THE IMPETUS FOR TEACHING ALGEBRA IN THE EARLY GRADES

Assist. Prof. Hülya Kılıç

Abstract

Algebra is one of the core subjects of secondary school mathematics. Having weak conceptual understanding of algebra and a low level of algebraic thinking skills causes low student performance in mathematics courses. Therefore, some scholars suggest introducing algebraic concepts in the elementary level to help students succeed in mathematics. The goal of this paper is to examine some of the current practices and studies on teaching algebra in the elementary grades and discuss their implications on curriculum development and teaching.

Keywords: Algebra, algebraic thinking, elementary mathematics.

Özet

Cebir, ortaöğretim matematiğinin temel konularından biridir. Cebiri kavramsal olarak anlamadaki eksiklik ve cebirsel düşünme becerilerinin zayıflığı, öğrencilerin matematik derslerindeki performanslarının düşük olmasına neden olmaktadır. Bu nedenle bazı akademisyenler, öğrencilerin matematikte başarılı olmasına yardımcı olacağı için cebirsel kavramların ilköğretim birinci kademede verilmesini önermektedir. Bu çalışmanın amacı, ilköğretim birinci kademede cebir öğretimi ile ilgili varolan uygulamaları ve yapılan çalışmaları incelemek ve bunların öğretim programı geliştirmeye ve öğretime yüklediği anlamı tartışmaktır.

Anahtar sözcükler: Cebir, cebirsel düşünme, ilköğretim birinci kademe

Given its important role in mathematics as well as its role as a gatekeeper to future educational and employment opportunities, algebra has become a focal point of research efforts in mathematics education (Knuth, Stephens, McNeil, & Alibali, 2006). Having increased number of students struggle with understanding algebra and obtaining lower scores from the related parts of the international assessment studies such as TIMSS and PISA urge researchers, teachers, policymakers, and curriculum developers of the countries to investigate the causes of the failure in understanding and learning algebra and to figure out possible actions to be taken to eliminate them. The results of several research on improving students' performance on algebra and promoting algebraic thinking imply that teaching algebra in the early years of schooling might be one of the major steps to enhance students' mathematical understanding and algebraic thinking (Bastable & Schifter, 2008; Blanton & Kaput, 2005; Carraher, Schliemann, & Schwartz, 2008; Ferrini-Mundy, Lappan, & Phillips, 1997; Kieran, 2004; Yackel, 1997).

Many scholars argued that algebra should become a part of elementary education (Carraher, Schliemann, Brizuela, & Earnest, 2006) despite of the opposite views of others that young children are incapable of learning algebra because they do not have the cognitive ability to handle algebraic concepts like variables and functions (Tierney & Monk, 2008). However, teaching algebra in elementary school is not a new idea. In a few countries like Japan, China, Singapore, and Russia, some algebraic concepts, at least implicitly, are taught in the elementary grades. Furthermore, in recent years, many countries revised their elementary and middle school mathematics curricula with an intention of improving students' mathematical understanding, in particular understanding of algebra (Cai, 2004a). In this paper, I present a few examples from the countries that algebra is already taught in the elementary school, then give examples from such curricular reforms and discuss the effectiveness of suggested activities on learning and understanding algebra in the early grades. Beforehand, I briefly explain how scholars view algebra and algebraic thinking and how such views are conveyed in recent secondary mathematics curricula.

Algebra and Algebraic Thinking

Algebra is one of the major branches of mathematics whose origin is based on the studies of arithmetic and geometry in ancient times. Kieran (1992) defined algebra as "the branch of mathematics that deals with symbolizing general numerical relationships and mathematical structures and with operating on those structures" (p.391). Similarly, Sfard and Linchevski (1994) identified operational and structural phases of algebra such that operational algebra "like arithmetic, deals (at least at its early stages) with numbers and with numerical computations, but it asks questions of a different type and treats the algorithmic manipulations in a more general way" (p. 196). A structural algebra, on the other hand, entails excessive use of symbols and algebraic notations.

Kieran (2007) also identified three types of school algebra activities. First, algebra involves "generational" activities where situations are generated into equations or expressions. For instance, writing equations containing an unknown to represent problem situations or deriving a rule for the relationships embedded in given numerical sequences could be counted as generational activities. Second, there are "transformational" or rule-based activities such as collecting similar terms, factoring and simplifying expressions. Third, there are "global, meta-level activities" where algebra is used as a tool. For instance, problem solving, modeling, generalizing, analyzing relationships, and justifying are meta-level activities and they are also essential to other activities of algebra. Similarly, Kaput (2008) stated that there are three strands of algebra which are compatible with Kieran's view of school algebra activities. The first strand includes generalizing arithmetic operations and their properties and more general relationships and their forms. The second strand includes the study of functions, relations, and joint variation. The third strand includes modeling of different situations. Furthermore, he noted that at more advanced levels the first strand leads to abstract algebra and the second strand leads to calculus and analysis.

