ÖHÜ Müh. Bilim. Derg. / OHU J. Eng. Sci.

ISSN: 2564-6605
doi: 10.28948/ngumuh.387230
Ömer Halisdemir Üniversitesi Mühendislik Bilimleri Dergisi, Cilt 7, Sayı 1, (2018), 390-398
Omer Halisdemir University Journal of Engineering Sciences, Volume 7, Issue 1, (2018), 390-398

Araştırma / Research

BENDING STRESS ANALYSIS OF AXIALLY LAYERED FUNCTIONALLY GRADED BEAMS

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***Geliş / Received:** 15.09.2017*
***Kabul / Accepted:** 20.11.2017*

ABSTRACT

In this study, the bending stress analysis of axially layered functionally graded beams subjected to their own weight were evaluated under clamped-free (C-F) boundary condition. The beams have four layers and each layer consist of different aluminium (Al)/monotungsten carbide (WC) systems based on increasing of the 6% WC. The layer positions in the beams were performed based on Taguchi L16 (4*4) orthogonal array design. The layers were considered as control factors and each layer has four levels. The analysis of signal-to-noise (S/N) ratios were used to obtain the optimum layer levels. Analyses were performed using finite element software ANSYS. In addition, analysis of variance (ANOVA) was performed to determine the important levels and percent contributions of the layers on the responses. The numerical results show that the increasing of the layer levels increases the bending stress and percent contributions of Layer 1, Layer 2, Layer 3 and Layer 4 on the bending stress were obtained as 1.12%, 11.83%, 29.54% and 57.48%, respectively.

Keywords: Functionally graded materials, bending stress, beam, finite element method

EKSENEL TABAKALI FONKSİYONEL DERECELENDİRİLMİŞ KİRİŞLERİN EĞİLME GERİLME ANALİZİ

ÖZ

Bu çalışmada, ankastre-serbest (C-F) sınır şartı altında kendi ağırlığına maruz kalmış eksenel fonksiyonel derecelendirilmiş tabakalı kirişlerin eğilme gerilme analizleri değerlendirilmiştir. Kirişler dört tabakaya sahip ve her tabaka %6 monotungsten karbür (WC) artışına bağlı olarak farklı alüminyum (Al)/WC sistemlerinden oluşmaktadır. Kirişlerdeki tabaka pozisyonları Taguchi L16 (4*4) ortogonal dizi tasarımına bağlı gerçekleştirilmiştir. Tabakalar kontrol faktörü olarak düşünüldü ve her tabaka dört seviyeye sahiptir. Sinyal gürültü oranları analizi optimum tabaka seviyelerini elde etmek için kullanıldı. Analizler ANSYS sonlu elemanlar programı kullanılarak gerçekleştirildi. Ayrıca varyans analizi sonuçlar üzerinde tabakaların önem seviyeleri ve yüzde katkıları karar vermek için gerçekleştirildi. Sayısal sonuçlar tabaka seviyelerindeki artışın eğilme gerilmesini artırdığını göstermektedir. Eğilme gerilmesi üzerinde Tabaka 1, Tabaka 2, Tabaka 3 ve Tabaka 4'ün katkıları sırasıyla %1,12, %11,83, %29,54 ve %57,48 olarak elde edildi.

Anahtar Kelimeler: Fonksiyonel derecelendirilmiş malzemeler, eğilme gerilmesi, kiriş, sonlu elemanlar yöntemi

