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On the Generalization of Soft Dimonoids

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Highlights:
- Soft Set Theory
- Hyperstructures
- Soft Hyperstructures

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- Soft Sets Hyperdimonoids
- Soft Hyperdimonoids
- Soft Subhyperdimonoids

ABSTRACT:

Different models have been developed for solving complex problems involving uncertainty in many fields, especially in mathematics, engineering, medicine and computer sciences. Soft set theory, proposed by the Russian mathematician Molodtsov, is one of these models. Soft sets have reached a wide potential by being applied to various areas. In this study, soft hyperdimonoids are introduced by presenting a soft approach to the concept of hyperdimomoids, which is a generalisation of dimonoids. The substructures of soft hyperdimonoids, which are defined as a generalisation of soft dimonoids, are examined and some structural properties are also studied in detail.

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INTRODUCTION

Developing models to explain phenomena in the real world is important for solving complex problems. In particular, some mathematical modelling is presented for problems involving uncertainty and incomplete information. Probability theory, rough set theory, fuzzy set theory are some of these models and they contain some problems arising from parameters. At this point, soft set theory was initiated by Molodtsov in 1999 as an approach independent of these problems (Molodtsov, 1999). This theory has been studied both in theoretical and applied fields (Maji et al., 2002; Aktas & Cagman, 2007; Kazanci et al., 2010; Shabir & Naz, 2011; Oguz, 2020a). First of all, Maji et al. investigated the application of soft set theory in decision making problems, introducing several operations on soft sets (Maji et al., 2002). The notion of soft groups was first defined by Aktas and Cagman, who subsequently conducted a detailed analysis of their fundamental operational characteristics. (Aktas & Cagman, 2007). Further algebraic studies on soft set theory are available (Oguz et al, 2019a; Oguz et al, 2019b, Oguz et al, 2020). This theory, which is combined with many branches of mathematics, has also been applied to algebraic hyperstructures (Yamak et al., 2011; Wang et al., 2011; Selvachandran & Salleh, 2013; Oguz, 2020b; Oguz, 2020c; Oguz & Davvaz, 2021; Oguz, 2023) . Algebraic structures are fundamental mathematical objects that study the relationships and operations between sets and elements within those sets (Davvaz et al., 2015). Algebraic hyperstructures are mathematical structures that generalize traditional algebraic structures like groups, rings, and semigroups (Golzio, 2018). In 1934, Marty initiated hyperstructure theory At the Eighth Congress of Scandinavian Mathematicians. Hyperstructures play a pivotal role in numerous branches of mathematics, cryptography, computer science, and combinatorics for modeling and solving mathematical problems and understanding the properties of different mathematical systems (Marty, 1934). While traditional algebraic structures like groups have binary operations (e.g., addition or multiplication), hyperstructures allow for more than two operands in their operations. In other words, hyperstructures are concerned with n-ary operations, where n can be a number greater than two (Davvaz et al., 2007; Corsini & Leoreanu, 2013). One of the hyperstructures is the hyperdimonoids, presented by Akan and Ekiz as a generalisation of the dinomoids (Akin & Ekiz, 2019).

In recent years, there have been multiple studies investigating the relations between soft sets and hyperstructures. The notions of soft hypergroupoids and soft subhypergroupoids were proposed by Yamak et al., the first of these (Yamak et al., 2011). Afterwards, the notions of soft (normal) polygroups and (normal) soft subpolygroups were introduced by Wang et al. (Wang et al., 2011). Selvachandran and Salleh examined the notions of soft hypergroups and soft subhypergroups (Selvachandran & Salleh, 2013). By studying polygroups and hypergroups with a soft topological approach, Oguz presented the concepts of soft topological polygroups and soft topological hypergroups (Oguz, 2020b; Oguz, 2020c). It is in this context that this study introduces the notion of soft hyperdimonoids as a soft approach to hyperdimonoids. Thus, a new connection between soft set theory and hyperstructures is established and important results are obtained. Subsequently, some features of soft hyperdimonoids are studied. Additionally, the category of soft hyperdimonoids and the concept of soft subhyperdimonoids are introduced.