There are different views about what algebraic thinking refers to in school algebra. Blanton and Kaput (2005) conceived algebraic thinking as students' activity of generalizing given data and mathematical relationships, establishing those generalizations through conjecture, and arguing and expressing them in increasingly formal ways. They discussed different forms of algebraic thinking such that (1) it might be using arithmetic as a domain for expressing and formalizing generalizations (generalized arithmetic), (2) it might be generalizing numerical patterns to describe patterns and functional relationships (functional thinking), (3) it might be modeling as a domain for expressing and formalizing generalizations, and (4) it might be generalizing about operations and properties associated with numbers. Their definition for algebraic thinking emphasizes both the importance and the ability of understanding variations and functional relations of variables.

Although generalizing number patterns, recognizing relationships and the similarities and differences between mathematical representations are conceived as involved in algebraic thinking (e.g., Curcio & Schwartz, 1997; Ferrini-Mundy, Lappan, & Phillips, 1997; Slavit, 1999), Kieran (1989) disagreed with the idea that generalization is equivalent to algebraic thinking rather; algebraic thinking is a necessary component for the use of algebraic symbolism in order to reason about and express that generalization. Kieran (2004) argued that algebraic thinking can be interpreted as an approach to quantitative situations that emphasizes the general relational aspects with tools that are not necessarily letter symbolic, but which can be used as cognitive support for introducing and for sustaining more traditional discourses of school algebra. She noted that students do not need to use letter symbolic algebra to analyze relationships between quantities, notice structures, justify their reasoning or prove conjectures.

Briefly, in school settings, algebra is studied in the form of generalizing, forming and solving equations, and working with functions and formulas (Bell, 1995). Teachers put emphasis on simplifying algebraic expressions, solving equations, inequalities, and the systems of equations and factoring polynomials and rational numbers (Kaput, 1999; Kieran 2007). Hence, algebraic thinking refers to students' ability to understand algebraic concepts and to deal with all related procedures and facts both in deductive and inductive manner.

Algebra in Elementary School Curricula

Traditional elementary school mathematics involves only teaching arithmetic procedures and students are introduced to algebra in the middle school (Cai & Knuth, 2005; Fujii & Stephens, 2008; Johanning, 2004; Kaput, 2008; Kieran, 1992; Tierney & Monk, 2008). However, teaching algebra separately from arithmetic is found to be unsuccessful practice in terms of student achievement in algebra (Blanton & Kaput, 2005; Carraher, Schliemann, & Schwartz, 2008; Herscovics & Linchevski, 1994). In the countries discussed below, algebraic concepts and arithmetic are taught simultaneously to emphasize the relationships between arithmetic and algebra and to facilitate students' understanding of more complex algebraic concepts taught in later grade levels.

Watanabe (2008) stated that a smooth transition from arithmetic to algebra is the core idea of Japanese elementary curriculum. Students begin to discuss fundamental algebraic concepts such as variables and functions implicitly during the second grade. The function concept is first introduced when the students learn about multiplication. Teachers encourage students to explore the relationship between a multiplicand and the product such that they want students to pay attention to how the product changes as one of the multiplicand changes. Thus, the students not only practice with the arithmetic of multiplication operation but also realize how multiplication function works. Moreover, in the upper elementary level, students are asked to figure out the relationship between two varying quantities. Teachers give concrete examples such as how the depth of a cup changes with respect to the amount of water in the cup changes or how the length of a rectangle changes with respect to its width providing that the area remains the same. Watanabe also stated that a special attention is given to expressing ideas and relationships embedded in the problems by using mathematical notations. He noted that writing and interpreting mathematical expressions involving arithmetic operations and also using symbols like □, Δ, x in mathematical expressions are emphasized in the Japanese curriculum. For instance, students are expected to interpret 3 + 4 as “4 objects are added to 3 objects” or “4 objects more than 3 objects.” Similarly, they are expected to interpret “3 + □ = 5” as “adding 3 to a number makes 5.” Furthermore, Japanese teachers emphasize expressing mathematical expressions in words. For instance, a 4-by-6 rectangle, the area is found by 4 x 6 = 24. The teacher asks students to write what each number represents, that is “length x width = area.” Students’ ability to make such interpretations can be thought as an example of what Blanton and Kaput (2005) suggested for the forms of algebraic thinking described in the previous section. Both examples are about *generalizing about operations and properties associated with numbers* because in the former example, students are expected to know what addition operation means and in the latter one, they are expected to make connections between the numbers and what each of them represents for in a rectangle. Watanabe indicated that studying such fundamental algebraic concepts in the elementary level helps students gain a deeper understanding of algebra and be successful in secondary school mathematics.

