



POLİTEKNİK DERGİSİ

JOURNAL of POLYTECHNIC

ISSN: 1302-0900 (PRINT), ISSN: 2147-9429 (ONLINE)

URL: <http://dergipark.org.tr/politeknik>



Effect of weighting schemes on weighted kappa coefficients in multi-rater agreement studies with ordinal categories

Çoklu değerlendiriciye sahip sıralanabilir düzeyli uyum çalışmalarında ağırlıklandırma şemalarının ağırlıklı kappa katsayıları üzerindeki etkisi

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To cite to this article: Yilmaz A.E., “Effect of weighting schemes on weighted kappa coefficients in multi-rater agreement studies with ordinal categories”, *Journal of Polytechnic*, 28(5): 1375-1397, (2025).

Bu makaleye şu şekilde atıfta bulunabilirsiniz: Yilmaz A.E., “Effect of weighting schemes on weighted kappa coefficients in multi-rater agreement studies with ordinal categories”, *Politeknik Dergisi*, 28(5): 1375-1397, (2025).

Erişim linki (To link to this article): <http://dergipark.org.tr/politeknik/archive>

DOI: 10.2339/politeknik.1568563

Effect of Weighting Schemes on Weighted Kappa Coefficients in Multi-Rater Agreement Studies with Ordinal Categories

Highlights

- ❖ The extensions of the quadratic, ridit linear, ridit quadratic, exponential linear, and exponential quadratic weighting schemes are proposed.
- ❖ The accuracy of the multi-rater weighted kappa coefficients and weighting schemes is discussed.
- ❖ A combination of coefficients and weighting schemes for use with various data compositions is discussed.

Graphical Abstract

The results are presented in three parts: 1) Comparison of the weighted kappa coefficients, 2) Comparison of the weighting schemes, and 3) Effect of the number of categories on weighting schemes.

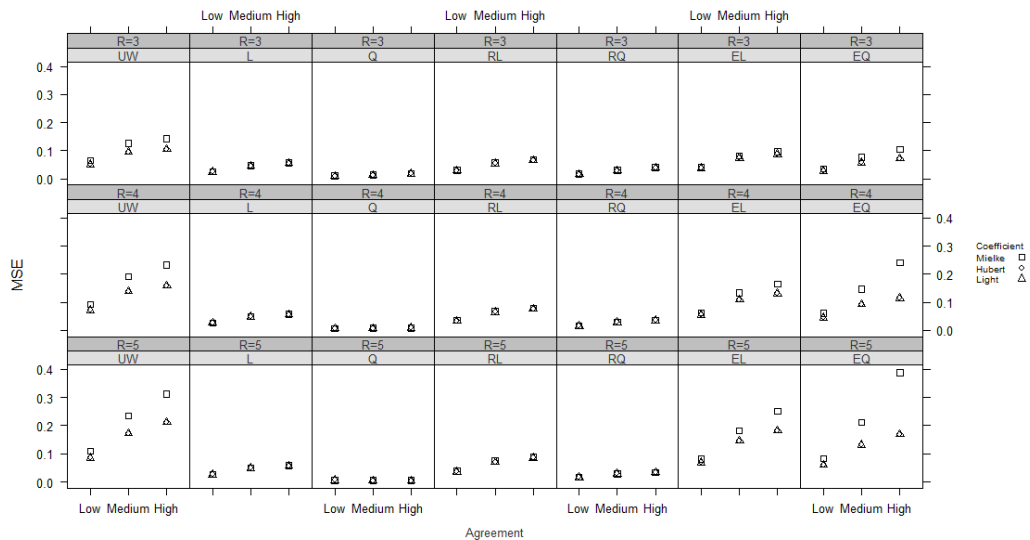


Figure. The MSE results of the balanced tables with three raters and $n = 100$ by weighted kappas

Aim

The aim of this study is to investigate the accuracy of weighted kappa coefficients and the effects of linear, quadratic, ridit type, and exponential type weighting schemes on these coefficients in the multi-rater agreement studies with ordinal categories.

Design & Methodology

A Monte Carlo simulation is performed to compare three weighted kappa coefficients and the weighting schemes for multi-raters with ordinal scales. The simulation study uses 216 different combinations.

Originality

This study provides an extended comparison of weighted kappa coefficients in terms of weighting scheme, the level of true agreement, and the structure of the table. Also, the extension of the quadratic, ridit linear, ridit quadratic, exponential linear, and exponential quadratic weighting schemes are proposed.

Findings

Hubert's and Light's weighted kappas have similar results in most situations. The unweighted kappa coefficients yield poorer results compared to other weighting schemes.

Conclusion

The results indicate that the structure of the table, the level of agreement, and the number of categories also have an effect on the coefficients.

Declaration of Ethical Standards

The author of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Effect of Weighting Schemes on Weighted Kappa Coefficients in Multi-Rater Agreement Studies with Ordinal Categories

Araştırma Makalesi / Research Article

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(Geliş/Received : 16.10.2024 ; Kabul/Accepted : 20.02.2025 ; Erken Görünüm/Early View : 27.02.2025)

ABSTRACT

Weighted kappa and kappa-like coefficients are used for the calculation of inter-rater agreement in cases where raters classify objects into ordinal categories. Weighted kappa coefficients are extended for use in studies with multiple raters. It is crucial to select appropriate weighting schemes as they can significantly impact the value of the coefficient. In this study, the accuracy of weighted kappa coefficients and the effects of linear, quadratic, ridit type, and exponential type weighting schemes on these coefficients are discussed in the multi-rater agreement studies with ordinal categories. The accuracy of the coefficients is investigated by an illustrative data and a simulation study.

