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The novel numerical solutions of conformable fractional Cahn-Hilliard equation in the presence of advection and reaction terms via the novel hybrid method

Adveksiyon ve reaksiyon terimlerinin varlığında uyumlu kesirli Cahn-Hilliard denkleminin yeni hibrit yöntemle yeni sayısal çözümleri

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Abstract

In this paper, the novel numerical method is used to analyze the conformable fractional Cahn-Hilliard equation. The Cahn-Hilliard equation is a mathematical model employed as a crucial tool in mathematical physics, specifically for understanding phase separation phenomena such as spinodal decomposition in systems with multiple phases. This study investigates the convergence and error of the proposed future scheme. The proposed technique produces h-curves that show the series solution's convergence interval. To ascertain the efficacy and appropriateness of this technique, the error analysis has been conducted. Also, 2D and 3D graphs of the solutions were drawn. Additionally, the behavior of the graphs was commented. This technique has been shown to be simple, effective and fast.

Keywords: Cahn-Hilliard equation, Shehu transform, Error analysis

Öz

Bu makalede, uyumlu kesirli Cahn-Hilliard denklemini analiz etmek için yeni sayısal yöntem kullanılmıştır. Cahn-Hilliard denklemi, matematiksel fizikte, özellikle çoklu fazlı sistemlerde spinodal ayrışma gibi faz ayırma olaylarını anlamak için önemli bir araç olarak kullanılan matematiksel bir modeldir. Bu çalışma, önerilen gelecekteki şemanın yakınsaklığını ve hatasını araştırmaktadır. Önerilen teknik, seri çözümünün yakınsama aralığını gösteren h-eğrileri üretir. Bu tekniğin etkinliğini ve uygunluğunu belirlemek için hata analizi gerçekleştirilmiştir. Ayrıca, çözümlerin 2B ve 3B grafikleri çizilmiştir. Ek olarak, grafiklerin davranışı yorumlanmıştır. Bu tekniğin basit, etkili ve hızlı olduğu gösterilmiştir.

Anahtar kelimeler: *Cahn-Hilliard denklemi, Shehu dönüşümü, Hata analizi*

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1. Introduction

Fractional calculus (FC) is a mathematical framework that expands the traditional calculus, which deals with integer order derivatives and integrals, to include derivatives and integrals of arbitrary order. Scientists have recently started to explore FC due to its capacity to offer precise descriptions for different types of nonlinear phenomena. Fractional differential equations (FDEs) exhibit nonlocal and hereditary material property effects. Several distinguished scholars conducted thorough research and developed specific definitions for FC, proposing innovative ideas that laid the groundwork for the discipline of FC. Nowadays, FC are widely employed in the development of nonlinear modelling. Various phenomena, such as chaotic processes, noisy environments. economic models, a noisy environment, physics, and others, have been linked to the theory of fractional-order calculus. The solutions of FDEs play a vital role in characterizing the properties of natural nonlinear systems. The scientists utilize several analytical and numerical methods to derive exact solutions for FDEs that describe nonlinear events (Miller & Ross, 1993; Baleanu et al., 2012; Baleanu et al., 2017; Esen et al., 2018; Veeresha et al., 2019).

The conformable fractional derivative (CFD) is a main and efficient concept. The CFD is an effective tool for tackling complex issues. Numerical solutions for differential equations that have CFD are comparatively simpler than those involving Caputo fractional derivative. This enables the application of the CFD, which is valuable to model various physical phenomena. Various fractional models are employed in natural sciences. A multitude of scholars have already employed CFD in a diverse array of disciplines. The CFD circumvents certain limitations associated with the current fractional operators (Khalil et al., 2014; Abdeljawad, 2015; Gao and Chi, 2020).