The idea of teaching algebraic concepts and arithmetic simultaneously in the elementary level is also seen in other countries such as China, Singapore, Russia, and the Netherlands. Cai and Moyer (2008) stated that the main goal of Chinese and Singaporean elementary school curricula is to make connections between arithmetic and algebra to facilitate students’ algebraic thinking abilities. They provided some examples from both curricula to show how they would achieve such a goal. They noted that in Chinese elementary schools, the first graders are introduced addition and subtraction operations simultaneously. The students are asked to solve equations written in the form of “1 + ( )= 3”. They are expected to find the value inside the parentheses by doing inverse operations. The same format is used for division and multiplication during the second grade. Cai and Moyer also indicated that because in Chinese elementary schools teachers use both arithmetic and algebraic approaches to solve the problems, students could attain a better understanding of quantitative relationships and have opportunity to explore the similarities and differences between arithmetic and algebra.

In the Singaporean elementary schools, students are expected to solve problems by using pictorial representations. The most common representation is strip diagrams. For instance, students use the pictorial representation shown in Figure 1 to solve the problem: I had $51. After buying 3 watermelons, I had $30 left. Find the cost for 1 watermelon.

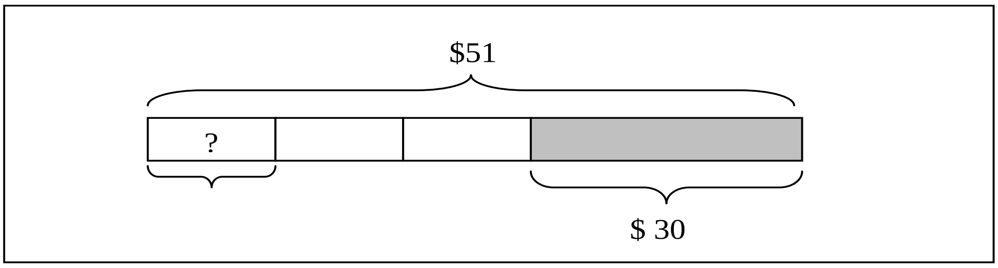

Figure 1. Example of a strip diagram

The students draw a strip to represent whole money and then shade a part which is not spent. Then they divide the remaining part 3 equal rectangles to represent 3 watermelons. They work in backwards to solve the problem. First, they subtract the amount which is not spent and then divide the remaining amount by 3 to find the price of one watermelon. The students are given a bit difficult problems in the fourth and fifth grade but they could solve them by using appropriate strip diagrams because strip diagrams serve as a concrete representation that helps students visualize the problems. Using strip diagrams would definitely contributes to students' algebraic thinking because students do not use formal algebraic notations but organize the given information to model the problem situation and understand the relationships between the quantities given in the problem (Ferrucci, Kaur, Carter & Yeap, 2008). Cai and Moyer (2008) stated that the students in later grade levels could easily write the algebraic equation represented in strip diagrams by replacing “x” for the unknown value. For instance, the students could find the algebraic equation for the problem given above as $3x + 30 = 51$ by replacing “x” for the value of small rectangle.

In the Netherlands, one of the major goals of mathematics curriculum is to provide opportunities for students to understand the connections between mathematics and reality by applying mathematics in practical situations (van den Huevel-Panhuizen & Wijers, 2005). The elementary students are expected to understand the pattern embedded in a set of numbers or shapes and the mathematical language that includes formal and informal notations, representations, tables, and graphs. The students are given problems that they first solve arithmetically and then explain the reason underlying those arithmetic operations. For instance, the students can solve the problem “I had 5 Euro. I bought a chocolate for 2 Euro. How much money is left?” as “5 – 2 = 3” and then explain that they use subtraction because when they buy something the amount of money they have decreases so they need to take the spent amount away from the initial amount. The students are also asked to make generalizations for given number patterns or repeated situations (e.g. the relationship between the numbers of chocolate bar is bought and how much is paid for the total).

