PLANNING SECTORAL INVESTMENTS WITH ECONOMIES OF SCALE: A COMPUTATIONALLY EFFICIENT MIXED BIVALENT PROGRAMMING APPROACH

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Abstract: In this paper, a sectoral investment planning model is examined from computational point of view. It is a well known fact that conventional mixed bivalent programming models with complex combinatorial structures are generally intractable. The paper exploits such a model to determine what capacity, if any, should be maintained at the various geographical regions during the planning period in order to meet regional demand and minimise total cost of the entire system. A Lagrangean relaxation-based procedure is developed to decompose the model into submodels by each geographical region and planning year. Following that, computationally efficient algorithms to solve the submodels are presented and computational results are discussed.

Key Words: Investment Planning, Integer Programming, Lagrangean Relaxation, Modelling.

1. INTRODUCTION

Empirical evidences show that most sectors of process industry are subject to economies of scale[1]. In such sectors the independent evaluation of investment projects causes to build suboptimal production capacities and therefore incur higher unit production costs. Because of such deficiencies of conventional project evaluation techniques, sectoral investment planning models are exploited to simultaneously assess plant locations, timing of investments, and production capacity expansions. The reader is referred to Hacihasanoglu[2] and its references for theory and practice of economies of scale and sectoral investment planning.

In this paper, a sectoral investment planning model is examined from computational point of view. The original mixed bivalent programming model is given in Ref.[2] with some extensions (consideration of export etc.). It is a well known fact that conventional mixed bivalent programming models of complex combinatorial problems are generally intractable. The aim of this paper is to develop a new computationally efficient algorithm to solve the aforementioned sectoral investment planning problem.

The paper is organised as follows: In §2 the model notation and the sectoral investment planning model are presented. §3 is devoted to the decomposition of the model into smaller subproblems by means of the Lagrangean relaxation method. Finally, conclusions and directions for further research are presented in §4.

2. MATHEMATICAL MODEL

Notation associated with the investment planning model is as follows:

I	Geographical region as a supplier, $I=1,\ldots,\beta$
j	Geographical region as a consumer, $j=1,\ldots,\beta$
τ, t	Year indices, $\tau=1,\ldots,\alpha$; $t=1,\ldots,\alpha$
x	Amount transported between the geographical regions
y	y=0 denotes rejection and y=1 denotes acceptance of investment

h Production capacity to be invested if investment is approved
d Regional demand
k Invested production capacity
p Unit production and transportation costs
v Variable investment cost component
w Fixed investment cost component
H Maximum production capacity
ρ Discount rate
σ Capital recovery factor

The formulation of the investment planning problem is given below at four steps. The first step deals with the formulation of the demand constraints. The relation between the production capacity and the delivery is considered in the second step. The third step takes account of production capacity limitations as well as investment decisions. Finally, the fourth step aims at tackling objective function.

It may be impossible (or possible, but too costly) to guarantee that demand will be met under all circumstances, especially when future demands are uncertain. However, as it is aforementioned, demand in subsequent time periods is regarded as known (i.e., dynamic deterministic demand). Therefore, by means of the inequality given below it is certain that demand does not exceed supply.

$$\sum_{i=1}^{\beta} x_{ij\tau} \geq d_{j\tau} \qquad j=1,\ldots,\beta \;;\; \tau=1,\ldots,\alpha \tag{1}$$

The total amount of delivery from a region may not exceed the total production of that region. Therefore, in order to limit the delivery by the production capacity the following inequality is used.

$$\sum_{j=1}^{\beta} x_{ij\tau} \leq k_{i\tau} + \sum_{t=1}^{\tau} h_{it} \qquad i=1,\ldots,\beta \;;\; \tau=1,\ldots,\alpha \tag{2}$$

The following inequality forces the production level to zero if there is not any investment decision. However, if the investment decision is positive then there is an upper limit on production capacity imposed by the same inequality.

$$h_{i\tau} \preceq H_{i\tau}\, y_{i\tau} \qquad i=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha \tag{3}$$

The objective function comprises three cost components. These are total fixed investment, variable investment, and unit production and transportation costs respectively from left to right.

$$Min.C = \sum_{\tau=1}^{\alpha}\sum_{i=1}^{\beta} \rho_{\tau}\left\{ \sigma w_{i\tau} y_{i\tau} + \sigma v_{i\tau} h_{i\tau} + \sum_{j=1}^{\beta} p_{ij\tau} x_{ij\tau} \right\} \tag{4}$$

The entire model is given below for the sake of convenience.