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1. INTRODUCTION

Scientists who are interested in material science introduced Functionally Graded Materials (FGMs) in the Sendai area, Japan, 1984 [1]. This concept (FGMs) was actually improved for application in aerospace field to continue height thermal barrier coating [2]. After that, the concept was used many different areas such as space solar power systems [3], engine piston rings [4], sensor and energy applications [5], thermal, wear and corrosion barriers [6], dental implant [7] etc. In the following years, in the literature, many studies consisting of static and dynamic analyses of beams made of FGMs have been published. One of these studies is the bending stress analyses. Li et al. [8] presented a study based on bending, buckling and vibration analyses of beams consisting of FGM in the axial direction by using nonlocal strain gradient theory. Thai and Vo [9] reported a study related to the bending and free vibration behaviours of the beams which having functionally graded materials and they also used the different higher-order shear deformation beam theories. Aldousari [10] published a study about the bending behaviour of the various material distribution of the beam prepared using functionally graded materials. Li et al. [11] presented a study with bending analyses of Timoshenko beams consisting of FGM according to different boundary conditions. Kang and Li [12] evaluated the bending behaviour of the FG cantilever beam based on power-law non-linearity subjected to an end force. Sallai et al. [13] reported a study consisting of the analytical bending analysis of the beam formed using functionally graded materials. In addition, literature survey shows that there are many studies with the static analysis of beams with FGM. Kadoli et al. [14] carried out the static behaviour of the beams made of FGM and they used higher order shear deformation theory. Vo et al. [15] performed a work about the static analysis of sandwich beams consisting of FGM according to a quasi-3D theory. Li et al. [16] performed a study consisting of static and dynamic behaviours of the beam formed using functionally graded materials and they used a higher-order theory. As stated in the literature, there are a lot of static and dynamic studies consisting of beams made of FGM. In this study, the bending stress analysis of the axially FG beams with four layers consisting of different percent volume fractions of Al and WC materials was investigated using finite element software ANSYS according to Taguchi L16 orthogonal array design with four control factors and four levels.

2. MATERIAL AND METHODS

2.1. Material

In the analyses, different % volume fractions of aluminium (Al) and monotungsten carbide (WC) materials were used. The mechanical properties of the Al and WC materials was given in Table 1. Table 1 shows that Poisson's ratio values of both materials were equal and it was taken as 0.3.

Table 1. Mechanical properties [17]

Material Properties	**Symbol**	**Materials**	
		Al	**WC**
Young's modulus (GPa)	E	70	696
Poisson's ratio	υ	0.3	0.3
Density (kg/m^3)	ρ	2707	15,600

The layers were considered using different Al/WC systems based on increasing of 6% WC and so each layer has various mechanical properties. The effective material properties (P_{Ef}) of the layers of the axially layered FG beams, such as Young's modulus (E_{ef}) and density (ρ_{ef}), were calculated using a simple rule of mixture of composite materials as given Equation 1 [18],

$$P_{Ef} = \sum_{j=1} P_j V_{f_j} \tag{1}$$

where, mechanical properties and volume fraction of the constituent material j were expressed as P_j and V_{fj} respectively. In addition, the sum of the volume fractions of all the constituent materials was calculated as one according to Equation 2 [18].

$$\sum_{j=1} V_{f_j} = 1 \tag{2}$$

The effective mechanical properties of the Al/WC systems in the layers were calculated using Equation 1 and 2 and the results calculated were tabled in Table 2.

2.2. Methods

Taguchi L16 orthogonal array design was used to determine the layer positions of the axially layered FG beams. The array has 16 runs and four control factor and so sixteen analyses were carried out. The mechanical properties of the Al/WC systems in layers were used as control factor and each control factor has four levels. The control factors and theirs levels were given in Table 2.

Table 2. Control factors and their levels

Levels	**Mechanical Properties**	**Control Factors**			
		Layer 1	**Layer 2**	**Layer 3**	**Layer 4**
Level 1	E (GPa)	107.56	257.80	408.04	558.28
	ρ (kg/m^3)	3480.58	6574.90	9669.22	12763.54
Level 2	E (GPa)	145.12	295.36	445.60	595.84
	ρ (kg/m^3)	4254.16	7348.48	10442.8	13537.12
Level 3	E (GPa)	182.68	332.92	483.16	633.40
	ρ (kg/m^3)	5027.74	8122.06	11216.38	14310.70
Level 4	E (GPa)	220.24	370.48	520.72	670.96
	ρ (kg/m^3)	5801.32	8895.64	11989.96	15084.28