Keywords: Inter-rater agreement, multi-raters, ordinal scales, weighted kappa, weighting schemes.

Çoklu Değerlendiriciye Sahip Sıralanabilir Düzeyli Uyum Çalışmalarında Ağırlıklandırma Şemalarının Ağırlıklı Kappa Katsayıları Üzerindeki Etkisi

ÖZ

Ağırlıklı kappa ve kappa benzeri katsayılar, değerlendiricilerin gözlemleri sıralanabilir düzeyler halinde sınıflandırdığı durumlarda, değerlendiriciler arası uyumun hesaplanmasında kullanılır. Ağırlıklı kappa katsayıları, çoklu değerlendiriciye sahip çalışmalarda kullanılmak üzere genişletilmiştir. Katsayının değerini doğrudan etkileyebileceğinden dolayı, uygun ağırlıklandırma şemalarının seçilmesi çok önemlidir. Bu çalışmada, çoklu değerlendiriciye sahip sıralanabilir düzeyli çalışmalarda ağırlıklı kappa katsayılarının doğruluğu ve doğrusal, karesel, ridit tipi ve üstel tipi ağırlıklandırma şemalarının bu katsayılara etkisi tartışılmıştır. Katsayıların doğruluğu, örnek bir veri ve simülasyon çalışması üzerinden araştırılmıştır.

Anahtar Kelimeler: Değerlendiriciler arası uyum, çoklu değerlendiriciler, sıralanabilir ölçekler, ağırlıklı kappa, ağırlıklandırma şemaları.

1. INTRODUCTION

Inter-rater agreement is essential in the medical, social, and behavioral sciences. For example, when pathologists categorize patients based on cancer stage, the agreement between their categorizations is examined. A high level of agreement is expected to yield more reliable results if there is no discrepancy among the raters. Agreement coefficients are employed to assess the level of agreement. Different coefficients may be used depending on the type of scale and the number of raters. While the kappa coefficient is well-known for inter-rater agreement on nominal scales [1,2], the weighted kappa is often preferred for ordinal scales [3].

Recent studies aim to increase the reliability of results by involving more raters. In diagnostic tests, using more than two doctors is recommended to classify patients accurately according to stage and disease severity. These studies evaluate the agreement level among raters. Despite the various options available for two rater

studies, there is a lack of literature examining the agreement of more than two raters who have classified subjects into ordinal categories. Agreement among multiple raters can be defined as 2-agreement or m-agreement [4,5,6,7]. In a 2-agreement or pairwise agreement, the agreement is accepted if the ratings of two of the raters or the agreement of all possible pairs of raters are consistent [6]. A weighted version of the kappa coefficients by Light [8] and Hubert [9] are the weighted kappa coefficients based on 2-agreement. The weighted kappa coefficients by Mielke et al. [10] are proposed for analyzing the agreement between multiple raters' ordered classifications based on 3-agreement. Additionally, several more specific weighted kappas for multiple raters have been proposed by [6,11,12,13].

The weight matrices employed by classical methods are symmetric and are used for continuous ordinal scale data. According to Kvalseth [14], cumulative probabilities can be utilized to compute weighted kappa coefficients and

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provide additional information from ordinal variables. Yilmaz & Aktas [15] recommends the use of ridit and exponential type weighting schemes for ordinal data, given the prevalent applications of ordinal scales in practice. These weighting schemes are based on the score values and are adapted to the weighted kappa coefficients to conduct weights for ordinal scales. The authors analyze the impact of different weighting schemes through various numerical examples. They conclude that the ridit weighted kappa coefficient is more accurate when the true agreement is slight. The coefficient with quadratic exponential weights is more accurate when the true agreement is moderate, and the coefficient with classical linear and quadratic weights is more accurate when the true agreement is almost perfect. Warrens [16] examines the practical applications of kappa and finds that, in most cases, the level of weighted kappa increases when comparing unweighted and weighted (linear and quadratic) kappa applied to the same data. Specifically, the weighted kappa increases from unweighted to linearly weighted and from linearly weighted to quadratically weighted. Tran et al. [17] provide an extensive comparison of weighted kappa-like coefficients and weighting schemes for the two-rater scenario. They compare six coefficients, including unweighted, linear, quadratic, ordinal, radical, and ratio weighting schemes. Their simulation study demonstrates that the precision of the coefficients is influenced by the weights used when the table is unbalanced and the true agreement is low. When the table is balanced, they perform similarly. Vanbelle et al. [18] provide an information about how to choose inter-rater agreement coefficients for ordinal raters. Demirhan & Yilmaz [19] discuss the effects of gray zones on the ordinal agreement coefficients. Yilmaz & Demirhan [20] discuss the effects of weighted kappa-like coefficients and weighting schemes in machine learning studies. de Raadt et al. [21] provides an extended comparison of the unweighted, linearly weighted, and quadratically weighted kappa coefficients, as well as Pearson, Spearman, and Kendall correlations, and inter-class correlation coefficients. Their findings reveal that the unweighted kappa and linearly weighted kappa coefficients generally yield lower values compared to the other coefficients. Also, the simulation study indicates that when there is moderate agreement, these coefficients tend to have closer values than if the agreement is high. The results support that the quadratically weighted kappa functions as a correlation coefficient. Additionally, the unweighted kappa and linearly weighted kappa generally have lower values than the other coefficients.