Minimise

$$C = \sum_{\tau=1}^{\alpha}\sum_{i=1}^{\beta} \rho_{\tau}\sigma w_{i\tau} y_{i\tau} + \sum_{\tau=1}^{\alpha}\sum_{i=1}^{\beta} \rho_{\tau}\sigma v_{i\tau} h_{i\tau} + \sum_{\tau=1}^{\alpha}\sum_{i=1}^{\beta}\sum_{j=1}^{\beta} \rho_{\tau} p_{ij\tau} x_{ij\tau}$$

Subject To

$$h_{i\tau} \preceq H_{i\tau} y_{i\tau} \qquad i=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha$$

$$\sum_{j=1}^{\beta} x_{ij\tau} \preceq k_{i\tau} + \sum_{t=1}^{\tau} h_{it} \qquad i=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha$$

$$\sum_{i=1}^{\beta} x_{ij\tau} \succeq d_{j\tau} \qquad j=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha$$

Figure 1. Sectoral Investment Planning Model

3. LAGRANGEAN RELAXED MODEL

Mainly due to their complex combinatorial structure, investment planning problem seems to be extremely difficult from computational point of view. Especially, the formulation given above is intractable for conventional Integer Programming techniques. In this section, a new solution approach is presented to deal with the investment planning problem.

One of the most computationally useful ideas of the 1970s is the observation that many hard problems can be viewed as easy problems complicated by a relatively small set of side constraints. Dualising the side constraints produces a Lagrangean problem that is easy to solve and whose optimal value is a lower bound on the optimal value of the original problem. The Lagrangean problem can thus be used in place of a linear programming relaxation to provide bounds in a Branch and Bound algorithm. Geoffrion[3] coined the perfect name "Lagrangean Relaxation" for this approach. The reader is referred to Geoffrion[3], Fisher[4], [5] and Shapiro[6] for theory and survey of Lagrangean relaxation.

Lagrangean relaxation is used to decompose the formulation into smaller subproblems. Multiplying the second constraint set by Lagrange multiplier vector $\lambda_{it}\geq 0$ and adding it to the objective function yields the following relaxed problem.

Minimise

$$\sum_{\tau=1}^{\alpha}\sum_{i=1}^{\beta}\left\{\rho_\tau\sigma\left(w_{i\tau}y_{i\tau}+v_{i\tau}h_{i\tau}\right)+\sum_{j=1}^{\beta}\left(\rho_\tau p_{ij\tau}+\lambda_{i\tau}\right)x_{ij\tau}-\lambda_{i\tau}\left(\sum_{t=1}^{\tau}h_{it}+k_{i\tau}\right)\right\}$$

Subject To

$$h_{i\tau}\le H_{i\tau}y_{i\tau} \qquad i=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha$$

$$\sum_{i=1}^{\beta}x_{ij\tau}\ge d_{j\tau} \qquad j=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha$$

Figure 2. Lagrangean Relaxed Model

The relaxed problem is decomposed into subproblems of the form SP1 and SP2 given below.

$$Min.C1=\sum_{\tau=1}^{\alpha}\sum_{i=1}^{\beta}\sum_{j=1}^{\beta}\left(\rho_\tau\ p_{ij\tau}+\lambda_{i\tau}\right)x_{ij\tau}$$

Subject To $\sum_{i=1}^{\beta}x_{ij\tau}\ge d_{j\tau} \qquad j=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha$

Figure 3. Subproblem SP1

$$Min.C2=\sum_{\tau=1}^{\alpha}\sum_{i=1}^{\beta}\left\{\left(\sigma\rho_\tau w_{i\tau}\right)y_{i\tau}+\left(\sigma\rho_\tau v_{i\tau}-\sum_{t=\tau}^{\alpha}\lambda_{it}\right)h_{i\tau}\right\}$$

Subject To $h_{i\tau}\le H_{i\tau}y_{i\tau} \qquad i=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha$

