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COUPLED SYSTEM OF MIXED HYBRID DIFFERENTIAL EQUATIONS: LINEAR PERTURBATIONS OF FIRST AND SECOND TYPE

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Abstract. This paper studies the existence of solutions for a mixed system of hybrid diferential equations, it is a coupled hybrid diferential equations of first and second type. We make use of the standard tools of the fixed point theory to establish the main results. The existence and uniqueness result is elaborated with the aid of an example.

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1. Introduction

Hybrid differential equations is a rich field of differential equations. Tt is quadratic perturbations of non linear differential equations. It has lately years been an object of increasing interest because of its vast applicability in several fields. For more details about hybrid differential equations, we refer to [1, 2, 3, 4, 5, 6].
Motivated by [2, 3]. The propose of this paper is to study the following coupled system of hybrid differential equations with perturbations of first and second type.

$$\begin{cases} \dfrac{d}{dt}\left[\dfrac{x(t)}{f_1(t,x(t),y(t))}\right] = h_1(t,x(t),y(t)),\ t \in J = [0,a], \\ \dfrac{d}{dt}\left[y(t) - f_2(t,x(t),y(t))\right] = h_2(t,x(t),y(t)),\ t \in J, \\ x(0) = x_0, \\ y(0) = y_0, \end{cases} \tag{1.1}$$

where $f_1 \in \mathcal{C}(J \times \mathbb{R} \times \mathbb{R}; \mathbb{R} \setminus \{0\})$ and $f_1,\ h_1,\ h_2 \in \mathcal{C}(J \times \mathbb{R} \times \mathbb{R}; \mathbb{R})$.
We first start by recalling Lery Schauder alternative.

Lemma 1.1. *(Lery Schauder alternative, [38,page 4])*
Let $\Gamma : X \longrightarrow X$ be a completely continuous operator (i.e, map that is restricted to any bounded set X is compact). Let $\mathcal{P}_\Gamma = \{x \in X : x = \delta\Gamma x$ for some $0 < \delta < 1\}$. Then either the set $\mathcal{P}_\Gamma$ is unbounded or Γ has at least one fixed point.

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2. Existence result

In this section, we prove the existence and uniqueness result for (1.1) by using Banach fixed point theorem. Then we discuss the existence of solutions for this problem by means of Lery Schauder alternative.

To establish our results, we introduce the following assumptions:

($\mathbf{A}_0$) $(a)-$ The map $x \longrightarrow \dfrac{x}{f_1(t,x,y)}$ is increasing in $\mathbb{R}$ for each $t \in J$, $y \in \mathbb{R}$.

$(b)-$ The map $y \longrightarrow y - f_2(t,x,y)$ is increasing in $\mathbb{R}$ for each $t \in J$, $x \in \mathbb{R}$.

($\mathbf{A}_1$) There exists positive numbers μ_1, μ_2, ν such that

$$\nu \leq |f_1(t,x,y)| \leq \mu_1, \quad |f_2(t,x,y)| \leq \mu_2,$$

for all $(t,x,y) \in J \times \mathbb{R} \times \mathbb{R}$.

($\mathbf{A}_2$) There exists positive numbers λ_1, λ_2, λ_3, λ_4 such that

$$|f_1(t,x,y) - f_1(t,x',y')| \leq \lambda_1|x-x'| + \lambda_2|y-y'|,$$

$$|f_2(t,x,y) - f_2(t,x',y')| \leq \lambda_3|x-x'| + \lambda_4|y-y'|,$$

for all $t \in J$ and $x, y, x', y' \in \mathbb{R}$.

($\mathbf{A}_3$) There exists a constants η_1, η_2, ξ_1, ξ_2 such that

$$|h_1(t,x,y) - h_1(t,x',y')| \leq \eta_1|x-x'| + \eta_2|y-y'|,$$

$$|h_2(t,x,y) - h_2(t,x',y')| \leq \xi_1|x-x'| + \xi_2|y-y'|,$$

for all $t \in J$ and x, y, x', $y' \in \mathbb{R}$.

The following Lemmas are useful in what follows.

