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Application of the Two Stage Network Data Envelopment Analysis Model Using the Bootstrap Simulation Model: Case of Turkish Banks

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Abstract

This study aims to assess the efficiency of the banking sector in Türkiye. To achieve this objective, a two-stage Network Data Envelopment Analysis (NDEA) model was applied. To enhance the accuracy of the results and calculate the bias in each stage, the bootstrap method was used in the NDEA. Specifically, an input-oriented under the constant return to scale (CRS) model was employed to evaluate efficiency. The study included data from the 2022 fiscal year for 13 commercial banks operating in Türkiye. In the first stage, three input variables were considered: total assets, number of employees, and number of branches. The output for this stage was the general collected resources, which simultaneously served as the input for the second stage. In the second stage, two output variables were used: Net interest profit and other operating incomes. According to the results, the average efficiency of the banking sector in Türkiye was 88.9% in the production stage and 80.7% in the intermediation stage, while the overall average efficiency was also 70.9%, as determined by the two-stage NDEA model. When applying the bootstrap method for the NDEA analysis, the average efficiencies for the three stages were 78.5%, 73.5%, and 56.4%, respectively. This indicates weak performance in the overall efficiency of the banking sector.

Keywords: *Network Data Envelopment, Turkish Banks, Bootstrap, Efficiency.*

JEL Classification: G21, C67, L25.

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1. Introduction

Banks and financial institutions have become very essential elements of life. They play a major role in the global economy due to the rapid economic progress witnessed worldwide (Alfaiate, Özdemir and Alp, 2023). Given their important role in economic development, meticulous attention is required when dealing with them, as any negative impact on the banking sector can be reflected throughout an entire country's economy. In addition, the presence of an efficient banking system is a prerequisite for achieving progress at the economic level. An effective banking system, which allocates resources efficiently, promotes rapid economic growth in every country. On the other hand, a strong banking system encourages investment by financing productive work and facilitating business activities. The evaluation of banks' performance gains critical because if these institutions operate more efficiently, they will earn more profits and enhance the liquidity of the economy. Without the presence of effective banking institutions, there will be a challenge to maintain economic growth within a country (Kamau, 2011).

Nowadays, with diminishing resources and escalating competition, the concepts of effectiveness and efficiency have become more important for the banking sector. The main responsibility of banks is to use their resources most effectively. Therefore, bank managers should set goals for the future policies of their banks by comparing their activities with those of competitors in the sector (Eken and Kale, 2011). The efficiency of the banking sector is crucial because it affects not only the success of the monetary system, but also affects the stability of the same banking sector.

Data Envelopment Analysis (DEA) is a methodology used to identify how effectiveness and efficiency Decision Making Units (DMUs) are using their resources. DEA is used to measure the relative efficiency of similar organizations (DMUs) that producing multiple outputs using multiple inputs. DEA is one of the most popular methods used to measure efficiency in recent years in various fields, including the banking sector. In other words, DEA is a non-parametric method for analyzing the performance of homogeneous units (in our study, banks) based on linear programming developed by Charnes, Cooper, and Rhodes in 1978 (CCR) and Banker, Charnes, and Cooper (BCC) in 1984 (Banker, Charnes and Cooper, 1984; Charnes, Cooper and Rhodes, 1978).

The scholarly literature indicates that from 1978 to 1995 there was a steady growth in DEA publications. But from 1995 and onwards there was an exponential increase in DEA publications, both in terms of theoretical development and various applications such as banking (Goyal, Singh and Aggarwal, 2019), transportation (Mahmoudi, Emrouznejad and Shetab-Boushehri, 2020), healthcare (Yeşilyurt and et al, 2021), education (Jauhar, Pant and Nagar, 2017), tourism (Leal Paço and Cepeda Pérez, 2013), finance (Moon and Min, 2020), sports (Cooper, Ruiz and Sirvent, 2009) and many more. This trend is clearly reflected in Figure 1 (Panwar and et al, 2022).

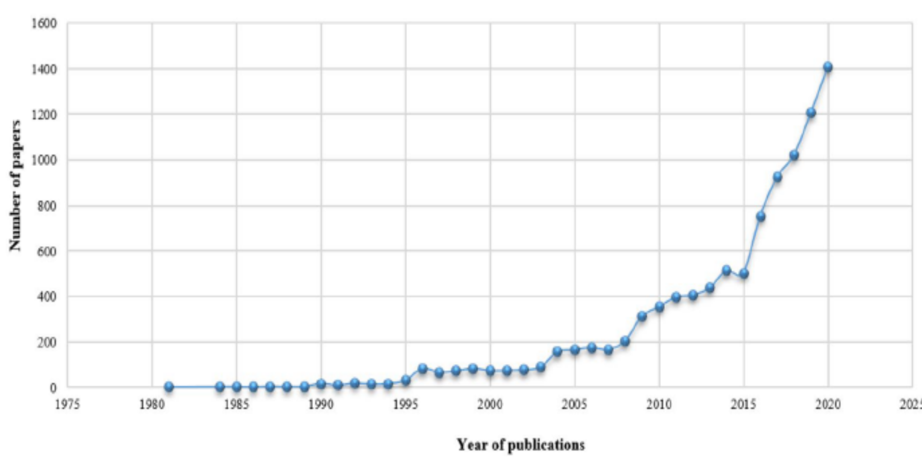

Figure 1. *DEA Publications Growth Trend Since 1980*

Through this technique enables the assessment of effective and inefficient units, providing insights for improving the effectiveness of inefficient units through suggested changes in inputs and outputs. The Traditional DEA is a method used to assess the effectiveness of DMUs by transforming inputs into outputs in a single step. This implies that it does not take into account the details of the internal process. Consequently, this method is called "black box" because it is not necessary to know the structures and process links within the production process. To overcome this drawback of the traditional approach, the network data envelopment analysis (NDEA) approach was introduced by Färe ve Grosskopf in 2000 (Färe ve Grosskopf, 2000). NDEA provides a more comprehensive perspective by analyzing a unit as a network of interconnected processes divided into multiple overlapping stages. This approach allows for a deeper understanding of the sources of inefficiency, allowing the stages responsible for suboptimal performance to be identified. Furthermore, NDEA offers additional flexibility compared to traditional methods as it can accommodate internally interconnected processes. This makes it an ideal tool for analyzing organizations with complex and interdependent structures.

