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The Role of Mathematical Methods in the Formation of Scientific Concepts (Elementary and Mathematics)

Abstract

It is known that mathematical justification, with all its rigor and in accordance with the reality being studied, attests to the role of mathematical methods in the formation of scientific concepts. Any analysis of the idea of elementarity that claims to be adequate must take into account the fact that in real scientific knowledge, we never encounter the concept of an elementary object—be it a material point, a point event, or a process as such—but always within a system of specific knowledge (scientific theories, models, hypotheses) used to solve well-defined scientific problems. Such an analysis must consider the inextricable connection between certain ideas *about elementarity and fundamental methodological principles of a systemic nature, such as simplicity, symmetry, invariance, and correspondence. In this study, an attempt was made to identify the methodological role of mathematical methods in the formation of scientific concepts.*

Keywords: *mathematics, elementarity, concept, principle, science, mathematical formalism*

Bilimsel Kavramların Oluşumunda Matematiksel Yöntemlerin Rolü (İlköğretim ve Matematik)

Öz

Matematiksel gerekçelendirmenin, tüm titizliğiyle ve incelenen gerçekliğe uygun olarak, matematiksel yöntemlerin bilimsel kavramların oluşumundaki rolüne tanıklık ettiği bilinmektedir. Yeterli olduğunu iddia eden unsursallık fikrine ilişkin her türlü analiz, gerçek bilimsel bilgide, ister maddi bir nokta, ister noktasal bir olay ya da bir süreç olsun, hiçbir zaman temel bir nesne kavramıyla karşılaşmadığımız, her zaman iyi tanımlanmış bilimsel sorunları çözmek için kullanılan belirli özel bilgiler (bilimsel teoriler, modeller, hipotezler) sistemiyle karşılaştığımız gerçeğini dikkate alarak gerçekleştirilmelidir. Böyle bir analiz, basitlik, simetri, değişmezlik ve tekabüliyet ilkesi gibi esasen sistemik nitelikteki temel metodolojik ilkelerle unsuriyete ilişkin belirli fikirlerin ayrılmaz bağlantısını dikkate almalıdır. Böyle bir analiz, belirli bilimsel fikirlerin basitlik, simetri, değişmezlik, tekabüliyet gibi özünde sistemik bir doğaya sahip temel metodolojik ilkelerle ayrılmaz bağlantısını dikkate almalıdır. Bu çalışmada, bilimsel kavramların oluşumunda matematiksel yöntemlerin metodolojik rolü tanımlanmaya çalışılmıştır.

Anahtar Kelimeler: *Matematik, Elementerlik, Kavram, İlke, Bilim, Matematiksel Formalizm*

Introduction

One of the most important requirements for the organization of modern education is to achieve high results in a short time without spending too much mental and physical effort. It is up to the teacher to provide students with specific theoretical knowledge in a short period of time, to develop their skills and competencies in a particular activity, as well as to monitor the activities of students, assess the level of knowledge, skills and abilities acquired by them. As it can be understood, it requires high pedagogical skills and a new approach to the educational process. We can also conduct the math lessons for our elementary grades based on the mathematical methods. This also explains the need to include in the analysis a consideration of the role of philosophical ideas in the process of forming certain ideas about elementality. At the same time, in our discussion of the idea of elementarity in scientific knowledge, we have not yet directly touched upon the role of mathematical methods in the process of "elementarization" of its initial concepts. This issue can be approached in different ways, bearing in mind the fundamental, the prestigious role that mathematics plays in the science of modern times, starting with Galileo and up to the present day. One of these approaches is associated with revealing the role of idealization in scientific knowledge. However, we will choose a slightly different path, which lies through consideration of a specific example - the role of mathematics in the formation of ideas about an elementary object. We are talking about quantum mechanics. We chose this example because the history of the emergence of quantum mechanics is well known, which saves us, on the one hand, from the need to spend a lot of time retelling all sorts of details, and on the other, makes it possible to pay attention to another important aspect of the concept of elementarity in scientific knowledge as methodological principle

The theoretical explanation and justification of new experimental results that do not fit into previous schemes ultimately requires a restructuring of the foundations of theoretical knowledge. Mathematics plays a key role in this process. It turns out to be an effective means of deriving some experimental data from others and identifying patterns hidden within these experiments. Mathematical justification means constructing a physical theory in accordance with the standards of mathematical rigor (Ismayılov, 2023). An elementary mathematics specialist develops and regularly monitors teachers' understanding and instructional implementation. These specially prepared professionals are regularly involved in mentoring novice and experienced teachers; coaching, which often includes co-planning and co-teaching; planning and providing professional development opportunities; and planning for and helping to maintain professional learning communities.