Lemma 2.1. *([3]) Assume that hypothesis $x \longrightarrow \dfrac{x}{f_1(t,x)}$ is increasing in $\mathbb{R}$ for each $t \in J$. Then for any $h \in L^1(J,\mathbb{R}_+)$, the function $x \in \mathcal{AC}(J,\mathbb{R}_+)$ is a solution of the hybrid Differential Equation*

$$\begin{cases} \dfrac{d}{dt}\left[\dfrac{x(t)}{f_1(t,x(t))}\right] = h_1(t), \ t \in J, \\ x(0) = x_0, \end{cases}$$

if and only if x satisfies the hybrid integral equation

$$x(t) = f_1(t,x(t))\left[\frac{x_0}{f_1(0,x_0)} + \int_0^t h_1(s)ds\right], \ t \in J.$$

Lemma 2.2. *([2]) Assume that hypothesis $y \longrightarrow y - f_2(t,y)$ is increasing in $\mathbb{R}$ for each $t \in J$. Then for any $h : J \longrightarrow \mathbb{R}_+$, the function $y \in \mathcal{C}(J,\mathbb{R}_+)$ is a solution of the hybrid Differential Equation*

$$\begin{cases} \dfrac{d}{dt}\left[y(t) - f_2(t,y(t))\right] = h_2(t), \ t \in J, \\ y(0) = y_0 \in \mathbb{R}. \end{cases}$$

if and only if y satisfies the hybrid integral equation

$$y(t) = y_0 - f_2(0,y_0) + f_2(t,y(t)) + \int_0^t h_2(s)ds, \ t \in J.$$

Denote $E = \mathcal{C}(J,\mathbb{R}) \times \mathcal{C}(J,\mathbb{R})$, equipped with the norm

$$\|(x,y)\| = \|x\| + \|y\|,$$

where $\|x\| = \sup_{t\in J} |x(t)|$.
Notice that the space E with this norm is a Banach space. Seeing that as in lemma 2.1 and 2.2 we define on E the operator Γ by

$$\Gamma(x,y)(t) = \begin{pmatrix} \Gamma_1(x,y)(t) \\ \\ \Gamma_2(x,y)(t) \end{pmatrix}$$

where

$$\Gamma_1(x,y)(t) = f_1(t,x(t),y(t)) \left[\frac{x_0}{f_1(0,x_0,y_0)} + \int_0^t h_1(s),x(s),y(s))ds \right],$$

$$\Gamma_2(x,y)(t) = y_0 - f_2(0,x_0,y_0) + f_2(t,x(t),y(t)) + \int_0^t h_2(s,x(t),y(t))ds.$$

As a results $\Gamma(x,y)$ is a solution to our problem under the assumption $(\mathbf{A}_0)$.
Let us set

$$\alpha_1 = a\mu_1\eta_1 + \lambda_1 \left(\frac{|x_0|}{\nu} + a\left(\eta_1 r + \eta_2 r + k_1\right) \right),$$

$$\beta_1 = a\mu_1\eta_2 + \lambda_2 \left(\frac{|x_0|}{\nu} + a\left(\eta_1 r + \eta_2 r + k_1\right) \right),$$

$$\alpha_2 = \lambda_3 + a\xi_1,$$

$$\beta_2 = \lambda_4 + a\xi_2,$$

$\gamma_1 = max(\alpha_1,\beta_1)$ and $\gamma_2 = max(\alpha_2,\beta_2)$, where $k_1 = \sup_{t\in J} |h_1(t,0,0)|$, $k_2 = \sup_{t\in J} |h_2(t,0,0)|$, and

$$r \geq \frac{\mu_1 \frac{|x_0|}{\nu} + |y_0| + 2\mu_2 + a(\mu_1 k_1 + k_2)}{1 - a(\mu_1(\eta_1+\eta_2) + \xi_1 + \xi_2)}. \tag{2.1}$$

Now we are in a position to state our first existence results. This result is based on Banach fixed point theorem.

Theorem 2.3. *Let assumptions $(\boldsymbol{A}_0)-(\boldsymbol{A}_3)$ be satisfied. Suppose, in addition that the following property is verified:*

$$\gamma_1 + \gamma_2 < 1$$

Then the problem (1.1) has a unique solution.

Proof. Let us define the closed ball

$$B_r = \{(x,y) \in E : \ \|(x,y)\| \leq r\},$$

Then we shall check that $\Gamma B_r \subseteq B_r$.