According to Simard and Wilson (1998, 2000, 2008), DEA estimators tend to bias especially when the DMU of analysis may not fully represent the problem (DMUs are a small) and the results are sensitive to the composition of the sample (Simar and Wilson, 2000a; Simar and Wilson, 1998; Simar and Wilson, 2000b). Additionally, both traditional DEA and NDEA have statistical limitations one notable limitation the data distribution is unknown. To address these challenges, Simar and Wilson have developed the bootstrap technique, initially proposed by Efron (1979), as an effective statistical tool such as these cases. The bootstrap is a resampling method in which from the original data samples is repeatedly reselected with replacement, creating numerous "pseudo-samples". This process effectively increases the sample size available for analysis, enabling more robust statistical inference. By applying the bootstrap technique to NDEA models, researchers can calculate bias in efficiency estimates, improve the precision of results, and account for the variability inherent in the data.The importance of the bootstrap technique lies in its ability to generate more accurate and reliable estimates. This reduces the influence of outliers and mitigates the impact of sample-specific anomalies. Consequently, the conclusions drawn from the analysis are more robust and generalizable. Furthermore, by the bootstrap technique the estimation of confidence intervals for efficiency scores, providing decision-makers with a clearer understanding of the range of possible outcomes and the reliability of the results.

Combining NDEA with the bootstrap technique introduces a new dimension to efficiency analysis by leveraging the advantages of network analysis alongside the statistical robustness of bootstrap. This hybrid approach facilitates a deeper and more accurate understanding of complex processes, enabling informed strategic decision-making based on precise data and realistic problem representations. This combination proves to be a valuable tool for performance evaluation in fields characterized by complex operations and numerous variables. More

over, the integration of these two techniques significantly impacts the banking sector by improving the efficiency of banks through more accurate and reliable analyses of internal operational performance within the production system. It allows banks to optimize resource allocation and enhance productivity, which positively influences profitability and sustainability. Additionally, more precise estimates bolster confidence in banking performance among investors and clients, contributing to financial stability and increased competitiveness in the market.

In context of the banking sector and measuring its system efficiency, many studies have been conducted in recent years about this important field. This is because the efficiency of the banking system constitutes a basic pivotal in financial markets due to its direct impact on the stability of the banking sector and thus on the effectiveness of the country's monetary policy. DEA is considered one of the most important models used to measure the performance of banks.

According to Wang et al. (2014) conducted a comparison between the two-stage NDEA approach and the traditional DEA model in measuring the operational efficiency of the Chinese banking system. The banking system was divided into two sub-processes: deposit producing and profit earning. the work including of 16 Chinese banks, both the state-owned commercial banks and the joint-stock commercial banks. Efficiency was evaluated during the period from 2003 to 2011, taking input-oriented variables returns to scale. The main results of this study showed that the two-stage DEA model is more effective than the traditional black box DEA model in identifying the inefficiency of the banking system. In addition, it proved that the inefficiency of the Chinese banking system is due to the sub-process of deposit production.

Similarly, Muhammad and Ali (2019) applied the three -stage NDEA in evaluating the operational efficiency, service effectiveness, and social effectiveness of 37 branches of one of the largest commercial banks in Iran. The overall performance assessment of the banking system is designed in three sections: production intermediation, and social welfare approaches. It has been shown that the proposed model has high robustness compared to traditional black box models.

Dia et al. (2020) proposed a three-stage division of the banking system into production, intermediation, and revenue generation stages. Using a bootstrap NDEA approach, they assessed the performance of the six largest Canadian banks for the period 2000–2017. The efficiency scores were calculated using the Simar and Wilson (1998, 2000) bootstrapping model with 2000 iterations to ensure accuracy. The study findings indicate that the financial crisis in Canada in 2007 led to a decrease in efficiency in the performance of Canadian banks in the revenue generation stage, although this decline was not significant for the production and investment stages.

Additionally, the authors compared the NDEA model with the traditional DEA black box model and concluded that the NDEA provides more insightful and accurate results regarding bank efficiency.Traditional data envelopment analysis models with bootstrap have been utilized also in studies within the banking sector.

According to LI (2020) This study uses data envelopment analysis (DEA) based on the bootstrap methodology developed by Simard and Wilson (1998) to measure the efficiency of Chinese banks. The study covers a sample of 101 (75 local and 26 foreign owned) banks for the period 2015-2017, with 2000 replications for the bootstrapped procedure. However, there are studies in other countries to measure the efficiency of the banking sector using this methodology, such as Gulf Cooperation Council (GCC) countries (Maghyereh, and Awartani, 2012), the Middle East and North Africa (Bahrini, 2017), Australia (Moradi-Motlagh and Saleh,2014), Türkiye (Diler, 2011) etc.

Outside the realm of banking, the bootstrapping principle has also been used in various fields. For instance, in the field of airports, Cifuentes-Faura and Faura-Martínez (Cifuentes-Faura and Faura-Martínez, 2023) were applied the bootstrap approach to evaluate the efficiency of 37 Spanish airport in 20018. To evaluate the efficiency of 43 turkish airports in 2013, the bootstrap DEA approach was used by özsoy and örkcü (Özsoy and Örkcü, 2021). According to Nwaogbe et al. (2018) used a bootstrap DEA approach to evaluate the efficiency of 30 Nigeria airports in the period 2003–2013.

Yan et al. (2023) evaluated the performance of transportation services in Taiwan using Simar and Wilson's bootstrap methodology. According to Yang et al. (2017) analyzed the regional technical efficiency of the iron and steel industry for seven regions in China. For the analysis, a two-stage DEA network procedure based on the bootstrap model was used from 1996 to 2003.

According to Vaseei et al. (2023) evaluated the performance of the sustainable supply chain in Iran using basic tow-stage NDEA models and bootstrap simulation model. A total of 25 tomato paste production companies were evaluated for the year 2021. The final results showed that 16 DMU_s were efficient and 9 DMU_s were inefficient. Using bootstrap simulation models, it was revealed that 4 DMU_s were efficient while 21 DMU_s were inefficient. The accuracy of the bootstrap model in evaluating the DMU was found to be better than the basic models.