The Cahn–Hilliard equation (CHE) was formulated by Cahn and Hilliard. The mathematical model in question is a crucial tool in mathematical physics, specifically for understanding phase separation phenomena such as spinodal decomposition in systems with multiple phases. The nonlinearity inherent in the CHE poses challenges in determining its exact solution. The complexity increases when it involves diffusion, advection, interface thickness parameters, and reaction terms. A crucial and significant feature of CHE is the finite thickness of the interface between two phases. Researchers and scientists in the literature investigate and analyze many variations of CHE, along with its applications in various engineering and scientific domains. Homotopy perturbation method (HPM) has been utilized to examine fractional CHE. The resolution of the time-fractional CHE with a reaction term by employing the homotopy analysis method has been analyzed. Homotopy analysis method (HAM) has been employed to analyze the resolution of the time-fractional CHE with a reaction term. Zhu et al. presented the resolution of the CHE by incorporating variable mobility. The solution behavior of the CHE using degenerate mobility has been examined. The generalized CHE, which incorporates temperature and mixing influences has been analyzed. The adaptive neural networks methodology has been employed to numerically solve CHEs. The viscous CHE has been solved via the variational iteration method. CHE has been resolved utilizing HPM and variational iteration method. Obtaining an exact analytical solution of the CHE is challenging because of the narrow interface width and abrupt change at the contact. The literature analysis indicates that CHE, which involves parameters of physical significance, has not been substantially investigated due to its intricate nature (Cahn & Hilliard, 1958; Kim et al., 2016; Bouhassoun & Hamdi Cherif, 2015; Tripathi et al., 2017; Shah & Siddiqui, 2012; Ugurlu & Kaya, 2008; Hussain et al., 2022).

Due to the presence of an integral in the description of the fractional operator in the nonlinear equations, solving these challenges often becomes more complex. The precise and quantitative solutions to the fractional equations are explored utilizing various computational methods that have been developed. The methods used include Adomian decomposition method (ADM) (Ray & Bera, 2006; Wazwaz & Gorguis, 2004), variational iteration method (VIM)) (Das, 2009), homotopy analysis method (HAM)) (Liao, 2004; Alkan, 2022, 2024), differential transform method (DTM) (Merdan et al., 2019), homotopy perturbation method (HPM) (He, 1998, 1999, 2003), residual power series method (RPSM) (Kurt et al., 2019), q-homotopy analysis transform method (q-HATM) (Şenol et al., 2019; Akınyemi et al., 2020), fractional reduced differential transform method (FRDTM) (Akınyemi, 2020), fractional natural transform decomposition method (FNTDM) (Alkan & Anaç, 2024), conformable Elzaki Adomian decomposition method (CEADM) (Avit & Anaç, 2024).

The innovation of this study is to get the novel numerical solutions of the conformable fractional nonlinear Cahn-Hilliard equation via the conformable q-Shehu analysis transform method (Cq-SHATM).

This paper is constructed as follows. Part 2 presents the fundamental of the CFD, its fundamental definitions and significant features. Part 3 introduces the Cq-SHATM. The offered method is utilized to solve the conformable fractional Cahn-Hilliard equation in Part 4. The outcomes are presented in part 5. The conclusion is presented in Section 6.

2. The fundamental definitions and theorems

In the Part, the fundamental definitions and theorems are given.

Definition 2.1. Assume that the function $h:[0,\infty)\to\mathbb{R}$. Thus, CFD for h of order μ is defined as (Khalil et al., 2014; Abdeljawad, 2015).

$$D_{\mu}(h)(x)=\lim_{\varepsilon\to 0}\frac{h(x+\varepsilon x^{1-\mu})-h(x)}{\varepsilon}, \tag{1}$$

for $\forall x>0, 0<\ \mu\leq 1$.

Theorem 2.1. Assume that $0<\ \mu\leq 1, \rho,\sigma$ are $\mu-$differentiable at the point $x\in(0,\infty)$. Thus, the following properties exist (Khalil et al., 2014; Abdeljawad, 2015).

(i) $D_{\mu}(c_1\rho+c_2\sigma)=c_1D_{\mu}(\rho)+c_2D_{\mu}(\sigma)$, for $\forall\ c_1,c_2\in\ \mathbb{R}$, (2)

(ii) $D_{\mu}(x^p)=px^{p-1}$, for $\forall p\in\mathbb{R}$, (3)

(iii) $D_{\mu}(c)=0$, for $\forall\rho(t)=c$, (4)

(iv) $D_{\mu}(\rho\sigma)=\rho D_{\mu}(\sigma)+\sigma D_{\mu}(\rho)$, (5)

(v) $D_{\mu}\left(\frac{\rho}{\sigma}\right)=\frac{\sigma D_{\mu}(\rho)-\rho D_{\mu}(\sigma)}{\sigma^2}$. (6)

Definition 2.2. Assume that the function $h:[0,\infty)\to\mathbb{R}$. Thus, conformable Shehu transform (CST) of h order α is defined as (Khalil et al., 2014; Abdeljawad, 2015).

$${}^{C}\mathcal{S}_{\alpha}[h(t)]=V_{\alpha}(s;u)=\int_{0}^{\infty}exp\left(\frac{-st^{\alpha}}{u\alpha}\right)h(t)t^{\alpha-1}dt. \tag{7}$$