Regardless of their role, when elementary mathematics specialists interact with groups of teachers, they can leverage powerful professional learning techniques such as doing math together, co-planning and co-teaching, examining student work, and analyzing video or live lessons. Doing so regularly builds a continuity, and it maintains the momentum for refining practice (Ismayilov & Khalafova, 2023). In addition, focusing group meetings on the issues of practice is crucial, such as students' learning and best teaching practices. Elementary mathematics specialists recognize the seemingly constant intersection of their content, pedagogical, and leadership knowledge and skills as they navigate relationships, conflicting initiatives, and their responsibilities related to improving teacher practice and student learning.

As is known, the search for an adequate mathematical scheme of quantum mechanics led to the conclusion that matrix calculus is a suitable mathematical form for describing quantum systems. V. Heisenberg built this formalism, refusing any model ideas about microphenomena; he proceeded from the so-called principle of observability, i.e. from the fact that the relations of the theory under construction must be relations between experimentally observed quantities. Each observable quantity corresponds to some physical property through which a quantum system can detect itself. Measuring any one observable quantity causes the quantum system to transition to a state for which there is a certain range of values of the second observable quantity that does not commute with the first.

These, according to Heisenberg, are only the frequencies and intensities characteristic of an atom of spectral lines, as well as the energy level of atoms. W. Heisenberg considered the task of his mechanics primarily to be the formation from spectral quantities of quantities suitable for calculations that could replace the coordinates and velocities of electrons. Next, it was necessary to establish relationships between these quantities in such a way that quantization rules could then be found; the latter would need to be done as closely as possible in agreement with experiments with a wider area of application than Bohr's theory allowed. And at that time, W. Heisenberg understood the impossibility of absolutely exact implementation of the principle of observability. To create an absolutely reliable basis for physical theories, it seems necessary to require that only concepts based entirely on experience be used to describe phenomena. This requirement, however, is completely impossible to carry out, since then everyday concepts would have to be revised, and It's hard to say what would be left of our language after this (Nadir & Oruj, 2022). B. Heisenberg came up with a truly revolutionary idea: to split each quantum quantity, so to speak, by presenting it in the form of a table of numbers similar to the (infinite) matrices used in mathematics.

The classical periodically changing coordinate of an electron is associated with a set of oscillations, the frequencies of which coincide with the frequencies of spectral lines: these lines, according to Bohr's theory, were emitted when the corresponding coordinates changed. Since each frequency corresponds to a transition between two states, the set of such frequencies can be expressed in the form of a table - a square matrix. Each element of the matrix qil is associated with a value characterizing the probability of a transition between i and l states. The square of this value determines the intensity of the spectral line caused by this transition. "The matrix elements," Heisenberg concludes, "must therefore stand in the same relation to the radiation of the atom as the coefficients of the Fourier series of the classical model to its radiation" (Nadir & Oruj, 2022). Subsequently, M. Born, P. Jordan and P. Dirac achieved even greater progress, making it possible to operate with matrices whose elements are not arranged according to stationary states, but in which the permissible values of any set of variables can be included as indices of matrix elements. Just as in the original theory "diagonal elements" associated with only one stationary state could be interpreted as time averages of the displayed quantities, the general theory of matrix transformation allows for the representation of such average values of some quantity. Moreover, when calculating these averages, a certain set of variables characterizing the "state" of the quantum system has these values, while canonically conjugate variables can take all possible values. The rigor and accuracy of the proposed formalism of matrix mechanics and its agreement with experimental data inspired confidence in the correctness of the chosen path.

But soon wave mechanics, proposed by E. Schrödinger, who, unlike W. Heisenberg, proceeded from different premises, also achieved the same success in describing quantum systems. However, there was something common inherent in the construction of these formalisms and distinguishing them from the construction of the formalism of classical theories; they were built before a corresponding physical interpretation had been developed, which remained the subject of heated debate for a long time. When constructing each of the mentioned formalisms, a situation unusual for classical physics arose when it first turned out to be better to derive a mathematical equation, not paying attention to physical models and descriptions, and only then look for a simple physical concept that would cover the content of these equations.