The table 1 shows a literature summary on bank efficiency and other fields using non-parametric DEA and NDEA techniques based bootstrap simulation models. It indicates also the cities of interest in each study, along with the methods used for assessing efficiency. A notable diversity is clearly in the utilization of this contemporary approach, according to the specified objectives.

Table 1. *Literature Summary on Bank Efficiency*

Authors	Year	DMU_s	Field	Country	Method
Vaseei et al. (2023).	2021	25	The sustainable supply	China	Two-stage bootstrap NDEA
Yang et al. (2017)	1996-2003.	7	İron and steel industry	China	Two-stage bootstrap NDEA
Yan et al. (2023).	2017	36	transport services.	Taiwan	Bootstrap DEA
Nwaogbe et al.(2018).	2003-2013.	30	Airport	Nigeria	Bootstrap DEA
Özsoy and Örkcü (2021).	2013	37	Airport	Türkiye	Bootstrap DEA
Cifuentes-Faura and Faura-Martínez (2023).	20018	37	Airport	Spania	Bootstrap DEA
Diler (2011).	2003-2010	22	banks	Türkiye	Bootstrap DEA
Moradi-Motlagh and Saleh, (2014).	1997–2005	10	banks	Australia	Bootstrap DEA
Bahrini, (2017).	2007–2012.	33	Islamic Banks	the Middle East and North Africa	Bootstrap DEA
Awartani and Maghyereh, (2012).	70	1998–2009	Bank	Gulf Cooperation Council (GCC)	Bootstrap DEA
LI (2020).	2015-2017	101	Bank	China	Bootstrap DEA

Dia et al, (2020).	2000-2017.	6	Bank	Canada	Three-stage Bootstrap NDEA
Muhammad and Ali (2019).	-	37	Bank	Iran	Three-stage NDEA
Wang et al. (2014).	2003 to 2011	16	Bank	China	Two-stage NDEA

2.Methodology: Nonparametric Approaches

2.1. DEA

Data Envelopment Analysis (DEA) is a modern, non-parametric method based on linear programming to measure the efficiency of homogeneous (DMU_s), DMU_sin our case are "banks". The main goal of this method is to determine which (DMU_s) showes the best performance. These units are classified "efficient units" and it is forming the limits of efficiency. In addition, units that are not on the efficiency frontier are classified as "inefficient units". DEA is highly flexible as it only requires information about inputs and outputs. Additionally, this method determines whether the poor in DMU_s performance is due to an excess of inputs, a shortage of outputs, or both. We say that a unit is efficient compared to others if the performance score equals 1, otherwise the unit is inefficient. Farrell's research in 1957 demonstrated the possibility of evaluating efficiency between a single input and single output without making any assumptions on the form of the production function. In 1978, Charnes et al. Introduced the constant return to scale (CCR) model to include measuring efficiency in casesthat include multiple inputs and multiple outputs. After that, in 1984, Banker, Charnes and Cooper developed a model assuming variable returns to scale (VRS). However, this model is created by adding a constraint to the CCR model. Usually there are two main models in DEA through which the efficiency value of the (DMU_s) is obtained. The first model is called the input-oriented and the second model is called the output-oriented. The input-oriented model minimizes inputs as much as possible while keeping output constant. On the other hand, in the output-oriented model maximizes output while keeping inputs constant. The choice of direction for the models and the choice of inputs and outputs vary according to the company's objectives and the point of view of the analyst who is measuring efficiency.

For mathematical formula of DEA method according to input-oriented CCR model is:

$$\mathrm{E} = \mathrm{Min}\,\theta - \varepsilon\Big(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+\Big)$$

$$\sum\nolimits_{j=1}^{n} x_{ij}\,\lambda_j + S_i^- = \theta\, x_{i0}\,; i = 1 \dots m. \qquad (1)$$
$$\sum\nolimits_{j=1}^{n} y_{rj}\,\lambda_j - S_r^+ = y_{r0}\,; r = 1 \dots s.$$

$$\lambda_j\,, S_r^+\,, S_i^- \geq 0\ \forall i, j,\ \ j=1\dots n.$$

In the above model, x_{ij} denote the ith input factor and y_{rj} denote the r^{th} output factor of the DMU_j under evaluation. θ is the efficiency score of the j^{th}DMU. $S_i^-$and S_r^+ respectively refer to slack variables for inputs and slack variables for outputs. λ_j is coefficient that allows determining the reference set for inefficient DMUs.

By adding $\sum_{j=1}^{n} \lambda_j = 1$ convexity constraint to the previous model, we obtain the Banker-Charnes model assuming variable returns to scale (VRS).

2.2.Network Data Envelopmen Analysis (NDEA)

DEA is called a black box because it does not take into account the knowledge of structures and connections within the production process. Indeed, it is important to consider the operations of production process when evaluating the efficiency of an overall system. Overlooking these internal processes can lead to inaccurate conclusions.

NDEA, the concerned performance process is considered as a network composed of interconnected sub-processes. One of the most important features of this approach is that it accurately reflects the internal processes. Therefore, it provides more representative and accurate results compared to traditional DEA models. Additionally, Network DEA models provide more comprehensive information than traditional DEA models. This is done by dividing the production system into two or more subsystems\process\ structure (depending on the requirements of the production system). Subsystems are interconnected through what are called intermediate products. Intermediate products are outputs of one subsystem and at the same time serve as inputs for the next subsystem. By introducing the NDEA to any production system, it enables managers to determine what any part of the production system is responsible for inefficiency. In this study, the terms subsystems, process, structure, and stage will be used interchangeably.

NVZA systems have different types of structures which can be classified into serial structures, parallel structures, mixed structures, hierarchical structures, and dynamic structures. In this study, we will consider two-stage structures, which are special cases of simple serial structures. The structure of the two-stage production system with two subprocesses is shown in Figure 2. The first stage consumes all the exogenous inputs X_i, i = 1, ..., m, to produce the intermediate products Z_g, g = 1, ..., h, which serve as inputs are consumed by the second stage to produce the final outputs Y_r, r = 1..., s. Here, Z_g is simultaneously the output of the first subprocess and the input of the second subprocess.