In order to obtain optimum layers of the axially layered FG beams for maximum bending stress, the numerical results were converted to S/N ratios. The S/N ratio values were calculated according to “larger is better” characteristic $(S/N)_{HB}$ as shown in Equation 3 [20]. Analysis of S/N ratio was performed using Minitab R15 software according to “larger is better” quality characteristic.

$$(S/N)_{HB} = -10.log\left(n^{-1}\sum_{i=1}^{n}(y_i^2)^{-1}\right) \tag{3}$$

in which, n refers to number of analysis in a trial and y_i is evaluated i_{th} data.

2.2.1. Numerical Analysis

Numerical analyses for bending stress were carried out using finite element software named ANSYS (Mechanical APDL) and the beams were modeled according to BEAM189 element type which is suitable for analyzing slender to moderately stubby/thick beam structures. In addition, this element is a quadratic three-node beam element in three dimensions. The software help menu can be used for BEAM189 element description. Gravitational acceleration value in the vertical direction was taken as 9.81 m/s^2. The bending stress results were observed based on von Mises stress. Numerical bending stress analyses were performed according to the mechanical properties, positions of the layers and boundary condition of the axially layered FG beams. The axially layered FG beams consist of four layers. Each layer was equal to each other and assumed as 70 mm cross section height, 100 mm cross section width and 450 mm length. So the length of axially layered FG beams was considered as 1800 mm totally. Clamped-Free (C-F) boundary condition was selected for the analyses. Clamped end of the beam was modelled as fixed for UX, UY, UZ, ROTX, ROTZ and ROTZ. Mesh of a total 80 elements under NDIV (No. of element divisions) was used for each layer according to lines and global as element attributes. The axially layered FG beams with C-F boundary condition was shown in Figure 1.

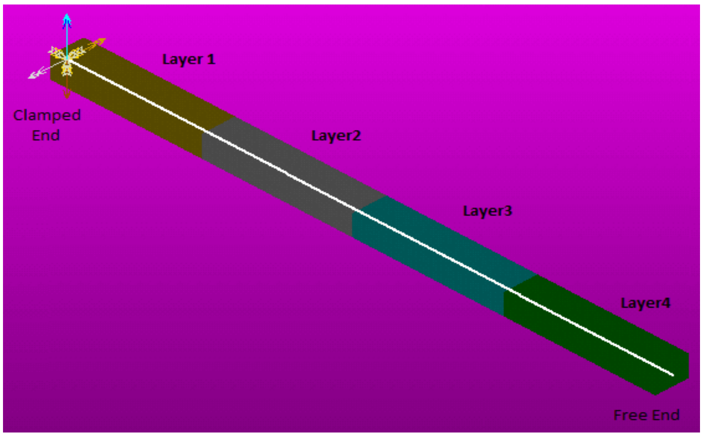


Figure 1. Axially layered FG beam with C-F boundary condition

3. RESULTS AND DISCUSSION

The aim of this study is to evaluate the maximum bending stress of the axially layered FG beams, subjected to their own weight, with four layers consisting of various Al/WC systems based on the increasing of 6% WC and decreasing of the 6% Al and for C-F boundary condition using finite element software ANSYS. The layer arrangements of the axially layered FG beams were conducted based on L16 Taguchi orthogonal array design and S/N ratios of numerical results obtained according to von Mises stresses were calculated using Equation 3. The numerical results and their S/N ratios were shown in Table 3.