This situation is well described by L. N. Mandelstam:

"*... it will not be a mistake to say that both de Broglie and Schrödinger were looking primarily for a mathematical apparatus, i.e. they were looking for such a construction of the mathematical side of the theory that would be inherent isolating discrete values, caring little at this stage... about what physical meaning will need to be assigned to those quantities that will be included in this mathematical apparatus, with the exception of the most discrete quantities, which, obviously, had to be interpreted as E (energy) or as H (frequency) - after all, this is what everything was done for"* (Ismayilov & Khalafova, 2023, p. 106-110). In his search for the wave equation, E. Schrödinger proceeded from the ideas of A. Einstein and Louis de Broglie. In his own words, his work was stimulated by de Broglie's dissertation and Einstein's brief but extremely profound remarks (Ismaylov, Makhamedli and Gasimli, 2023). E. Schrödinger admitted that there is some as yet unknown mechanics, covering both the region of "ordinary" mechanical phenomena and typical quantum phenomena: the region of atomic order processes, in which the presence of wave properties in particles is reflected. Moreover, this new mechanics should be related to the "old" classical mechanics, just as wave optics is to geometric ones. And since there is a connection between geometric optics and classical mechanics, the existence of a similar connection was proposed by E. Schrödinger for wave optics and new mechanics, i.e. he extended Hamilton's analogy to new mechanics and wave optics. "Perhaps our classical mechanics," writes E. Schrödinger, represents a complete analogy with geometric optics and, like the latter, refuses to serve and is not consistent with the actual state of affairs with sizes and radii of curvature of trajectories approaching in magnitude a certain wavelength, which now takes in q space real meaning (Ismayilov, Mahammadli and Gasimli, 2023). Reasoning in this way, E. Schrödinger came to the conclusion that real quantum mechanical phenomena should be understood as certain wave processes; in his opinion, it was quite natural to associate them with some oscillatory process in the atom. And although this assumption, as it soon became clear, turned out to be erroneous, the main task posed by E. Schrödinger - to find the fundamental equation of new mechanics, similar to the fundamental equations of Newton in classical mechanics, or Maxwell's in electrodynamics - was successfully solved by him.

E. Schrödinger later showed that his wave mechanics was mathematically equivalent to the earlier formalism of quantum matrix mechanics. Thus, recalls W. Heisenberg, we finally received a consistent mathematical formalism, which can be expressed in two equally valid ways: either using matrix relations, or using wave equations (Ptashne, Backman, Humayun, Jeffrey, Maurer, Meyer and Sauer, 1976). The solutions of the Schrödinger equation in matrix mechanics appear as auxiliary functions that determine the transformation of matrices with indices representing the energy values of a quantum system into other matrices whose indices are possible values of the coordinates of microparticles. Namely, it was found that from the solutions given by the Schrödinger equation, it is possible to calculate the elements of the matrices of the corresponding problem of Heisenberg matrix mechanics. So, the elements of one formalism could be calculated on the basis of the elements of another, and vice versa. This gives grounds for asserting the equivalence of both formalisms. Schrödinger wave mechanics and Heisenberg matrix mechanics can be considered as different forms of recording objectively the same relationships.

At first glance it seemed," Louis de Broglie wrote, expressing the opinions of physicists of this period, that both theories were completely opposite both in appearance and in the formalism used. These theories, so different in appearance, should in fact be considered one and the same because each of them is only a translation of the other into a different mathematical language. These initially so different attempts to build a new mechanics, truly saturated with quantum concepts, eventually merged into a single whole, into a theory that can be called a new quantum theory (Broglie, 1953).

1.The Role of Mathematics in Intellectual Development

Mathematics plays an important role in intellectual development by developing problem solving skills (Toraman, Orakci & Aktan, 2020). The process of solving mathematical problems involves critical thinking, analysis and the application of various problem solving strategies. Engaging in activities that involve solving mathematical and logic problems as well as brain teasers can significantly improve analytical intelligence. In addition, strategy games and brain teasers involving logical reasoning can further develop analytical thinking skills, providing individuals with the ability to analyze information, make informed decisions and identify patterns or relationships (Gönül, İzgeç, Geçen, İzgeç, Gönül & Uslu, 2024). By developing logical and analytical thinking through mathematical activities, individuals can strengthen their problem-solving abilities and cognitive flexibility.

