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Research Article

Mathematics in Environmental Education: An Investigation of Preservice Teachers' Skills in Drawing and Interpreting Population Size Graphs

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Abstract – Drawing and interpretation graphs, as key mathematical skills, are widely used in teaching various subjects within science education. Population ecology, a topic in the environmental education course within the elementary education curriculum of Türkiye, is one such subject. In the context of this environmental education course, graphs are employed to teach and interpret the factors influencing the growth and decline of populations. This qualitative study aimed to examine the graph drawing and interpretation skills of preservice elementary school teachers in relation to population size. Fifty-seven preservice elementary school teachers from the elementary education department of a university in the Central Anatolia region of Türkiye participated in the study. These preservice teachers were provided with growth rate vs. time graphs of different populations and were asked to draw and interpret graphs representing change numbers of individuals vs. time. The findings revealed that many preservice teachers struggled with both drawing and interpreting population graphs. Specifically, most participants encountered difficulties while drawing and interpreting linear and particularly curvilinear graphs, as they failed to account for the simultaneous changes in two variables. The study's findings are expected to raise awareness about the importance of mathematical skills and the need for interdisciplinary collaboration in environmental education, as well as providing direction for future research.

Keywords: Environmental education, mathematics education, graph drawing skills, graph interpretation skills, population ecology, preservice teachers.

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Introduction

The teaching of many subjects in science requires the use of mathematics (Ministry of National Education [MoNE], 2018). Graph drawing and interpretation are among the essential mathematical skills used in science teaching and are crucial for science literacy, which involves interpreting and understanding scientific data (Osborne & Allchin, 2024). Graphs, as visual representations of quantitative and qualitative data, depict relationships between variables and enable comparisons by visualizing the data. Additionally, graphs help in summarizing, organizing, interpreting, and presenting data (Ateş et al., 2019). In this context, drawing graphs that align with specific research purposes based on observation and measurement results, as well as using tools such as frequency distributions, bar graphs, tables, and physical models, are integral parts of the science curricula currently and previously implemented in Türkiye (MoNE, 2005; MoNE, 2013; MoNE, 2018; MoNE, 2024). In current teaching practices within the updated science curriculum, students are often asked to create graphs based on data or interpret existing ones (MoNE, 2024).

Graph drawing and interpretation can be classified as mathematical and logical skills that contribute to the development of scientific processes, reasoning, visual literacy, and scientific reasoning skills, as noted in the literature (Ateş et al., 2019; Coştu et al., 2017; Krell et al., 2020). Since graph creation involves considering how two variables change together, covariational thinking/reasoning skills have been highlighted in recent studies (Altındiş et al., 2024; Basu & Panorkou, 2019; González, 2021, 2024). Covariational thinking is a complex cognitive process that requires understanding how two quantities change simultaneously (Carlson et al., 2002). In international exams, covariational reasoning skills involving graph drawing and interpretation are necessary to answer certain mathematics and science questions (Gant et al., 2023). Although research on covariational reasoning has traditionally focused on mathematics education, recent studies have extended that research into science education (Altındiş et al., 2024; González, 2021, 2024). Carlson et al. (2002) categorized covariational reasoning into five mental action (MA) levels. At the lowest level, MA1, students typically coordinate changes in one variable with changes in another. At the MA2 level, students recognize the direction of change; at MA3, they understand how much the dependent variable changes in relation to the independent variable; at MA4, they grasp the rate of change concerning the independent variable; and at the highest level, MA5, students comprehend how the instantaneous rate of change varies with consistent changes in the independent variable.

Many studies have been conducted to assess the graph reading and interpretation skills of students at various educational levels (Aydan & Dönel Akgül, 2021; Aydın & Tarakçı, 2018; Coştu et al., 2017; Erbilgin et al., 2015; Sezek, 2022; Sülün & Kozcu, 2005; Şahinkaya & Aladağ, 2013). Research has shown that students at the secondary school level (Erbilgin et al., 2015; Sülün & Kozcu, 2005; Tairab & Khalaf Al-Naqbi, 2004) and even at the university level (Aydan & Dönel Akgül, 2021; Aydın & Tarakçı, 2018; Coştu et al., 2017; Şahinkaya & Aladağ, 2013) encounter difficulties in drawing and interpreting graphs, skills that should be developed beginning in primary school. In the study by Coştu et al. (2017), it was found that preservice science teachers were more proficient in reading graphs in chemistry than interpreting them. Other researchers (Aydan & Dönel Akgül, 2021; Tairab & Khalaf Al-Naqbi, 2004; Taşar et al., 2002) have also indicated that students struggle more with drawing graphs than with reading or interpreting them. In a graphic drawing, difficulties may be experienced in issues such as determining and naming the axes, assigning numerical values to the axes, scaling and naming the graphic, determining the intersection points, starting the graphic axis from the appropriate place and continuing it (Aydan & Dönel Akgül, 2021; Aydın & Tarakçı, 2018). Research suggests that graph construction should be mastered before learning to read and interpret graphs (Ateş et al., 2019). Domain knowledge plays a critical role in interpreting scientific graphs, as the use of quantities during the interpretation process helps to form mental images and facilitates the development of covariational reasoning skills, particularly in cases of complex topics (Altındiş et al., 2024).