Table 3. Numerical results and their S/N ratios

Test	**Control Factors**				**Results**	**S/N Ratios**
	Layer 1	**Layer 2**	**Layer 3**	**Layer 4**	**σ (MPa)**	**η (dB)**
1	6%WC/94%Al	30%WC/70%Al	54%WC/46%Al	78%WC/22%Al	13.7	22.7344
2	6%WC/94%Al	36%WC/64%Al	60%WC/40%Al	84%WC/16%Al	14.7	23.3463
3	6%WC/94%Al	42%WC/58%Al	66%WC/34%Al	90%WC/10%Al	15.7	23.9180
4	6%WC/94%Al	48%WC/52%Al	72%WC/28%Al	96%WC/4%Al	16.7	24.4543
5	12%WC/88%Al	30%WC/70%Al	60%WC/40%Al	90%WC/10%Al	15.0	23.5218
6	12%WC/88%Al	36%WC/64%Al	54%WC/46%Al	96%WC/4%Al	15.3	23.6938
7	12%WC/88%Al	42%WC/58%Al	72%WC/28%Al	78%WC/22%Al	15.1	23.5795
8	12%WC/88%Al	48%WC/52%Al	66%WC/34%Al	84%WC/16%Al	15.5	23.8066
9	18%WC/82%Al	30%WC/70%Al	66%WC/34%Al	96%WC/4%Al	15.9	24.0279
10	18%WC/82%Al	36%WC/64%Al	72%WC/28%Al	90%WC/10%Al	15.9	24.0279
11	18%WC/82%Al	42%WC/58%Al	54%WC/46%Al	84%WC/16%Al	14.7	23.3463
12	18%WC/82%Al	48%WC/52%Al	60%WC/40%Al	78%WC/22%Al	14.8	23.4052
13	24%WC/76%Al	30%WC/70%Al	72%WC/28%Al	84%WC/16%Al	15.3	23.6938
14	24%WC/76%Al	36%WC/64%Al	66%WC/34%Al	78%WC/22%Al	14.8	23.4052
15	24%WC/76%Al	42%WC/58%Al	60%WC/40%Al	96%WC/4%Al	16.0	24.0824
16	24%WC/76%Al	48%WC/52%Al	54%WC/46%Al	90%WC/10%Al	15.4	23.7504
Average Value					15.28	

3.1. Analysis of Effects of Al/WC Systems

Main effects plot was created by using S/N ratios of the average numerical values for all the layers at level 1, level 2, level 3 and level 4 according to “larger is better” characteristic in order to see the influences of the

Al/WC systems in layers on the bending stress. Figure 2 shows that each layer has four levels and different percent volume fractions based on the increasing of 6% WC. According to Figure 2, all the layers have positive effect on the bending stress. In order word, the increment of %WC and the decreasing of the %Al in layers increase the bending stress.