Although arithmetic and algebra is taught together in Russian elementary schools, the sequence of the topics is different than the countries discussed above. In Russia, algebra is introduced before arithmetic. Schmittau and Morris (2004) stated that the students study algebraic generalizations first and they use arithmetic as a concrete application of these algebraic generalizations. For instance, students compare the length of two objects and identify the relationships between them as A=B or A<B or A>B, then they are expected to use such relationships when they are given numerical values for length.

Briefly, the examples given above revealed that the goal of elementary school algebra is to raise students' awareness about the relationship between arithmetic and algebra. The scholars indicated that students are able to understand the relationship and use it to solve problems. They also noted that learning algebraic concepts in the early grades contribute to the development of students' algebraic thinking skills.

Teaching Algebra in the Early Grades

Teaching algebra in elementary level refers to elaboration of students' ability of algebraic thinking and reasoning rather than emphasizing complicated algebraic activities. The studies on elementary school mathematics revealed that elementary students are capable of learning fundamental unifying ideas that are the foundations of both arithmetic and algebra (Carpenter, Franke, & Levi, 2003; Clements & Sarama, 2007). The scholars noted that learning and articulating these ideas both enhance students' understanding of arithmetic and provide them with a concrete basis extending their knowledge of arithmetic to learn algebra. This conclusion is compatible with the main goal of the elementary school algebra discussed in the previous section. However, in recent studies the scholars are not only discussing how to achieve a smooth transition from arithmetic to algebra but also investigating whether elementary students are able to make generalizations, understand the concepts of variable and function, and use algebraic notations. In this section, I present examples from such studies and discuss their findings.

Many scholars investigated whether students are able to recognize the relationships in a given pattern and make a generalization in the early grades (e.g., Curcio & Schwartz, 1997; Threlfall, 1999; Warren & Cooper, 2008; Willoughby, 1997). The studies revealed that even in the kindergarten level students are able to recognize the patterns, extend them, and construct their own patterns. For instance, Warren and Cooper (2008) indicated that 5-year old children participated in their study were capable of recognizing growing patterns such that given geometric representations of number pattern 2, 4, and 6 as a group of small squares, they could recognize the next group would consist of 8 squares, the next one would 10 squares, etc. They also noted that 7-year old children could find the total number of figures or letters in the given pattern and 8-year old children could answer complex questions about the nature of the pattern by making inferences about the given piece of the pattern. For instance, when they were given a pattern like RRGGGRRGGGRR, they could find how many R would be in the set of 60 letters or how many R would be in the set when the pattern was repeated 100 times. Additionally, Warren and Cooper asked students to generalize the pattern for *n* repeats. They stated that some of the students were able to find the answer. They noted that as students practice more on the deconstruction and reconstruction of the given pattern they generalize the given pattern more easily.

The studies on algebraic thinking skills revealed that these skills could be improved by providing opportunities for students work on different subjects of elementary mathematics. Carraher, Schliemann, and Schwartz (2008) conducted a longitudinal study to investigate characteristics of early algebra and development of algebraic thinking skills through the observation of four classes from the second half of the second grade to the end of the fourth grade. They prepared activities related with fractions, ratio, proportion, four operations and negative numbers and each semester the students participated in six to eight activities. They emphasized that throughout one and half a year the students' algebraic thinking had been improved. One of their activities aimed to achieve transition from a particular situation to generalization. They presented a "candy box problem" to the third grade students such that one of the researchers held two boxes in his hand and said that the box in his left hand was John's and the box in his right hand was Mary's. He threw away three candies from Mary's box and put them on the top of the box, thus the number of the candies in each box became equal. The researchers gave students a box of candies and asked them to guess the number of candies in each box without opening the boxes. Then the researcher made a table of students' answers most of which had the same pattern: "The number of Mary's candies is three more than John's candies." Then the researcher asked that what would be the number of Mary's candies if John had N candies where N can be any number. His way of phrasing the question puzzled students since they thought that the number of Mary's candies would be N because N was "any number". Then the researcher rephrased his statement as N could stand for any number so that some of the students were able to figure out that the number of Mary's candies, which is N+3. Although students did get confused about using variables and making generalizations in this problem, they performed better when they were asked to work on a similar problem at the beginning of the second semester of the fourth grade. Carraher et al. asked the following problem to the students:

> Mike has $8 in his hand and the rest of his money is in his wallet; Robin has exactly 3 times as much money as Mike has in his wallet. What can you say about the amounts of money Mike and Robin have? (p.248).