Even there are several weighting schemes in the literature for two-raters, there are only a few study for the multi-rater case. Mielke et al. [10,22] and Mielke & Berry [23] discuss the extended linear and quadratic weights for the three raters, and Warrens [6,24] for the multi-rater case.

The previous studies have demonstrated the significance of choosing an appropriate weighting scheme for weighted kappas. The weights have a direct impact on the

agreement coefficients. In this study, we focus on the coefficients derived from combinations of 2- and m-agreement and weighting schemes to yield accurate and reliable inferences for multi-rater studies involving ordinal categories. We consider the Light's, Hubert's and Mielke's weighted kappa coefficients, exhibiting linear and quadratic versions of classical, ridit type, and exponential type weighting schemes. We examine a total of 21 combinations and compare them based on the mean square error (MSE) and mean absolute error (MAE), incorporating differing sample sizes, number of categories, number of raters, agreement levels, and table structure (balanced and unbalanced). The study's contributions include: i) the extension of the quadratic, ridit linear, ridit quadratic, exponential linear, and exponential quadratic weighting schemes, ii) an investigation into the accuracy of the multi-rater weighted kappa coefficients, iii) an investigation into the accuracy of the weighting schemes, iv) identification of a combination of coefficients and weighting schemes for use with various data compositions.

The general information on weighted inter-rater agreement coefficients is presented in Section 2. Linear and the proposed weighting schemes are presented in Section 3. The results of the illustrative data and synthetic data are presented in Sections 4 and 5. General conclusions are given in Section 6.

2. INTER-RATER AGREEMENT COEFFICIENTS

Cohen [1] unweighted kappa and Cohen [3] weighted kappa coefficients are suggested to calculate the inter-rater agreement for nominal and ordinal classifications of two rates, respectively. Suppose a square contingency table with R categories in the row and column variables. n_{ij} denote the number of subjects and π_{ij} denote the cell probabilities where $i, j = 1, \dots, R$. $\pi_{i.}$ and $\pi_{.j}$ are the row and column totals. Cohen's unweighted kappa coefficient is

$$\kappa = \frac{\sum_{i=1}^R \pi_{ii} - \sum_{i=1}^R \pi_{i.} \pi_{.i}}{1 - \sum_{i=1}^R \pi_{i.} \pi_{.i}}, \quad (1)$$

weighted kappa coefficient is

$$\kappa_w = \frac{\sum_{i=1}^R \sum_{j=1}^R w_{ij} \pi_{ij} - \sum_{i=1}^R \sum_{j=1}^R w_{ij} \pi_{i.} \pi_{.j}}{1 - \sum_{i=1}^R \sum_{j=1}^R w_{ij} \pi_{i.} \pi_{.j}}. \quad (2)$$

For the classifications of more than two raters, Light [8], Hubert [9], and Mielke et al. [10] weighted kappa coefficients can be refereed. Hubert's κ coefficient is reformulated for ordinal tables [25]. Suppose three raters rate n subjects into R categories. n_{ijk} is the number of subjects and π_{ijk} is the cell probabilities. The marginal probabilities p_i , q_j , and r_k are

$$p_i = \sum_{j=1}^R \sum_{k=1}^R \pi_{ijk}, \quad q_j = \sum_{i=1}^R \sum_{k=1}^R \pi_{ijk}, \quad (3)$$

$$r_i = \sum_{j=1}^R \sum_{k=1}^R \pi_{jki}.$$

The sub-tables $A = \{a_{ij}\}$, $B = \{b_{ij}\}$, and $C = \{c_{ij}\}$ are

$$\begin{aligned} a_{ij} &= \sum_{k=1}^R \pi_{ijk}, & b_{ij} &= \sum_{k=1}^R \pi_{ikj}, \\ c_{ij} &= \sum_{k=1}^R \pi_{kij}. \end{aligned} \quad (4)$$

Hubert's weighted kappa coefficient can be calculated by Eq.5 and w_{ij} is the weight, ranges $0 \leq w_{ij} \leq 1$.

Hubert's weighted kappa coefficient can be extended for more than three raters. Let $n_{(1i)(2i)\dots(M_i)}$ be the number of subjects at category i where M is the number of raters and $i = 1, \dots, R$. $\pi_{(1i)(2i)\dots(M_i)}$ represents the cell probabilities. a_{mij} represents the $R \times R$ sub-tables and p_{mi} represents the marginal probabilities where $m = 1, \dots, M$ and $i, j = 1, \dots, R$. Then Hubert's

generalized weighted kappa coefficient based on 2-agreement is given in Eq. 6.

Mielke et al. [9] suggest a weighted kappa coefficient for three rater studies with ordinal categories. Mielke's weighted kappa coefficient as 3-agreement is given in Eq. 7. Here w_{ijk} is the weight, ranges $0 \leq w_{ijk} \leq 1$.

Light's [8] kappa coefficient is an arithmetic mean of the kappa coefficients calculated for every possible rater pair. The weighted version of Light's κ coefficient is calculated using weighted kappas. Suppose there are M raters and κ_w^{ij} is the weighted kappa coefficient among i th and j th raters where $i < j$. κ_w^{ij} can be calculated from Eq. 2 for each sub-table in Eq. 4. Then, Light's weighted kappa coefficient is given in Eq. 8.