2.The Role of Mathematics in Vocational Development

The primary goal of education was to aid students in earning a living and becoming self-sufficient.

3.The Role of Mathematics in Moral Development

Morality is a vital aspect of life that is influenced by time, people, situations, and places. Mathematics, as a discipline, could contribute to a student's moral growth because mathematical knowledge aids in forming personality and character. It cultivates all qualities a person of pleasing personality must-have. Cleanliness and realism are attributes that a child acquires.

4.The Role of Mathematics in Spiritual Development

Educators interested in advanced elementary mathematics preparation and leadership must have both incentives to participate in and access to professional learning opportunities and continued support. Recommended actions that should be taken by stakeholders from educators, administrators, and district leaders to teacher educators, state/province supervisors, and policymakers are presented below.

Conclusion

When everything mentioned above is taken into consideration, it would be approtiate to state that we were not talking here about a random coincidence; behind this coincidence of results, there was a deeper reason - the actual identity was strictly shown, the methods of both mechanics were combined, and together with the theory of representations of P. A. Dirac they formed the modern theory of quantum mechanics. For us, it is only important to note that without analyzing the formation of mathematical knowledge, it is simply impossible to deeply understand the very history of the development and formation of elementary ideas and concepts in physical theories. This is what the history of the emergence of quantum mechanics actually testifies to.

Unlike a material point of classical mechanics, the state of which is determined by coordinates and momentum, for a quantum system it is impossible to determine both coordinates and momentum with absolute accuracy: the more accurately one characteristic is recorded, the less accurately the value of the other can be determined. The uncertainty relation in quantum mechanics does not simply limit the concept of a material point as an elementary object of the theory, but qualitatively transforms it. The elementary object of the theory becomes a quantum system, which can be considered as an ensemble of possible quantum events that are realized in the process of its interaction with a device of a certain type (Ismayilov & Khalafova, 2023). An important feature of quantum mechanics was that it combined the idea of discreteness, characteristic of the physical atomism of classical mechanics, and continuity, characteristic of the concept of the force field of classical electrodynamics. And this unification was an important step towards the formation of the concept of an elementary process in quantum mechanics and an event as its observable result (Ismayilov & Aliyeva, 2023). Like any physical theory, quantum mechanics has its own well-defined sphere of action, correlated with certain objective conditions, and within this sphere it reflects the essential properties of objects. In the course of cognition, reality is divided into separate aspects and studied separately. The concept of an elementary event and process in quantum mechanics is based on certain idealizations and covers the invariant, necessary, essential aspects of the phenomena of the microworld (Ismayılov, 2023). Elementary objects of quantum mechanics, quantum systems, let us repeat, are idealized abstract systems of potentially possible events. The history of the creation of quantum mechanics shows that the process of the formation of an elementary object that deeply and adequately reflects the patterns of behavior of microparticles proceeded in close connection with the process of finding those new mathematical objects (Heisenberg matrices and Schrödinger wave functions) that were able to quantitatively express these essentially new patterns, and thereby "connecting" to this process the entire system of regulatory methodological principles: symmetry, invariance, conservation, correspondence, simplicity (Ismayilov and Khalafova, 2023). For example, the role of the conservation principle in the formation of an elementary object of quantum mechanics is revealed through a mathematical consideration of the symmetry properties of the wave function, which is based on a specific identity, namely the identity (indistinguishability) of particles. In this case, that is, if we are talking about symmetry, how is it reflected (Ismailov, Makhamedli and Gasymli, 2023). In the mathematical apparatus of the theory, according to the accepted terminology, the researcher deals with transformations in which the preservation of one or another abstract object of the theory is revealed. In other words, in a theoretical system, symmetry appears as transformations with corresponding invariants. In this sense, it can be argued that the formation of a mathematical representation of an elementary object of quantum mechanics is carried out using the regulatory principle of conservation, namely, as a consistent reflection of the specific symmetry of the wave properties of microparticles, manifested in the features of their wave motion. As a result, the function itself turns out to have the properties of symmetry and invariance. These properties seem to be the most important properties underlying the description of quantum mechanical laws (Ismayilov, Mahammadli & Gasimli, 2023).

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