416	7.31312	17.64494
	***c* = 5**		1.29622	0.55202	2.07236	0.95889	2.75166	1.69695
	***c* = 10**		1.36218	0.44724	2.00697	1.05143	2.55366	2.17880
Mle		***n* = 70**	2.02797	0.05776	3.04745	0.14047	4.04641	0.24060
Self			1.92921	0.05087	2.82030	0.13190	3.64807	0.27840
Elf			1.90184	0.05420	2.78030	0.14507	3.59633	0.31314
Plf			1.94284	0.04978	0.12654	0.12654	3.67385	0.26311
Linexlf	***c* = -10**		2.58868	0.42674	3.89315	1.06752	5.41010	2.67484
	***c* = 5**		1.06035	0.92584	1.88711	1.31728	2.60159	2.06323
	***c* = 10**		1.31867	0.49517	2.01267	1.02877	2.59702	2.03842
Nlinexlf	***c* = -10**		2.75354	0.65812	4.16136	1.67429	5.85061	4.30444
	***c* = 5**		1.30860	0.52177	2.15374	0.80062	2.90059	1.32891
	***c* = 10**		1.42043	0.36915	2.14727	0.78773	2.77220	1.58935
Mle		***n* = 90**	2.02507	0.04677	3.02781	0.10255	4.04501	0.18797
Self			1.94772	0.04171	2.85082	0.10119	3.72882	0.20581
Elf			1.92620	0.04357	2.81932	0.10984	3.68762	0.22695
Plf			1.95846	0.04114	2.86653	0.09762	3.74936	0.19656
Linexlf	***c* = -10**		2.55689	0.36793	3.74179	0.71043	5.09393	1.57450
	***c* = 5**		1.05983	0.92237	1.92404	1.22483	2.71174	1.75989
	***c* = 10**		1.35875	0.44017	2.10215	0.85470	2.76497	1.59458
Nlinexlf	***c* = -10**		2.70919	0.56603	3.96453	1.11530	5.43521	2.51891
	***c* = 5**		1.31351	0.50986	2.18883	0.72840	3.0023	1.10430
	***c* = 10**		1.45691	0.32548	2.22693	0.65075	2.92561	21.23268
Mle		***n* = 110**	2.01365	0.037582	3.02873	0.08849	4.04084	0.15622
Self			1.95074	0.03501	2.88234	0.08498	3.77911	0.16572
Elf			1.93309	0.03647	0.09052	0.09052	3.74491	0.17989
Plf			1.95955	0.03452	2.89535	0.08273	3.79617	0.15953
Linexlf	***c* = -10**		2.53318	0.32799	3.68057	0.58632	4.93338	1.13273
	***c* = 5**		1.04366	0.94787	1.95337	1.15882	2.77927	1.58430
	***c* = 10**		1.37505	0.41641	2.17095	0.73494	2.88021	1.32190
Nlinexlf	***c* = -10**		2.67878	0.50751	3.88013	0.91301	5.22194	1.79988
	***c* = 5**		1.30283	0.51913	2.21879	0.67584	3.06494	0.97472
	***c* = 10**		1.47100	0.30676	2.28951	0.55600	3.03003	1.01606

4) The results show that estimators of the different methods are closer to each other as the sample size increases, except for Linexlf and Nlinexlf.
5) Moreover, it is seen that when the sample size increases, the MSE decreases significantly.

CONCLUSION

This study deals with the investigation of the mle and the Bayesian estimator for the scale parameter of IWD when the shape parameter is known. For Bayesian inference, five different loss functions are used respectively: Self, Plf, Elf, Linexlf, and Nlinexlf. In the simulation study, the Bayesian estimates and the mle are computed. To compare the results of the estimations, the MSE values are calculated. The results show that the Bayesian method of estimation for the Gamma prior under the Self, Plf, and Elf gives better results than the mle method. Also, the Bayesian estimators that are obtained under the Plf have the smallest MSE as compared with the Bayesian estimators that are obtained under the other loss functions. Also, Bayesian estimators, which are obtained under Linexlf and Nlinexlf, are worse than the other loss functions and the mle.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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