A literature review revealed that a limited number of studies have focused on the integration of mathematical skills in environmental education courses (Altındiş et al., 2024; Aydan & Dönel Akgül, 2021; Basu & Panorkou, 2019; González, 2021, 2024; Mumu et al., 2021; Özdemir, 2021; Sülün & Kozcu, 2005). For instance, in the study by Altındiş et al. (2024), university students were provided with two graphs representing the growth of two populations over time (exponential and logistic growth) and shown a video illustrating how the number of individuals changed over time. After watching the video, students were asked to explain the relationships depicted in each graph, compare the exponential and logistic graphs, and draw graphs showing how the number of individuals changed over time for each scenario. The results indicated that students with prior knowledge of graphing made more accurate predictions and interpretations. Another experimental study focused on 6th-grade students. It found that using simulations of the greenhouse effect was beneficial for exploring covariational relationships and developing complex reasoning (Basu & Panorkou, 2019).

González (2021, 2024) investigated the development of preservice mathematics teachers' covariational reasoning skills while modeling the relationship between carbon dioxide levels and global warming. The findings showed that as preservice teachers completed the tasks, their covariational reasoning improved, and using these skills enhanced their understanding and modeling of climate change. These results suggest that science topics can be better understood and explained when supported by mathematical skills. In a study by Mumu et al. (2021), middle school students' abilities to solve mathematical problems related to environmental education were examined. Solving problems about toxic waste, clean water, and flooding required knowledge of decimals, exponents, and fractions. Students' responses were categorized into four groups: (1) students who could solve mathematical problems and had environmental awareness, (2) students who could solve mathematical problems but had no interest in environmental issues, (3) students who could not solve mathematical problems but were interested in environmental issues, and (4) students who could not solve mathematical problems and were indifferent to environmental issues. The study found that the largest group consisted of students in the fourth category. In Özdemir's (2021) study, activities were developed that integrated environmental and mathematics education for 5th-grade students and the impact of those activities on students' views of sustainability was examined. The findings showed that combining mathematical and environmental concepts deepened students' superficial knowledge.

Population size, a topic covered in the environmental education course of the classroom teaching program, requires the use of covariational thinking and mathematical skills such as performing calculations and interpreting tables and graphs. Population size refers to the number of individuals that make up a population at a given time, and it is influenced by several factors, including birth rate, death rate, and migration. The population growth rate is determined by the sum of births and in-migration minus deaths and out-migration. In a population with a constant growth rate, the number of individuals increases steadily. In a population with a zero growth rate, the population size remains stable. If the growth rate increases smoothly over time, the population will experience an accelerating increase in the number of individuals per unit of time. Conversely, if the growth rate decreases linearly over time, the population size-and thus the rate of increase in the number of individuals-will decline per unit of time.

The development of preservice teachers' covariational reasoning skills related to graph drawing and interpretation is crucial, both for them to become scientifically literate individuals and to enhance the quality of their future classroom applications in this area. Integrating mathematics into environmental education not only fosters a deeper understanding of the subject matter but also improves mathematical skills and raises environmental awareness. As noted earlier, the existing literature highlights the need for more research on graph drawing and interpretation, often referred to as covariational reasoning, within environmental education. In light of this gap, the present study aimed to examine the population graph drawing and interpretation skills of preservice elementary school teachers. Specifically, the study sought to answer the following question: "*How proficient are preservice teachers in drawing and interpreting individuals vs. time graphs for populations based on given growth rate vs. time graphs?*"

Method

Research Model

In this study, preservice elementary school teachers were provided with growth rate-time graphs of different populations and asked to draw and interpret corresponding graphs of the number of individuals vs. time. The findings obtained from the qualitative data were interpreted. Therefore, this research was conducted using a basic qualitative approach (Merriam, 2013).

Participants

The study involved 57 first-year preservice teachers enrolled in the elementary education department at a university situated in a small city center in Türkiye's Central Anatolia region. The average age of the participants was 18 years. These preservice teachers were taking courses related to science and mathematics at the university, such as Basic Mathematics in Elementary School, Basic Science in Elementary School, and Environmental Education.

Data Collection Tool and Process

A form prepared by the researchers was used as the data collection tool. Data were collected at the end of the courses in the spring semester of 2024. The form presented the participating preservice teachers with growth rate-time graphs for four distinct populations and asked them to draw and interpret the corresponding graphs of number of individuals vs. time. It was initially explained to the participants that the populations did not start with zero individuals. The first graph depicted a constant growth rate, the second a zero growth rate, the third a linearly increasing growth rate, and the fourth a linearly decreasing growth rate. To ensure the validity of the form, it was reviewed by three field experts, one language expert, and one expert in measurement and evaluation.