Carraher et al. stated that 16 students out of 63 represented the amount of Mike's money as N+8 and the amount of Robin's money as N+N+N or 3N or N*3 while others used wallet symbols to represent that relation or used algebraic notation but omitted the signs between them. For instance, they wrote "N 8" for Mike's money and "N N N" for Robin's money. Later in the term, Carraher et al. revisited this problem and modified it to discuss solving equations and graphs. The researcher used a table to show what might be some of the points of the graph and then the students plotted the graph. In order to show how to solve an equation, the researcher wrote the equation for the modified problem as "W+8=3W". Some of the students guessed that the answer would be 4. However, the aim of the researcher was to show them how to simplify the equation by eliminating like terms. At the end of the lesson, students were able to figure out the answer by solving the equation.

Two conclusions could be drawn from Carraher and his colleagues' study. The first one is the same as what Warren and Cooper (2008) concluded about patterning activities. The elementary graders are capable of recognizing patterns and make generalizations. However, teachers should be careful about expressing the mathematical terms. In this case, using phrase "any number" was confusing for the students. Instead, the teacher might rephrase it as what the number of candies in Mary's box would be if there were N candies in John's box. The second conclusion is that in the upper elementary level students are more capable of working with algebraic notations and symbols. In this case, fourth graders were able to represent Mike's money as 3N. However, only 25% of the students represented it correctly. To increase the number of students who represent the problem correctly, Singaporean strip diagrams could be used by the teachers. Because strip diagram help students visualize the problem and understand the reasoning underlying the algebraic expressions.

Bastable and Schifter (2008) also investigated the development of algebraic thinking skills by using different tasks in elementary classrooms. They indicated that when the students were given opportunity to discuss their answers for given arithmetic questions or geometric representations they were able to formulate and test generalizations. For instance, in a fourth grade class they observed that the students figured out some properties of square numbers like "if you multiply a square number by a square number, you'll get a square number". Additionally, one of the students found out that "if you take two consecutive numbers, add the lower number and its square to the higher number, you get the higher number's square." His initial example was $2+2^2+3=3^2$, and then his friends found other examples to confirm his conjecture. Bastable and Schifter stated that such activities not only contribute to the development of students' algebraic thinking abilities but also facilitate transition from numbers to algebraic notation. The examples given above could be represented as $a^2 \cdot b^2 = c^2$ where $c = a \cdot b$, and $n + n^2 + (n+1) = n^2 + 2n + 1 = (n+1)^2$, respectively. Although the students may not be able to figure out these generalizations in the elementary level, such problems can be revisited in the middle school while teaching algebraic notations and identities.

Carraher and his colleagues (Carraher, Schliemann, Brizuela, & Earnest, 2006) indicated that algebraic notation can play a supportive role in learning mathematics in the early grades. They stated that symbolic notation, number lines, function tables and graphs are powerful tools that students use to understand and express functional relationships across a wide variety of problem context. They argued that students could achieve the transition from arithmetic to algebra when they were introduced with tables, graphs and algebraic symbolic notations gradually. Tierney and Monk (2008) investigated how the fifth graders made sense of “change” through graphs and tables. One of the tasks they gave students was about comparing graphs of growth of two plants to decide which one was growing faster. The first line started from the origin with a slope approximately 1 and the second line started from a point on the y-axis with a slope approximately 1/2. The students realized that the first plant reached to the same height with the second one within the same amount of time although its height was approximately zero at the beginning. They also interpreted that the steepness of a line shows how fast it grows. It was evident that the students were able to make inferences about the relationships between two varying quantities, in this case, time and height. Another task was about creating a table for the given story problem for a trip and then constructing its graph. One of the problems was follows: Walk very slowly about a quarter of the distance, stop for about 6 seconds, and then walk fast to end. The students draw a time-distance table for the given story. All students were able to fill out the table according to the given information and then draw its graph. Because these students were able to make tables fit to given problems and interpret the graphs of linear lines, Tierney and Monk suggested developing a curriculum that facilitates transition from arithmetic to algebra through representation of varying quantities in stories or graphs. Indeed, working on varying quantities is common in Japanese curriculum such that students practice on the relationships between two varying quantities in different contexts.

In this section, I have given a few examples from the studies on teaching algebra in the early grades. Although those studies were administered on a limited number of students, the findings support the view that students could learn algebraic concepts in the early grades and teaching some basic algebraic facts in the elementary level contributes to the development of algebraic thinking skills of students.