Definition 2.3. Assume that $\mu\in(0,1]$, $h:[0,\infty)\to\mathbb{R}$ is the function. The CST for the CFD of $h(t)$ is identified as (Benattia & Belghaba, 2021).

$$V_{\mu}[D_{\mu}h(t)](v)=\frac{s}{u}V_{\mu}(s;u)-h(0). \tag{8}$$

3. The numerical method

Examine the conformable time-fractional partial differential equations (CTFPDEs) (Erol et al., 2023).

$$D_t^{\alpha}w(x,t)+Aw(x,t)+Hw(x,t)=\xi(x,t), t>0, n-1<\alpha\leq n, \tag{9}$$

where A and H are respectively linear and nonlinear operators, $\xi(x,t)$ is a source term and D_t^{α} is the CFD operator.

Applying the CST to this equation with the initial condition (IC), then, it is acquired as

$$\frac{s}{u}\ {}^{C}\mathcal{S}_{\alpha}[\varpi(x,t)]-\varpi(x,0)+\ {}^{C}\mathcal{S}_{\alpha}[A\varpi(x,t)]+\ {}^{C}\mathcal{S}_{\alpha}[H\varpi(x,t)]=\ {}^{C}\mathcal{S}_{\alpha}[\xi(x,t)]. \tag{10}$$

Simplifying the equation (10), equation (20) is derived as

$$\begin{aligned}&{}^{C}\mathcal{S}_{\alpha}[\varpi(x,t)]-\frac{u}{s}\varpi(x,0)+\frac{u}{s}\ {}^{C}\mathcal{S}_{\alpha}[A\varpi(x,t)]+\frac{u}{s}\ {}^{C}\mathcal{S}_{\alpha}[H\varpi(x,t)]\\&-\frac{u}{s}\ {}^{C}\mathcal{S}_{\alpha}[\xi(x,t)]=0.\end{aligned} \tag{11}$$

Using the homotopy analysis method, the nonlinear operator to $\rho(x,t;q)$ is described by

$$N[\rho(x,t;q)] = {}^{C}\mathcal{S}_{\alpha}[\rho(x,t;q)] - \frac{u}{s}\rho(x,t;q)\,(0^{+}) + \frac{u}{s}\left({}^{C}\mathcal{S}_{\alpha}[A\rho(x,t;q)] + {}^{C}\mathcal{S}_{\alpha}[H\rho(x,t;q)] - {}^{C}\mathcal{S}_{\alpha}[\xi(x,t)]\right), \tag{12}$$

where $q\epsilon\left[0,\frac{1}{n}\right]$.

A homotopy is produced by

$$(1-nq)\,{}^{C}\mathcal{S}_{\alpha}[\rho(x,t;q)-\varpi_0(x,t)] = hqH^{1,*}(x,t)H[\rho(x,t;q)], \tag{13}$$

where, $h \neq 0$ is the auxiliary parameter. For $q=0$ and $q=\frac{1}{n}$, the outcomes of this equation are obtained in the following

$$\rho(x,t;0) = \varpi_0(x,t), \rho\left(x,t;\frac{1}{n}\right) = \varpi(x,t). \tag{14}$$

Therefore, while increasing the value of q from 0 to $1/n$, the solution $\rho(x,t;q)$ approaches the solution $\varpi(x,t)$ by beginning at $\varpi_0(x,t)$. By using the Taylor theorem around q and thereafter expanding $\rho(x,t;q)$, the resulting expression is produced as

$$\rho(x,t;q) = \varpi_0(x,t) + \sum_{i=1}^{\infty}\varpi_m(x,t)q^m, \tag{15}$$

where,

$$\varpi_m(x,t) = \frac{1}{m!}\frac{\partial^m\rho(x,t;q)}{\partial q^m}|_{q=0}. \tag{16}$$

Eq. (15) approaches at $q=\frac{1}{n}$ for the convenient $\varpi_0(x,t)$, n and h. Subsequently, we obtain the solution for the Eq. (9) in the specified format

$$\varpi(x,t) = \varpi_0(x,t) + \sum_{m=1}^{\infty}\varpi_m(x,t)\left(\frac{1}{n}\right)^m. \tag{17}$$

By taking the $m-th$ derivative of Eq. (13) with respect to q and dividing by the factorial of m, when $q=0$, we obtain

$${}^{C}\mathcal{S}_{\alpha}[\varpi_m(x,t) - k_m\varpi_{m-1}(x,t)] = hH^{1,*}(x,t)\mathcal{R}_*(\vec{\varpi}_{m-1}), \tag{18}$$

where the vectors are given by

$$\vec{\varpi}_m = \{\varpi_0(x,t), \varpi_1(x,t), \dots, \varpi_m(x,t)\}. \tag{19}$$