Data Analysis

In the analysis process, an existing conceptual framework for classifying covariational thinking (e.g., Carlson et al., 2002; Thompson & Carlson, 2017) was not used. Instead, the preservice teachers' ability to draw and interpret graphs of number of individuals vs. time based on the given population size graphs was examined. During the data analysis, patterns were identified in the participants' responses (Merriam, 2013). In this process, data were coded for each graph given to the preservice teachers and categories were established. The types of graphs drawn and interpreted based on the given graphs were accepted as themes. Finally, the findings were interpreted.

The forms were coded by assigning numbers to each participant (PsT1, PsT2, ..., PsT57). Two researchers, one being an expert in science education and the other in mathematics education, independently evaluated the graphs drawn and the explanations provided by the preservice teachers. For each graph and its corresponding explanation, the researchers recorded whether the response was correct or incorrect and noted the reasoning behind their evaluation in a table. Any discrepancies between their evaluations were discussed until a consensus was reached, ensuring reliability. To enhance the credibility of the research, examples of the graphs drawn by the participants and their explanations are presented in the findings. The graphical drawings and explanations made by the preservice teachers regarding the given growth rate-time graph were analyzed independently of each other.

Findings

In this study, preservice teachers were provided with four population growth rate-time graphs. The data obtained from their drawings (D) and explanations (E) were organized based on those four graphs and findings are presented below under the corresponding headings.

Findings Related to the Drawing and Explanation of the First Graph

For the first question, preservice teachers were provided a graph showing a constant growth rate and were asked to draw a corresponding graph of number of individuals vs. time and explain their drawing. The findings obtained from the drawings and explanations for the first question are presented in Table 1.

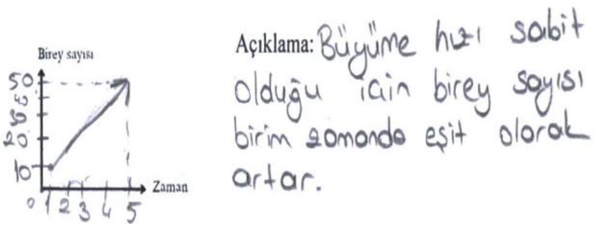
Table 1 Findings Regarding Drawings and Explanations of Number of Individuals vs. Time Graph for the Graph with Constant Growth Rate*

D/E	Categories	Frequency	Codes
Drawings	Correct	6	A positively sloped linear graph drawn starting from the y-axis (6)
	Incorrect	46	A graph drawn parallel to the x-axis (23) A positively sloped linear graph drawn from the origin (16) A graph that is not linear (4) A graph drawn parallel to the y-axis (1) A negatively sloped linear graph (1) A graph for multiple time periods (1)
	No drawing	5	No drawing
Explanations	Correct	26	It was stated that the number of individuals would increase steadily over time (25) It was explained that the number of individuals should initially be at a certain value and increase steadily over time (1)
	Incorrect	30	The number of individuals is balanced or constant (24) Type 1 growth (3) The number of individuals decreases (2) Type 2 growth (1)
	No explanation	1	No explanation

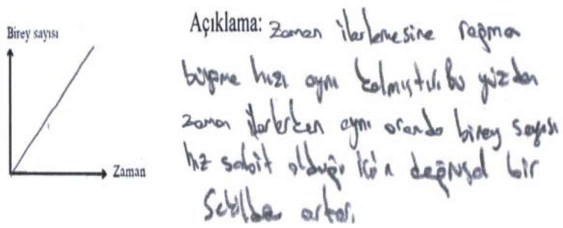
* Graphical drawings and explanations were analyzed independently of each other.

As can be seen in Table 1, 6 of the preservice teachers drew the graph of number of individuals vs. time correctly for the graph with a constant growth rate. The participants who drew it correctly depicted the graph as a positively sloped linear graph starting from the y-axis. A total of 46 participants produced incorrect drawings. Their inaccuracies were caused by drawing the graph parallel to the x-axis (n=23), starting it from the origin (n=16), not drawing it as a linear graph (n=4), drawing it parallel to the y-axis (n=1), drawing it as a negatively sloped linear graph (n=1), or combining different graphs (n=1). Five participants did not produce any drawings. Figure 1 shows examples of the drawings and explanations provided by these preservice teachers. Of the explanations given for the graphs, 26 were correct while 30 were incorrect. One participant did not provide an explanation.