Implications for Curriculum Reforms

Teaching algebra is one of the most popular issues in mathematics education because many students still suffer from learning and understanding algebraic concepts. There is an emerging consensus that reformative actions on teaching algebra require reconceptualizing the nature of algebra in school mathematics (Cai, 2004b). Many mathematics educators advocate that children should be introduced to algebraic concepts and be given opportunities to improve their algebraic thinking skills in the early years of schooling rather than waiting for the middle school years (Carraher et al., 2006).

As discussed above, in a few countries algebra is already taught as a part of elementary school curriculum. It is noted that, in those countries, students are able to recognize patterns, make generalizations about simple patterns and solve simple algebraic problems by using representations or symbols. The effectiveness of those practices could be thought as impetus for curriculum developers of other countries where students struggle with understanding algebra in the middle school (Cai & Moyer, 2008). They should either suggest similar practices for their elementary school students or design new activities that would be more appropriate for their students and aligned with the requirements of their secondary school curriculum.

In Turkey, the new elementary mathematics curriculum was launched in 2006. The curriculum is aimed to foster students' mathematical thinking and learning through various activities. Although algebra is not identified as a major subject area in the elementary level, students are introduced with algebraic concepts such as finding relationships in number patterns during the fourth grade. Then, students are formally introduced with algebra in the sixth grade. Previously, the students began to learn algebra in the seventh grade although teachers were using some symbols like □ and Δ to represent unknown values when solving arithmetic or simple word problems in the fourth or the fifth grade. Then, students were used to replace such symbols with letters like *x* and *y* in the seventh grade. In Turkey, there are not large scale studies investigating effectiveness of the new elementary curriculum. Some small scale investigations (e.g., Akkan, Çakıroğlu, & Güven, 2009; Gürbüz & Akkan, 2008; Yenilmez & Teke, 2008) revealed that elementary students are able to understand some algebraic concepts like variables but they have difficulty with using them in different contexts such as carrying out operations between variable expressions or writing a problem statement for given algebraic equation. Therefore, there is an immediate need for large scale studies on the new curriculum to elicit whether it contributes to the development of students' algebraic thinking.

The results of the studies presented in the previous section support the fact that elementary students are able to recognize the rule of the patterns and make inferences about them. The scholars noted that activities about number patterns may facilitate transition from arithmetic to algebra (e.g., Warren & Cooper, 2008). For instance, when students are asked to find the relationship between the entries of ordered pairs (2, 5), (4, 7), (8, 11), … they are able to conclude that the second number is 3 more than the first number and represent the second entry as $n+3$ when the first entry is given as n. Students can also find one of the missing numbers in such pairs because they know the relationship between the entries.

Elementary grade students could be successful at patterning activities but they might have difficulty in understanding algebraic notations. In the countries mentioned above, although elementary students are introduced with the idea of using letters to represent unknown values and scholars noted that students are able to deal with variables and solve equations (Cai, 2004a), students' understanding might be procedural rather than conceptual. For instance, they may overgeneralize the solution of algebraic equations. If they have learned that to find the value of *a* in "a + 2 = 8" they subtract 2 from 8 then they may apply the same operation for "2 x a = 8". They may not differentiate the meaning of "a + 2" and "2 x a." In some textbooks variables are used to represent rules or identities such as A=lw (Area=length x width) or a+b = b+a (commutative property of addition) but such representations before giving away the definition of a variable may not be meaningful for students. They either try to memorize the rules without paying attention to what letters stands for or totally neglect them (Driscoll, 1999). Furthermore, they may assume that an unknown or variable, say *n*, has a single value. For instance, if they have found that *n* is 3 for "2n + 4 = 10" then they may assume that it is still 3 for "3n – 1 = 5." Therefore, curriculum developers and teachers should design or select activities that enable elementary students understand the meaning of algebraic notations. They particularly should pay attention to the language they use because students' language skills may not be developed yet. As I indicated in the previous section, when Carraher et al. (2008) told the students that the number of John's candies is "N" where "N" can be "any number", students replied him back that the number of Mary's candies would also be "N" because "N" could be any number. In that case, Carraher focused on the letter "N" rather than emphasizing the relationship between the John's and Mary's candies such that "N" is used to generalize that relationship. In order not to suffer from such misinterpretations, teachers should assign simple word problems for students and use appropriate phrases that students could understand *the* mathematics involved in the problem correctly. For instance, to address the misconception that an unknown or a variable has a single value, the teacher may tell that there are people with the same name as another (namesake) but each person has different characteristics. Therefore, value of a letter that represents an unknown or a variable may be different in each problem setting.