MATERIALS AND METHODS

In this section, for the sake of completeness, some foundational descriptions and features of soft sets and hyperdimonoids will be reviewed.

Let $\mathcal{M}$ represent the initial universe set and V denote the set of parameters. In addition, let $P(\mathcal{M})$ be defined as the power set of $\mathcal{M}$ and $S \subseteq V$. The description of a soft set as first presented by Molodtsov is the following:

Definition 1 *A couple* $(ð, S)$ *is referred to as a soft set over* $\mathcal{M}$, *where* ð *represents a mapping defined by*

$ð: S \longrightarrow P(\mathcal{M})$

It should be noted that a soft set over $\mathcal{M}$ can be regarded as a parametrized collection of subsets of the universe $\mathcal{M}$ (Molodtsov, 1999).

Definition 2 *The support of a soft set* $(ð, S)$ *is characterised by*

$Supp(ð, S) = \{\omega \in S: ð(\omega) \neq \emptyset\}$

In the event that $Supp(ð, S)$ is not equal to the empty set, then $(ð, S)$ is designated as non-null (Kazanci et al., 2010).

Definition 3 *Let* $(ð, S)$ *and* $(ð_1, R)$ *be two soft sets, both defined over the common universe* $\mathcal{M}$. *It is asserted that* $(ð_1, R)$ *constitutes a soft subset of* $(ð, S)$, *henceforth designated as* $(ð_1, R) \tilde{\subseteq} (ð, S)$, *provided the subsequent conditions are satisfied:*

• $R \subseteq S$;

• $ð_1(\omega)$ and $ð(\omega)$ are identical approximations for all $\omega \in R$.

Here are some generalisations for the nonempty collection $\{(ð_i, S_i) | i \in I\}$ of soft sets over the common universe X (Molodtsov, 1999).

Definition 4 *The restricted intersection of the collection* $\{(ð_i, S_i) |\ i \in Ẋ\}$ *is defined by a soft set* $(ð, S) = \tilde{\cap}_{i \in Ẋ} (ð_i, S_i)$ *such that* $S = \bigcap_{i \in Ẋ} S_i \neq \emptyset$ *and* $ð(\omega) = \bigcap_{i \in Ẋ} ð_i(\omega)$ *for all* $\omega \in S_i$ (Kazanci et al., 2010).

Definition 5 *[3] The extended intersection of the collection* $\{(ð_i, S_i) \mid i \in Ẋ\}$ *is a soft set* $(ð, S) = \bigcap_{ð_{i \in Ẋ}} (ð_i, S_i)$ *such that* $S = \bigcup_{i \in Ẋ} S_i$ *and* $ð(\omega) = \bigcap_{i \in Ẋ(\alpha)} ð_i(\omega)$, $Ẋ(\omega) = \{i \in Ẋ \mid \omega \in S_i\}$ *for all* $\omega \in S_i$ (Kazanci et al., 2010).

In the following, we provide an overview of the descriptions and characteristics of hyperoperations and hyperdimonoids.

Definition 6 *A dimonoid* $\mathfrak{D}$ *is defined as a triple* $(\mathfrak{D}, \circ, \diamond)$ *comprising two binary operations two binary operations* $\circ$ *and* $\diamond$, *which satisfy the following properties:*

$ꭓ \circ (ꞵ \diamond ꙁ) = (ꭓ \circ ꞵ) \diamond ꙁ$

$ꭓ \diamond (ꞵ \circ ꙁ) = (ꭓ \diamond ꞵ) \circ ꙁ$

$ꭓ \diamond (ꞵ \diamond ꙁ) = (ꭓ \circ ꞵ) \diamond ꙁ$

$ꭓ \circ (ꞵ \circ ꙁ) = (ꭓ \circ ꞵ) \circ ꙁ$

$ꭓ \diamond (ꞵ \diamond ꙁ) = (ꭓ \diamond ꞵ) \diamond ꙁ$

for all $x, y, z \in \mathcal{D}$. Furthermore, a dimonoid $\mathfrak{D}$ is designated as commutative if both of its binary operations are commutative (Loday, 2001).