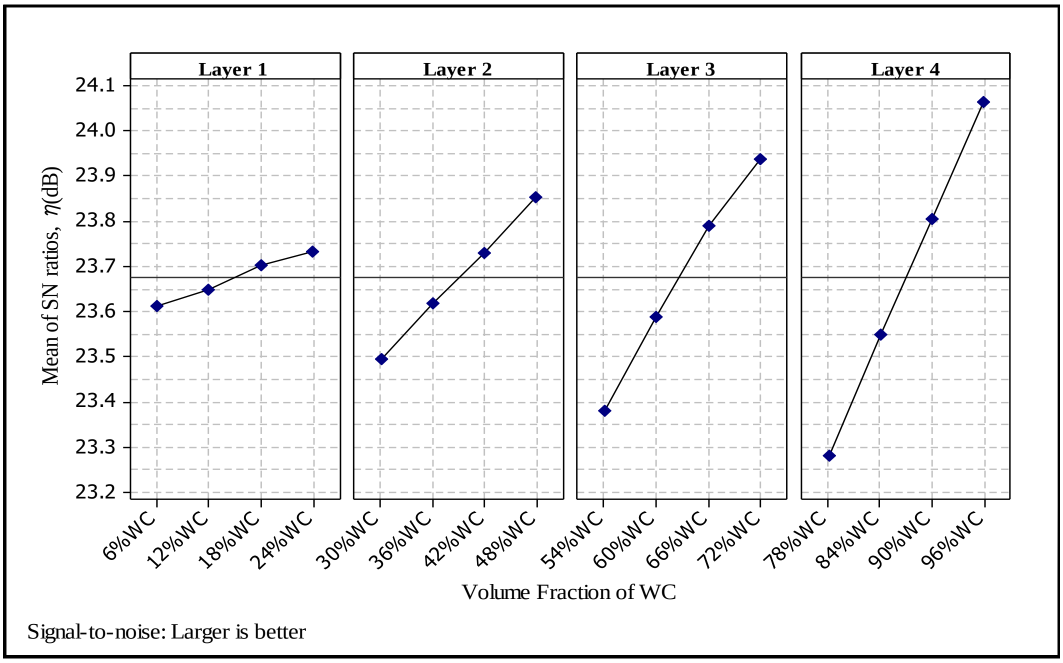


Figure 2. Main effects plot of S/N ratios for maximum bending stress

3.2. Analysis of Optimum Layers and Their Levels

Average numerical result and S/N ratio of each level of each layer were calculated to obtain optimum layer levels. These results were given in Table 4. According to this table, the optimum levels for Layer 1, Layer 2, Layer 3 and Layer 4 were determined as level 4. Optimum layers and their levels were marked by (*).

Table 4. Response results of S/N ratios and means

Level	S/N ratios				Means			
	Layer 1	Layer 2	Layer 3	Layer 4	Layer 1	Layer 2	Layer 3	Layer 4
1	23.61	23.49	23.38	23.28	15.20	14.98	14.78	14.60
2	23.65	23.62	23.59	23.55	15.23	15.18	15.13	15.05
3	23.70	23.73	23.79	23.80	15.32	15.38	15.48	15.50
4	23.73*	23.85*	23.94*	24.06*	15.38*	15.60*	15.75*	15.98*
Delta	0.12	0.36	0.56	0.78	0.18	0.63	0.97	1.38
Rank	4	3	2	1	4	3	2	1

3.3. Analysis of Variance

Analysis of Variance (ANOVA) was investigated to obtain the important levels and percent contribution of the layers on the maximum bending stress and ANOVA results obtained for R-Sq = 99.97% and R-Sq(adj) = 99.87% were given in Table 5 based on raw data. It can be seen from Table 5 that all the layers have significant effects on the bending stress for p values <0.05. However, Layer 1 has less effect than other layers such as Layer 2, Layer 3 and Layer 4 since p value of the Layer 1 is 0.006. In addition, the percent contributions of the Layer 1, Layer 2, Layer 3 and Layer 4 on the maximum bending stress are 1.12 %, 11.83 %, 29.54 % and 57.48 %, respectively. So the maximum percent contribution was obtained for Layer 4 and the minimum percent contribution was found for Layer 1.

Table 5. ANOVA results for bending stress

Source	DF	Seq SS	Adj MS	F	P	% Contribution
Layer 1	3	0.08188	0.02729	43.67	0.006	1.12
Layer 2	3	0.86187	0.28729	459.67	0	11.83
Layer 3	3	2.15187	0.71729	1147.67		29.54
Layer 4	3	4.18688	1.39563	2233.00		57.48
Error	3	0.00187	0.00062			0.03
Total	15	7.28438				

3.4. Analysis of Estimated Optimum Bending Stress

Optimum levels of the significant control factors were used to estimation the optimum bending stress result for maximum value. Optimum levels of Al/WC systems for all the layers were determined as fourth levels and so estimated mean of the bending stress was calculated by Equation 4 [19].