Although one of the reasons underlying the curricular reforms in elementary mathematics was getting lower scores in international assessment studies, there are no large-scale studies investigating either the effects of teaching algebra in the early grades on the students' performance in international exams or the effects on students' understanding and learning algebra in the later grades (Cai & Knuth, 2005). The researchers should investigate the effectiveness of such intervention on students' performance in international exams by analyzing students' algebra scores in those exams. However, obtaining reliable data about the effectiveness of new elementary curriculum on students' achievement in algebra in later grades entails examination of year-by-year records of students who have begun to learn algebra in the elementary school. It is hard to keep that much information for large group of students therefore many scholars preferred working with small groups. Because the studies with small groups revealed that teaching algebra in the early grades contributes to the development of students' algebraic thinking skills (e.g., Bastable & Schifter, 2008; Ferrini-Mundy, Lappan, & Phillips, 1997; Warren & Cooper, 2008), similar results may be obtained from the large groups when the curriculum is applied as intended.

The implementation of a new curriculum in a way that it is intended entails time for developing appropriate curriculum materials and professional training for teachers. Teachers should be given inservice training about teaching with the new curriculum. Therefore, some scholars study on such training programs to guide elementary teachers how to teach algebraic concepts in the early grades (e.g., Blanton & Kaput, 2008; Franke, Carpenter & Battey, 2008). Otherwise, teachers would either ignore the new curriculum to continue teaching in a way that they are used to teach or choose teaching activities that might not be appropriate for the students' level of readiness. Not only inservice teachers but also preservice teachers should be informed about the new elementary curriculum. Because many preservice teachers do not know much about the new curriculum, they might have a tendency to teach in a way that they were taught in the school. During the teacher education programs, the preservice teachers should be given opportunities to discuss the philosophical, psychological, and educational foundations of the new curriculum and the requirements for effective implementation.

Briefly, teaching algebra in the early grades is not a new idea in mathematics education because it is a part of elementary school mathematics in some countries for many years. But investigating the effects of teaching algebra in the early grades on the development of students' mathematical understanding and their performance in algebra is a recent research problem. The researchers stated that elementary students are capable of understanding some algebraic concepts such as variables and generalizations and they advocated that teaching algebra in the early grades contributes to the development of students' mathematical understanding and algebraic thinking. Although elementary students can understand simple algebraic concepts, the curriculum developers and policy makers should pay attention to the following facts when designing and implementing a new curriculum. First, teaching activities and materials should be appropriate for level of readiness of elementary students such that they should neither lead to root memorization nor misconceptions (Bastable & Schifter, 2008). The activities should help students make a smooth transition from arithmetic to algebra. Second, the curriculum should be piloted in many schools for at least two years and then revised (if necessary) before launch it at national level. The decision about the effectiveness of the curriculum should be given by investigating how it works for a diverse group of students (representative group of the all students in the country) rather than for a specific group of students. Third, elementary teachers should be offered professional development programs about how to implement the new curriculum. The new curriculum would be meaningless and ineffective when the teachers do not know how to implement it.

References

Akkan, Y., Çakıroğlu, Ü., & Güven, B. (2009). İlköğretim 6. ve 7. sınıf öğrencilerinin denklem oluşturma ve problem kurma yeterlilikleri. *Mehmet Akif Ersoy Üniversitesi Egitim Fakültesi Dergisi, 17,* 41-55.

Bastable, V., & Schifter, D. (2008). Classroom stories: Examples of elementary students engaged in early algebra. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp.165-184). New York: Lawrence Erlbaum Associates.

Bell, A. (1995). Purpose in school algebra. *Journal of Mathematical Behavior, 14*, 41-73.

Blanton, M. L., & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education, 36(5),* 412-446.

Cai, J. (2004a). Developing algebraic thinking in the early grades: A case study of the Chinese elementary school curriculum. *The Mathematics Educator [Singapore], 8,* 107-130.

Cai, J. (2004b). Introduction. *The Mathematics Educator [Singapore], 8,* 1-5.

Cai, J. & Knuth, E. J. (2005). The development of students' algebraic thinking in earlier grades from curricular, instructional, and learning perspectives. *Zentralblatt für Didaktik der Mathematik*, *37,* 1-4.

Cai, J. & Moyer, J. (2008). Developing algebraic thinking in earlier grades: Some insights from international comparative studies. In C. E. Greenes, & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp.169-180). Reston, VA: NCTM.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth, NH: Heinemann.