Inverse CST is applied to Eq. (18), we get

$$\varpi_m(x,t) = \chi_m\varpi_{m-1}(x,t) + h\left({}^{C}\mathcal{S}_{\alpha}\right)^{-1}\left[H^{1,*}(x,t)\mathcal{R}_{1,m}(\vec{\varpi}_{m-1})\right], \tag{20}$$

where

$$\mathcal{R}_{1,m}(\vec{\varpi}_{m-1}) = {}^{C}\mathcal{S}_{\alpha}[\varpi_{m-1}(x,t)] - \left(1-\frac{\chi_m}{n}\right)\frac{u}{s}\varpi_0(x,t) + \frac{u}{s}\,{}^{C}\mathcal{S}_{\alpha}[A\varpi_{m-1}(x,t) + H^{1,*}{}_{m-1}(x,t) - \xi(x,t)], \tag{21}$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ n, & m > 1. \end{cases} \tag{22}$$

Also, $H_m^{1,*}$ is homotopy polynomial and introduced by

$$H_m^{1,*} = \frac{1}{m!}\frac{\partial^m \psi(x,t;q)}{\partial q^m}|_{q=0} \text{ and } \psi(x,t;q) = \psi_0 + q\psi_1 + q^2\psi_2 + \cdots. \tag{23}$$

Utilizing Eqs. (20)-(21), one gets

$$\varpi_m(x,t) = (\chi_m + h)\varpi_{m-1}(x,t) - \left(1 - \frac{\chi_m}{n}\right)\frac{u}{s}\varpi_0(x,t)$$
$$+h\left({}^{C}\mathcal{S}_\alpha\right)^{-1}\left[\left(\frac{u}{s}\,{}^{C}\mathcal{S}_\alpha[A\varpi_{m-1}(x,t) + H^{1,*}{}_{m-1}(x,t) - \xi(x,t)]\right)\right]. \tag{24}$$

Via Cq-SHATM, then we have

$$\varpi(x,t) = \sum_{m=0}^{\infty} \varpi_m(x,t). \tag{25}$$

4. Application

Let us analyze the conformable fractional nonlinear CH equation (CFNCHE) (Ugurlu & Kaya, 2008; Hussain et al., 2022)

$$D_t^\alpha w(x,t) - 6w(x,t)\left(\frac{\partial w(x,t)}{\partial x}\right)^2 - 3w^2(x,t)\frac{\partial^2 w(x,t)}{\partial x^2} + \frac{\partial^2 w(x,t)}{\partial x^2} + \frac{\partial^4 w(x,t)}{\partial x^4}$$
$$-\frac{\partial w(x,t)}{\partial x} = 0, 0 < \alpha \leq 1, \tag{26}$$

with IC

$$w(x,0) = \tanh\left(\frac{x}{\sqrt{2}}\right). \tag{27}$$

Applying the CST to Eq. (26) and using the IC, we have

$$\frac{s}{u}\,{}^{C}\mathcal{S}_\alpha[w(x,t)] - w(x,0) + {}^{C}\mathcal{S}_\alpha\left[-6w(x,t)\left(\frac{\partial w(x,t)}{\partial x}\right)^2 - 3w^2(x,t)\frac{\partial^2 w(x,t)}{\partial x^2}\right.$$
$$\left.+\frac{\partial^2 w(x,t)}{\partial x^2} + \frac{\partial^4 w(x,t)}{\partial x^4} - \frac{\partial w(x,t)}{\partial x}\right] = 0. \tag{28}$$

By simplifying the Eq. (28), we get

$${}^{C}\mathcal{S}_\alpha[w(x,t)] - \frac{u}{s}w(x,0) + \frac{u}{s}\,{}^{C}\mathcal{S}_\alpha\left[-6w(x,t)\left(\frac{\partial w(x,t)}{\partial x}\right)^2 - 3w^2(x,t)\frac{\partial^2 w(x,t)}{\partial x^2}\right.$$
$$\left.+\frac{\partial^2 w(x,t)}{\partial x^2} + \frac{\partial^4 w(x,t)}{\partial x^4} - \frac{\partial w(x,t)}{\partial x}\right] = 0. \tag{29}$$