$$\mu_\sigma = \bar{T}_\sigma + (\overline{Layer1}_4 - \bar{T}_\sigma) + (\overline{Layer2}_4 - \bar{T}_\sigma) + (\overline{Layer3}_4 - \bar{T}_\sigma) + (\overline{Layer4}_4 - \bar{T}_\sigma) \tag{4}$$

where, average results of the bending stress at fourth levels of Layer 1, Layer 2, Layer 3 and Layer 4 are 15.38, 15.60, 15.75 and 15.98 respectively. These values were taken from Table 4. In addition, $\bar{T}_\sigma$ = 15.28 refers to average result of the bending stress values based on Taguchi L16 orthogonal array design and was taken from Table 3. The 95 % confidence interval of confirmation bending stress ($CI_{c\sigma}$) was carried out based on Equation 5 [20].

$$CI_{c\sigma} = \left(F_{\alpha;1;n_2} V_{error} \left[\frac{1}{n_{eff}} + \frac{1}{R}\right]\right)^{1/2} \tag{5}$$

where, R is repetition number of verification analyses and was taken as 1. V_{error} is error value of the variance and was taken as 0.00062 from Table 5. n_2 is error result of DF in Table 5 and was taken as 3. Therefore $F_{\alpha;1;n_2}$ was taken as $F_{0.05;1;3}$ and was taken as 10.128 based on listed F values [19]. n_{eff} is known as the effective number of repetitions and was calculated based on Equation 6 [20],

$$n_{eff} = N/[1+T_{DF}] \tag{6}$$

where, N refers to the total number of runs and was taken as 16 from Table 3. Total value of the DF for significant layers is 12 and was taken from Table 5. Therefore n_{eff} is calculated as 1.2307 and so $CI_{c\sigma} = \pm 0.1067$. Predicted optimum bending stress for 95% confidence interval was calculated using Equation 7 [20].

$$\text{Mean } \mu_\sigma - CI_{c\sigma} < \mu_\sigma < CI_{c\sigma} + \text{Mean } \mu_\sigma \tag{7}$$

Comparisons of the results obtained from optimum levels for 95% confidence interval were listed in Table 6. In addition, S/N ratio results for 16.7633 and 16.9767 values were calculated using Equation 3 and so 24.4872 and 24.5971 values were found respectively.

Table 6. Predicted and ANSYS results

Symbol	Unit	ANSYS Result	Predicted Result	% Difference	Predicted Confidence Intervals at 95% Confidence Level
σ	MPa	16.90	16.87	0.18	$16.7633 < \mu_\sigma < 16.9767$
η	dB	24.5577	24.5423	0.06	$24.4872 < \mu_\eta < 24.5971$

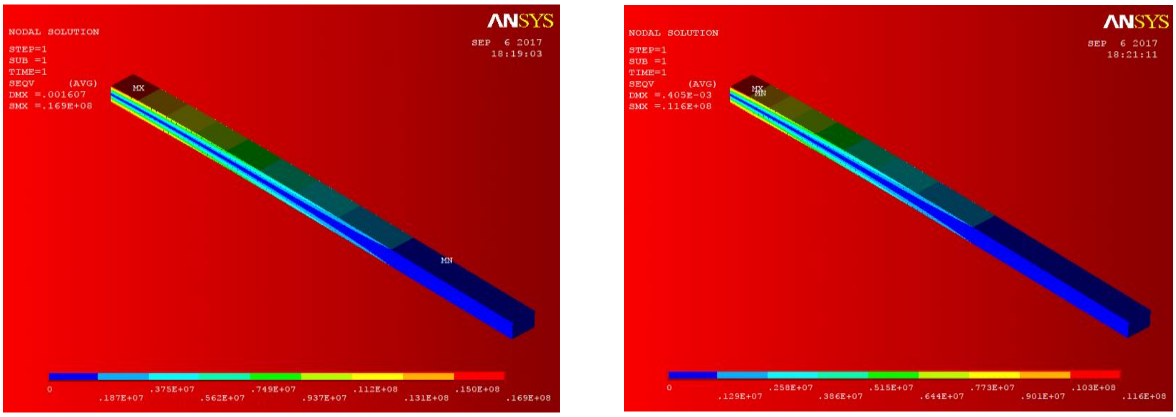
3.5. Influences of Positions of AL/WC Systems

The optimum layers were arranged in opposite direction based on percent contributions to investigate the effects of the positions of optimum layers on the bending stress of the axially layered FG beams. Results obtained were given in Table 7.