Example 1 *Choose* $\mathfrak{D} = \{ꭓ, ꞵ\}$. *Define a semigroup* $(\mathfrak{D}, \diamond, \diamond)$ *with the binary operation as follows:*

$\diamond$	ꭓ	ꞵ
ꭓ	ꭓ	ꞵ
ꞵ	ꞵ	ꞵ

Definition 8 *Let* $\mathcal{H}$ *be a non-empty set, and let* $P^*(\mathcal{H})$ *be defined as the collection of non-empty subsets of* $\mathcal{H}$. *In this case, the mapping* $\cdot: \mathcal{H} \times \mathcal{H} \longrightarrow P^*(\mathcal{H})$ *is referred to as a hyperoperation, and the couple* $(\mathcal{H}, \cdot)$ *is also designated a hypergroupoid* (Davvaz et al, 2015).

Definition 9 *A hyperdimonoid* $\mathcal{H}$ *is a triple* $(\mathcal{H},\circ,\diamond)$ *together with two hyperoperations* $\circ$ *and* $\diamond$ *that satisfies the following properties:*

$$ӽ \circ (ƀ \diamond ⱬ) = (ӽ \circ ƀ) \diamond ⱬ$$

$$ӽ \diamond (ƀ \circ ⱬ) = (ӽ \diamond ƀ) \circ ⱬ$$

$$ӽ \diamond (ƀ \diamond ⱬ) = (ӽ \circ ƀ) \diamond ⱬ$$

$$ӽ \circ (ƀ \circ ⱬ) = (ӽ \circ ƀ) \circ ⱬ$$

$$ӽ \diamond (ƀ \diamond ⱬ) = (ӽ \diamond ƀ) \diamond ⱬ$$

for all $ӽ, ƀ, ⱬ \in \mathcal{H}$. Furthermore, a hyperdimonoid $\mathcal{H}$ is defined as commutative when the two hyperoperations are commutative (Akin & Ekiz, 2019).

Example 2 *Choose* $\mathcal{H} = \{ӽ, ƀ\}$. *Define a semigroup* $(\mathcal{H},\circ,\diamond)$ *with the binary operation as follows:*

$\circ$	$ӽ$	$ƀ$
$ӽ$	$\mathcal{H}$	$\mathcal{H}$
$ƀ$	$\mathcal{H}$	$\mathcal{H}$

$\diamond$	$ӽ$	$ƀ$
$ӽ$	$\{ƀ\}$	$\mathcal{H}$
$ƀ$	$\mathcal{H}$	$\mathcal{H}$

It is then evident that $(\mathcal{H},\circ,\diamond)$ constitutes a hyperdimonoid (Akin and Ekiz, 2019).

Definition 11 *A subset* $\mathcal{G}$ *of a hyperdimonoid* $\mathcal{H}$ *is defined as a subhyperdimonoid of* $\mathcal{H}$ *if it is a hyperdimonoid under the same hyperoperations as that of* $\mathcal{H}$.

Definition 12 *A strong homomorphism* $ϥ: \mathcal{H}_1 \longrightarrow \mathcal{H}_2$ *between two hyperdimonoids* $\mathcal{H}_1$ *and* $\mathcal{H}_2$ *is a mapping from* $\mathcal{H}_1$ *to* $\mathcal{H}_2$ *such that* $ϥ(ӽ \circ ƀ) = ϥ(ӽ) \circ ϥ(ƀ)$ *and* $ϥ(ӽ \diamond ƀ) = ϥ(ӽ) \diamond ϥ(ƀ)$ *for all* $ӽ, ƀ \in \mathcal{H}_1$ (Akin & Ekiz A, 2019).

