Sigma J Eng Nat Sci, Vol. 42, No. 5, pp. 1500−1510, October, 2024

Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2024.00051

Research Article

Reliability and availability analyses of an industrial system with two subsystems arranged in series-parallel

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ARTICLE INFO

Article history
Received: 26 June 2023
Revised: 23 September 2023
Accepted: 04 October 2023

Keywords:
Availability; Copula; Exponential Distribution; Failure Rate; Subsystems; Reliability

ABSTRACT

The research focuses on the reliability analysis of a complex system comprising two interconnected subsystems. Each subsystem consists of three identical units arranged in parallel. The operational policy employed is the 1-out-of-3: G policy, which means that as long as at least one unit is operational in each subsystem, the system as a whole remains functional. The failure rates of the units within the subsystems are consistent and follow an exponential distribution. To address unit failures and repair them, the Gumbel-Hougaard copula repair method is employed. The research investigates various reliability metrics, including system availability, system reliability, mean time to failure (MTTF), and sensitivity analysis. The researchers employ stochastic theory, differential equations, and supplementary variables to model and analyze the reliability behavior of the system. Moreover, the model's findings can guide decision-making processes related to system design, component selection, and maintenance strategies. System engineers and managers can utilize the insights gained from the reliability analysis to optimize the system's performance, enhance its reliability, and reduce costs associated with maintenance and repair.

Cite this article as: Maihulla AS, Yusuf I, Koo MBC. Reliability and availability analyses of an industrial system with two subsystems arranged in series-parallel. Sigma J Eng Nat Sci 2024;42(5):1500−1510.

INTRODUCTION

The researchers V. V. Singh et al. [1] conducted a study in 2013 that focused on the reliability and accessibility of repairable systems under the k-out-of-n policy. This policy is significant in maintaining the reliability of repairable systems, where the failure of a certain number of components within a system does not render the entire system inoperable. In their analysis, the researchers considered an engineering system composed of two subsystems arranged in a series configuration. The cost analysis of the system would typically involve evaluating various factors such as repair costs, maintenance costs, system downtime costs, and the costs associated with any losses or disruptions caused by system failures. The utilization of the Gumbel-Hougaard

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This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkilic

Published by Yıldız Technical University Press, İstanbul, Turkey

family copula distribution and the k-out-of-n: G policy is an important approach to enhance the reliability of repairable systems. The increasing reliance on computing systems in critical applications and the commercial world necessitates a focus on their reliability. Failure in systems such as nuclear reactor systems, hospital monitoring systems, or traffic control systems can have severe consequences. As a result, researchers have devoted significant efforts to develop various models for repairable systems, aiming to improve their reliability and performance in real-world industrial settings. A reliable evaluation of computing systems is desired in order to improve their performance. Over the past few decades, authors Kullstam [2]. have made significant efforts to formulate and address the reliability characteristics of k-out-of-n systems, such as availability, MTBF, and MTTR for a repairable system. By creating various types of models of repairable systems, various researchers have presented a highly significant work to increase the reliability of actual industrial systems. A reliable evaluation of computing systems is desired in order to improve their performance. As a result, a critical component of any system's ability to survive is its component reliability. One might encounter a few complex systems in real life where the failure of one component lowers the effectiveness of the entire system. In order to establish the reliability of a system to the desired level, the reliability of the entire system is dependent on the reliability allocations to each part or unit that makes up the system. A reliability allocation-based-integrated factor method approach to the aerospace system was discussed by Di Bona, Forcina, and Sivestri [3]. Di Bona, Forcina, Petrillo, and De Felice [4] proposed a new reliability allocation method as a critical flow method for a thermonuclear system as a result of the reliability allocation method study. Liang et al. [5] showed the precise reliability formula for consecutively repairable k-out-of-n-:G type operational systems. A cost analysis of an engineering system with two subsystems in a series configuration, controllers, and human failure has been studied by authors like Singh et al. [6] using the k-out-of-n: G policy. It's important to note that general repair can be used if the device is already in use and experiencing minor or major partial failure. For example, Singh et al. [6], Gulati et al. (2016)[7], Ibrahim et al. [8], Jia et al. [9], Kumar et al. [10] and Maihulla et al. [11] investigated the reliability measures of systems made up of subsystems in series configurations and k-out-of-n: G/F policy with implications of a joint probability distribution. Filtration system of reverse osmosis was studied using the k-out of-n: G policy using the Gumbel Copular approach by Maihulla and Yusuf [12]. Complex repairable system was investigated by Maihulla et al. [13]. Reliability, Availability, Maintainability, and dependability (RAMD) analyses were used by [14] to evaluating the strength of the system at components level. Gumbel-Hougaard family copula for reliability modeling and performance assessment of solar photovoltaic system by Isah et al. [15]. An industrial system, reverse osmosis system in particular was analyses by [16] using modified Weibull distribution. Gumbel Haugaard family Copula distribution, with the help of supplementary variable technique was used in evaluating the performance of small solar photovoltaic system by [11].