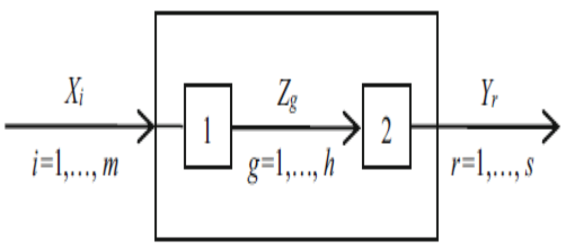

Figure 2. *NDEA* Two-Stage System

Through the input-oriented NVZA models proposed by Kao and Huang (2008), the overall system efficiency and the efficiency of system sub-processes can be achieved. Where model (2) represents the overall efficiency of the system E_K, while models (3) and (4) represent the efficiency of the first stage E_k^1 and the second stage E_k^2, respectively (Kao and Hwang, 2008).

$$E_K = max \sum_{r=1}^{s} u_r y_{rk} \qquad (2)$$

S.T $\quad \sum_{i=1}^{m} v_i x_{ik} = 1$

System constraints:

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \dots, n$$

Division constraints:

$$\sum_{g=1}^{h} w_g z_{gj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \dots, n$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{g=1}^{h} w_g z_{gj} \leq 0, j = 1, \dots, n$$

$$u_r, w_g, v_i \geq \epsilon \; r = 1, \dots, s, \quad i = 1, \dots, m \,, g = 1, \dots, h$$

Efficiency of the second stage $E_{\,k}^{\,1}$.

$$E_{\,k}^{\,1} = max \sum_{g=1}^{h} w_g z_{gk} \qquad (3)$$

S.T $\sum_{i=1}^{m} v_i x_{ik} = 1$

$$\sum_{g=1}^{h} w_g z_{gj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \dots, n$$

$$u_r, w_g, v_i \geq \epsilon \; i = 1, \dots, m \,, g = 1, \dots, h$$

Efficiency of the second stage $E_{\,k}^{\,2}$.

$$E_{\,k}^{\,2} = max \sum_{r=1}^{s} u_r y_{rk} \qquad (4)$$

S.T $\sum_{g=1}^{h} w_g z_{gk} = 1$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{g=1}^{h} w_g z_{gj} \leq 0, j = 1, \dots, n$$

$$u_r, w_g, v_i \geq \epsilon \; r = 1, \dots, s, \;\; , g = 1, \dots, h$$

Overall efficiency is the product of the efficiencies of the two sub-processes:

$$E_K = E_{\,k}^{\,1} \times E_{\,k}^{\,2} \qquad (5)$$

2.3. Bootstrap Network Data Envelopment Analysis

DEA is a widely used non-parametric approach that does not need a predefined functional formula that links inputs to outputs. However, there are some disadvantages to this method. Firstly, the efficiencies estimated by this method are only point estimates, which means that they efficiency value of the DMU represent a specific point rather than a possible range of values. Second, efficiency results depend heavily on the given data set, which means that any change in the data can cause efficiency values to change. Finally, it is difficult to apply statistical inferences to the efficiency results produced by this method, making it difficult for decision makers to draw accurate statistical conclusions (Mahajan, Mogha, and Pannala, 2024). To overcome these issues, a smooth bootstrap methodology integrated with DEA has been proposed by Simard and Wilson (Simard and Wilson, 1998; Simard and Wilson, 2000).

Bootstrap is a statistical method based on simulation of original data first proposed by Bradley Efron in 1979 (Efron, 1979). The main idea of Bootstrap is to resample the available data by generation replicates from the original sample with replacement. This process is repeated a certain number of times on the available data set. This method is useful for generating multiple estimations used for statistical inferences. Bootstrap requires computer-intensive calculations. Therefore, the use of this method has increased with the development of computers, especially since the 1990s (Özdemir and Navruz, 2016). This technique has been used in both parametric and non-parametric methods. In non-parametric methods, bootstrap-based DEA was first used by Simar and Wilson as mentioned previously. The bootstrap DEA method provides efficient bias correction and confidence intervals for efficiency scores, making it more accurate and reliable.

In this paper we will use two-stage Bootstrap NDEA method developed by Simar and Wilson (1998) and its step can be summarized as follows:

1. Using the available sample data (X_i, Z_i, Y_i) (inputs, outputs and intermediate products), solutions are obtained with the linear programming model of NVZA. The original efficiency scores for each DMU are $\hat{\theta}_i$; i = 1,2, …, n is calculated.
2. For each i where, i = 1, …, n a bootstrap efficiency $\hat{\theta}^*_{ib}$, i =1, ... n is generated. Through taking random Samples of size n with replacement from the set of efficiency scores calculated in step 1. In this way, a set of bootstrap efficiency scores is created.
3. The Bootstrap simulation data set (X_i^*, Y_i^*, Z_i^*) is created in the following format.
    - If the input oriented, $(X_i^*, Y_i^*, Z_i^*) = \left(x\frac{\hat{\theta}_i}{\hat{\theta}^*_{ib}}, z\frac{\hat{\theta}_i}{\hat{\theta}^*_{ib}}, y\right)$; i = 1,2, …, n. (6)
    - If the output oriented, $(X_i^*, Y_i^*, Z_i^*) = \left(x, z\frac{\hat{\theta}_i}{\hat{\theta}^*_{ib}}, \frac{\hat{\theta}_i}{\hat{\theta}^*_{ib}}y\right)$; i = 1, …, n. (7)
4. Calculating Bootstrap efficiency estimates are obtained using the NVZA method with the simulation data set calculated in step 1.
5. Repeating step 2 and 4 for a large number of B we will have a set of estimated score for each DMU.
6. The amount of bias of the estimated efficiency scores obtained from Bootstrap NDEA is calculated with Equation (1)
   $Bias\,(\hat{\theta}^*_{ib}) = E\left(\hat{\theta}^*_{ib}\right) - \hat{\theta}_i = \frac{1}{B}\sum_{b=1}^{b} \hat{\theta}^*_{ib} - \hat{\theta}_i$. (8)

3. Method, Data sources, Variables, and Decision-making Units (DMU)

It is well known that the operational process of banks is compatible with a network system consisting of interconnected sub-processes. For this reason, the operational process of banks cannot be considered as a black box that just transforms inputs into outputs. Rather, the sub-processes must be carefully looked at in order to accurately assess the efficiency and uncover the sources of inefficiency in the components of this process. In this way, we can understand how the bank is performing accurately, which is of great importance for improving banking performance.