In the literature, numerous interpretations of the kappa statistic have been proposed. Landis & Koch [26] propose the following levels of interpretation. Additionally, Altman [27]'s and Fleiss et al. [28]'s interpretation levels are available (see [29], Table 5).

$$H. \kappa_w = \frac{\sum_{i=1}^R \sum_{j=1}^R w_{ij} (a_{ij} + b_{ij} + c_{ij}) - \sum_{i=1}^R \sum_{j=1}^R w_{ij} (p_i q_j + p_i r_j + q_i r_j)}{3 - \sum_{i=1}^R \sum_{j=1}^R w_{ij} (p_i q_j + p_i r_j + q_i r_j)}, \quad (5)$$

$$H. \kappa_w = \frac{\sum_{m=1}^M \sum_{i,j=1}^R w_{ij} a_{mij} - \sum_{m < m'=1}^M \sum_{i,j=1}^R w_{ij} p_{mi} p_{m'j}}{\frac{M(M-1)}{2} - \sum_{m < m'=1}^M \sum_{i,j=1}^R w_{ij} p_{mi} p_{m'j}}. \quad (6)$$

$$M. \kappa_w = \frac{\sum_{i=1}^R \sum_{j=1}^R \sum_{k=1}^R w_{ijk} \pi_{ijk} - \sum_{i=1}^R \sum_{j=1}^R \sum_{k=1}^R w_{ijk} p_i q_j r_k}{1 - \sum_{i=1}^R \sum_{j=1}^R \sum_{k=1}^R w_{ijk} p_i q_j r_k}. \quad (7)$$

$$L. \kappa_w = \frac{2}{M(M-1)} \sum_{i=1}^{M-1} \sum_{j=i+1}^M \kappa_w^{ij}. \quad (8)$$

3. WEIGHTING SCHEMES

According to the weighting scheme, each cell is attributed with a defined value of weight. The most commonly used weight schemes are the linear weights [30] in Eq. 10 and the quadratic weights [31] in Eq. 11. The linear weights consist of equally spaced numbers. Furthermore, the quadratic weights are a function that increases quadratically as the number of steps increases, whereas the linear weights increase linearly with the number of steps.

- Unweighted:

$$w_{ij} = \begin{cases} 1, & i = j \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

- Linear weights:

$$w_{ij} = 1 - \frac{|i - j|}{R - 1}. \quad (10)$$

- Quadratic weights:

$$w_{ij} = 1 - \frac{(i - j)^2}{(R - 1)^2}. \quad (11)$$

Although the literature offers numerous suggestions for weighting schemes in two-rater studies, there is a lack of options for multi-rater studies. The linear weight version for three raters, in its generalized form, is discussed by [23].

$$w_{ijk} = 1 - \frac{|i - j| + |i - k| + |j - k|}{2(R - 1)}, \quad (12)$$

where $w_{iii} = 1$ for $i, j, k = 1, \dots, R$.

Mielke's weighted kappa coefficient is calculated using the 3-agreement and requires a three-dimensional matrix. This study expands on the classical quadratic weights and introduces the linear and quadratic versions of the rdit and exponential weights to compute Mielke's weighted kappa coefficient.

The unweighted weighting scheme is the matrix of one. The unweighted scheme is

$$w_{ijk} = \begin{cases} 1, & i = j = k \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

The three-dimensional generalized quadratic weights matrix to calculate Mielke's weighted kappa coefficient is

$$w_{ijk} = 1 - \frac{(i-j)^2 + (i-k)^2 + (j-k)^2}{2(R-1)^2}. \quad (14)$$

The weighting matrices for classical methods are symmetric and applicable to continuous-ordinal scale data. Yilmaz & Aktas [15] suggest using ridit and exponential weighting schemes for ordinal scale data. These schemes consider cumulative probabilities rather than constant values of i and j . These schemes consider cumulative probabilities rather than constant values of i and j . As a result, they exhibit an asymmetric structure.

In this study, Light's, Hubert's, and Mielke's weighted kappa coefficients are discussed with the weighting schemes for three or more raters. Since Mielke's weighted kappa coefficient is proposed for three raters, the ridit and exponential weights are proposed for the cases of three raters. On the other hand, a two-dimensional weighting matrix is used to calculate Hubert and Light's kappa coefficients. Thus, two-dimensional ridit and exponential weights are also suggested.

Ridit type scores are suggested by Bross [32], adapted for square contingency tables by Iki et al. [33], and used as weighing schemes in Yilmaz & Aktas [15]. The three-dimensional ridit weights matrix is extended to calculate Mielke's weighted kappa coefficient. Suppose there are the classifications of three raters named A, B, C. The i th ridit score of rater A (r_i^A), the j th ridit score of rater B (r_j^B), and the k th ridit score of rater C (r_k^C) are shown in Eq. 15.

$$r_i^A = \frac{F_{i-1}^A - F_i^A}{2}, \quad r_j^B = \frac{F_{j-1}^B - F_j^B}{2}, \quad r_k^C = \frac{F_{k-1}^C - F_k^C}{2}, \quad (15)$$

where the cumulative distribution function of A, B, and C are

$$F_i^A = \sum_{m \leq i} \pi_{m..}, \quad F_j^B = \sum_{m \leq j} \pi_{.m.}, \quad F_k^C = \sum_{m \leq k} \pi_{..m}. \quad (16)$$

Then, score values are

Hubert and Light's weighted kappa coefficients are calculated using two-dimensional weighting matrices

$$u_{ij}^{ABr} = \frac{r_i^A + r_j^B}{2}, \quad u_{ik}^{ACr} = \frac{r_i^A + r_k^C}{2}, \quad u_{jk}^{BCr} = \frac{r_j^B + r_k^C}{2}. \quad (17)$$

The ridit type linear and quadratic weights are calculated from Eqs. 18 and 19, respectively.