The preceding literature provided their work on reliability and performance analysis of several serial systems and declared a higher system performance. Little is known about the reliability and performance evaluation of a serial system staffed by two human operators, with partial failures repaired by repair machines. However, a reliability examination of the serial system attended by the Copula repair machines is still required. In this study, we provide a new model. This model consists of two serial subsystems. One operator and one repair machine are assigned to each subsystem. Each repair machine is allocated to one subsystem to fix a partially failed unit. The repair machines are systems that are doomed to fail. A system of partial differential equations is constructed and solved using the transition diagram to obtain system strong reliability characteristics such as reliability, availability, mean time to failure (MTTF), sensitivity analysis, and profit function. The goal of this work is to create dependability models in order to assess the system's strength. The findings of this research will be useful to plant managers, industries, and manufacturing systems that want to deploy repair machines. The present analyses can be used to evaluate the performance of an arbitrary industrial system having two subsystems arranged in series-parallel, regardless of the number of redundant units in any of the subsystems.

The fact that no work on the performance of an industrial system using the use of supplemental variable technique working with the Gumbel Copula distribution has been published is one of the distinctive features of the current research over the literature. The modified result from this study can be used to evaluate any industrial system at random.

The Supplementary Variables Technique is a multivariate statistical method that is commonly employed in the context of Principal Component Analysis (PCA) and related techniques. This technique entails inserting supplementary variables (also known as supplementary data points or supplementary observations) into an existing dataset in order to examine their correlations with the original variables.

In the current paper, we have taken into account a mathematical modeling of a system that consists of two subsystems in a series configuration for further study. Unit A1, A2 & B1 and Unit B2 are the two parallel units that make up each subsystem. When the system first enters state S0, both subsystems are in excellent working order and the system is fully operational. When a unit in subsystem 1 fails, the system enters the state S1, at which point the parallel unit loads the failed unit and assigns it for repair. After one unit in subsystem 2 failed, the system entered state S2, where it is in a degraded state. One subsystem1 unit is in good shape and the other is undergoing repair in the operational state

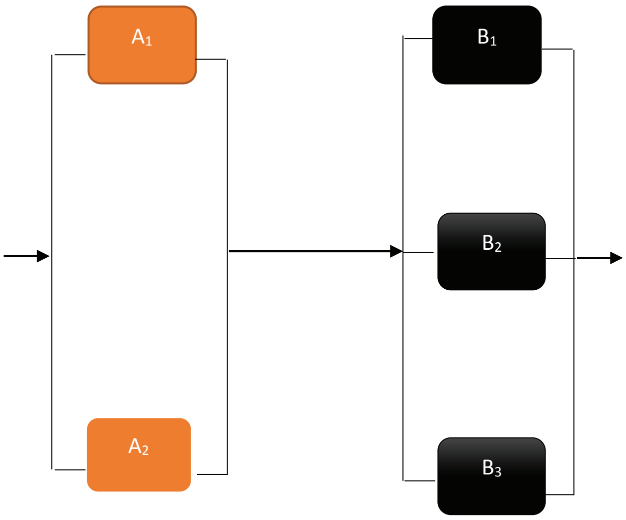

Figure 1. System structure.

S3, which is. Completely failed states are S4, S6, and S9, while operational states are S5, S7, and S8. Utilizing Laplace transforms and the supplementary variable technique, the system's performance is investigated.

Different values of failure and repair rates have been computed for the various reliability metrics, including availability, reliability, mean time to failure (MTTF), sensitivity for MTTF, and cost analysis. Tables and graphs have highlighted critical analyses of the results.

MODEL DESCRIPTION AND NOTATIONS

System Description

In order to address the gap in evaluating the k-out-of-n: G scheme, the researchers focused on studying the performance of a repairable hot standby (redundancy or failover configuration, In a hot standby system, there are two or more identical components or systems running in parallel, where one is actively handling the workload and the other is on standby) system with two subsystems, specifically subsystems 1 and 2. Subsystem one consists of two parallel units operating under a 2-out-of-2 and subsystem two consists of three parallel units operating under a 1-out-of-3 good policy. This policy ensures that as long as at least one unit remains operational in each subsystem, the system as a whole can continue functioning properly. The units in both subsystems are connected to a switch, which allows for immediate switching if necessary. This immediate switching is essential for maintaining the system's proper operation. Whenever an operating unit fails, a standby unit is immediately switched in to replace it. In addition to the possibility of unit failures, there is also the consideration of unexpected human failures that may occur during system operation if the subsystem is mishandled. The researchers assumed that the failure rates of both the active and standby units follow exponential distributions. This assumption allows for a probabilistic analysis of the system's reliability and performance. The system can be in one of four possible states: perfect operation, minor partial failure, major partial failure, or maximum failure. By considering these different states and the dynamics of unit failures, standby unit switching, and potential human failures, the researchers aimed to evaluate the performance of the repairable hot standby system under the considered scheme. Through their analysis, the researchers aimed to provide insights into the reliability, availability, and performance characteristics of the system, taking into account various failure scenarios and operational conditions.