Based on this information, this study aims to analyze the efficiency of commercial banks in Türkiye using two-stage network data envelopment analysis (NDEA). Additionally, the study will apply the bootstrap two-stage network data envelopment analysis (BNDEA) techniquet to bias calculation, which will significantly increase increase the accuracy and reliability of the banking performance analysis results. Efficiency is measured on the basis that the bank structure consists of two interrelated processes, namely production and intermediation. The production approach focuses on the creation of financial products and services, while the intermediation approach the role of banks in channeling funds from savers to borrowers. Both stages are integral to the overall operation and efficiency of banking institutions

In this analysis, data from 2022 was used, and the sample data was collected from the Data Query System of the Banks Association of Türkiye. Due to the positivity requirements in Data Envelopment Analysis (DEA), which necessitate that all numbers must be non-negative and preferably strictly positive, banks with missing or negative data were excluded from the analysis. Consequently, Performance analysis was conducted on 13 decision-making units, which in this study are the banks. The analysis includes 3 state-owned banks, 6 privately-owned banks, and 4 foreign-owned banks. The names of the banks included in the analysis are shown in Table 2 below. All mathematical formulations were coded using MATLAB programming language.

Table 2. *Names of Banks (DMU) İncluded in The Two-Stage Analysis*

DMU	Bank Name	Bank Trip
1	Türkiye Cumhuriyeti Ziraat Bank	state-owned banks
2	Türkiye Halk Bank	state-owned banks
3	Türkiye Vakıflar Bank	state-owned banks
4	Akbank	privately-owned banks
5	Anadolubank	privately-owned banks
6	Şekerbank	privately-owned banks
7	Türk Ekonomi Bankası	privately-owned banks
8	Türkiye İş Bankası	privately-owned banks
9	Yapı Kredi Bank	privately-owned banks
10	Denizbank	foreign-owned banks.
11	ING Bank	foreign-owned banks.
12	QNB Finansbank	foreign-owned banks.
13	Türkiye Garanti Bank	foreign-owned banks.

Furthermore, the variables were selected to reflect the nature of each stage in the process of evaluating banking performance. In the first stage, representing the production stage, the bank aims to employ its essential resources to achieve its production goals. This is accomplished through the use of labor resources, including the number of employees and branches, along with physical capital represented by fixed assets such as buildings and equipment. These resources work together to generate what is known as general collected resources. In our study, the general collected resources variable is defined as a combination of three main variables: deposits, loans received and issued securities. This comprehensive definition aims to provide an integrated view of the resources used by the bank, enabling a deeper, more accurate, and inclusive analysis of banking performance. Accordingly, fixed assets, employees, and the number of branches can be considered inputs for this stage, while the general collected resources represent its outputs. At the same time, this variable serve as input for the second stage, which is the intermediation stage.

In the second stage, the intermediation stage, the bank uses the general collected resources, which represent the sole output of the production stage, to generate net interest profit and other operating incomes. The net interest profit variable is defined as the difference between interest revenue and interest expenses. Consequently, general collected resources can be viewed as an input for this stage, while net interest profit and other operating incomes are considered outputs of the second stage. Table 5 provides a summary of the inputs and outputs variables used in both the first and second stages of the analysis.

Table 3. *The Input and Output Variables Used in The First and Second Stages*

The first stage (production).	intermediate product.	The second stage (intermediation).
Variable name	Variable name	Variable name
Input: Fixed assets, Employees, The number of branches,	General collected resources,	Output: Net interest profit, Other operating, incomes,

The descriptive statistics of the input, intermediate product and output variables for the year 2022 is presented in Table 4.

Table 4. *The Descriptive Statistic For Variables*

	Input			intermediate product	Output	
	Fixed assets (million)	The number of Employees	The number of branches	General collected resources (million)	net interest profit (million)	other operating incomes (million)
Min	39296	1671	116	50388.18	1675.47	301.65

Max	2311665	24484	1758	1877242.91	121482.90	11973.74
Mean	902556.18	13354	714	693514.65	51716.57	3854.91
Standard deviation	703728.27	7585.49	456.54	550302.36	37823.00	3900.01

4.Experimental Results

4.1.Results Of the Tow Stage NDEA Model and Traditional CCR VZA MODEL

In the study, the overall efficiency values of the banks were calculated separately for each bank by solving with model (2). Subsequently, the optimal efficiency values related to the sub-processes were calculated using model (3) and model (4). Since the NVZA model operates under the assumption of constant returns to scale (CRS), the input-oriented assumption was adopted in the study. The AVZA model results for the banks are presented in Table 8.

Table 5. *Production, Intermediation and General Efficiency Scores for NDEA Model.*

Banks		Tow stage NVZA			Traditional CCR VZA	Difference
Num	DMU	E_0: General Efficiency Scores	E_1: Production Efficiency Scores	E_2: Intermediation Efficiency Scores	E_0	$E_{VZA}\_E_{NVZA}$
1	Türkiye Cumhuriyeti Ziraat Bank	0.646	1.000	0.646	0.866	0.220
2	Türkiye Halk Bank	0.591	0.940	0.628	0.680	0.089
3	Türkiye Vakıflar Bank	0.558	1.000	0.558	0.935	0.377
4	Akbank	0.880	0.880	1.000	1.000	0.120

5	Anadolubank	0.814	1.000	0.814	1.000	0.186
6	Şekerbank	0.609	0.628	0.969	1.000	0.391
7	Turk Ekonomi Bank	0.724	0.944	0.767	1.000	0.276
8	Türkiye İş Bank	0.636	0.872	0.730	0.738	0.102
9	Yapı Kredi Bank	0.841	0.859	0.979	0.966	0.125
10	Denizbank	0.576	0.819	0.703	0.743	0.166
11	ING Bank	0.710	0.789	0.900	0.833	0.123
12	QNB Finansbank	0.753	0.936	0.803	0.860	0.107
13	Türkiye Garanti Bank	0.884	0.884	1.000	1.000	0.116
Average		0.709	0.889	0.807	0.894	0.185

