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# COGNITIVE SYSTEMS AND QUANTUM COMPUTATION

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**This review presents an overview of cognitive systems, quantum phenomena and possible connections between them. The focus will be the artificial cognitive systems and briefly touch the discussion of possible benefits from quantum counterparts. The non-classical features of Quantum Theory introduced as quantum resources which enables possible speed ups or advantages over classical computational tasks. Quantum computation is introduced as a powerful computational tool over its classical counterparts by also covering possible applications of cognitive phenomena in the framework of quantum cognition. Also different attempts in order to implement decision making processes for cognitive purposes mentioned.**

***Index Terms*** — Quantum computation, quantum machine learning, cognitive systems, quantum resource

## I. INTRODUCTION

COGNITIVE science is an inter-disciplinary study of decision making, intelligence including artificial intelligence and artificial neural networks and also human memory linguistics and anthropology [1]. On the other hand, quantum cognition adopts quantum probability theory instead of classical probability theories derived from Kolmogorov axioms [2] obey the Boolean axioms of logic. Since the quantum logic is able to explain some discrepancies between experiment and the classical probability principles such as in the 'disjunction effect' [3]; quantum logic becomes a generalization of classical logic and quantum probability theory.

Cognition can be defined as the ability to process information of perception knowledge acquired through experience. Though perception and experience look like humanoid concepts, the term 'learning' is widespread used for algorithmic processes as 'machine learning' in the context of artificial intelligence which is a field of computer science deals with intelligent machines mimic human behaviors. The term artificial intelligence (AI) first coined by john McCarthy in 1956 in a workshop referred to as the official birth of artificial intelligence [4]. Frank Rosenblatt developed 'perceptron' an early artificial neural network based on a two-layer computer learning network [5]. Today AI finds applications of our everyday life from pattern recognition implementations, driverless cars to the humanoid robots mimicking physical human actions. The progress of machine learning algorithms advances the AI current state of art as an underlying fact.

Since the volume of processed global data has reached a persistent annual increment [6] the idea to benefit the potential of quantum computing and information emerges by the physics society [7]. To this end, there are several proposals exploring the gains of unification of machine learning algorithms with quantum computing, particularly with efforts to develop quantum versions of artificial neural networks[8-10]. Another proposal is to reformulate the machine learning subroutines in order to implement on a quantum computer [11-13]. This review presents a general survey of possible proposals to relate machine learning in the context of cognitive systems. Future prospects and open problems also addressed.

## II. WHY QUANTUM?

Quantum theory is the theory (QT) of matter and energy based on the quantization nature with successful mathematical descriptions. The historical development of QT is full of debates and scientific discussions due to its counter-intuitive nature [14]. Today, QT appears to be one of the most successful theories ever with the large number of experimental verifications. Moreover, the subject of early discussions such as 'quantum superposition' or 'quantum entanglement' is now resources of current quantum technologies. Quantum superposition is the existence of linear combinations of distinct eigenstates representing a physical system. Superposition is a direct consequence of Schrödinger equation which is a differential equation representing the temporal evolution of a quantum system as wave mechanics formulation. Though this fact, Erwin Schrödinger criticized the result of his own equation by quoting a cat in a superposition of states 'alive' and 'dead' in which later become famous as Schrödinger's cat [15]. These results triggered an enhancement of already existing discussion in the physics community about the interpretations of QT including uncertainty, measurement problem and wave function collapse [16-17]. One discussion side embraced QT with its own counter-intuitive nature by describing a quantum system

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as not corresponding to a physical quantity before a measurement. This view of QT is known as Copenhagen interpretation devised by Neils Bohr and Werner Heisenberg reflecting the uncertain and probabilistic nature of QT. On the other hand, Einstein and Schrödinger refused the probabilistic nature or wave function collapse and tried to express the results in a deterministic way. In 1935 Einstein attacked the fundamentals of QT with a paper interrogating the completeness of QT by introducing a thought experiment expressing that by the admittance of the validity of Copenhagen interpretation QT is in a clear contradiction with locality, special relativity and causality principles [18]. John Bell invented an inequality in 1964 [19] which is not possible to be violated under Einstein's deterministic assumptions of QT. By this inequality discussions were able to be carried into the laboratory. Experimental studies examining the inequalities resulted by the violations of the inequalities [20]. By these results non-local correlations of QT , 'quantum entanglement' [21] has been experimentally proven. Quantum entanglement is now being used as a quantum resource and enables quantum teleportation [22], quantum dense coding [23] and quantum cryptography [24] possible. Non-classical quantum resource is not the only compelling reason for quantum technologies. In 1960's Gordon Moore noticed that the number of transistors in a circuit doubles every two years [25].