RESULTS AND DISCUSSION

Soft Hyperdimonoids

This section introduces the notion of soft hyperdimonoids and examines their associated characteristics.

Definition 13 *Let* $\mathcal{H}$ *be a hyperdimonoid and let* $\mathcal{P}(\mathcal{H})$ *denotes the power set of* $\mathcal{H}$. *A pair* $(ð, S)$ *is defined as a soft hyperdimonoid over* $\mathcal{H}$, *where* ð *is a mapping given by*

$$ð: S \longrightarrow \mathcal{P}(\mathcal{H})$$

and S is a set of parameters, if $S(ω)$ is a subhyperdimonoid of $\mathcal{H}$ for all $ω \in Supp(ð, S)$.

In this instance, it is evident that if each $ð(ω)$ is commutative as a hyperdimonoid, the soft hyperdimonoid $\mathcal{H}$ is said to be commutative. Furthermore, a soft hyperdimonoid $\mathcal{H}$ can be considered as a parameterized *collection* of subhyperdimonoids of the hyperdimonoid $\mathcal{H}$.

In the following sections, the notation $(\mathcal{H}, ð, S)$ will be used to refer to the soft hyperdimonoid $(ð, S)$ over the hyperdimonoid $\mathcal{H}$.

Example 2 *Take* $\mathcal{H} = \{ӽ, ƀ\}$. *Consider a hyperdimonoid* $(\mathcal{H},\circ,\circ)$ *with the binary operation as follows:*

$\circ$	$ӽ$	$ƀ$
$ӽ$	$\mathcal{H}$	$\mathcal{H}$
$ƀ$	$\mathcal{H}$	$\mathcal{H}$

Now define the mapping $ð: S = \mathbb{N} \longrightarrow \mathcal{P}(\mathcal{H})$ letting $ð(\varpi) = \{ẍ\}$ for all $\varpi \in \mathbb{N}$It is a straightforward process to verify that $(\mathcal{H}, ð, \mathbb{N})$ is a soft hyperdimonoids.

Example 3 *Let $(ð, S)$ be an soft hyperdimonoid over $\mathcal{H}$ and $\mathcal{G} \subseteq \mathcal{H}$. Then, $(ð|_{\mathcal{G}}, S)$ is a soft hyperdimonoid over $\mathcal{G}$ if it is non-null.*

Definition 14 *Let $(\mathcal{H}_1, ð_1, S)$ and $(\mathcal{H}_2, ð_2, V)$ be two soft hyperdimonoids. The product of them is defined as $(\mathcal{H}_1, ð_1, S) \times (\mathcal{H}_2, ð_2, V) = (\mathcal{H}_1 \times \mathcal{H}_2, ð, S \times V)$, where $ð(ẍ, ẞ) = ð_1(ẍ) \times ð_2(ẞ)$ for all $(ẍ, ẞ) \in S \times V$.*

In light of the aforementioned definition, the following proposition can be put forth.

Proposition 1 *The product of any two soft hyperdimonoids is itself a soft hyperdimonoid.*

Proof. Consider the soft hyperdimonoids $(\mathcal{H}_1, ð_1, S)$ and $(\mathcal{H}_2, ð_2, V)$. Then,

$ð_1: S \longrightarrow \mathcal{P}(\mathcal{H}_1)$

$\varpi \mapsto ð_1(\varpi)$

and

$ð_2: V \longrightarrow \mathcal{P}(\mathcal{H}_2)$

$\omega \mapsto ð_2(\omega)$

such that $ð_1(\varpi)$ is a subhyperdimonoid of the hyperdimonoid $\mathcal{H}_1$ for all $\varpi \in S$ and $ð_2(\omega)$ is a subhyperdimonoid of the hyperdimonoid $\mathcal{H}_2$ for all $\omega \in V$. Using these mappings, we define ð by

$ð: S \times V \longrightarrow P(\mathcal{H}_1 \times \mathcal{H}_2)$

$(\varpi, \omega) \mapsto ð(\varpi, \omega) = ð_1(\varpi) \times ð_2(\omega)$

It can be observed that $ð_1(\varpi) \times ð_2(\omega)$ is also a subhyperdimonoid of the product hyperdimonoid $\mathcal{H}_1 \times \mathcal{H}_2$ for all $(\varpi, \omega) \in S \times V$. This implies that $(\mathcal{H}_1 \times \mathcal{H}_2, ð, S \times V)$ is a soft hyperdimonoid.