- Ridit linear weights:

$$w_{ijk} = 1 - \frac{\frac{|r_i^A - r_j^B|}{u_{ij}^{ABr}} + \frac{|r_i^A - r_k^C|}{u_{ik}^{ACr}} + \frac{|r_j^B - r_k^C|}{u_{jk}^{BCr}}}{3(R-1)}, \quad (18)$$

where $i, j, k = 1, \dots, R$.

- Ridit quadratic weights:

$$w_{ijk} = 1 - \frac{\frac{(r_i^A - r_j^B)^2}{(u_{ij}^{ABr})^2} + \frac{(r_i^A - r_k^C)^2}{(u_{ik}^{ACr})^2} + \frac{(r_j^B - r_k^C)^2}{(u_{jk}^{BCr})^2}}{3(R-1)^2}, \quad (19)$$

where $i, j, k = 1, \dots, R$.

The exponential scores of Bagheban & Zayeri [34] are used as exponential weights [15]. The exponential score values of multi-rater tables are

$$u_{ij}^{ABe} = \frac{i^a + j^b}{2}, \quad u_{ik}^{ACe} = \frac{i^a + k^c}{2}, \quad u_{jk}^{BCE} = \frac{j^b + k^c}{2}. \quad (20)$$

Here, a , b , and c are the power parameters ($a, b, c > 0$) which are calculated directly from the cumulative distribution functions. The power parameters are

$$a = \left[\prod_{i=1}^{R-1} \alpha_i \right]^{R-1}, \quad b = \left[\prod_{j=1}^{R-1} \beta_j \right]^{R-1}, \quad c = \left[\prod_{k=1}^{R-1} \gamma_k \right]^{R-1}, \quad (21)$$

where $\alpha_i = F_{i+1}^A / F_i^A$, $\beta_j = F_{j+1}^B / F_j^B$, and $\gamma_k = F_{k+1}^C / F_k^C$ where $i, j, k = 1, \dots, R$. The exponential type linear and quadratic weights can be calculated from Eqs. 22 and 23.

- Exponential linear weights:

$$w_{ijk} = 1 - \frac{\frac{|i^a - j^b|}{u_{ij}^{ABe}} + \frac{|i^a - k^c|}{u_{ik}^{ACe}} + \frac{|j^b - k^c|}{u_{jk}^{BCE}}}{3(R-1)}, \quad (22)$$

where $i, j, k = 1, \dots, R$.

- Exponential quadratic weights:

$$w_{ijk} = 1 - \frac{\frac{(i^a - j^b)^2}{(u_{ij}^{ABe})^2} + \frac{(i^a - k^c)^2}{(u_{ik}^{ACe})^2} + \frac{(j^b - k^c)^2}{(u_{jk}^{BCE})^2}}{3(R-1)^2}, \quad (23)$$

where $i, j, k = 1, \dots, R$.

based on 2-agreement, as shown in Eqs. 10 and 11. Hubert and Light's weighted kappa coefficients are

calculated using two-dimensional weighting matrices based on 2-agreement, as shown in Eqs. 10 and 11. These matrices utilize the information from the three-way table to compute ridit and exponential weights for the coefficients. Eqs. 24 and 25 present the two-dimensional versions of the ridit weights.

- Ridit linear weights:

$$w_{ij} = 1 - \frac{\frac{|r_i^A - r_j^B|}{u_{ij}^{ABr}} + \frac{|r_i^A - r_j^C|}{u_{ij}^{ACr}} + \frac{|r_i^B - r_j^C|}{u_{ij}^{BCr}}}{3(R-1)}, \quad (24)$$

where $i, j, k = 1, \dots, R$.

- Ridit quadratic weights:

$$w_{ij} = 1 - \frac{\frac{(r_i^A - r_j^B)^2}{(u_{ij}^{ABr})^2} + \frac{(r_i^A - r_j^C)^2}{(u_{ij}^{ACr})^2} + \frac{(r_i^B - r_j^C)^2}{(u_{ij}^{BCr})^2}}{3(R-1)^2}, \quad (25)$$

where $i, j, k = 1, \dots, R$.

The two-dimensional versions of the exponential weights are given in Eqs. 26–27.

- Exponential linear weights:

$$w_{ij} = 1 - \frac{\frac{|i^a - j^b|}{u_{ij}^{ABe}} + \frac{|i^a - j^c|}{u_{ij}^{ACe}} + \frac{|i^b - j^c|}{u_{ij}^{BCE}}}{3(R-1)}, \quad (26)$$

where $i, j, k = 1, \dots, R$.

- Exponential quadratic weights:

$$w_{ij} = 1 - \frac{\frac{(i^a - j^b)^2}{(u_{ij}^{ABe})^2} + \frac{(i^a - j^c)^2}{(u_{ij}^{ACe})^2} + \frac{(i^b - j^c)^2}{(u_{ij}^{BCE})^2}}{3(R-1)^2}, \quad (27)$$

where $i, j, k = 1, \dots, R$.