According to this observation the number of atoms represent one bit of information decrease logarithmically means that miniaturization process of electronic devices will end up with the entrance of the quantum region with a few or less atoms where the classical circuit theory is no longer valid [26]. By these facts, first arguments about implementing computational logic governed by quantum systems which is known as the first building blocks of quantum computation coined by Paul Benioff [27]. A milestone appeared by Richard Feynman's report based on the idea of the potential of the efficient simulation of quantum systems could be implemented by other quantum systems [28]. The first quantum logical algorithm reported by David Deutsch in 1985 considering the solution of a decision problem with a speed up over classical algorithms by exploiting quantum superposition principle [29]. A breakthrough of interest to quantum computation occurred after the introduction Shor's factoring algorithm which can factorize long digit prime numbers in polynomial time [30] followed by Simon's algorithm [31] for period finding in polynomial time and Grover's search algorithm in an unstructured database [32].

Though quiet much number of quantum algorithms developed to date, these algorithms are mostly the variants of the quantum algorithms considered above. The reasons of the difficulty of developing novel pure quantum algorithms are twofold. First, developing a pure quantum algorithm is not sufficient alone since the necessity to have better performance over any classical algorithm among its classical counterparts. Second, since the developers of the algorithms live in a classical world, it's difficult to lift the barriers arising from the classical intuitions to reach pure quantum algorithms. For a comprehensive introduction to quantum computation see [33-34].

Undoubtedly, a natural question arises about the possibility of building a feasible quantum computer. There are several physical systems as candidates to implement quantum algorithms [35]. Di Vincenzo reported some criterias for quantum systems should fulfill in order to be

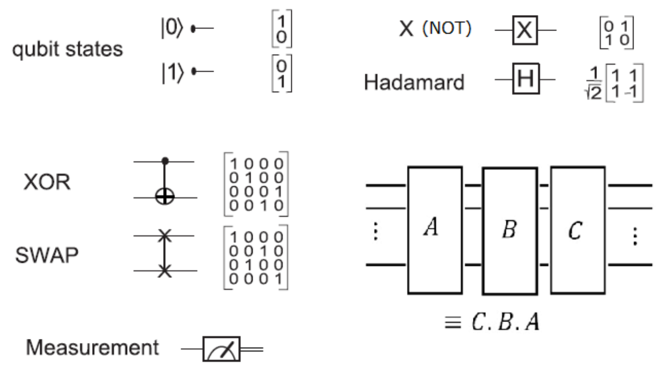


Fig.1 Circuit and matrix representations of some quantum logic gates. Each line represents quantum bits while double lines (after measurement) represent classical bits. Time flows from left to right in a quantum circuit and matrix multiplication order to the qubit states is from right to left.

considered as a universal quantum computer [36]. After two decades of research experience no single physical quantum system appears to fulfill these criterias in a complete manner. However, hybrid systems such as semi-conductor and superconductor systems are promising for future applications [37-38].

## III. QUANTUM COMPUTATION

Quantum computation (QC) [33-34] is a computational method using the mathematical abstractions defining the nature of quantum mechanics. The elementary processing unit of QC is quantum bits or 'qubits' in short, defined by a unit bi-dimensional vector

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

in a $\mathbb{C}^2$ complex vector space where $|0\rangle$ and $|1\rangle$ are the computational basis in dirac notation and $\alpha, \beta \in \mathbb{C}^2$ such that $|\alpha|^2 + |\beta|^2 = 1$. Computational basis are represented by column vectors as

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \quad , \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \ . \quad (2)$$

Larger complex vector spaces required in order to define multi-qubit systems in the context of QC. For instance, $N$ qubit system can be expressed as

$$|\Phi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes .. \otimes |\phi_N\rangle \tag{3}$$

or as $|\Phi\rangle = |\phi_1 \phi_2 .. \phi_N\rangle$ in short where $\otimes$ stands for a tensor product of vectors.

Logical operators acting on computational basis of $N$ qubits are the $2^N \times 2^N$ dimensional unitary operators represented by unitary matrices $U$, obeying $U^\dagger U = \mathbb{1}$ where $U^\dagger$ is the transpose conjugate and $\mathbb{1}$ is the unitary matrix with a convenient dimension. Fig.1 depicts the single and multi-qubit gates and a circuit representation and relevant symbols of quantum circuits.

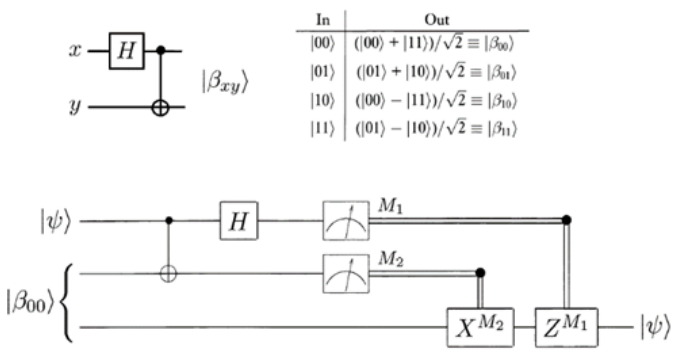


Fig.2. Bell state generator (top panel) and the quantum teleportation circuit (bottom panel). Bell states are the entangled states and are widely used in QC circuits.