Let us give the following theorem, whose proofs are easily demonstrated:

Theorem 1 *Let $(\mathcal{H}, ð, S)$ be a soft hyperdimonoid with the hyperoperation $\circ$ and $\diamond$, and $G, F, U \in \mathcal{P}(ð(\varpi)$ for all $\varpi \in S$. Then*

i. $G \diamond (F \diamond U) = (G \circ F) \diamond U$

ii. $G \diamond (F \circ U) = (G \diamond F) \circ U$

iii. $G \diamond (F \diamond U) = (G \diamond F) \diamond U$

iv. $G \circ (F \diamond U) = (G \circ F) \circ U$

v. $G \circ (F \circ U) = (G \circ F) \circ U$

Some generalisations for a non-empty family of soft hyperdimonoids are given in the following theorems.

Theorem 2 *Let $\{(ð_i, \mathcal{S}_i) \mid i \in \mathfrak{X}\}$ be a non-empty collection of soft hyperdimonoids over $\mathcal{H}$.*

ii. The restricted intersection of the *collection* $\{(ð_i, \mathcal{S}_i) \mid i \in \mathfrak{X}\}$ with $\bigcap_{i\in\mathfrak{X}} \mathcal{S}_i \neq \emptyset$ is a soft hyperdimonoid over $\mathcal{H}$ if it is non-null.

iii. The extended intersection of the *collection* $\{(ð_i, \mathcal{S}_i) \mid i \in \mathfrak{X}\}$ is a soft hyperdimonoid over $\mathcal{H}$ if it is non-null.

Proof. **i.** The restricted intersection of the *collection* $\{(ð_i, S_i) \mid i \in \mathfrak{X}\}$ with $\bigcap_{i\in\mathfrak{X}} S_i \neq \emptyset$ defined by the soft set $\widetilde{\bigcap_{i\in\mathfrak{X}}} (ð_i, S_i) = (ð, S)$ such that $\bigcap_{i\in\mathfrak{X}} ð_i(\varpi)$ for all $\omega \in \mathcal{S}$. Choose $\omega \in Supp(ð, S)$. Together with the hypothesis, $\bigcap_{i\in\mathfrak{X}} ð_i(\varpi) \neq \emptyset$, implies that $ð_i(\varpi) \neq \emptyset$ for all $i \in \mathfrak{X}$. Since $\{(ð_i, S_i \mid i \in \mathfrak{X}\}$ is a non-empty family of soft hyperdimonoids over $\mathcal{H}$, it is then straightforward to ascertain that $ð_i(\varpi)$ is a subhyperdimonoid of $\mathcal{H}$ for all $i \in \mathfrak{X}$. In addition, $\bigcap_{i\in\mathfrak{X}} ð_i(\varpi)$ is a subhyperdimonoid of $\mathcal{H}$ too. Hence, $(ð, S)$ is a soft hyperdimonoid over $\mathcal{H}$.

ii. The proof is analogous to that of the previous case.

Soft subhyperdimonoids

Definition 15 *Let* $(ð, S)$ *be a soft hyperdimonoid over* $\mathcal{H}$ *. Then,* $(ð_1, V)$ *is defined as a soft subhyperdimonoid of* $(ð, S)$ *if* $V \subseteq S$ *and* $ð_1(\omega)$ *is a subhyperdimonoid of* $ð(\omega)$ *for all* $\omega \in Supp(ð_1, V)$*.*

Example 4 *Take* $\mathcal{H} = \{ꭓ, ꞵ\}$ *. Consider a hyperdimonoid* $(\mathcal{H}, \circ, \circ)$ *with the binary operation as follows:*