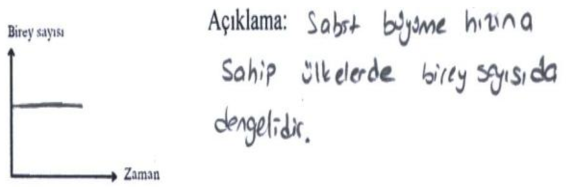
As shown in Figure 1, PsT10 provided both a correct drawing and explanation. PsT10 drew the graph as linear with a positive slope starting above the +y-axis. PsT25, however, drew the graph linearly with a positive slope but started from the origin, making PsT25's answer incorrect. The participants were informed that the population did not start with zero individuals. Therefore, the drawings of those who produced a linear graph with a positive slope starting from the origin, like PsT25, were marked incorrect, and this mistake was made by 16 of the preservice teachers, as seen in Table 1. Additionally, it was observed that PsT10 carefully considered the variables of population size and time while drawing the graph. Most participants who gave incorrect explanations believed that the population size remained in equilibrium or was constant (n=24). They assumed that with a constant growth rate, the population size would stay the same, attributing this to equal birth and death rates, which influenced both their drawings and explanations. The drawing and explanation provided by PsT21 in Figure 1 illustrate this common misunderstanding. Some participants also confused this graph with survival curves, interpreting it as a type 1 (n=3) or type 2 (n=1) survival curve. For instance, PsT49 stated that the population size would not be significantly affected by environmental factors, linking this to a type 1 survival curve in the provided explanation and drawing. In contrast, two participants thought that the population size would decrease in a population with a constant growth rate. PsT6's drawing and explanation exemplify this mistake, as PsT6 assumed that since the growth rate was constant, no new individuals were added, leading to a decrease in the population size.



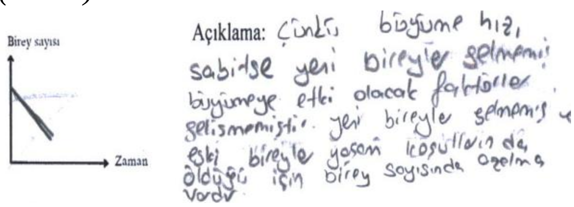
" Since the growth rate is constant, the number of individuals increases equally in unit time." (PsT10)



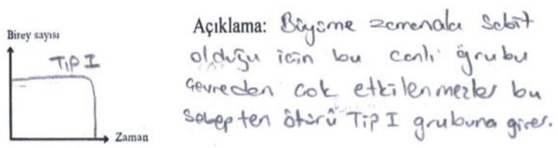
"Despite the progression of time, the growth rate remains the same. Therefore, as time progresses, the number of individuals at the same rate increases linearly because the rate is constant." (PsT25)



"In countries with a stable growth rate, the number of individuals is also balanced." (PsT21)



"Because if the growth rate is constant, new individuals have not arrived and the factors that will affect growth have not developed. There is a decrease in the number of individuals because new individuals have not arrived and old individuals have died in living conditions." (PsT6)



" Since growth is constant over time, this group of organisms are not affected much by the environment, so they are included in the Type 1 growth." (PsT49)

Figure 1 Examples of Drawings of Preservice Teachers Who Correctly and Incorrectly Drew Graphs of the Number of Individuals vs. Time for a Population with Constant Growth Rate

Findings Related to the Drawing and Explanation of the Second Graph

For the second question, the preservice teachers were given a population graph with a zero growth rate and were asked to draw and explain the related graph for number of individuals vs. time. The findings obtained from the preservice teachers' answers to the second question are presented in Table 2.

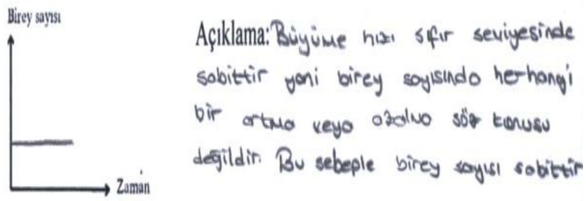
Table 2 Findings Regarding Drawings and Explanations of Number of Individuals vs. Time Graph for the Graph With Zero Growth Rate*

D/E	Categories	Frequency	Codes
Drawings	Correct	22	Graph drawn parallel to the x-axis (22)
	Incorrect	31	Graph coinciding with the x-axis (17) Linear graph with negative slope (8) Nonlinear graph (4) Linear graph with positive slope (1) Graph with multiple time intervals (1)
	No drawing	4	No drawing
Explanations	Correct	26	The number of individuals remains constant (26)
	Incorrect	28	The number of individuals decreases over time (13) The number of individuals is zero because the growth rate is zero (6) J-type growth (4) No individuals are born (1) The number of individuals increases over time (1) Type 2 growth (1) Type 3 growth (1) Explanation unrelated to the number of individuals (1)
	No explanation	3	No explanation

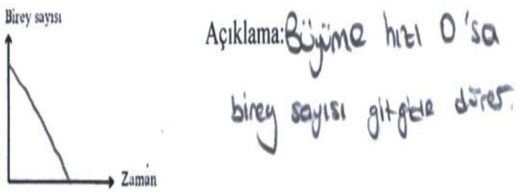
*Graphical drawings and explanations were analyzed independently of each other.

Twenty-two of the participants correctly drew the graph of number of individuals vs. time as a line parallel to the x-axis based on the graph with zero growth rate. The majority of those who drew the graph incorrectly assumed that the number of individuals would also be zero, reflecting this in their graphs (n=17). Some participants drew linear graphs with a negative slope, indicating a decrease in the number of individuals over time, since they interpreted a zero growth rate as causing a decline in population (n=8). Additionally, the responses of participants who did not draw a linear graph for the number of individuals vs. time (n=4), who drew a linear graph with a positive slope (n=1), or who combined different graphs (n=1) were also deemed incorrect. Based on the graph with zero growth rate, 26 of the preservice teachers drew the graph of number of individuals vs. time graph correctly, indicating that the number of individuals would remain constant over time.