The analysis results, which pertain to the efficiency scores of each bank according to the two-stage NDEA and the standard DEA, are presented in Table 8. According to the 2022 data, the average overall efficiency score of the two-stage NDEA for the 13 banks is 0.709. In other words, the average inefficiency in commercial banks stands at approximately 23%. The results indicate that the efficiency scores in the first stage were lower than in the second stage, mean that there is inefficiency in the Production stage within the banking sector. In the first stage, 3 banks (Türkiye Cumhuriyeti Ziraat Bank, Türkiye Vakıflar Bank, Anadolubank) were relatively efficient, while only 2 banks (Akbank, Türkiye Garanti Bank) were efficient in the second stage. Additionally, the average production stage efficiency score of the banks was 0.889, while the average intermediation stage efficiency score was 0.807. The production stage efficiency scores ranged within the range [0.62 1], while the intermediation stage efficiency scores ranged within the range [0.558 1]. On the other side of the analysis, according to the results of the Traditional CCR DEA model, the average efficiency score for banks is 0.894, indicating an inefficiency of approximately 10% in commercial banks. Additionally, 5 banks were identified as being on the efficiency frontier in the DEA model. In general, it's clear that the DEA model considers a larger number of banks as efficient units compared to the two-stage NDEA model, where the efficiency scores in the DEA is higher. Where A difference of 0.185 on average was observed between the scores of the two models. This means that in the DEA model, the sub-processes within the banking production system are overlooked due to the adoption of the "black box" approach. Consequently, the efficiency scores of banks tend to be overestimated, reflecting values higher than their actual efficiency values. Moreover, unlike the findings of the two-stage NDEA model, the scores from the Traditional DEA model do not provide insights into the underlying sources of inefficiency within the banks.

To better observe the similarities and differences between the banks, they were grouped into four quadrants based on the efficiency scores for both the first and second stages. Figure 3 presents the scatter plot of intermediation efficiency scores versus production efficiency scores for all banks.

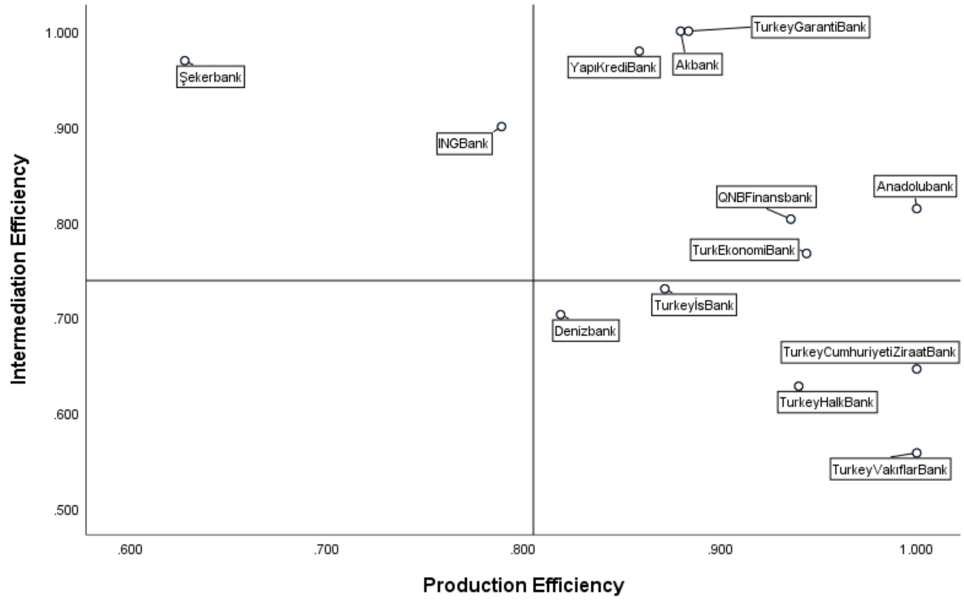

Figure 3. *Distribution of First and Second Stage Efficiency Scores of Banks*.

1. Those positioned at high levels of production efficiency and intermediation efficiency: Yapı Kredi Bank, Anadolubank, QNB Finansbank, Türkiye Garanti Bank, Akbank and Turk Ekonomi Bank.
2. Those positioned at high levels of production efficiency and low levels of intermediation efficiency: Türkiye Vakıflar Bank, Türkiye Cumhuriyeti Ziraat Bank, Denizbank, Türkiye is banka and Türkiye Halk Bank.
3. Those positioned at low levels of production efficiency and high levels of intermediation efficiency: Şekerbank and ING Bank Türkiye Garanti Bank.
4. Those positioned at low levels of production efficiency and low levels of intermediation efficiency: None of the banks.

4.2.Results Of the Bootstrap Tow Stage NDEA Model

The efficiency scores were calculated using the bootstrap model proposed by Simar and Wilson (1999), following the steps summarized in the previous section. This model was applied to the performance of banks to measure and analyze efficiency, allowing for a comprehensive evaluation of performance and an analysis of differences between banks in their ability to manage resources and operations. By repeating the proposed bootstrap algorithm 2000 times (B = 2000), accurate estimates of overall efficiency scores as well as sub-process efficiency for each bank were obtained. Subsequently, the bias for each stage was calculated using Equation 1. These results are clearly presented in Table 6.