For $(x, y) \in B_r$ and $t \in J$, we have

$$\begin{aligned}
|\Gamma_1(x,y)(t)| &= |f_1(t,x(t),y(t))| \left| \frac{x_0}{f_1(0,x_0,y_0)} + \int_0^t h_1(s,x(s),y(s))ds \right| \\
&\leq \mu_1 \left(\frac{|x_0|}{|f_1(0,x_0,y_0)|} + \int_0^t \left(|h_1(s,x(s),y(s)) - h_1(s,0,0)| + |h_1(s,0,0)|\right) ds \right) \\
&\leq \mu_1 \left(\frac{|x_0|}{\nu} + \int_0^t \left(\eta_1|x(s)| + \eta_2|y(s)| + k_1\right) ds \right) \\
&\leq \mu_1 \left(\frac{|x_0|}{\nu} + a\left(\eta_1\|x\| + \eta_2\|y\| + k_1\right) \right).
\end{aligned}$$

Hence

$$|\Gamma_1(x,y)(t)| \leq \mu_1 \left(\frac{|x_0|}{\nu} + a\left(\eta_1 r + \eta_2 r + k_1\right) \right). \tag{2.2}$$

On the other hand, we have

$$\begin{aligned}
|\Gamma_2(x,y)(t)| &= \left| y_0 - f_2(0,x_0,y_0) + f_2(t,x(t),y(t)) + \int_0^t h_2(s,x(s),y(s))ds \right| \\
&\leq |y_0| + |f_2(0,x_0,y_0)| + |f_2(t,x(t),y(t))| + \int_0^t |h_2(s,x(s),y(s))|ds \\
&\leq |y_0| + 2\mu_2 + \int_0^t \left(|h_2(s,x(s),y(s)) - h_2(s,0,0)| + |h_2(s,0,0)|\right) ds \\
&\leq |y_0| + 2\mu_2 + \int_0^t \left(\xi_1|x(s)| + \xi_2|y(s)| + k_2\right) ds \\
&\leq |y_0| + 2\mu_2 + a\left(\xi_1\|x\| + \xi_2\|y\| + k_2\right).
\end{aligned}$$

Hence

$$|\Gamma_2(x,y)(t)| \leq |y_0| + 2\mu_2 + a\left(\xi_1 r + \xi_2 r + k_2\right). \tag{2.3}$$

From (2.1), (2.2) and (2.3), we deduce that

$$\|\Gamma(x,y)\| \leq r.$$

Thus $\Gamma B_r \subseteq B_r$.

For (x,y), $(x',y') \in B_r$ and $t \in J$, we have

$$\begin{aligned}
|\Gamma_1(x,y)(t) - \Gamma_1(x',y')(t)| &= \Bigg| f_1(t,x(t),y(t)) \left(\frac{x_0}{f_1(0,x_0,y_0)} + \int_0^t h_1(s,x(s),y(s))ds \right) - \\
&\quad f_1(t,x(t),y(t)) \left(\frac{x_0}{f_1(0,x_0,y_0)} + \int_0^t h_1(s,x'(s),y'(s))ds \right) + \\
&\quad f_1(t,x(t),y(t)) \left(\frac{x_0}{f_1(0,x_0,y_0)} + \int_0^t h_1(s,x'(s),y'(s))ds \right) - \\
&\quad f_1(t,x'(t),y'(t)) \left(\frac{x_0}{f_1(0,x_0,y_0)} + \int_0^t h_1(s,x'(s),y'(s))ds \right) \Bigg| \\
&\leq |f_1(t,x(t),y(t))| \int_0^t |h_1(s,x(s),y(s)) - h_1(s,x'(s),y'(s))| ds + \\
&\quad \left| \frac{x_0}{f_1(0,x_0,y_0)} + \int_0^t h_1(s,x'(s),y'(s))ds \right| |f_1(t,x(t),y(t)) - f_1(t,x'(t),y'(t))| \\
&\leq a\mu_1 \left(\eta_1 \|x - x'\| + \eta_2 \|y - y'\| \right) \\
&+ \left(\frac{|x_0|}{\nu} + a \left(\eta_1 \|x'\| + \eta_2 \|y'\| + k_1 \right) \right) \left(\lambda_1 \|x - x'\| + \lambda_2 \|y - y'\| \right) \\
&\leq a\mu_1 \left(\eta_1 \|x - x'\| + \eta_2 \|y - y'\| \right) + \left(\frac{|x_0|}{\nu} + a \left(\eta_1 r + \eta_2 r + k_1 \right) \right) \left(\lambda_1 \|x - x'\| + \lambda_2 \|y - y'\| \right) \\
&\leq \left(a\mu_1\eta_1 + \lambda_1 \left(\frac{|x_0|}{\nu} + a \left(\eta_1 r + \eta_2 r + k_1 \right) \right) \right) \|x - x'\| + \\
&\quad \left(a\mu_1\eta_2 + \lambda_2 \left(\frac{|x_0|}{\nu} + a \left(\eta_1 r + \eta_2 r + k_1 \right) \right) \right) \|y - y'\| \\
&= \alpha_1 \|x - x'\| + \beta_1 \|y - y'\| \\
&\leq \gamma_1 \left(\|x - x'\| + \|y - y'\| \right),
\end{aligned}$$