As considered before, quantum superposition is one of the main resources of QT which makes QC more powerful. Benefiting quantum superposition of QC processes is known as quantum parallelism. Therefore Hadamard operator H, is of central importance since its action is to put single computational basis $|0\rangle$ , $|1\rangle$ into superposition such as

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad , \quad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \tag{4}$$

Also multi-qubit gates are necessary to implement multi-qubit operations. For instance, a two- qubit quantum gate C-NOT gate can be represented by a generalization of XOR gate with the action $|A, B\rangle \rightarrow |A, B \oplus A\rangle$. Then the effect of C-NOT gate can be represented by actions $|00\rangle \rightarrow |00\rangle$ , $|01\rangle \rightarrow |01\rangle$, $|10\rangle \rightarrow |11\rangle$ and $|11\rangle \rightarrow |10\rangle$. Here the first qubit is the control qubit and the second qubit is the target qubit. If the state of the control qubit is 0 then the gate leaves the target qubit left alone, if the target qubit is 1 then $X$ (Not gate) is applied to the target qubit. In fact this is a conditional operation and the state of the target qubit flipped depended on the condition of the control qubit. These conditional operations can be generalized by any two-qubit operator $U$ and the matrix representation can be decomposed as

$$CU = |0\rangle\langle 0| \otimes \mathbb{1} + |0\rangle\langle 0| \otimes U \tag{5}$$

where $\langle . |$ is the transposition of any qubit state. Another important two-qubit gate is the swap gate in which swaps the two qubit state. The action of this gate can be summarized as $|01\rangle \rightarrow |10\rangle$ , $|10\rangle \rightarrow |01\rangle$ for instance. Another gate operation worth mentioning is the 'Bell state generator' which is a two-qubit operation with an Hadamard gate applies to the first qubit followed by a C-Not gate to the both qubits. By this operation, entangled states are obtained for different possible input states (Fig.2 top panel). A general expression for Bell states is

$$|\beta_{xy}\rangle \equiv \frac{|0,y\rangle + (-1)^x |1,\bar{y}\rangle}{\sqrt{2}}. \tag{6}$$

A simple application of entangled states is the quantum teleportation circuit (Fig.2 Bottom panel) which teleports an unknown quantum state from one qubit to another one.

The final stage of the quantum computational tasks is the measurement. According to the measurement postulate of QT, quantum measurements are described by a set of measurement operators $\{M_m\}$ acting on the states of the relevant systems being measured. Here the index $m$ is the measurement outcome in the experiment where the probability result $m$ occurs is

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle. \tag{7}$$

The state of the system after the measurement is

$$|\varphi'\rangle = \frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}. \tag{8}$$

Here, the linear sum of measurement operators is equal to unity $\sum_m M_m^\dagger M_m = I$ implying that the probabilities sum to one. Before the measurement the evolution of the system is unitary which means that the system has no contact with the environment. Measurements are the interactions with the environment or the measurement apparatus in which the details of the measurement process is the out of the scope of QC.

## IV. TOWARDS QUANTUM COGNITIVE COMPUTATION

Direction of computer science evolved more rapidly towards cognitive computation in the past decade. Cognitive features of a computer, in other words ability to learn and implementing decisive processes will be a more efficient assistant for humans. IBM researchers underline the shift in technology with new advances with the cognitive abilities by their experience based on the first cognitive system Watson [39]. According to IBM Watson CTO, quantum computing would advance artificial intelligence by orders of magnitude [40]. By this point of view, recent ideas on improving machine learning algorithms by exploiting the advantages of quantum computing have been reported [41-45]. Clustering unstructured or sorting labeled data are the central problems of unsupervised or supervised learning which are important topics of machine learning.

Quantum version of specific classical algorithms for pattern classification has been reported for different goals. For instance swap test [46] was introduced in order to identify the similarity between two quantum states. Inspired by the swap test Lloyd et al proposed a routine to recover classical distance between two vectors via quantum measurement [47]. On the other hand, a pure quantum pattern recognition aimed algorithm was developed by Trugenberger [44]. The procedure was based on the measurement of hamming distance between two binary quantum states. Lloyd et al also developed a quantum support vector machine for supervised machine learning [48] and quantum principle component analysis [49] as applying classical ML procedures to quantum register. Beyond the efforts of implementing QC tasks in terms of cognitive duties a comprehensive theory of quantum learning is still missing. Therefore cognitive studies exploiting the advantages of QT make an active research area including the past two decades' useful discussions about the efforts on understanding of origin of human consciousness linked by QT [50].

## CONCLUSIONS

The current direction of quantum cognitive processes appears to be divided into two main paths; first, application of cognitive processes to quantum computation with already existing quantum algorithms; second, implementing cognitive tasks to quantum systems. Though yet there is no current convincing theory of quantum learning, the capacity of quantum registers with valuable quantum resources makes quantum cognitive systems a promising candidate to strengthen the capacity of cognitive systems.

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