System reliability is assessed to test various characteristics like transition state probabilities, system availability, system reliability, MTTF, and benefits analysis using the additional variables and effects of the Laplace transformation. The system of this essay is set up as follows.

It is evident from the provided information that the research study is structured into several sections, each addressing specific aspects of the system's analysis and evaluation. Here is a breakdown of the different sections mentioned: Section 1: In this section, the researchers reviewed relevant papers and existing research related to the topic of system reliability and performance analysis. It served as an introduction to the research study, establishing the existing knowledge and identifying the research gap. Section 2: This section provides a summary of the system description, including details about the system's components, configuration, and operational policies. Assumptions and notations used in the analysis are also outlined here, setting the foundation for subsequent sections. Section 3: The state of the system is described in this section. It explains the different operational states or conditions that the system can assume during its functioning. This information is crucial for understanding the system's behavior and analyzing its reliability and performance. Section 4: The system configuration and transition diagram are presented in this section. The configuration describes how the system components are interconnected and organized, while the transition diagram illustrates the possible state transitions that the system can undergo. Section 5: This section discusses the mathematical modeling approach used in the analysis, which is based on differential equations. Section 6: Specific cases or scenarios related to the system under investigation are considered in this section. These cases are chosen to assess the system's performance in different operating conditions or scenarios. The researchers perform simulations and analyze the reliability, availability, mean time to failure (MTTF), and expected profit margin based on the results obtained. Section 7: This section summarizes the findings of the research study. The researchers likely present the explicit expressions derived for reliability characteristics using the MAPLE software. This section may provide an overview of the key insights and conclusions drawn from the analysis.

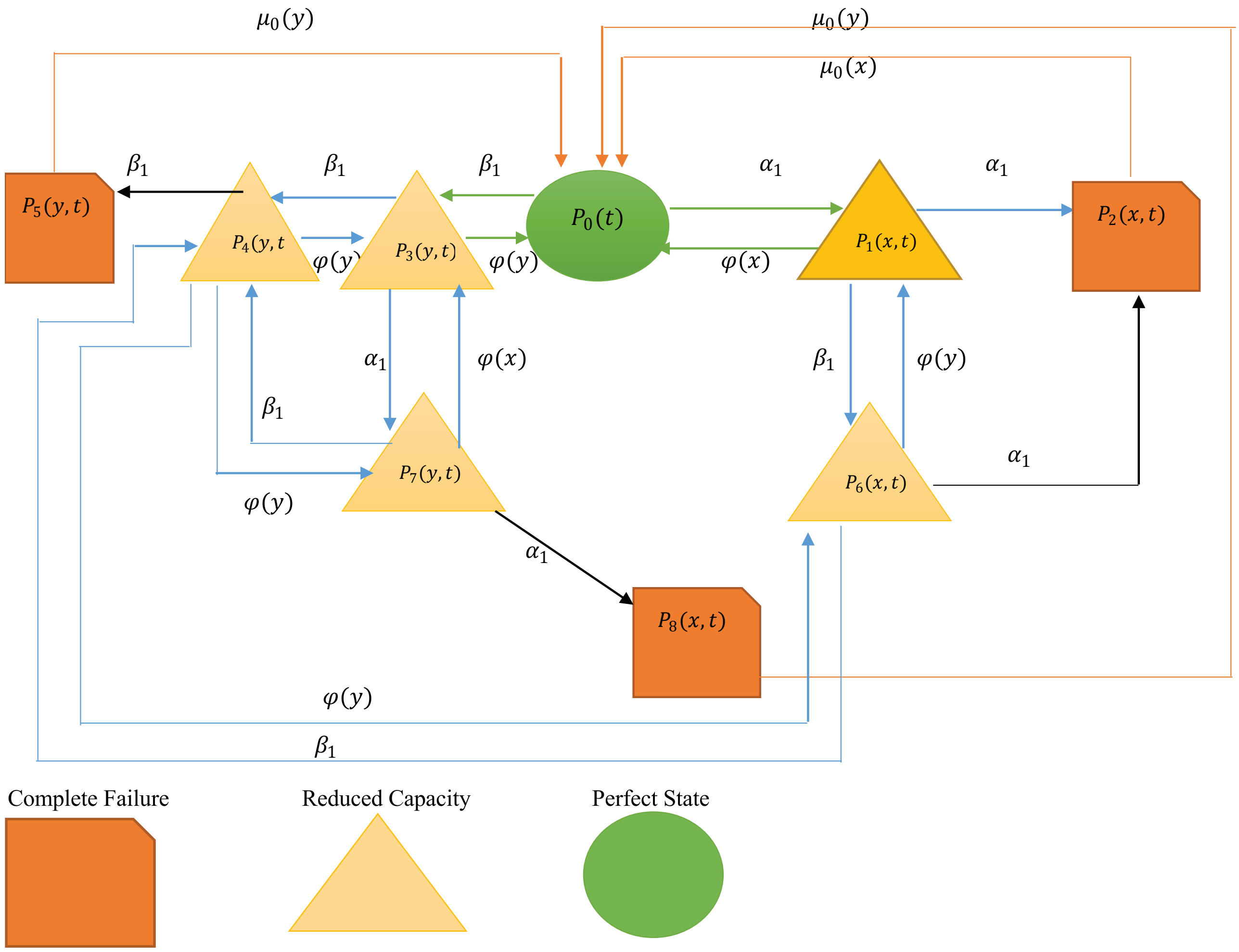

Figure 2. State transition diagram of the model.