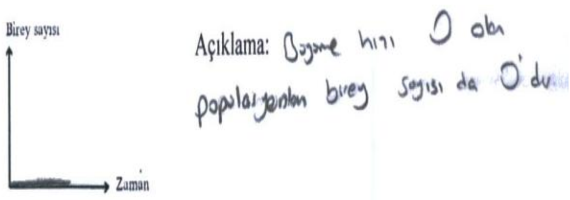
Figure 2 shows the answer of PsT16, one of the participants who produced both a correct drawing and explanation. PsT16 explained that the number of individuals would remain constant over time and drew the graph parallel to the x-axis. Most participants who provided incorrect explanations stated that the number of individuals would decrease over time (n=13). Figure 2 includes PsT20's drawing and explanation as an example. Some participants, such as PsT30, thought that the number of individuals would be zero because the growth rate was zero (n=6), thus drawing the graph of number of individuals vs. time to coincide with the x-axis (n=17). It was also observed that some participants whose answers were considered incorrect made explanations referencing type 2 and type 3 growth curves, as seen in survival curves of populations, like PsT49, or referenced J-type growth, a pattern seen in populations with exponential growth. Three participants did not provide any explanation. Interestingly, 4 participants drew the graph incorrectly but provided a correct explanation for the zero growth rate.



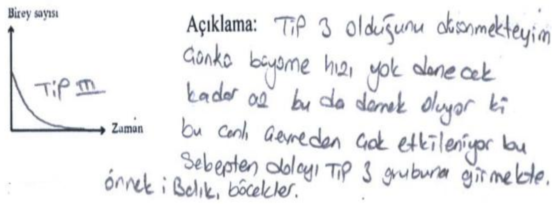
" The growth rate is constant at zero level, that is, there is no increase or decrease in the number of individuals. Therefore, the number of individuals is constant." (PsT16)



"If the growth rate is zero, the number of individuals gradually decreases." (PsT20)



"If the population growth rate is 0, the number of individuals will also be 0." (PsT30)



" I think it is Type 3. Because its growth rate is almost non-existent. This means that this creature is greatly affected by the environment. For this reason, it falls into the type 3 group. Example: Fish, insects." (PsT49)

Figure 2 Examples of Drawings of Preservice Teachers who Correctly and Incorrectly Drew Graphs of the Number of Individuals vs. Time for a Population with Zero Growth Rate

Findings Related to the Drawing and Explanation of the Third Graph

For the third question, the preservice teachers were given a graph where the growth rate increased linearly over time and were asked to draw and explain the graph of number of individuals vs. time accordingly. The findings are shown in Table 3.

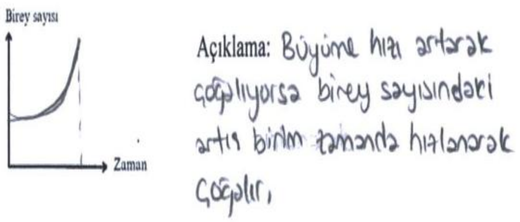
Table 3 Findings Regarding Drawings and Explanations of Number of Individuals vs. Time Graph for the Graph with Linearly Increasing Growth Rate*

D/E	Categories	Frequency	Codes
Drawings	Correct	5	Exponentially drawn graph starting from the +y-axis (5)
	Incorrect	51	Linearly drawn graph with positive slope (36) Graph covering multiple time periods (9) Linearly drawn graph with negative slope (3) Exponentially drawn graph starting from the origin (2) Graph parallel to the x-axis (1)
	No drawing	1	No drawing made
Explanations	Correct	10	The number of individuals increases more over time (10)
	Partially correct	32	The number of individuals increases over time (32)
	Incorrect	14	S-shaped growth (4) The number of individuals decreases (2) The number of individuals increases, remains constant, then decreases (1) The number of individuals is in equilibrium or remains constant (1) The number of individuals increases then remains constant (1) The number of individuals increases logarithmically (1)

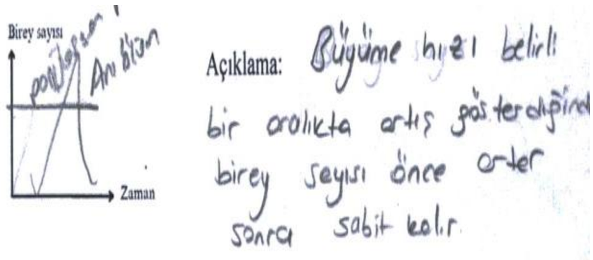
*Graphical drawings and explanations were analyzed independently of each other.

As can be seen in Table 3, only 5 of the preservice teachers drew the graph correctly. These participants represented the graph exponentially on the +y-axis. It was observed that the majority of participants (n=51) incorrectly drew the graph of number of individuals vs. time based on the graph with linearly increasing growth rate. Most of those who drew it incorrectly (n=36) represented both the growth rate and the number of individuals as increasing linearly over time. The answers of participants who combined different graphs (n=9), drew a linear graph with a negative slope (n=3), drew an exponential graph starting from the origin (n=2), or drew a graph parallel to the x-axis (n=1) were also considered incorrect. One participant did not make a drawing.