For all the two-way weighting schemes, $w_{ii} = 1$ for $i, j = 1, \dots, R$ where $0 \leq w_{ij} \leq 1$ and for all the three-way weighting schemes, $w_{iii} = 1$ for $i, j, k = 1, \dots, R$ where $0 \leq w_{ijk} \leq 1$.

4. ILLUSTRATIVE EXAMPLE

The illustrative data are taken from [35]. The patients who were either discharged from the hospital or treated and released from the emergency room (ER) for suspected or confirmed ischemic heart disease in the past three months were examined [35]. An interview was conducted with each patient, where they were asked about their perceptions of the hospital, what information they received regarding their heart condition, the treatment options offered, etc. Then, three evaluators (A, B, C) independently reviewed transcribed audiotapes of patients 765 patients and classified them into three categories: (1) Not a factor, (2) Minor factor, (3) Major factor. The data are given in Table 1 and contains $M = 3$ raters, $R = 3$ categories, and $n = 765$ subjects.

Table 1. The classifications of three raters

A	B	C	Observed Frequency
1	1	1	266
1	1	2	59
1	1	3	164
1	2	3	47
2	1	1	14
2	2	3	29
3	3	1	68
3	3	2	44
3	3	3	74

The overall agreement coefficients computed by the raters are detailed in Table 2, with values range between 0.259 and 0.418. The agreement level is similar when Mielke's and Hubert's weighted kappas with linear and quadratic weights are used. Light's kappa coefficient indicates greater agreement values than the other coefficients, except for ridit linear weights. The lowest agreement is observed with exponential quadratic weights when using Mielke's weighted kappa. When Hubert's and Lights's kappa coefficients are used, the highest agreement is observed with ridit quadratic weights. When Mielke's kappa coefficient is used, the highest agreement is observed with ridit linear weights.

This illustrative example demonstrates that different combinations of coefficients and weighting schemes give different results. These results strongly indicate the need for a detailed simulation study to discuss the accuracy of the coefficients and also the weighting schemes.

Table 2. The calculated overall agreement coefficients among the raters

Coefficients	Weighting Schemes						
	UW	L	Q	RL	RQ	EL	EQ
Mielke	0.279	0.320	0.337	0.394	0.342	0.313	0.259
Hubert	0.295	0.320	0.337	0.370	0.405	0.318	0.319
Light	0.318	0.353	0.377	0.389	0.418	0.356	0.366

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

5. NUMERICAL EXPERIMENTS WITH SYNTHETIC DATA

In this section, we discuss the numerical studies with synthetic data. We conduct a simulation study to compare the accuracy of weighted kappa coefficients with different weighting schemes introduced in Sections 2 and 3. In this section, simulation design and its results are given.

Table 3. The simulation design and the abridgments

	Level
Coefficient	Light (L), Hubert (H), Mielke (M)
Weighting scheme	Unweighted (UW), Linear (Li), Quadratic (Q), Redit Linear (RL), Redit Quadratic (RQ), Exponential Linear (EL), Exponential Quadratic (EQ)
Sample size	100, 500, 1000, 5000
Number of categories	3, 4, 5
Number of raters	3, 4, 5
True level of agreement	Low, Medium, High

Tran et al. [17] used the correlation coefficient as the true value of the inter-rater agreement. To generate ordinal squared contingency tables, they generated ordinal variables by underlying normally distributed continuous variables [36]. They use quantiles to categorize the generated values. In this study, we used the method presented in Sertdemir et al. [37] to generate the ordinal tables. The intraclass correlation coefficient (ICC) is used as the true value of agreement. Normal and skewed distribution settings are used. The data are generated from the following model equation.

5.1. The Simulation Design

We perform a Monte Carlo simulation to compare three weighted kappa coefficients and the weighting schemes for multi-raters with ordinal scales. The generalized versions of the classical linear and quadratic weights are considered. The simulation study uses 216 different combinations (see Table 3).

$$Y_{ij} = Z_i + \varepsilon_{ij}, \quad (28)$$

where $i = 1, \dots, n$ and $j = 1, \dots, M$. Here, $Z_i \sim N(0, \sigma_p^2)$ indicates the true value of object i and $\varepsilon_{ij} \sim N(0, \sigma_e^2)$ indicates the stochastic measurement error. The distribution of Z_i and ε_{ij} are summarized in Table 4 [37]. While the normal distribution produces a balanced table, the skewed distribution produces an unbalanced table. From this point on, the normal distribution will be referred to as the balanced table structure and the skewed distribution will be referred to as the unbalanced table structure.

Table 4. The simulation settings by distributions

Distribution	Z_i	ε_{ij}		
		Low	Medium	High
Normal	$N(0,1)$	$N(0,1.5)$	$N(0,0.6667)$	$N(0,0.25)$
Skewed	χ_3^2	$N(0,9)$	$N(0,4)$	$N(0,1.5)$

The true agreement level is considered as 0.40, 0.60, and 0.80 which refer to low, medium, and high agreement among the raters. The variance of stochastic measurement error term (σ_e^2) for each agreement level is calculated by $ICC = \sigma_p^2 / (\sigma_p^2 + \sigma_e^2)$ where $\sigma_p^2 = 1$. Then, σ_e^2 is calculated as 1.5, 0.6667, and 0.25, respectively. The continuous Y_{ij} is transformed to X_{ij} by using Eq. 29 which is an ordinal scale. Here, RND is the nearest integer [36]

$$X_{ij} = \text{RND} \left(\frac{R + 0.5 \exp(Y_{ij}) + 0.5}{1 + \exp(Y_{ij})} \right). \quad (29)$$

The accuracies of the agreement coefficient-weighting scheme combinations are assessed using MAE and MSE as

$$\begin{aligned} MAE &= \frac{1}{r} \sum_{i=1}^r |\kappa - \hat{\kappa}_i| \quad \text{and} \\ MSE &= \frac{1}{r} \sum_{i=1}^r (\kappa - \hat{\kappa}_i)^2. \end{aligned} \quad (30)$$

Here, r is the number of replications, κ is the true agreement, and $\hat{\kappa}_i$ is the inter-rater agreement estimation in the i th replication. In the scenarios that involve three raters, three weighted kappas are calculated.