Notations and Assumption

s	Laplace transform variable for all expressions.
t	Time variable on a time scale.
α_1	Failure rate of the unit in subsystem 1
β_1	Failure rate of the unit in subsystem 2
$\varphi(x)$	Repair of the failed unit in subsystem 1
$\varphi(y)$	Repair of the failed unit in subsystem 2
$\mu_0(x)$	Copula repair of full failure of unit in subsystem 1
$\mu_0(y)$	Copula repair of full failure of unit in subsystem 2
$P_0(t)$	Is a state in which the system working perfectly and single failure
$P_1(x, t)$	Is a state in which the system is working with reduced capacity.The first unit from the first subsystem has failed.
$P_2(x, t)$	Is a complete failure state. This is due to the failure of the second unit of the first subsystem
$P_3(x, t)$	Is a state in which the system is working with reduced capacity.The first unit from the second subsystem has failed.
$P_4(x, t)$	This is partial failure state, due to the failure of the second unit of the second subsystem. No redundant unit remain.
$P_5(x, t)$	Is a complete failure state. This is due to the failure of the third unit of the second subsystem
$P_6(x, t)$	This is reduced capacity state due to the failure of one unit from first subsystem and two units from the second subsystem.
$P_7(x, t)$	This is reduced capacity state.
$P_8(x, t)$	Is a complete failure state.

Assumptions

The following assumption are taken throughout the discussion of the model:

1) Initially, both subsystems are in good working condition.
2) One unit from subsystem 1 and one unit from subsystem 2 in consecutive are necessary for operational mode.
3) The one unit out of two in subsystem 1 is necessary for operational mode.

Table 1. Description of the system

State	Description
S_0	Initial state, Unit A_1 is working. Unit A_2 is on Standby mode hotly. And the system is in operational condition. Unit B_1 in the sub-system B is in working state. B_2 and B_3 are on standby.
S_1	In this state, the unit A_1 failed and under repair. And the elapsed repair time is (x,t). While the units A_2 and B_1 are on operation. B_2 and B_3 are on standby.
S_2	In this state, the units A_1 and B_1 have failed and under repair. The units A_2 and B_2 are on operational states. Also B_3 is on standby.
S_3	Complete failure state, due to the failure of the second unit in the first subsystem.
S_4	State S_4 is degraded, but despite operational. The units $A_{1,}$ B_1, and B_2, has failed. While the units A_2 and B_3 are on operation. Therefore no unit in standby.
S_5	The state S_5 is complete failed state due to the failure of the second unit from subsystem 1.
S_6	S_6 is the partial working state. This is due to the failure of the first and second units from subsystem 2, and also failure of the first unit from subsystem 1.
S_7	State S_4 is degraded, but despite operational. No standby unit in all subsystems.
S_8	The state S_8 is complete failed state due to the failure of the third unit from subsystem 2.

4) The one unit out of three in subsystem 2 is necessary for operational mode.
5) The system will be inoperative if three units from subsystem 2 failed. Also if two units from subsystem 1 failed.
6) Failed unit of the system can be repaired when it is inoperative or failed state.
7) Copula repair follows a total failure of a unit in subsystem.
8) It is assumed that a repaired system by copula works like a new system and no damage appears during repair.
9) As soon as the failed unit gets repaired, it is ready to perform the task.

Formulation and Solution of Mathematical Model

By the probability of considerations and continuity of arguments, the following set of difference-differential equations are associated with the above mathematical model were generated using the method adopted by [12,14], and also by [15].

$$\left(\frac{\partial}{\partial t}+\alpha_1+\beta_1\right)P_0(t)=\int_0^\infty \varphi(x)P_1(x,t)\,dx+\int_0^\infty \varphi(y)P_3(y,t)\,dy +\int_0^\infty \mu_0(x)P_2(x,t)\,dx+\int_0^\infty \mu_0(y)P_5(y,t)\,dy +\int_0^\infty \mu_0(y)P_8(y,t)\,dy \quad (1)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_1+\beta_1+\varphi(x)\right)P_1(x,t)=0 \quad (2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_2(x,t)=0 \quad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\alpha_1+\beta_1+\varphi(y)\right)P_3(y,t)=0 \quad (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_1+2\varphi(y)\right)P_4(y,t)=0 \quad (5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(x)\right)P_5(y,t)=0 \quad (6)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_1+\beta_1+\varphi(x)\right)P_6(x,t)=0 \quad (7)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\alpha_1+\beta_1+\varphi(y)\right)P_7(y,t)=0 \quad (8)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)P_8(y,t)=0 \quad (9)$$