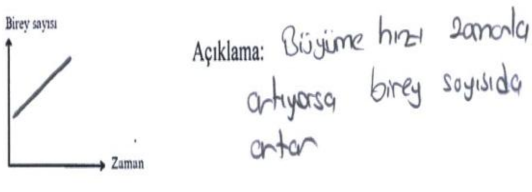
As seen in Table 3, 10 participants provided a correct explanation, stating that the number of individuals would increase exponentially over time. PsT52's response in Figure 3 is an example of a correct drawing and explanation. PsT52 drew the graph of number of individuals vs. time for the population with a linearly increasing growth rate as being exponential on the +y-axis, explaining that the number of individuals would increase more per unit of time. Responses of the participants that were considered partially correct (n=32) included accurate observations about the rise in the number of individuals over time. However, these increases were depicted as linear rather than exponential in their drawings. The explanations were generally similar to PsT3's response, shown in Figure 3, wherein the increase in the number of individuals was mentioned, but not in detail. A small number of participants (n=5) produced both correct drawings and comments. However, 14 participants provided incorrect explanations. Some explained and drew the number of individuals as decreasing when the growth rate increased over time, possibly because they were thinking about the later stages of S-shaped growth curves. Additionally, 5 participants made correct explanations but drew their graphs incorrectly, and 1 did not provide any explanation.



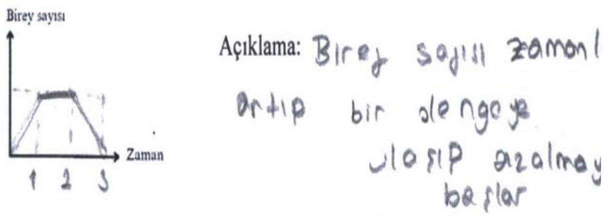
"If the growth rate increases, the number of individuals increases at an accelerating rate per unit time." (PsT52)



"Since the growth rate increases within a certain range, the number of individuals first increases and then remains constant." (PsT44)



"If the growth rate increases over time, the number of individuals also increases" (PsT3)



"The number of individuals increases over time, reaches a balance and begins to decrease" (PsT24)

Figure 3 Examples of Drawings of Preservice Teachers who Correctly and Incorrectly Drew Graphs of the Number of Individuals vs. Time for a Population whose Growth rate Increases Linearly over Time

Findings Related to the Drawing and Explanation of the Fourth Graph

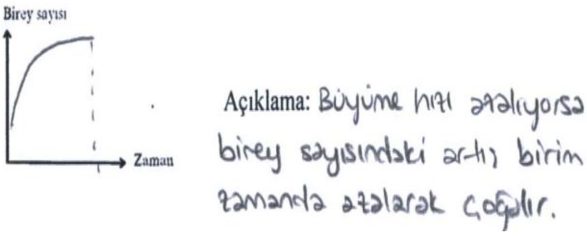
For the fourth question, the participating preservice teachers were asked to draw and explain the graph of number of individuals vs. time graph for a graph whose growth rate decreases linearly over time. The findings obtained from the participants' answers to the fourth question are shown in Table 4.

Table 4 Findings Regarding Drawings and Explanations of Number of Individuals vs. Time Graph for a Population whose Growth Rate Decreases Linearly Over Time*

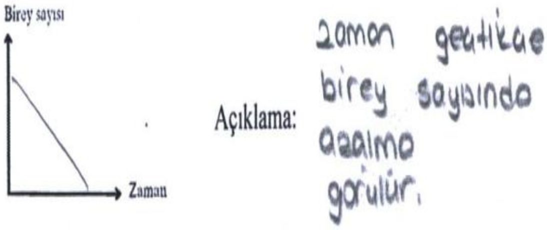
D/E	Categories	Frequency	Codes
Drawings	Correct	1	Logarithmic graph drawn starting from the +y-axis (1)
	Incorrect	55	Linear graph with a negative slope (22) Graph with multiple time intervals (13) Linear graph with a positive slope (9) Non-logarithmic graph starting from the +y-axis (8) Logarithmic graph starting from the origin (2) Graph parallel to the x-axis (1)
	No drawing	1	No drawing
Explanations	Correct	12	The number of individuals increases more slowly (12)
	Partially correct	2	The number of individuals increases over time (2)
	Incorrect	39	The number of individuals decreases (30) Number remains stable or constant (2) Sudden deaths occur (1) S-type growth (1) Decreases, then remains stable (1) Type 3 growth (1) Increases, then decreases (1) Increases very rapidly (1) Explanation unrelated to the number of individuals (1)

*Graphical drawings and explanations were analyzed independently of each other.