∘	ꭓ	ꞵ
ꭓ	$\mathcal{H}$	$\mathcal{H}$
ꞵ	$\mathcal{H}$	$\mathcal{H}$

Now define the mapping $ð: S = \mathbb{Z} \longrightarrow \mathcal{P}(\mathcal{H})$ letting $ð(\omega) = \mathcal{H}$ for all $\omega \in \mathbb{Z}$. Also define $ð_1: V = \mathbb{N} \longrightarrow \mathcal{P}(\mathcal{H})$ letting $ð_1(\omega) = \{ꭓ\}$ for all $\omega \in \mathbb{N}$. It is straightforward to check that $(\mathcal{H}, ð, \mathbb{Z})$ and $(\mathcal{H}, ð_1, \mathbb{N})$ are soft hyperdimonoids. Besides $(\mathcal{H}, ð_1, \mathbb{N})$ is a soft subhyperdimonoid of $(\mathcal{H}, ð, \mathbb{Z})$.

Theorem 3 *Let* $(ð, S)$, $(ð_1, R)$ *and* $(ð_2, K)$ *be any three soft hyperdimonoids over* $\mathcal{H}$*. If* $(ð_1, R)$ *is a soft subhyperdimonoid of* $(ð, S)$ *and* $(ð_2, K)$ *is a soft subhyperdimonoid of* $(ð_1, R)$*, then* $(ð_2, K)$ *is a soft subhyperdimonoid of* $(ð, S)$*.*

Proof. Straightforward.

Theorem 4 *Let* $(ð, S)$ *and* $(ð_1, V)$ *be two soft hyperdimonoids over* $\mathcal{H}$*. Then* $(ð_1, V)$ *is a soft subhyperdimonoid of* $(ð, S)$ *if* $(ð_1, V)$ *is a soft subset of* $(ð, S)$*.*

Proof. Assume that $(ð, S)$ and $(ð_1, V)$ are two soft hyperdimonoids over $\mathcal{H}$. Then, if $(ð_1, R)$ is a soft subset of $(ð, S)$, we have $V \subseteq S$ and $ð_1(\omega) \subseteq ð(\omega)$ for all $\omega \in Supp(ð_1, V)$. It follows that $ð_1(\omega)$ is a subhyperdimonoid of $ð(\omega)$. Hence, $(ð_1, V)$ is a soft subhyperdimonoid of $(ð, S)$.

The soft homomorphism between soft hyperdimonoids is now going to be defined.

Definition 16 *Let* $(ð, S)$ *and* $(ð_1, V)$ *be soft hyperdimonoids over* $\mathcal{H}$ *and* $\mathcal{H}_1$*, respectively. Let* $\lambda: S \longrightarrow V$ *and* $ꟼ: \mathcal{H} \longrightarrow \mathcal{H}_1$ *be two mappings. In this case, the pair* $(ꟼ, \lambda)$ *is defined as a soft homomorphism if the subsequent conditions hold:*

i. ꟼ is a strong hyperdimonoid homomorphism;

ii. $ð_1(\lambda(\omega)) = ꟼ(ð(\omega))$ for all $\omega \in Supp(ð, S)$.

Note that a soft homomorphism $(ꟼ, \lambda)$ is a mapping of soft hyperdimonoids. A new category is thus established, whose objects are soft hyperdimonoids and whose arrows are soft homomorphisms.