Table 7. Bending stress of the beam types

Beam Types	Bending Stress (MPa)
Layer1-Layer2-Layer3-Layer4	16.9
Layer4-Layer3-Layer2-Layer1	11.6

It can be seen from Table 7 that bending stress value of beam with Layer 4-Layer 3-Layer 2-Layer 1 is smaller than beam with Layer 1-Layer 2-Layer 3-Layer 4. Therefore layer positions of the axially layered FG beams with same mechanical properties on the bending stress play important role under own weight of the beam.



(a) (b)

Figure 3. Maximum bending stress results as a) Layer 1-Layer 2-Layer 3-Layer 4 and b) Layer 4-Layer 3-Layer 2-Layer 1

3.6. Effects of the Number of Element Divisions for Mesh

In order to determine the influences of the mesh element numbers on the bending stress, the axially layered FG beams were analyzed using various number of element divisions for each layer. In the analyses, the axially layered FG beam with optimum layers was used. The effects of the mesh factors on the bending stress were shown in Table 8.

Table 8. The influences of Number of Element Divisions

Number of Element Divisions	Bending Stress (MPa)
2	16.8 MPa
5	16.9 MPa
10	16.9 MPa
20	16.9 MPa
40	16.9 MPa
80	16.9 MPa

Table 8 shows that the number of element divisions between 5 and 80 for mesh has no effects on the maximum bending stress.

4. CONCLUSIONS

This study presents the investigation of the numerical bending stress behaviour of axially layered FG beams consisting of Al/WC systems based on the increasing of 6% WC and the decreasing of the 6% Al using finite element software named ANSYS according to von Mises stresses. The layer positions of the layered FG beams in axial direction were conducted using Taguchi L16 orthogonal array design consisting of 16 runs 4 control factor and 4 levels and the axially layered FG beams were subjected to their own weight. According to numerical results following conclusions can be drawn:

- The increasing of the percent volume fractions of WC material in layers increases the bending stress of the axially layered FG beams.
- The axially layered FG beam with optimum layers is obtained as Layer1 with 24%WC/76%Al, Layer2 with 48%WC/52%Al, Layer3 with 72%WC/28%Al and Layer4 with 96%WC/4%Al.
- Clamped and free end of the axially layered FG beams have maximum and minimum bending stress, respectively.
- All the layers have important effects on the bending stress since p value is smaller than 0.05.
- Percent contributions of the layers on the bending stress of axially layered FG beams are Layer 4 with 57.48 %, Layer 3 with 29.54 %, Layer 2 with 11.83 % and Layer 1 with 1.12 % respectively.
- According to p values, significant level of Layer 1 is smaller than other layers.
- ANSYS and predicted results are calculated between 16.7633 and 16.9767 MPa at 95% confidence level and results are 16.90 MPa and 16.87 MPa respectively.
- The densities and positions of the Al/WC systems in the axially layered FG beams play significant role on the bending stress.
- Bending stress decrease from clamped to free end.
- Bending stress value of beam with Layer 1-Layer 2-Layer 3-Layer 4 is higher than beam with Layer 4-Layer 3-Layer 2-Layer 1 under own weight of the beam and C-F boundary condition.
- Number of element divisions between 5 and 80 for line mesh operation has no influences on the bending stress.

REFERENCES

[1] KOIZUMI, M., "FGM Activities in Japan". Composites Part B: Engineering, 28(1), 1-4, 1997.

[2] UDUPA, G., RAO, S.S., GANGADHARAN, K.V., "Functionally Graded Composite Materials: An Overview". Procedia Materials Science, 5, 1291-1299, 2014.

[3] NIINO, M., KISARA, K., MORI, M., "Feasibility Study of FGM Technology in Space Solar Power Systems (SSPS)", Material Science Forum, 492, 163–168, 2005.