In three-rater scenarios, three of the weighted kappa coefficients are calculated. As Mielke's weighted kappa coefficient is only proposed for cases with three raters, Hubert's and Light's coefficients are calculated for cases with more than three raters.

The simulation program is developed by the author in R version 3.6.1. The results are based on 1000 replications. The lattice package is used to visualize the results [38].

5.2. Results

The results are discussed in terms of the weighted kappa coefficients, the weighting scheme, the level of true agreement, and the structure of the table. The results are presented in three parts: 1) Comparison of the weighted kappa coefficients, 2) Comparison of the weighting schemes, and 3) Effect of the number of categories on weighting schemes.

The MSE results are summarized in Tables 5-13 by the number of raters, number of categories, sample size, true level of agreement, coefficient, and weighting scheme. Tables 5-7 show the results for the three raters, Tables 8-

10 for the four raters, and Tables 11-13 for the five raters. Not all results are tabulated here due to space limitations. The MAE results can be found in Tables A1-A9 of the Appendix.

Table 5. MSE values computed for low agreement tables with three raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Mielke	0.0646	0.0250	0.0098	0.0300	0.0166	0.0411	0.0351	0.0287	0.0274	0.0210	0.0190	0.0182	0.0257	0.0198
		Hubert	0.0509	0.0250	0.0098	0.0290	0.0163	0.0376	0.0282	0.0355	0.0274	0.0210	0.0198	0.0179	0.0249	0.0196
		Light	0.0509	0.0250	0.0098	0.0290	0.0164	0.0376	0.0282	0.0358	0.0276	0.0213	0.0200	0.0181	0.0251	0.0198
	500	Mielke	0.0620	0.0224	0.0066	0.0270	0.0131	0.0366	0.0259	0.0242	0.0224	0.0142	0.0137	0.0119	0.0199	0.0133
		Hubert	0.0484	0.0224	0.0066	0.0264	0.0131	0.0350	0.0244	0.0311	0.0224	0.0142	0.0140	0.0118	0.0195	0.0132
		Light	0.0484	0.0224	0.0066	0.0264	0.0131	0.0350	0.0244	0.0311	0.0224	0.0142	0.0140	0.0119	0.0196	0.0133
	1000	Mielke	0.0618	0.0222	0.0063	0.0266	0.0128	0.0358	0.0247	0.0236	0.0217	0.0133	0.0130	0.0111	0.0191	0.0124
		Hubert	0.0482	0.0222	0.0063	0.0262	0.0128	0.0348	0.0240	0.0305	0.0217	0.0133	0.0132	0.0110	0.0188	0.0124
		Light	0.0482	0.0222	0.0063	0.0262	0.0128	0.0348	0.0240	0.0305	0.0217	0.0133	0.0132	0.0111	0.0188	0.0124
	5000	Mielke	0.0616	0.0220	0.0059	0.0262	0.0125	0.0351	0.0238	0.0233	0.0212	0.0126	0.0127	0.0106	0.0185	0.0118
		Hubert	0.0481	0.0220	0.0059	0.0260	0.0125	0.0346	0.0237	0.0301	0.0212	0.0126	0.0127	0.0106	0.0184	0.0118
		Light	0.0481	0.0220	0.0059	0.0260	0.0125	0.0346	0.0237	0.0301	0.0212	0.0126	0.0127	0.0106	0.0184	0.0118
	100	Mielke	0.0923	0.0263	0.0068	0.0351	0.0161	0.0626	0.0611	0.0226	0.0176	0.0100	0.0085	0.0074	0.0154	0.0088
		Hubert	0.0713	0.0263	0.0068	0.0336	0.0158	0.0542	0.0447	0.0317	0.0176	0.0100	0.0089	0.0072	0.0147	0.0087
		Light	0.0712	0.0263	0.0069	0.0336	0.0159	0.0542	0.0447	0.0319	0.0178	0.0101	0.0090	0.0073	0.0148	0.0088
	500	Mielke	0.0906	0.0238	0.0034	0.0321	0.0123	0.0555	0.0440	0.0197	0.0142	0.0049	0.0046	0.0027	0.0114	0.0042
		Hubert	0.0694	0.0238	0.0034	0.0311	0.0122	0.0515	0.0397	0.0288	0.0142	0.0049	0.0048	0.0027	0.0111	0.0042
		Light	0.0694	0.0238	0.0034	0.0312	0.0122	0.0515	0.0397	0.0288	0.0143	0.0049	0.0048	0.0027	0.0111	0.0042
	1000	Mielke	0.0906	0.0236	0.0030	0.0317	0.0119	0.0543	0.0413	0.0192	0.0137	0.0042	0.0040	0.0020	0.0107	0.0035
		Hubert	0.0693	0.0236	0.0030	0.0310	0.0119	0.0516	0.0392	0.0282	0.0137	0.0042	0.0041	0.0020	0.0105	0.0035
		Light	0.0693	0.0236	0.0030	0.0310	0.0119	0.0516	0.0392	0.0282	0.0137	0.0042	0.0041	0.0020	0.0105	0.0035
	5000	Mielke	0.0904	0.0234	0.0027	0.0312	0.0117	0.0528	0.0393	0.0190	0.0134	0.0037	0.0037	0.0016	0.0102	0.0031
		Hubert	0.0690	0.0234	0.0027	0.0308	0.0117	0.0516	0.0388	0.0279	0.0134	0.0037	0.0038	0.0015	0.0101	0.0031
		Light	0.0690	0.0234	0.0027	0.0308	0.0117	0.0516	0.0388	0.0279	0.0134	0.0037	0.0038	0.0015	0.0101	0.0031
4	100	Mielke	0.1097	0.0265	0.0053	0.0383	0.0161	0.0812	0.0837	0.0205	0.0121	0.0057	0.0048	0.0045	0.0103	0.0047
		Hubert	0.0848	0.0265	0.0053	0.0364	0.0160	0.0687	0.0604	0.0304	0.0121	0.0057	0.0047	0.0049	0.0097	0.0046
		Light	0.0848	0.0265	0.0053	0.0364	0.0160	0.0687	0.0604	0.0305	0.0122	0.0057	0.0048	0.0049	0.0097	0.0046
	500	Mielke	0.1090	0.0245	0.0022	0.0357	0.0126	0.0721	0.0631	0.0182	0.0103	0.0019	0.0016	0.0010	0.0076	0.0015
		Hubert	0.0840	0.0245	0.0022	0.0346	0.0126	0.0650	0.0544	0.0281	0.0103	0.0019	0.0016	0.0011	0.0073	0.0015
		Light	0.0840	0.0245	0.0022	0.0346	0.0126	0.0650	0.0545	0.0281	0.0103	0.0019	0.0016	0.0011	0.0073	0.0015
	1000	Mielke	0.1089	0.0243	0.0019	0.0353	0.0123	0.0701	0.0584	0.0179	0.0098	0.0014	0.0011	0.0005	0.0071	0.0010
		Hubert	0.0839	0.0243	0.0019	0.0345	0.0123	0.0652	0.0539	0.0276	0.0098	0.0014	0.0011	0.0005	0.0068	0.0010
		Light	0.0839	0.0243	0.0019	0.0345	0.0123	0.0652	0.0539	0.0276	0.0098	0.0014	0.0011	0.0005	0.0069	0.0010
	5000	Mielke	0.1088	0.0242	0.0016	0.0349	0.0121	0.0682	0.0547	0.0178	0.0095	0.0009	0.0008	0.0001	0.0067	0.0007
		Hubert	0.0839	0.0242	0.0016	0.0345	0.0121	0.0659	0.0536	0.0275	0.0095	0.0009	0.0008	0.0001	0.0066	0.0007
		Light	0.0839	0.0242	0.0016	0.0345	0.0121	0.0659	0.0536	0.0275	0.0095	0.0009	0.0008	0.0001	0.0066	0.0007