Boundary conditions

$$P_1(0,t)=\alpha_1P_0(t) \quad (10)$$

$$P_2(0,t)=\alpha_1^2P_0(t) \quad (11)$$

$$P_3(0,t)=\beta_1P_0(t) \quad (12)$$

$$P_4(0,t)=(\beta_1^2+\beta_1\alpha_1^2)P_0(t) \quad (13)$$

$$P_5(0,t)=\beta_1^3P_0(t) \quad (14)$$

$$P_6(0,t) = \alpha_1^2 P_0(t) \quad (15)$$

$$P_7(0,t) = \alpha_1 \beta_1 P_0(t) \quad (16)$$

$$P_8(0,t) = \alpha_1 \alpha_1^2 P_0(t) \quad (17)$$

Initial condition $P_0(t) = 1$ and other transition probability at t=0 are zero (18)

Taking Laplace transformation of equation (1) – (17) and using the equation with the help of (18), one can obtain

$$\left(\frac{\partial}{\partial t} + \alpha_1 + \beta_1\right) \bar{P}_0(s) = \int_0^\infty \varphi(x)\bar{P}_1(x,s)\,dx + \int_0^\infty \varphi(y)\bar{P}_3(y,s)\,dy + \int_0^\infty \mu_0(x)\bar{P}_2(x,s)\,dx + \int_0^\infty \mu_0(y)\bar{P}_5(y,s)\,dy + \int_0^\infty \mu_0(y)\bar{P}_8(y,s)\,dy \quad (19)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_1 + \beta_1 + \varphi(x)\right) \bar{P}_1(x,s) = 0 \quad (20)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right) \bar{P}_2(x,s) = 0 \quad (21)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \alpha_1 + \beta_1 + \varphi(y)\right) \bar{P}_3(y,s) = 0 \quad (22)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \beta_1 + 2\varphi(y)\right) \bar{P}_4(y,s) = 0 \quad (23)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_0(x)\right) \bar{P}_5(x,s) = 0 \quad (24)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_1 + \beta_1 + \varphi(x)\right) \bar{P}_6(x,s) = 0 \quad (25)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \alpha_1 + \beta_1 + \varphi(y)\right) \bar{P}_7(y,s) = 0 \quad (26)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_0(y)\right) \bar{P}_8(y,s) = 0 \quad (27)$$

Boundary conditions

$$P_1(0,S) = \alpha_1 P_0(s) \quad (28)$$

$$P_2(0,S) = \alpha_1^2 P_0(s) \quad (29)$$

$$\bar{P}_3(0,S) = \beta_1 \bar{P}_0(s) \quad (30)$$

$$\bar{P}_4(0,S) = (\beta_1^2 + \beta_1\alpha_1^2)\bar{P}_0(s) \quad (31)$$

$$\bar{P}_5(0,S) = \beta_1^3 \bar{P}_0(s) \quad (32)$$

$$\bar{P}_6(0,S) = \alpha_1^2 \bar{P}_0(s) \quad (33)$$

$$P_7(0,S) = \alpha_1\beta_1 P_0(s) \quad (34)$$

$$\bar{P}_8(0,S) = \alpha_1\alpha_1^2 \bar{P}_0(s) \quad (35)$$

Solving equation (20) to (27) with the help of boundary condition (28) to (35) and applying the below shifting properties of Laplace.

$$\bar{P}_1(S) = \alpha_1 \left\{\frac{1-\bar{S}_\varphi(S+\alpha_1+\beta_1)}{S+\alpha_1+\beta_1}\right\} \bar{P}_0(s) \quad (36)$$

$$\bar{P}_2(S) = \alpha_1^2 \left\{\frac{1-\bar{S}_\mu(S)}{S}\right\} \bar{P}_0(s) \quad (37)$$

$$\bar{P}_3(s) = \beta_1 \left\{\frac{1-\bar{S}_\varphi(S+\alpha_1+\beta_1)}{S+\alpha_1+\beta_1}\right\} \bar{P}_0(s) \quad (38)$$

$$\bar{P}_4(S) = (\beta_1^2 + \beta_1\alpha_1^2) \left\{\frac{1-\bar{S}_{2\varphi}(S+\beta_1)}{S+\beta_1}\right\} \bar{P}_0(s) \quad (39)$$

$$\bar{P}_5(S) = \beta_1^3 \left\{\frac{1-\bar{S}_\mu(S)}{S}\right\} \bar{P}_0(s) \quad (40)$$

$$\bar{P}_6(s) = \alpha_1^2 \left\{\frac{1-\bar{S}_\varphi(S+\alpha_1+\beta_1)}{S+\alpha_1+\beta_1}\right\} \bar{P}_0(s) \quad (41)$$

$$\bar{P}_7(s) = \alpha_1\beta_1 \left\{\frac{1-\bar{S}_\varphi(S+\alpha_1+\beta_1)}{S+\alpha_1+\beta_1}\right\} \bar{P}_0(s) \quad (42)$$

$$\bar{P}_8(S) = \alpha_1\alpha_1^2 \left\{\frac{1-\bar{S}_\mu(S)}{S}\right\} \bar{P}_0(s) \quad (43)$$