which yields

$$\|\Gamma_1(x,y) - \Gamma_1(x',y')\| \leq \gamma_1 \left(\|x - x'\| + \|y - y'\| \right). \tag{2.4}$$

And we have

$$\begin{aligned}
|\Gamma_2(x,y)(t) - \Gamma_2(x',y')(t)| &= \Bigg| y_0 - f_2(0,x_0,y_0) + f_2(t,x(t),y(t)) + \int_0^t h_2(s,x(s),y(s))ds - \\
&\quad y_0 + f_2(0,x_0,y_0) - f_2(t,x'(t),y'(t)) - \int_0^t h_2(s,x'(s),y'(s))ds \Bigg| \\
&\leq |f_2(t,x(t),y(t)) - f_2(t,x'(t),y'(t))| \\
&+ \int_0^t |h_2(s,x(s),y(s)) - h_2(s,x'(s),y'(s))| ds \\
&\leq \lambda_3 \|x - x'\| + \lambda_4 \|y - y'\| + a \left(\xi_1 \|x - x'\| + \xi_2 \|y - y'\| \right) \\
&\leq (\lambda_3 + a\xi_1) \|x - x'\| + (\lambda_4 + a\xi_2) \|y - y'\| \\
&= \alpha_2 \|x - x'\| + \beta_2 \|y - y'\| \\
&\leq \gamma_2 \left(\|x - x'\| + \|y - y'\| \right).
\end{aligned}$$

Then

$$\|\Gamma_2(x,y) - \Gamma_2(x',y')\| \leq \gamma_2 \left(\|x - x'\| + \|y - y'\| \right). \tag{2.5}$$

This implies by (2.4) and (2.5) that we have

$$\begin{aligned}
\|\Gamma(x,y) - \Gamma(x',y')\| &= \|\Gamma_1(x,y) - \Gamma_1(x',y')\| + \|\Gamma_2(x,y) - \Gamma_2(x',y')\| \\
&\leq (\gamma_1 + \gamma_2) \left(\|x - x'\| + \|y - y'\| \right).
\end{aligned}$$

Finally, we find that Γ is a contraction. This completes the proof. $\square$

Example 2.4. We give an example to illustrate our abstract results.

Consider the following coupled system

$$\begin{cases} \dfrac{d}{dt}\left[\dfrac{x(t)}{\frac{1}{4}+\frac{1}{10}\cos|x(t)|+\frac{1}{20}\frac{|y(t)|}{1+|y(t)|}}\right] = \dfrac{1}{7}+\dfrac{1}{9}x(t)+\dfrac{1}{10}y(t),\ t\in J=[0,1], \\ \dfrac{d}{dt}\left[y(t)-\left(\frac{1}{4}+\frac{1}{10}\sin|y(t)|+\frac{1}{20}\frac{|x(t)|}{1+|x(t)|}\right)\right] = \dfrac{1}{7}+\dfrac{1}{9}y(t)+\dfrac{1}{10}x(t),\ t\in J, \\ x(0)=0, \\ y(0)=0. \end{cases} \tag{2.6}$$

This problem can be abstracted into

$$\begin{cases} \dfrac{d}{dt}\left[\dfrac{x(t)}{f_1(t,x(t),y(t))}\right] = h_1(t,x(t),y(t)),\ t\in J=[0,1], \\ \dfrac{d}{dt}\left[y(t)-f_2(t,x(t),y(t))\right] = h_2(t,x(t),y(t)),\ t\in J, \\ x(0)=x_0, \\ y(0)=y_0, \end{cases}$$

where

$$f_1(t,x(t),y(t)) = \frac{1}{4}+\frac{1}{10}\cos|x(t)|+\frac{1}{20}\frac{|y(t)|}{1+|y(t)|},\ h_1(t,x(t),y(t)) = \frac{1}{7}+\frac{1}{9}x(t)+\frac{1}{10}y(t),$$

$$f_2(t,x(t),y(t)) = \frac{1}{4}+\frac{1}{10}\sin|y(t)|+\frac{1}{20}\frac{|x(t)|}{1+|x(t)|},\ h_2(t,x(t),y(t)) = \frac{1}{7}+\frac{1}{9}y(t)+\frac{1}{10}x(t),$$

and $x_0=0$, $y_0=0$.