Figure 4. Subproblem SP2

By means of SP1 and SP2 it is observed that the subproblems are separated by geographical regions and by planning years into smaller subproblems. Each of these subproblems can be solved just by checking the coefficients of the variables. Therefore, there is no need to use any of the time consuming standard algorithms (such as Simplex algorithm) to obtain a lower bound on the optimal solution. The most simplified forms of SP1 and SP2, and corresponding solution algorithms are given below.

$$\underset{\{j=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha\}}{Min.\overline{C1}} = \sum_{i=1}^{\beta}\left(\rho_\tau p_{ij\tau}+\lambda_{i\tau}\right) x_{ij\tau}$$

$$Subject\ To \quad \sum_{i=1}^{\beta} x_{ij\tau} \geq d_{j\tau}$$

Solution Algorithm:

i. Determine i, j, and τ which gives the smallest coefficient of Xs.

ii. $X_{ij\tau} = d_{j\tau}$

iii. All other Xs are netted out.

$$\underset{\{i=1,\dots,\beta\ ;\ \tau=1,\dots,\alpha\}}{Min.\overline{C2}} = \left(\sigma\rho_\tau w_{i\tau}\right) y_{i\tau} + \left(\sigma\rho_\tau v_{i\tau} - \sum_{t=\tau}^{\alpha}\lambda_{it}\right) h_{i\tau}$$

$$Subject\ To \quad h_{i\tau} \leq H_{i\tau} y_{i\tau}$$

Solution Algorithm:

For all i=1,...,β; τ=1,...,α do

i. If the coefficient of $h_{i\tau}$ is positive then $y_{i\tau}$ and $h_{i\tau}$ is netted out.

ii. If the coefficient of $h_{i\tau}$ is negative and

$$\left|\left(\sigma\rho_\tau w_{i\tau}\right)\right| < \left|\left(\sigma\rho_\tau v_{i\tau} - \sum_{t=\tau}^{\alpha}\lambda_{it}\right) H_{i\tau}\right| \Longrightarrow y_{i\tau}=1\ ,\ h_{i\tau}=H_{i\tau}$$

iii. If the coefficient of $h_{i\tau}$ is negative and

$$\left|\left(\sigma\rho_\tau w_{i\tau}\right)\right| > \left|\left(\sigma\rho_\tau v_{i\tau}-\sum_{t=\tau}^{\alpha}\lambda_{it}\right)H_{i\tau}\right| \Rightarrow \; y_{i\tau}=0\;,\; h_{i\tau}=0$$

One crucial point that should be made clear is the process of determination of Lagrange multipliers. It is well known that the optimal value of the relaxed problem is less than or equal to the optimal value of the mixed bivalent programming problem. As mentioned before, this fact allows Lagrangean relaxed problem to be used in place of linear programming relaxation to provide lower bounds in a Branch-and-Bound algorithm. It is clear that the best choice for Lagrange multipliers would be an optimal solution to the dual problem, Z_D, where $Z_D(\lambda)$ is the Lagrangean relaxed problem: Z_D=max $Z_D(\lambda)$. One of the schemes for determining λ is the subgradient method. Because the subgradient method is easy to program and has worked well on many practical problems, it has become the most popular method for the solution of Z_D. Computational performance and theoretical convergence properties of the subgradient method are discussed in Held et al.[7] and their references.

The Lagrangean relaxation approach is tested on randomly generated sectoral investment planning problems. The test results clearly show that on the average the Lagrangean relaxed models are solved to optimality 4.644 times faster than the corresponding LP relaxed models. However, it is noticed that the lower bounds produced by Lagrangean relaxation are 95.36% of the LP relaxation ones. Since our primary concern is the improvement of the solution time, it cannot be considered as a fatal flaw.

4. CONCLUSIONS

Lagrangean relaxation is an important new computational technique in the operational researcher's arsenal. In this paper we have developed algorithms that generate optimal solutions for sectoral investment planning models using Lagrangean relaxation method. It is observed that although the bounds generated by Lagrangean relaxation method is not as tight as the LP relaxation ones, the solution time is considerably improved. Two research areas that deserve further attention are the development and analysis of heuristics to determine step sizes, initial values of Lagrange multipliers and the upper bound, and the analysis (worst-case or probabilistic) of the quality of bounds produced by Lagrangean relaxation.

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