Table 6. *The Efficiency Scores Using Bootstrap Two Stage NDEA Model*

DMU	Banks	Bootstrap Tow Stage NVZA			Bais			Standard deviation
		E0: General Efficiency Scores	E1: Production Efficiency Scores	E2: Intermediation Efficiency Scores	Bais of general stage	Bais of first stage	Bais of second stage	
1	Türkiye Cumhuriyeti Ziraat Banka	0.654	0.821	0.797	0.008	-0.179	0.151	0.015
2	Türkiye Halk Bank	0.570	0.727	0.784	-0.021	-0.213	0.156	0.020
3	Türkiye Vakıflar Bank	0.654	0.994	0.658	0.096	-0.006	0.100	0.017
4	Akbank	0.563	0.760	0.741	-0.317	-0.120	-0.259	0.032
5	Anadolubank	0.714	0.714	1.000	-0.100	-0.286	0.186	0.010
6	Şekerbank	0.418	0.999	0.418	-0.191	0.371	-0.551	0.024
7	Turk Ekonomi Bank	0.609	0.684	0.889	-0.115	-0.260	0.122	0.021
8	Türkiye İş Bank	0.521	0.785	0.664	-0.115	-0.087	-0.066	0.018
9	Yapı Kredi Bank	0.529	0.727	0.728	-0.312	-0.132	-0.251	0.015
10	Denizbank	0.515	0.710	0.725	-0.061	-0.109	0.022	0.016
11	ING Bank	0.507	0.757	0.670	-0.203	-0.032	-0.230	0.018
12	QNB Finansbank	0.554	0.608	0.911	-0.199	-0.328	0.108	0.015
13	Türkiye Garanti Bank	0.521	0.921	0.566	-0.363	0.037	-0.434	0.019
Average		0.564	0.785	0.735	-0.146	-0.104	-0.073	0.018

According to the analysis presented in Table 6, the first three columns represent the values calculated using the Bootstrap-NDEA method for the first stage, second stage, and the overall stage. During the study period, it was found that none of the banks were efficient in the overall stage or the intermediation stage, while only one bank

(Anadolubank) was efficient in the production stage. The average efficiency scores for the three stages were 0.564, 0.785, and 0.735, respectively. The last three columns reflect the bias values for each stage, which were calculated using Equation (8). The results indicated both positive and negative biases. A positive bias signifies that the average Bootstrap efficiency exceeds the actual efficiency, while a negative bias indicates that the average Bootstrap efficiency is lower than the actual efficiency. The last column represents the standard deviation of the efficiency scores for the analyzed units. The standard deviation here indicates the level of homogeneity in the efficiency scores obtained through the BNVZA method. A smaller standard deviation value reflects higher homogeneity in the efficiency scores.

Accordingly, the results show that the smallest standard deviation value is 0.01, which corresponds to Anadolubank, indicating a high level of homogeneity in the efficiency scores obtained for this bank.

5.Conclusion

Given the increasing competition in the banking sector, measuring its efficiency is of great importance to ensure its ability to adapt to the ongoing global developments. Banking efficiency has received significant attention in academic literature, with numerous studies, such as those by Moradi-Motlagh and Saleh, (2014)., Bahrini, (2017)., and Awartani and Maghyereh, (2012)., analyzing bank performance using Data Envelopment Analysis (DEA) models. However, these studies focused on measuring bank efficiency through a single process without considering the interactions and interconnections between different internal processes. Accordingly, this study aims to address this gap by applying a network efficiency analysis model that takes into a

ccount the multiple structures of banking operations and more accurately reflects the links between various internal processes. This study expanded the selection of intermediate products compared to the studies by Wang et al. (2014), Fukuyama et al. (2011) and Dirik et al. (2022), highlighting key differences in how intermediate variables are defined. In our study, the intermediate variable "General collected resources" was defined to include three variables: deposits, loans received and issued securities. This allowed us to include additional variables in the analysis, providing a more comprehensive understanding of banks' financial efficiency.

In contrast, the previously mentioned studies focused on using only one intermediate variable, such as deposits. This difference in the approach to selecting intermediate variables significantly impacts efficiency measurement, enabling our method to offer deeper and more detailed insights into banks' performance, particularly in specific contexts.

In this study, the financial performance of 13 banks operating in Türkiye 3 of which are publicly owned, 6 privately owned, and the other 4 foreign-owned was analyzed using the input-oriented two-stage NVZA and two-stage Bootstrap-NDEA methods. Upon examining the results of the analysis using data from 2022, the results of the Data Envelopment Analysis indicated that no single bank achieved full efficiency in both stages. However, some banks were efficient in the production stage, while others were efficient in the intermediation stage. Additionally, the average performance in the financial intermediation stage was found to be very close to the average performance in the production stage, indicating a notable balance in performance between the two stages.

On the other hand, the results of the analysis using the bootstrap method also showed that no bank achieved full efficiency in both stages. The efficiency values in the overall stage ranged between 0.418 and 0.714, while in the production stage, they ranged between 0.608 and 0.999, and in the intermediation stage, they ranged between 0.418 and 1. The average biases in the three stages were -0.146, -0.104 and -0.073, respectively. Based on these results, it can be concluded that the bootstrap-NDEA analysis is more reliable in measuring the efficiency of banks, as it provides more accurate estimates of efficiency compared to NDEA. In addition, the results obtained from this study can significantly support the decision-making process of bank managers by providing a clear insight into the current performance efficiency and helping them identify strengths and weaknesses within the sector. Finally, a more detailed evaluation of the banking sector's efficiency in Türkiye can be suggested by increasing the number of internal processes to three or four steps, or even more, as a proposal for future research. This approach allows for a deeper and more comprehensive analysis of banks' performance and their ability to improve efficiency at different stages of banking operations.

References

Alfaiate, N. C., Özdemir, Y. A., & Alp, İ. (2023). Efficiency Assessment of Mozambican Banks: A Slacks-Based Measure of Efficiency Approach. *Journal of Economics and Business Issues*, *3*(2), 26-39.

Bahrini, R. (2017). Efficiency analysis of Islamic banks in the Middle East and North Africa region: A bootstrap DEA approach. *International Journal of Financial Studies*, *5*(1), 7.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis. Management Science, 30(9), 1078-1092.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the Efficiency of Decision-Making Units. European Journal of Operational Research, 2(6), 429-444

Cifuentes-Faura, J., & Faura-Martínez, U. (2023). Measuring Spanish airport performance: A bootstrap data envelopment analysis of efficiency. *Utilities Policy*, *80*, 101457.

Cooper WW, Ruiz JL, Sirvent I (2009) Selecting non-zero weights to evaluate efectiveness of basketball players with dea. Eur J Oper Res 195(2):563–574.

Dia, M., Golmohammadi, A., & Takouda, P. M. (2020). Relative efficiency of Canadian banks: A three-stage network bootstrap DEA. *Journal of Risk and Financial Management*, *13*(4), 68.

Diler, M. (2011). Efficiency, productivity, and risk analysis in Turkish banks: A bootstrap DEA approach. *BDDK Bankacılık ve Finansal Piyasalar Dergisi*, *5*(2), 71-133.