Using the homotopy analysis method, the nonlinear operator to the function $\rho(x,t;q)$ is described by

$$N[\rho(x,t;q)] = {}^{C}\mathcal{S}_{\alpha}[\rho(x,t;q)] - \frac{u}{s}\rho(x,t;q)\,(0^{+}) + \frac{u}{s}\,{}^{C}\mathcal{S}_{\alpha}\Bigg[-6\rho(x,t;q) \times \left(\frac{\partial\rho(x,t;q)}{\partial x}\right)^{2} - 3\rho^{2}(x,t)\frac{\partial^{2}\rho(x,t;q)}{\partial x^{2}} + \frac{\partial^{2}\rho(x,t;q)}{\partial x^{2}} + \frac{\partial^{4}\rho(x,t;q)}{\partial x^{4}} - \frac{\partial\rho(x,t;q)}{\partial x}\Bigg], \tag{30}$$

where $q\epsilon\left[0,\frac{1}{n}\right]$.

The application of the proposed algorithm yields the definition of the m order deformation equation as

$$ {}^{C}\mathcal{S}_{\alpha}[w_m(x,t) - \chi_m w_{m-1}(x,t)] = h\mathcal{R}_{1,m}(\vec{w}_{m-1}), \tag{31}$$

where

$$\mathcal{R}_{1,m}(\vec{w}_{m-1}) = {}^{C}\mathcal{S}_{\alpha}[w_{m-1}(x,t)] - \left(1-\frac{\chi_m}{n}\right)\frac{u}{s}w_0(x,t) + \frac{\text{u}}{s}\,{}^{C}\mathcal{S}_{\alpha}\Bigg[-6\sum_{r=0}^{m-1}\left(\sum_{j=0}^{r} w_j\frac{\partial w_{r-j}}{\partial x}\right)\frac{\partial w_{m-1-r}}{\partial x} - 3\sum_{r=0}^{m-1}\left(\sum_{j=0}^{r} w_j w_{r-j}\right)\frac{\partial^{2} w_{m-1-r}}{\partial x^{2}} + \frac{\partial^{2} w_{m-1}(x,t)}{\partial x^{2}} + \frac{\partial^{4} w_{m-1}(x,t)}{\partial x^{4}} - \frac{\partial w_{m-1}(x,t)}{\partial x}\Bigg]. \tag{32}$$

Inverse CST is applied to Eq. (31), it is obtained as

$$w_m(x,t) = \chi_m w_{m-1}(x,t) + h\left({}^{C}\mathcal{S}_{\alpha}\right)^{-1}[\mathcal{R}_m(\vec{w}_{m-1})]. \tag{33}$$

Using the IC, then we get

$$w_0(x,t) = \tanh\left(\frac{x}{\sqrt{2}}\right). \tag{34}$$

For $m = 1, m = 2$ in the Eq. (33) respectively, we obtain

$$w_1(x,t) = -\frac{h\sqrt{2}t^{\alpha}}{2\alpha}\operatorname{sech}^{2}\left(\frac{x\sqrt{2}}{2}\right), \tag{35}$$

$$w_2(x,t) = (n+h)\left(\frac{-h\sqrt{2}t^{\alpha}}{2\alpha}\operatorname{sech}^{2}\left(\frac{x\sqrt{2}}{2}\right)\right) + h\left(-\frac{3}{\alpha}\operatorname{sech}^{2}\left(\frac{x\sqrt{2}}{2}\right)\tanh\left(\frac{x\sqrt{2}}{2}\right)\right)\Bigg[-\tanh^{2}\left(\frac{x\sqrt{2}}{2}\right)t^{\alpha} + \frac{ht^{2\alpha}\sqrt{2}}{2}\left(\cosh^{2}\left(\frac{x\sqrt{2}}{2}\right) - \frac{3}{2}\right) \times \tanh\left(\frac{x\sqrt{2}}{2}\right) + \frac{ht^{2\alpha}}{6}\Bigg]. \tag{36}$$