It is easy to check that

$$\eta_1=\frac{1}{9},\ \eta_2=\frac{1}{10},\ \xi_1=\frac{1}{10},\ \xi_2=\frac{1}{9},\ k_1=k_2=\frac{1}{7},\ \mu_1=\mu_2=\frac{2}{5},\ \lambda_1=\lambda_4=\frac{1}{10},\ \lambda_2=\lambda_3=\frac{1}{20}.$$

We have

$$\frac{\mu_1\frac{|x_0|}{\nu}+|y_0|+2\mu_2+a(\mu_1k_1+k_2)}{1-a(\mu_1(\eta_1+\eta_2)+\xi_1+\xi_2)} \simeq 1.419.$$

Then, for $r=1.5$, we have

$$\alpha_1\simeq 0.09,\ \beta_1\simeq 0.06,\ \alpha_2\simeq 0.15,\ \beta_2\simeq 0.21.$$

Consequently

$$\gamma=\gamma_1+\gamma_2\simeq 0.3<1.$$

Thus, all assumptions in Theorem 2.3 are satisfied and the problem (2.6) has a unique solution on J.

By using Lery Schauder alternative we obtain the second existence result

Theorem 2.5. *Let assumptions* $(\boldsymbol{A}_0)$ *and* $(\boldsymbol{A}_1)$ *be satisfied. Suppose in addition that*

$$|h_1(t,x,y)|\le \rho_0+\rho_1\|x\|+\rho_2\|y\|,\ |h_2(t,x,y)|\le \sigma_0+\sigma_1\|x\|+\sigma_2\|y\|,$$

for each $(t,x,y)\in J\times\mathbb{R}\times\mathbb{R}$.
And $a(\mu_1\rho_1+\sigma_1)<1$, $a(\mu_2\rho_2+\sigma_2)<1$. *Then the problem (1.1) has at least one solution.*

Proof. Let $\mathcal{M} \subseteq E$ be bounded.

Then we can find positive constants N_1, N_2 such that

$$|h_1(t, x(t), y(t))| \leq N_1, \ |h_2(t, x(t), y(t))| \leq N_2$$

for each $(x, y) \in \mathcal{M}$ and $t \in J$.

For $(x, y) \in \mathcal{M}$, $t \in J$, we have

$$\begin{aligned} |\Gamma_1(x,y)(t)| &= |f_1(t, x(t), y(t))| \left| \frac{x_0}{f(0, x_0, y_0)} + \int_0^t h_1(s, x(s), y(s)) ds \right| \\ &\leq \mu_1 \left(\frac{|x_0|}{\nu} + aN_1 \right). \end{aligned}$$

Then

$$\|\Gamma_1(x, y)\| \leq \mu_1 \left(\frac{|x_0|}{\nu} + aN_1 \right). \tag{2.7}$$

In a similar manner, we obtain

$$\|\Gamma_2(x, y)\| \leq |y_0| + 2\mu_2 + aN_2.. \tag{2.8}$$

From (2.7) and (2.8), we deduce that Γ is uniformly bounded.

We will show that the operator Γ is equicontinuous.

Let t_1, $t_2 \in J$ with $t_1 < t_2$. For $(x, y) \in E$, we have

$$\begin{aligned} |\Gamma_1(x,y)(t_2) - \Gamma_1(x,y)(t_1)| &= \Bigg| f_1(t_2, x(t_2), y(t_2)) \left(\frac{x_0}{f(0,x_0,y_0)} + \int_0^{t_2} h_1(s,x(s),y(s)) ds \right) - \\ & \quad f_1(t_2, x(t_2), y(t_2)) \left(\frac{x_0}{f(0,x_0,y_0)} + \int_0^{t_1} h_1(s,x(s),y(s)) ds \right) + \\ & \quad f_1(t_2, x(t_2), y(t_2)) \left(\frac{x_0}{f(0,x_0,y_0)} + \int_0^{t_1} h_1(s,x(s),y(s)) ds \right) - \\ & \quad f_1(t_1, x(t_1), y(t_1)) \left(\frac{x_0}{f(0,x_0,y_0)} + \int_0^{t_1} h_1(s,x(s),y(s)) ds \right) \Bigg| \\ &\leq \mu_1 \int_{t_1}^{t_2} |h_1(s,x(s),y(s))| ds + \\ & \quad \left(\frac{|x_0|}{\nu} + \int_0^{t_1} |h_1(s,x(s),y(s))| ds \right) |f_1(t_2,x(t_2),y(t_2)) - f_1(t_1,x(t_1),y(t_1))| \\ &\leq \mu_1 N_1 (t_2 - t_1) + \left(\frac{|x_0|}{\nu} + aN_2 \right) |f_1(t_2,x(t_2),y(t_2)) - f_1(t_1,x(t_1),y(t_1))| \to 0 \end{aligned}$$