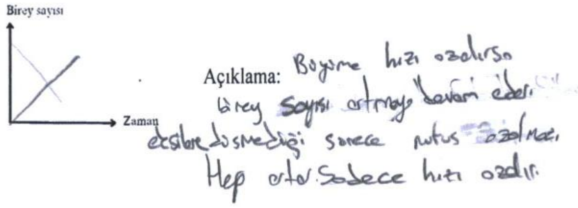
As seen in Table 4, only one participant produced a correct drawing. The majority of participants who produced incorrect drawings (n=22) drew the graph of number of individuals vs. time based on a decreasing linear growth rate as a linearly decreasing graph. Additionally, some participants combined different graphs in their drawings (n=13). As can be seen in Table 4, only 12 of the participants stated that the number of individuals would increase but the increase would occur at a decreasing rate. Some participants who correctly stated that the number of individuals would decrease over time drew this decrease linearly rather than logarithmically (n=11). Only one participant (n=1) produced both a correct drawing and explanation. Two participants whose answers were considered partially correct stated that the number of individuals would decrease over time and did not elaborate on the nature of the decrease. Most of the participants whose answers were deemed incorrect explained that the number of individuals decreased (n=30). Additionally, 11 participants drew the graph incorrectly but provided a correct explanation. Some participants produced explanations and drawings suggesting that the number of individuals would increase as the growth rate decreased over time. Furthermore, some interpreted the graph as a J-shaped graph. Figure 4 provides examples of the drawings and explanations of the participants who correctly and incorrectly drew the graph of number of individuals vs. time for a population whose growth rate decreases linearly over time.



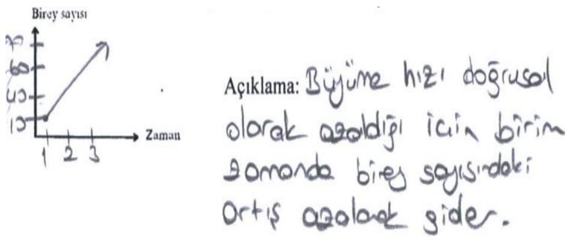
"If the growth rate decreases, the increase in the number of individuals increases by decreasing in unit time." (PsT52)



"Over time, the number of individuals decreases." (PsT43)



"If the growth rate decreases, the number of individuals continues to increase. The population does not decrease unless it is negative. It always increases. It just decreases in speed." (PsT25)



"Since the growth rate decreases linearly, the increase in the number of individuals per unit time decreases." (PsT10)

Figure 4 Examples of Drawings of Preservice Teachers who Correctly and Incorrectly Drew Graphs of the Number of Individuals vs. Time for a Population whose Growth Rate Decreases Linearly Over Time

Only PsT52 stated that the number of individuals in a population with a linearly decreasing growth rate increases at a decreasing rate over time and made a correct drawing. Most of the participating preservice teachers, like PsT43, thought that the number of individuals would decrease over time and drew a linear graph with a negative slope. From the drawings and explanations of PsT25 and PsT10, it can be understood that some preservice teachers predicted that the number of individuals would increase at a decreasing rate over time; they made correct explanations but could not accurately transfer those explanations to the graph.

Conclusions, Discussion, and Suggestions

In this study, preservice primary school teachers were given growth rate-time graphs of different populations and asked to draw and interpret the corresponding graphs for number of individuals vs. time. The results showed that these preservice teachers had difficulty in drawing and interpreting population graphs and particularly in interpreting linear and especially curvilinear graphs. Only one participant drew and interpreted all the graphs correctly. This participant is a graduate of a science high school where science and mathematics courses are predominant. In this respect, it can be said that this participant has a basic background. The most common mistakes can be listed as follows: (1) Starting from the origin when drawing the graph of the number of individuals vs. time, even though they were initially told that the number of individuals in the population is not zero; (2) believing that the number of individuals in a population with a constant growth rate will also remain constant; (3) thinking that the number of individuals in a population with a zero growth rate will also be zero; (4) assuming that the number of individuals in a population with a linearly increasing growth rate will also increase linearly over time; and (5) believing that the number of individuals in a population with a smoothly decreasing growth rate will also decrease over time. This may be due to the fact that the preservice teachers did not sufficiently understand and inquiry the graph, and lacked prior knowledge (Shah & Hoeffner, 2002). It is thought that the most common mistakes in graph drawing are due to the fact that the preservice teachers do not reflect the relationships between the data on the graph, but rather think of this relationship as in the first graph and transfer it to the second graph as a picture. This reveals that they have a misconception of graphs as pictures (Roth & Bowen, 2001). Also these results indicate that most of the preservice teachers could not account for the simultaneous change of two variables. They had more difficulty drawing curvilinear graphs than linear graphs. This may be due to the fact that preservice teachers tend to create linear graphs because they do not evaluate the relationship in the graph by not looking at the whole graph (Leinhardt vd., 1990). Similarly, some research results related to physics showed that students struggled to interpret rates of change in nonlinear or curvilinear graphs, which was associated with deficiencies in their mathematical knowledge (McDermott et al., 1987; Planinic et al., 2013). Furthermore, in a study conducted with calculus students, it was observed that the students had difficulty creating images of the rate of change and could not accurately represent or interpret the increasing and decreasing rates of functions (Carlson et al., 2002). In addition, as a result of this research, the difficulty experienced by preservice teachers in determining the starting point of the graph is similar to the study of Aydın and Tarakçı (2018).