The truncated series is used to approximate the analytical solution of $w(x,t)$:

$$w(x,t) = \lim_{\delta\to\infty}\mu_{\delta}(x,t), \tag{37}$$

where

$$\mu_{\delta}(x,t) = \sum_{m=1}^{\delta-1} w_m(x,t). \tag{38}$$

Therefore, the Cq-SHATM solution of Eq. (26) is obtained as

$$w(x,t) = w_0(x,t) + \sum_{m=1}^{\infty} w_m(x,t)\left(\frac{1}{n}\right)^m = \tanh\left(\frac{x}{\sqrt{2}}\right) - \frac{h\sqrt{2}t^\alpha}{2\alpha}\operatorname{sech}^2\left(\frac{x\sqrt{2}}{2}\right)$$
$$+(n+h)\left(\frac{-h\sqrt{2}t^\alpha}{2\alpha}\operatorname{sech}^2\left(\frac{x\sqrt{2}}{2}\right)\right) + h\left(-\frac{3}{\alpha}\operatorname{sech}^2\left(\frac{x\sqrt{2}}{2}\right)\tanh\left(\frac{x\sqrt{2}}{2}\right)\right)$$
$$\times\left[-\tanh^2\left(\frac{x\sqrt{2}}{2}\right)t^\alpha + \frac{ht^{2\alpha}\sqrt{2}}{2}\left(\cosh^2\left(\frac{x\sqrt{2}}{2}\right) - \frac{3}{2}\right)\tanh\left(\frac{x\sqrt{2}}{2}\right) + \frac{ht^{2\alpha}}{6}\right]. \quad (39)$$

Figs. 1-5 demonstrates 3D graphics representing the Cq-SHATM solution, with the distinct values of α at $n = 1$ and $h = -1$.