Example 5 *Choose* $\mathcal{H} = \{ꭓ, ꞵ\}$ *. Consider a hyperdimonoid* $(\mathcal{H}, \circ, \circ)$ *with the binary operation as follows:*

∘	ꭓ	ꞵ
ꭓ	$\mathcal{H}$	$\mathcal{H}$
ꞵ	$\mathcal{H}$	$\mathcal{H}$

Now define the mapping $ð: S = \mathbb{N} \longrightarrow \mathcal{P}(\mathcal{H})$ letting $ð(\omega) = \mathcal{H}$ for all $\omega \in \mathbb{N}$. Clearly, $(ð, S)$ is a soft hayperdimonoid on $\mathcal{H}$. Define the mapping $ꟼ: \mathcal{H} \longrightarrow \mathcal{H}$, where $ꟼ(ꭓ) = ꞵ$ and $ꟼ(ꞵ) = ꭓ$ so that ψ is a strong hyperdimonoid homomorphism. Further, define the identity mapping $\mathcal{I}d: S \longrightarrow S$. It can easily be checked that the pair $(ꟼ, \mathcal{I}d)$ is a soft homomorphism from $(ð, S)$ to $(ð, S)$.

Example 6 *Let* $(ð_1, V)$ *be a soft subhyperdimonoid of* $(ð, S)$ *over* $\mathcal{H}$*. Considering the inclusion map* $\lambda: V \longrightarrow S$ *and the identity mapping* $\mathcal{I}d: \mathcal{H} \longrightarrow \mathcal{H}$*, it can be reasonably deduced that pair* $(\mathcal{I}d, \lambda)$ *is a soft homomorphism from* $(ð_1, V)$ *to* $(ð, S)$*.*

The aforementioned definition leads directly to the following consequence:

Theorem 5 *Let* $(\eth,S)$, $(\eth_1,V)$ *and* $(\eth_2,K)$ *be soft hyperdimonoids over* $\mathcal{H}$, $\mathcal{H}_1$ *and* $\mathcal{H}_2$, *respectively. If* $(\eta,\lambda):(\eth,S)\longrightarrow(\eth_1,V)$ *and* $(\eta_1,\lambda_1):(\eth_1,V)\longrightarrow(\eth_2,K)$ *are two soft homomorphisms, then* $(\eta_1\circ\eta,\lambda_1\circ\lambda):(\eth,S)\longrightarrow(\eth_2,K)$ *is a soft homomorphism.*

Let us conclude by presenting the final corollary.

Theorem 6 *Let the couple* (η,λ) *be a soft homomorphism from the soft hyperdimonoids* $(\eth,S)$ *and* $(\eth_1,V)$ *over* $\mathcal{H}$ *and* $\mathcal{H}_1$, *respectively. Then* $(\eta^{-1}(\eth_1),S)$ *is a soft hyperdimonoid over* $\mathcal{H}$ *if it is non-null.*

Proof. Let $(\eth,S)$ and $(\eth_1,V)$ be two soft hyperdimonoids over $\mathcal{H}$ and $\mathcal{H}_1$, respectively. Then it is easy to check that

$\lambda(Supp(\eta^{-1}(\eth_1,R)=\lambda^{-1}\big(Supp(\eth_1,V)\big).$

Taking $\omega\in Supp(\eta^{-1}(\eth_1,S)$, we get $\lambda(\omega)\in Supp(\eth_1,V)$. So, the nonempty set $\eth_1(\lambda(\omega))$ is a subhyperdimonoid of $\mathcal{H}_1$. Also, since η is a strong hyperdimonoid homomorphism, we conclude that $\eta^{-1}(\eth_1(\lambda(\omega)))=\eta^{-1}(\eth_1(\omega))$ is a subhyperdimonoid of $\mathcal{H}$. This implies that $(\eta^{-1}(\eth_1),S)$ is a soft hyperdimonoid over $\mathcal{H}$.

CONCLUSION

Soft set theory, which is the subject of research of many mathematicians worldwide, is an effective approach to modelling uncertainty. The research topic of this manuscript is to introduce and study a new notion called soft hyperdimonoids, which is a generalisation of soft dimonoids. Furthermore, the notions of soft subhyperdimonoids and soft homomorphisms between soft hyperdimonoids are introduced. Characterisations and features of these notions are obtained. As a future direction, it is aimed to further extend these theoretical approaches and to investigate the application of the results to other hyperalgebraic structures.

Conflict of Interest

The article authors declare that there is no conflict of interest between them.

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