Where

$$\bar{P}_0(S) = \frac{1}{D(S)} \quad (44)$$

And

$$D(S) = \left[S + \alpha_1 + \beta_1 - (\alpha_1\bar{S}_\varphi(s+\alpha_1+\beta_1) + \beta_1\bar{S}_\varphi(S+\alpha_1+\beta_1) + (\beta_1^2 + \beta_1\alpha_1^2)\bar{S}_{2\varphi}(s+\beta_1) + \alpha_1^2\bar{S}_\varphi(S+\alpha_1+\beta_1) + \alpha_1\beta_1\bar{S}_\varphi(S+\alpha_1+\beta_1) + (\alpha_1^2 + \beta_1^3 + \alpha_1\alpha_1^2)\bar{S}_\mu(S)\right] \quad (45)$$

The state transition probabilities between the system's operational mode and failed state at any given time are represented by the following Laplace transformations:

$$\bar{P}_{up}(S) = \bar{P}_0(S) + \bar{P}_1(S) + \bar{P}_3(S) + \bar{P}_4(S) + \bar{P}_6(S) + \bar{P}_7(S) \quad (46)$$

$$P_{down}(S) = 1 - P_{up}(S) \quad (47)$$

ANALYTICAL STUDY OF THE MODELS FOR PARTICULAR CASES

Availability

In order to get the system availability, there must be a maximum repair, there fore

Setting all repairs to 1. i.e. $\varphi(x)=\varphi(y)=\mu_0(y)=\mu_0(x)=1$ (48)

$$\bar{S}_{\varphi}(S) = \frac{2.7183}{S+2.7183}, \quad \frac{1-S_{\phi}(S)}{S} = \frac{1}{S+\phi}$$

Considering the worldwide database of failures, taking the values of different parameters as $\alpha_1 = 0.001$, $\beta_1 = 0.002$. In (45) then taking the inverse Laplace transform, we can obtain, the expression for availability as:

$$\text{Availability} = \{0.0364e^{-2.765772t} + (-1.05931+0.034103)e^{(-1.045183-0.183546)t} + 1.107597e^{-0.071297t} - 0.0213641e^{-2.110t} - 0.012473e^{-2.051500t}\} \quad (49)$$

For different values of time t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

Unit of time, we may get different of $\overline{P}_{up}(t)$ with the help of (48) as shown in Table 2 and corresponding Figure 3.

Reliability Analysis

Taking all repair rate. ϕ. $\varphi(x) = \varphi(y) = \mu_0(y) = \mu_0(x) = 0$ In equation (45) and for same values of failure rate as $\lambda_1 = 0.001$, $\lambda_2 = 0.002$, $\lambda_3 = 0.003$ and $\lambda_4 = 0.004$

After applying the inverse Laplace transform, one may have the system reliability expression. The following is an expression for the system's dependability:

$$D(S) = S + 0.013 \quad (50)$$

$$\overline{P}_{up}(S) = [1 + \frac{0.001}{S+1.013} + \frac{0.003}{S+1.013} + \frac{0.008}{S+1.013} + \frac{0.000016}{S+2.11} + \frac{0.000052}{S+2.105} + \frac{0.000012}{S+2.015} + \frac{0.0000071}{S+1.214}]$$

For different values of time t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

Unit of time, we may get different of $\overline{P}_{up}(t)$ with the help of (50) as shown in Table 1 and corresponding Figure.

Table 2. Variation of availability with respect to time

Time (t)	Availability
0	0.99999
1	0.88737
2	0.79513
3	0.71247
4	0.63841
5	0.57204
6	0.51258
7	0.45929
8	0.41155
9	0.36877
10	0.33043

Table 3. Reliability analysis with respect to time

Time (t)	Reliability
0	1.00000
1	0.91673
2	0.81606
3	0.66661
4	0.54453
5	0.44480
6	0.36334
7	0.29680
8	0.24245
9	0.19804
10	0.16177

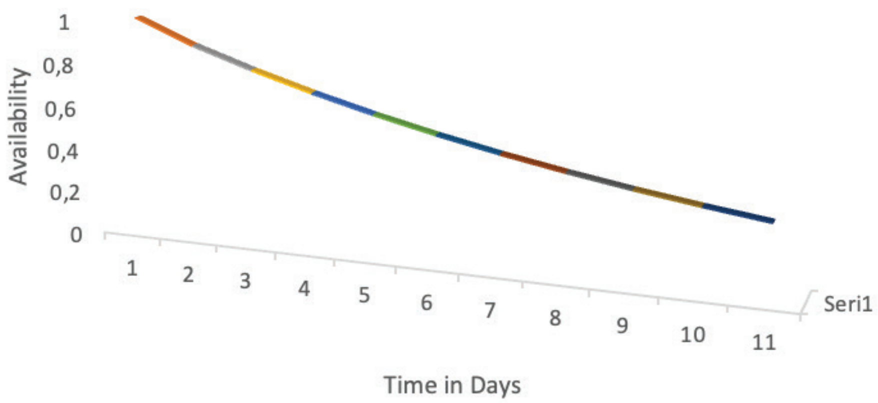

Figure 3. Availability with respect to time.