as $t_2 \to t_1$.

Similarly, we have

$$\begin{aligned} |\Gamma_2(x,y)(t_2) - \Gamma_2(x,y)(t_1)| &= \Bigg| y_0 - f_2(0,x_0,y_0) + f_2(t_2,x(t_2),y(t_2)) + \int_0^{t_2} h_2(s,x(s),y(s)) ds - \\ & \quad y_0 + f_2(0,x_0,y_0) - f_2(t_1,x(t_1),y(t_1)) - \int_0^{t_1} h_2(s,x(s),y(s)) ds \Bigg| \\ &\leq |f_2(t_2,x(t_2),y(t_2)) - f_2(t_1,x(t_1),y(t_1))| + \int_{t_1}^{t_2} |h_2(s,x(s),y(s))| ds \\ &\leq |f_2(t_2,x(t_2),y(t_2)) - f_2(t_1,x(t_1),y(t_1))| + N_2(t_2 - t_1) \to 0 \text{ as } t_2 \to t_1. \end{aligned}$$

Finally, we find that Γ is equicontinuous.

In the next step. Denote

$$\mathcal{P}_\Gamma = \{(x,y) \in E : \ (x,y) = \delta\Gamma(x,y), \ 0 \leq \delta \leq 1\}.$$

And

$$\varrho = \min\left(1 - a\mu_1\rho_1 - a\sigma_1, 1 - a\mu_1\rho_2 - a\sigma_2\right).$$

$\mathcal{P}_\Gamma$ is bounded. Indeed

Let $(x, y) \in \mathcal{P}_\Gamma$. Thus for any $t \in J$

$$\begin{cases} x(t) = \delta\Gamma_1(x, y) \\ y(t) = \delta\Gamma_2(x, y). \end{cases}$$

Then

$$|x(t)| \le \mu_1 \left(\frac{|x_0|}{\nu} + a(\rho_0 + \rho_1\|x\| + \rho_2\|y\|) \right).$$

Also

$$|y(t)| \le |y_0| + 2\mu_2 + a(\sigma_0 + \sigma_1\|x\| + \sigma_2\|y\|).$$

Which imply that

$$\|x\| \le \mu_1 \left(\frac{|x_0|}{\nu} + a(\rho_0 + \rho_1\|x\| + \rho_2\|y\|) \right).$$

And

$$\|y\| \le |y_0| + 2\mu_2 + a(\sigma_0 + \sigma_1\|x\| + \sigma_2\|y\|).$$

As a consequence, we have

$$\|x\| + \|y\| \le (a\mu_1\rho_1 + a\sigma_1)\|x\| + (a\mu_1\rho_2 + a\sigma_2)\|y\| + \mu_1 \left(\frac{|x_0|}{\nu} + a\rho_0 \right) + |y_0| + 2\mu_2 + a\sigma_0.$$

Thus we obtain

$$(1 - a\mu_1\rho_1 - a\sigma_1)\|x\| + (1 - a\mu_1\rho_2 - a\sigma_2)\|y\| \le \mu_1 \left(\frac{|x_0|}{\nu} + a\rho_0 \right) + |y_0| + 2\mu_2 + a\sigma_0.$$

It follows that

$$\|(x, y)\| = \|x\| + \|y\| \le \frac{\mu_1 \left(\frac{|x_0|}{\nu} + a\rho_0 \right) + |y_0| + 2\mu_2 + a\sigma_0}{\varrho}$$

Thus, all assumptions of Lemma (1.1) are satisfied and this permits us to conclud that Γ has at least one fixed point. Which is a solution of the problem (1.1). □

Conclusion

The aim of this paper is to discuss the existence of solutions for a mixed system of hybrid diferential equations. Our results improve and generalize some known results.

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