Dirik, C., & Göker, İ. E. K. (2022). Türkiye'deki Mevduat Bankalarının Üretim ve Aracılık Etkinlikleri: İki-Aşamalı Network VZA Uygulaması. *Ankara Hacı Bayram Veli Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi*, *24*(1), 386-409.

Efron, B. (1979) Bootstrap Methods: Another Look at the Jackknife. The Annals of Statistics, 7, 1-26.

Eken, M. H., & Kale, S. (2011). Measuring bank branch performance using Data Envelopment Analysis (DEA): The case of Turkish bank branches. *African Journal of Business Management*, *5*(3), 889-901.

Färe, R., & Grosskopf, S. (2000). Theory and application of directional distance functions. *Journal of productivity analysis*, *13*(2), 93-103.

Fukuyama, H., & Matousek, R. (2011). Efficiency of Turkish banking: Two-stage network system. Variable returns to scale model. *Journal of International Financial Markets, Institutions and Money*, *21*(1), 75-91.

Goyal RSJ, Singh M, Aggarwal A (2019) Efciency and technology gaps in Indian banking sector: application of meta-frontier directional distance function dea approach, The Journal of Finance and Data. Science 5:156–172

Jauhar SK, Pant M, Nagar AK (2017) Sustainable educational supply chain performance measurement through dea and diferential evolution: a case on indian hei. J Computat Sci 19:138–152

Kamau, A. W. (2011). Intermediation efficiency and productivity of the banking sector in Kenya. *Interdisciplinary Journal of Research in Business*, 1 (9), 12-26.

Kao, C., & Hwang, S.-N. N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. European Journal of Operational Research, 185(1), 418–429.

Leal Paço, C., & Cepeda Pérez, J. M. (2013). The use of DEA (Data Envelopment Analysis) methodology to evaluate the impact of ICT on productivity in the hotel sector. *Via. Tourism Review*, (3).

Li, Y. (2020). Analyzing efficiencies of city commercial banks in China: An application of the bootstrapped DEA approach. *Pacific-Basin Finance Journal*, *62*, 101372.

Maghyereh, A. I., & Awartani, B. (2012). Financial integration of GCC banking markets: A non-parametric bootstrap DEA estimation approach. *Research in International Business and Finance*, *26*(2), 181-195.

Mahajan, V., Mogha, S. K., & Pannala, R. P. K. (2024). Evaluation of efficiency and ranking of Indian hotels and restaurants: a bootstrap DEA approach. *Benchmarking: An International Journal*, *31*(1), 186-198.

Mahmoudabadi, M. Z., & Emrouznejad, A. (2019). Comprehensive performance evaluation of banking branches: A three-stage slacks-based measure (SBM) data envelopment analysis. *International Review of Economics & Finance*, *64*, 359-376.

Mahmoudi, R., Emrouznejad, A., Shetab-Boushehri, S. N., & Hejazi, S. R. (2020). The origins, development and future directions of data envelopment analysis approach in transportation systems. *Socio-Economic Planning Sciences*, *69*, 100672.

Moon H, Min D (2020) A dea approach for evaluating the relationship between energy efciency and fnancial performance for energy-intensive frms in korea. J Clean Prod 255:120283.

Moradi-Motlagh, A., & Saleh, A. S. (2014). Re-examining the technical efficiency of Australian banks: a Bootstrap DEA Approach. *Australian Economic Papers*, *53*(1-2), 112-128.

Nwaogbe, O. R., Wanke, P., Ogwude, I. C., Barros, C. P., & Azad, A. K. (2018). Efficiency driver in Nigerian airports: a bootstrap DEA–censored quantile regression approach. *Journal of Aviation Technology and Engineering*, *7*(2), 2.

Özdemir, A. F., & Navruz, G. (2016). Bootstrap-t ve yüzdelik bootstrap yöntemlerinde tekrar sayısı, budama yüzdesi ve dağılımın sonuçlara etkisi. *Nevşehir Bilim ve Teknoloji Dergisi*, *5*(2), 74-85.

Özsoy, V. S., & Örkcü, H. H. (2021). Structural and operational management of Turkish airports: a bootstrap data envelopment analysis of efficiency. *Utilities Policy*, *69*, 101180.

Panwar, A., Olfati, M., Pant, M., & Snasel, V. (2022). A review on the 40 years of existence of data envelopment analysis models: Historic development and current trends. *Archives of Computational Methods in Engineering*, *29*(7), 5397-5426.

Simar, L and Wilson, P. W. (2000). Statistical Inference in Nonparametric Frontier Models: The State of The Art. Journal of Productivity Analysis, 13(1), 49-78.

Simar, L. and Wilson, P.W. (1998). Sensitivity Analysis of Efficiency Scores: How to Bootstrap in Non-Parametric Frontier Models. Management Science 44(1), 49-61.

Simar, L. And Wilson. (2000). A General Methodology for Bootstrapping in Non-Parametric Frontier Models. Journal of Applied Statistics 27(6), 779-802.

Vaseei, M., Daneshmand-Mehr, M., Bazrafshan, M., & Kanafi, A. G. (2023). A network data envelopment analysis to evaluate the performance of a sustainable supply chain using bootstrap simulation. *Journal of Engineering Research*.

Wang, K., Huang, W., Wu, J., & Liu, Y. N. (2014). Efficiency measures of the Chinese commercial banking system using an additive two-stage DEA. *Omega*, *44*, 5-20.

Yang, W., Shi, J., Qiao, H., Shao, Y., & Wang, S. (2017). Regional technical efficiency of Chinese Iron and steel industry based on bootstrap network data envelopment analysis. *Socio-Economic Planning Sciences*, *57*, 14-24.

Yen, B. T., Mulley, C., & Yeh, C. J. (2023). Performance evaluation for demand responsive transport services: A two-stage bootstrap-DEA and ordinary least square approach. *Research in Transportation Business & Management*, *46*, 100869.

Yeşilyurt ME, Şahin E, Elbi MD, Kızılkaya A, Koyuncuoğlu MU, Akbaş-Yeşilyurt F (2021) A novel method for computing single output for dea with application in hospital efciency. Socioecon Plann Sci 76:100995.