[4] CARVALHO, O., BUCIUMEANU, M., MADEIRA, S., SOARES, D., SILVA, F.S., MIRANDA, G., "Optimization of AlSi–CNTs Functionally Graded Material Composites for Engine Piston Rings". Materials & Design, 80, 163-173, 2015.

[5] MÜLLER, E., DRAŠAR, Č., SCHILZ, J., KAYSSER, W.A., "Functionally Graded Materials for Sensor and Energy Applications". Materials Science and Engineering: A, 362(1), 17-39, 2003.

[6] SCHULZ, U., PETERS, M., BACH, F.W., TEGEDER, G., "Graded Coatings for Thermal, Wear and Corrosion Barriers". Materials Science and Engineering: A, 362(1), 61-80, 2003.

[7] WATARI, F., YOKOYAMA, A., SASO, F., UO, M., KAWASAKI, T., "Fabrication and Properties of Functionally Graded Dental Implant". Composites Part B: Engineering, 28(1), 5-11, 1997.

[8] LI, X., LI, L., HU, Y., DING, Z., DENG, W., "Bending, Buckling and Vibration of Axially Functionally Graded Beams based on Nonlocal Strain Gradient Theory". Composite Structures, 165, 250-265, 2017.

[9] THAI, H.-T. VO, T.P., "Bending and Free Vibration of Functionally Graded Beams using Various Higher-Order Shear Deformation Beam Theories". International Journal of Mechanical Sciences, 62(1), 57-66, 2012.

[10] ALDOUSARI, S.M., "Bending Analysis of Different Material Distributions of Functionally Graded Beam". Applied Physics a-Materials Science & Processing, 123(4), 1-9, 2017.

[11] LI, S.R., CAO, D.F., WAN, Z.Q., "Bending Solutions of FGM Timoshenko Beams from Those of The Homogenous Euler–Bernoulli Beams". Applied Mathematical Modelling, 37(10), 7077-7085, 2013.

[12] KANG, Y.-A. LI, X.-F., "Bending of Functionally Graded Cantilever Beam with Power-Law Non-Linearity Subjected To An End Force". International Journal of Non-Linear Mechanics, 44(6), 696-703, 2009.

[13] SALLAI, B., HADJI, L., DAOUADJI, T.H., ADDA, B., "Analytical Solution for Bending Analysis of Functionally Graded Beam". Steel and Composite Structures, 19(4), 829-841, 2015.

[14] KADOLI, R., AKHTAR, K., GANESAN, N., "Static Analysis of Functionally Graded Beams using Higher Order Shear Deformation Theory". Applied Mathematical Modelling, 32(12), 2509-2525, 2008.

[15] VO, T.P., THAI, H.-T., NGUYEN, T.-K., INAM, F., LEE, J., "Static Behaviour of Functionally Graded

Sandwich Beams using A Quasi-3D Theory". Composites Part B: Engineering, 68, 59-74, 2015.

[16] LI, X.-F., WANG, B.-L., HAN, J.-C., "A Higher-Order Theory for Static and Dynamic Analyses of Functionally Graded Beams". Archive of Applied Mechanics, 80(10), 1197-1212, 2010.

[17] BERNARDO, G.M.S., DAMÁSIO, F.R., SILVA, T.A.N., LOJA, M.A.R., "A Study on the Structural Behaviour of FGM Plates Static and Free Vibrations Analyses". Composite Structures, 136, 124-138, 2016.

[18] SHEN, H.-S., Functionally Graded Materials: Nonlinear Analysis of Plates and Shells, CRC Press, Boca Raton, London, New York, 2009.

[19] ROY, R.K., A Primer on the Taguchi Method, Van Nostrand Reinhold, New York, USA, 1990.

[20] ROSS, P.J., Taguchi Techniques for Quality Engineering, (2nd Edition), McGraw-Hill International Book Company, New York, USA, 1996.