In the present study, some of the participating preservice teachers interpreted the graphs given to them correctly, but they could not transfer their correct interpretations into the drawing and interpretation of a new graph. This was particularly evident in the responses to the first and second questions, which required drawing and interpreting linear graphs. From this perspective, it can be inferred that the majority of participants remained at the comprehension/understanding level in graph drawing and interpretation; they had not progressed to higher levels such as application, analysis, and synthesis. These results align with findings from several previous studies on graph interpretation and drawing. For instance, Tairab and Khalaf Al-Naqbi (2004) and Taşar et al. (2002) found that students had more difficulty drawing graphs than reading and interpreting them. In a study on preservice science teachers, it was determined that, similar to the results of this research, the preservice teachers' levels of reading and interpreting graphs were better than drawing graphs in most cases (Aydan & Dönel Akgül, 2021).

In light of the findings of this study, it may be beneficial to support preservice teachers in developing the ability to draw and interpret graphs through modeling. In this process, the initial number of individuals should be determined, tables should be created for the number of individuals at specific time intervals, and graphical drawings and interpretations should be made based on the created tables. Using numbers during the process of interpreting and creating graphs can help provide a more concrete understanding and promote correct thinking (Altındiş et al., 2024). As indicated in the findings of this study, some preservice teachers drew graphs of the number of individuals vs. time by assigning values to the growth-rate time graph. Additionally, there should be an emphasis on questioning the operations performed. Otherwise, the interpretation of graphs that are consistently explained and observed may be incorrect, resulting in a lack of skill development in this area. Furthermore, the results obtained from this study highlight the necessity of integrating different disciplines. Difficulties in graph interpretation and drawing may hinder the understanding of concepts that are intended to be taught (Coştu et al., 2017). In this context, it is essential to know and utilize the mathematical skills, methods, and techniques specific to mathematics education. When needed, lecturers working in mathematics and science disciplines can work together and support teaching practices, thus enabling interdisciplinary collaboration. Another suggestion based on the results of this research is that the environmental education course should be designed and implemented to provide not only knowledge but also practical skills. These skills could include not only the ability to draw and interpret graphs, as evaluated in this study, but also probabilistic thinking and proportional reasoning (Lawson et al., 2000). Some subjects may inherently be more suitable for teaching certain skills. Therefore, it is crucial to first determine which skills can be developed within the context of the environmental education course and to design appropriate teaching environments accordingly. Future research may focus on the design, implementation, and evaluation of a skills-based environmental education course.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

The authors declare that they have no competing interests.

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Research involving Human Participants and/or Animals

The study involves human participants. Ethics committee permission was obtained from Yozgat Bozok University, Social and Human Sciences Ethics Committee.

Çevre Eğitiminde Matematik: Öğretmen Adaylarının Popülasyon Büyüklüğü Grafiği Çizme ve Yorumlama Becerilerinin İncelenmesi

Özet:

Matematiksel bir beceri olan grafik çizme ve yorumlama, fen eğitiminde pek çok konunun öğretiminde kullanılmaktadır. Sınıf öğretmenliği programında yer alan çevre eğitimi dersindeki popülasyon ekolojisi bu konulardan biridir. Çevre eğitimi dersi kapsamında bir popülasyonun büyüme ve küçülme nedenlerinin öğretimi ve yorumlanmasında grafiklere başvurulmaktadır. Nitel olarak yürütülen bu çalışmada sınıf öğretmen adaylarının popülasyon grafiği çizme ve yorumlama becerilerinin incelenmesi amaçlanmıştır. Araştırmaya Türkiye'nin İç Anadolu Bölgesinde yer alan bir üniversitenin sınıf eğitimi bölümünde öğrenim gören 57 öğretmen adayı katılmıştır. Sınıf öğretmen adaylarına farklı popülasyonlara ait büyüme hızı-zaman grafikleri verilmiş, onlardan birey sayısındaki değişimleri zamana göre gösteren grafikler çizmeleri ve yorumlamaları istenmiştir. Sonuçlar öğretmen adaylarının popülasyon grafiği çizme ve yorumlama konusunda güçlük yaşadığını göstermiştir. Doğrusal ve özellikle eğrisel grafiklerin çizim ve yorumlanmasında öğretmen adaylarının çoğu iki değişkenin eş zamanlı değişimini hesaba katamamıştır. Araştırma sonuçlarının çevre eğitiminde matematiksel becerilerin kullanımına ve disiplinler arası işbirliğinin gerekliliğine ilişkin farkındalığı artıracağı, yapılacak çalışmalara rehber olacağı düşünülmektedir.

Anahtar kelimeler: Çevre eğitimi, matematik eğitimi, grafik çizme becerisi, grafik yorumlama becerisi, popülasyon ekolojisi, öğretmen adayları.

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