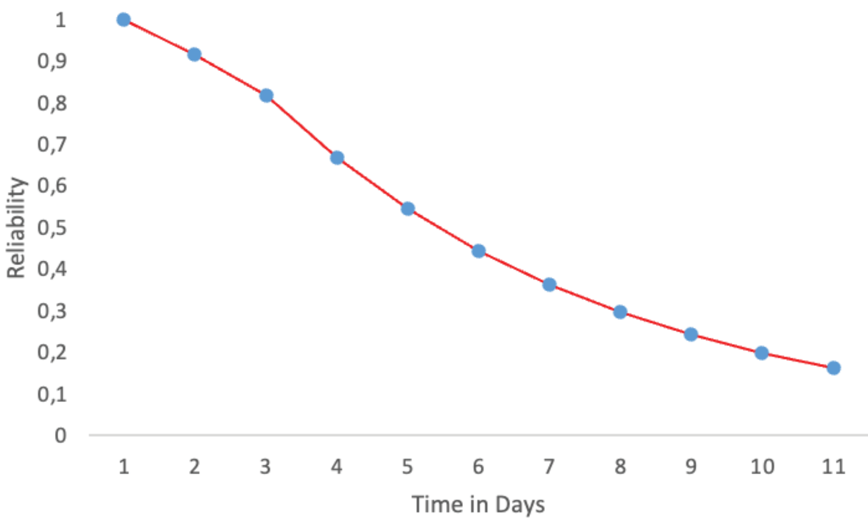

Figure 4. Reliability with respect to time.

Mean Time to Failure (MTTF)

Using the exponential distribution and the assumption that all repair rates are zero and the limit is (50), we can determine the MTTF as:

$$\text{MTTF} = \lim_{s\to 0} \bar{P}_{up}(S) \tag{50}$$

One can obtain the variation of MTTF with respect to failure rates as shown in table 3 corresponding to figure 5 by setting and varying the failure rates, one by one, respectively, as 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008 and 0.009 in (50).

Sensitivity analysis corresponding to (MTTF)

By partially differentiating MTTF in relation to system failure rates, it is possible to examine the sensitivity of the system's MTTF. Using the set of criteria as a guide α_1 = 0.0001, β_1 = 0.0002, The MTTF sensitivity in partial differentiation of MTTF may be calculated as shown in table 4 and related graphs in Figure 6 below:

Table 4. Variation of MTTF with failure rates α_k

Failure Rate	Subsystem 1	Subsystem 2
0.001	136.835	175.276
0.002	130.803	136.835
0.003	123.291	111.676
0.004	115.961	93.999
0.005	109.198	80.948
0.006	103.067	70.9490
0.007	97.537	63.0626
0.008	92.549	56.696
0.009	88.043	51.457

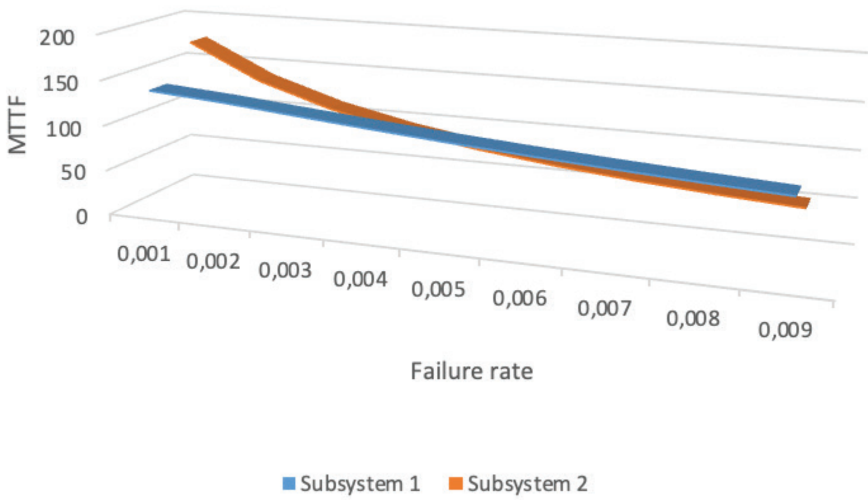

Figure 5. Variation of MTTF with failure rates.