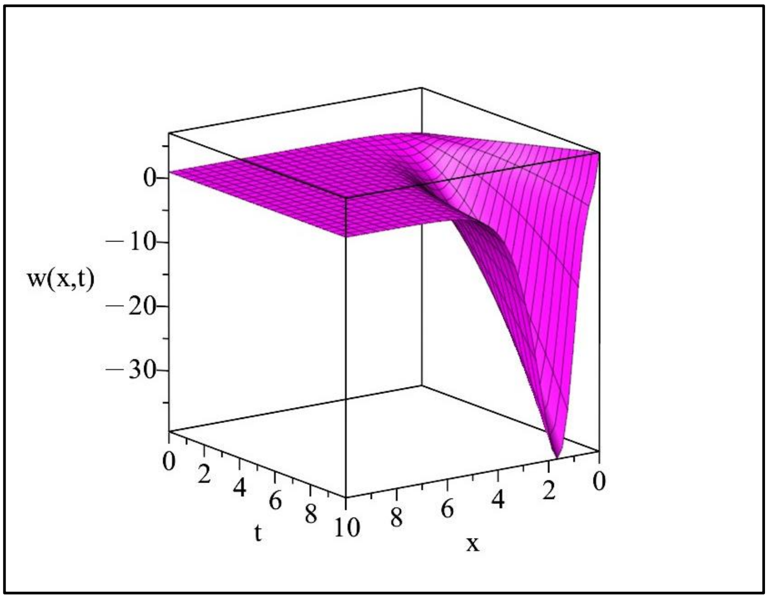

Figure 1. For $\alpha = 1$ Cq-SHATM solution

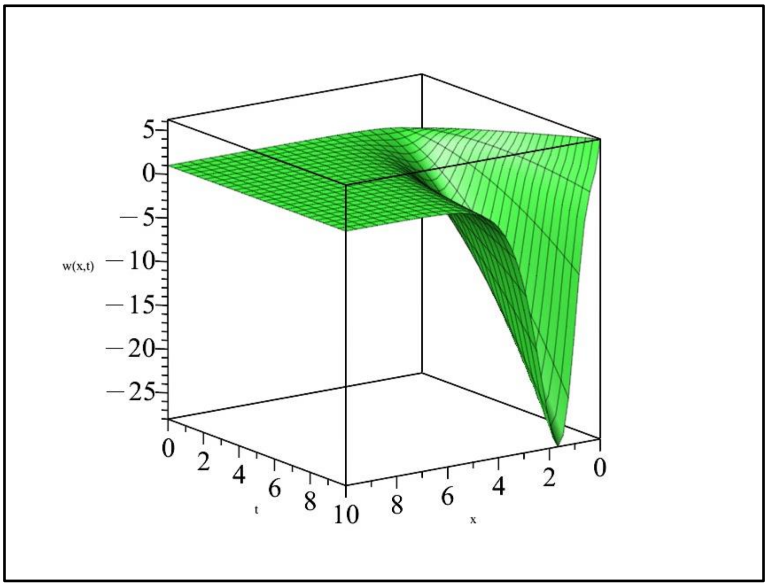

Figure 2. For $\alpha = 0.9$ Cq-SHATM solution

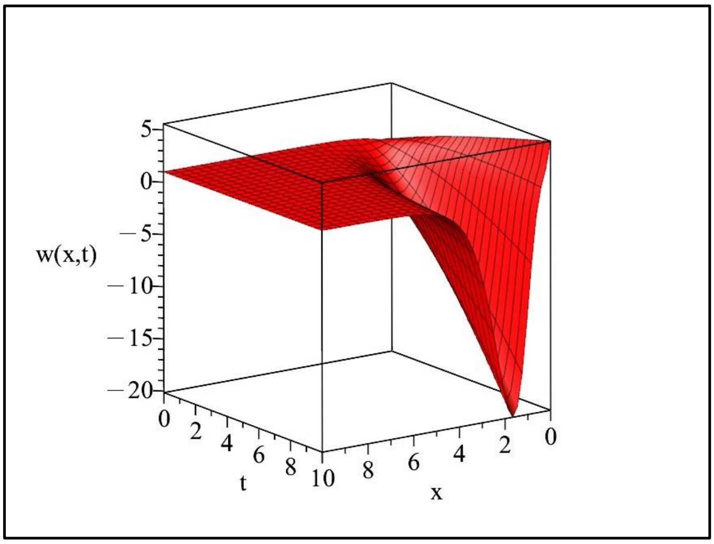

Figure 3. For $\alpha = 0.8$, Cq-SHATM solution

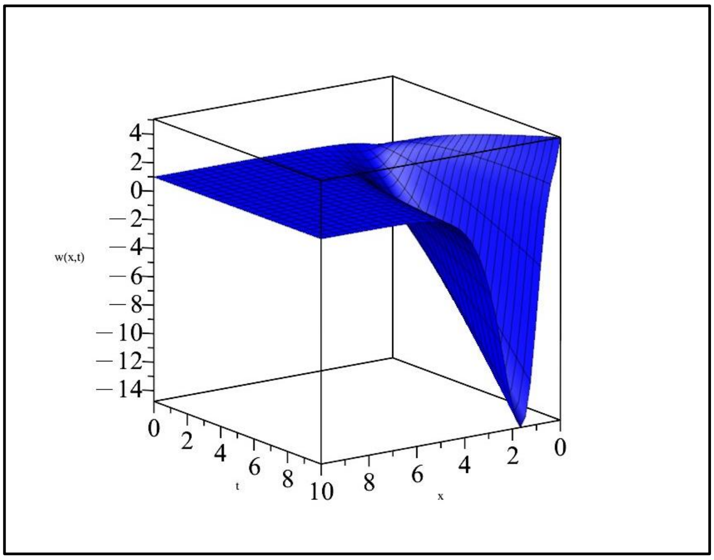

Figure 4. For $\alpha = 0.7$, Cq-SHATM solution

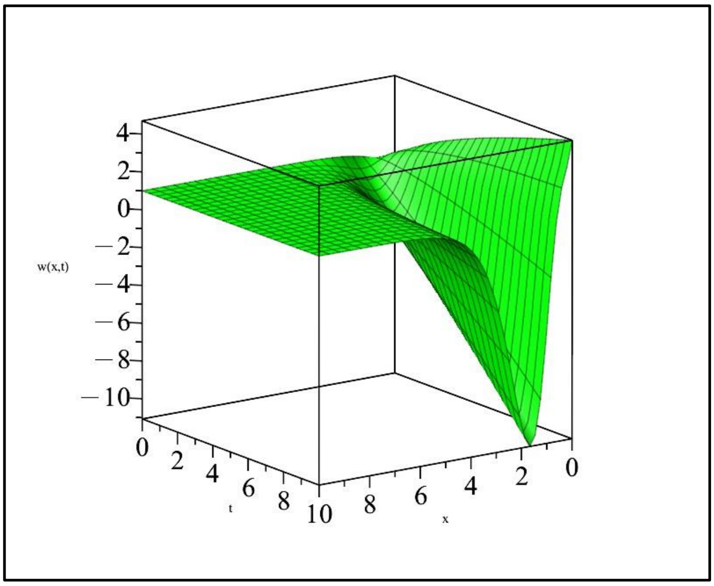

Figure 5. For $\alpha = 0.6$, Cq-SHATM solution

Figure 6 illustrates the two dimensional plots for Cq-SHATM solutions, each represented by various α values.