Carraher, D. W., Schliemann A. D., Brizuela B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education, 37(2),* 87-115.

Carraher, D. W., Schliemann, A. D., & Schwartz, J. L. (2008). Early algebra is not the same as algebra early. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp.235-272). New York: Lawrence Erlbaum Associates.

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks project. *Journal for Research in Mathematics Education, 38,* 136-163.

Curcio, F. R., & Schwartz, S. L. (1997). What does algebraic thinking look like and sound like with preprimary children. *Teaching Children Mathematics, 3,* 296-300.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, grade 6-10*. Portsmouth, NH: Heinemann.

Ferrucci, B. J., Kaur, B., Carter, J. A., & Yeap, B. (2008). Using a model approach to enhance algebraic thinking in the elementary school mathematics classroom. In C. E. Greenes, & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp.195-209). Reston, VA: NCTM.

Ferrini-Mundy, J., Lappan, G., & Phillips, E. (1997). Experiences in patterning. *Teaching Children Mathematics*, *3*, 282-288.

Franke, M. L., Carpenter, T. P., & Battey, D. (2008). Content matters: Algebraic reasoning in teacher professional development. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp.333-359). New York: Lawrence Erlbaum Associates.

Fujii, T. & Stephens, M. (2008). Using number sentences to introduce the idea of variable. In C. E. Greenes, & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp.127-140). Reston, VA: NCTM.

Gürbüz, R. & Akkan, Y. (2008). Farklı öğrenim seviyesindeki öğrencilerin aritmetikten cebire geçiş düzeylerinin karşılaştırılması: Denklem örneği. *Eğitim ve Bilim, 33,* 64-76.

Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics, 27,* 59-78.

Johanning, D. I. (2004). Supporting development of algebraic thinking in middle school: A closer look at students' informal strategies. *Journal of Mathematical Behavior, 23,* 371-388.

Kaput, J. J. (1999). Teaching and learning a new algebra. In E. Fennema, & T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 133-155). Mahwah, NJ: Erlbaum.

Kaput, J. J. (2008). What is algebra? What is algebraic reasoning? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp.5-17). New York: Lawrence Erlbaum Associates.

Kieran, C. (1989). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 33-56). Reston, VA: NCTM.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator [Singapore], 8,* 139-151.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Charlotte, NC: Information Age Publishing.

Knuth, E. J., Stephens, A. C., McNeil, N. M. & Alibali, M. W. (2006). Does understanding equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37(4),* 297-312.

Lew, H. C. (2004). Developing algebraic thinking in early grades: Case study of Korean elementary school mathematics. *The Mathematics Educator [Singapore], 8,* 88-106.

Schmittau, J., & Morris, A. (2004). The development of algebra in the elementary mathematics curriculum of V. V. Davydov. *The Mathematics Educator [Singapore], 8,* 60-87.

Sfard, A., & Linchevski, L. (1994). The gains and the pitfalls of reification: The case of algebra. *Educational Studies in Mathematics, 26,* 191-228.

Slavit, D. (1999). The role of operation sense in transitions from arithmetic to algebraic thought. *Educational Studies in Mathematics, 37,* 251-274.

Threlfall, J. (1999). Repeating patterns in the primary years. In A. Orton (Ed.). *Pattern in the teaching and learning of mathematics* (pp. 18–30). Cassell, London.

Tierney, C., & Monk, S. (2008). Children's reasoning about change over time. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp.185-200). New York: Lawrence Erlbaum Associates.

van den Heuvel-Panhuizen, M. & Wijers, M. (2005). Mathematics standards and curricula in the Netherlands. *Zentralblatt für Didaktik der Mathematik, 37,* 287-307.

Warren, E. & Cooper, T. J. (2008). Patterns that support early algebraic thinking in the elementary school. In C. E. Greenes, & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp.113-126). Reston, VA: NCTM.

Watanebe, T. (2008). Algebra in elementary school: A Japanese perspective. In C. E. Greenes, & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp.183-193). Reston, VA: NCTM.

Willoughby, S. S. (1997). Functions from kindergarten through six grade. *Teaching Children Mathematics, 3*, 314-318.

Yackel, E. (1997). A foundation for algebraic reasoning in the early grades. *Teaching Children Mathematics, 3,* 276-280.

Yenilmez, K., & Teke, M. (2008). Yenilenen matematik programının öğrencilerin cebirsel düşünme düzeylerine etkisi. *İnönü Üniversitesi Eğitim Fakültesi Dergisi, 9,* 229-246.