Table 5. MTTF sensitivity as function of time

Failure rate	$\frac{\partial(MTTF)}{\alpha_1}$	$\frac{\partial(MTTF)}{\beta_1}$
0.1	-637.499	-402.645
0.2	-102.666	-284.768
0.3	-1.85185	-215.558
0.4	19.89795	-170.600
0.5	23.94733	-139.362
0.6	22.18244	-116.457
0.7	21.10204	-99.3439
0.8	17.81667	-85.7718
0.9	15.7437	-74.9538

When the other parameters are taken as constant, Table 4, correspond to Figure 5 shows the mean-time-to-failure (MTTF) of the system for variations in α_1 and β_1, respectively. When MTTF is compared to failure rates α_1, the variation is almost very close, but when compared to failure rates β_1, the variation is very high, showing that both are crucial to the system's smooth operation.

Sensitivity analysis is a quantitative technique used in economics, engineering, finance, and decision-making to analyze how changes in input variables or assumptions affect the output or outcomes of a model, system, or decision. It is used to assess the robustness, dependability, and stability of models, projections, or judgments in the face of various scenarios or uncertainties.

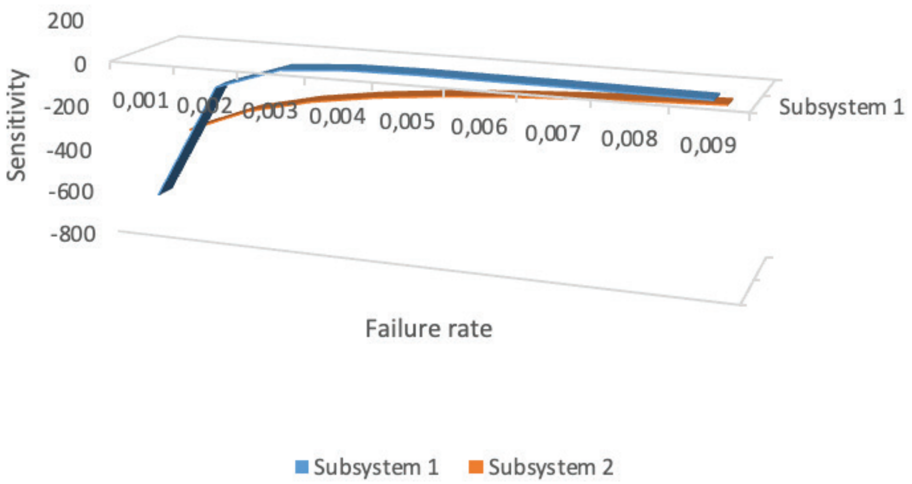

Figure 6. Sensitivity with respect to failure rate.

RESULTS AND DISCUSSION

The reliability measures for various failure and repair rates are critically analyzed in order to examine the performance of the system under investigation. Figure 3 shows how the availability of a complex repairable system varies over time as failure rates are fixed at various levels. When failure rates are set at lower levels, such as as $\alpha_1 = 0.001$, $\beta_1 = 0.002$, availability of the system declines gradually over time while the likelihood of failure rises, eventually stabilizing at zero after a sufficiently long period of time. Therefore, as shown by the graphical analysis of the model, one can confidently predict the future behavior of a complex system at any time for any given set of parametric values.

The analysis's Figure 4 concentrated on the system's reliability in the absence of a repair. The availability and reliability values in Tables 2 and 3 can be compared, and it is clear that when repairs are made, the system performs much better than when a replacement is made.

The analytical section of the paper performs a sensitivity analysis of the system. The variation in sensitivity with changes in parameter values is depicted in Table 5 correspond to the Figure 6.

CONCLUSION

This research studied the performance of a system composed of two subsystems A and B arranged in a series-parallel arrangement. Subsystem A is made up of two identical units that work as 2-out-of-2, whereas Subsystem B is made up of three identical units that operate as 1-out-of-3. When a unit in a subsystem fails, the system can continue to operate, however full failure happens when the minimum number of required operational units fails. Copula is used to repair the system when it entirely fails. To calculate the system's transient probabilities and reliability measurements of system performance and strength, the Markovian process, Laplace transformation, and additional variable approaches are utilized. The study's findings indicate that the reliability measurements described are time and failure sensitive. Expressions of reliability measures for measuring the strength and performance of the system, such as availability, reliability, mean time to failure, and cost function, are derived and confirmed through numerical experiments. MATLAB was used to simulate the effect of time and various system parameters on Metrics for dependability. Where the system's reliability strength is strong, it may assist the system to survive some of the hurdles, reducing system performance and improving system life span. These are the primary contributions of the study. The paper's findings show that reliability modeling can be used to evaluate the strength, efficiency, and performance of any arbitrary system with two series-parallel subsystems. As a result, the model's graphical depiction indicates that for any given set of parametric parameters, one can safely forecast the future behavior of a complex system at any point in time. This research will involve both online and offline preventative maintenance after the failure of the second unit in each subsystem. This will be looked into more in future studies.

ACKNOWLEDGMENTS

I am grateful to Professor Ibrahim Yusuf of Bayero University Kano and Professor Michael Khoo Boon Chong of School of Mathematical Sciences, Universiti Sains Malaysia for contributing to My Ph.D. work. I am also grateful to TETFund for the sponsorship.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

FUNDING

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

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