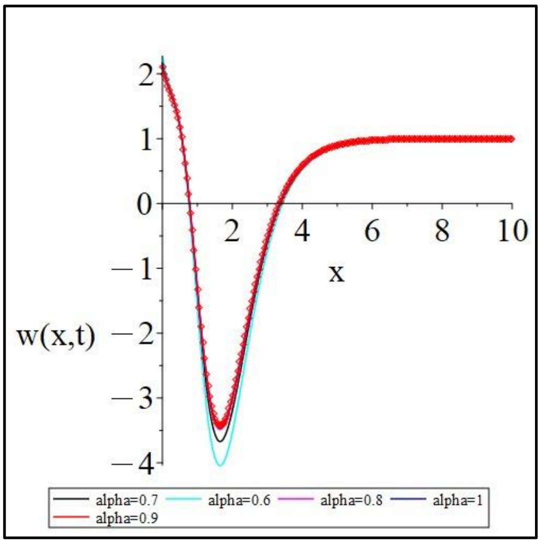

Figure 6. The two dimensional plot of the Cq-SHATM solutions for Eq. (26) at $t = 3$, $\mathrm{h} = -1, \mathrm{n} = 1$ via different values of α.

Table 1 demonstrates the absolute error (AE) between the exact solution and the numerical solution acquired by Cq-SHATM.

Table 1. AE between exact solution and approximate solution by Cq-SHATM for $n = 10$ and $h = -10.223$.

x/t	$\mathbf{0.01}$	$\mathbf{0.02}$	$\mathbf{0.03}$	$\mathbf{0.04}$	$\mathbf{0.05}$
0.1	9.8×10^{-6}	1.8×10^{-5}	2.3×10^{-5}	2.6×10^{-5}	2.5×10^{-5}
0.2	7.9×10^{-5}	1.5×10^{-4}	2.2×10^{-4}	2.8×10^{-4}	3.4×10^{-4}
0.3	2.5×10^{-4}	5.0×10^{-4}	7.4×10^{-4}	9.7×10^{-4}	1.2×10^{-3}
0.4	5.7×10^{-4}	1.1×10^{-3}	1.6×10^{-3}	2.2×10^{-3}	2.7×10^{-3}
0.5	1.0×10^{-3}	2.0×10^{-3}	3.0×10^{-3}	4.0×10^{-3}	5.0×10^{-3}

5. Results and discussion

The numerical solutions of the CFNCHE has been presented. The simulations were evaluated for various values of x, t and the conformable fractional order α. Figures 1-5 illustrates the three-dimensional graphs of the $w(x,t)$ solution obtained by Cq-SHATM for various α values for CFNCHE. Figure 6 depicts the behaviour of Cq-SHATM solution for CFNCHE, shown by 2D graphs, with different values of α. In Figure 6, the Cq-SAHAM solution of CFNCHE is seen to diverge from the analytical solution as α moves away from 1.Table 1 presents AE between the exact solution and Cq-SHATM solution for $\alpha = 1$ for Eq. (39). Table 1 shows that the absolute error is quite small. The numerical result obtained with Cq-SHATM is the same as the one obtained with the q-homotopy analysis method (Hussain et al., 2022) and better than the homotopy perturbation method in (Bouhassoun & Hamdi Cherif, 2015). Fractional derivatives facilitated the observation of memory effects during the phase separation procedure. The q-Shehu homotopy analysis transform method is highly successful for fractional derivative systems and demonstrates rapid convergence. The temporal and geographic evolution of phase structures varied according to the order of the fractional derivative. The fractional Cahn-Hilliard equation offers a notable benefit for modeling long-range interactions and anomalous diffusion phenomena. Three-dimensional graphics distinctly demonstrated the influence of fractional order on the morphological development of the system.

6. Conclusion

This paper presents the novel numerical solutions of CFNCHE using the Cq-SHATM. Moreover, the MAPLE software has constructed 2D and 3D graphics that indicate the solutions to CFNCHE for different values of α. In 2D graphics, it can be seen that it remains constant and overlaps after $x = 4$ for different α values. Table 1 demonstrates that the absolute inaccuracy is relatively insignificant. The conformable fractional Cahn-Hilliard equation provides a more accurate and flexible framework for modeling phase separation, diffusion, and complex dynamic systems. Its numerical solutions offer insights into real-world applications, from material science to biomedical engineering and climate modeling. Cq-SHATM offers that notable computational accuracy and simplicity when addressing and resolving complex occurrence for CTFPDEs, which play a vital role in numerous scientific and technological fields.

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Author contribution

AA: Conceptualization, Formal Analysis, Methodology, Software, Resources, Writing–original draft; TA: Investigation, Supervision, Validation, Visualization, Writing–review & editing. HB: Methodology, Writing–review & editing.

Conflicts of interest

The authors declare no conflicts of interest.

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