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Redit (R), Exponential (E).

Table 6. MSE values computed for medium agreement tables with three raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Mielke	0.1264	0.0462	0.0136	0.0566	0.0300	0.0818	0.0762	0.0714	0.0680	0.0552	0.0530	0.0520	0.0651	0.0541
		Hubert	0.0952	0.0462	0.0136	0.0548	0.0293	0.0737	0.0572	0.0792	0.0680	0.0552	0.0545	0.0511	0.0636	0.0537
		Light	0.0952	0.0463	0.0136	0.0548	0.0294	0.0737	0.0572	0.0796	0.0684	0.0555	0.0548	0.0515	0.0639	0.0540
	500	Mielke	0.1234	0.0435	0.0108	0.0532	0.0262	0.0744	0.0563	0.0666	0.0641	0.0515	0.0507	0.0487	0.0603	0.0503
		Hubert	0.0923	0.0435	0.0108	0.0521	0.0261	0.0708	0.0525	0.0733	0.0641	0.0515	0.0513	0.0484	0.0596	0.0501
		Light	0.0923	0.0435	0.0108	0.0521	0.0261	0.0708	0.0525	0.0734	0.0642	0.0517	0.0514	0.0486	0.0597	0.0503
	1000	Mielke	0.1231	0.0432	0.0104	0.0526	0.0258	0.0728	0.0537	0.0661	0.0636	0.0509	0.0503	0.0479	0.0595	0.0495
		Hubert	0.0920	0.0432	0.0104	0.0518	0.0257	0.0704	0.0519	0.0728	0.0636	0.0509	0.0506	0.0478	0.0590	0.0494
		Light	0.0920	0.0432	0.0104	0.0518	0.0257	0.0704	0.0519	0.0728	0.0637	0.0510	0.0507	0.0479	0.0591	0.0495
	5000	Mielke	0.1227	0.0429	0.0101	0.0519	0.0254	0.0713	0.0518	0.0656	0.0632	0.0504	0.0500	0.0474	0.0587	0.0489
		Hubert	0.0917	0.0429	0.0101	0.0515	0.0254	0.0702	0.0515	0.0722	0.0632	0.0504	0.0501	0.0473	0.0585	0.0489
		Light	0.0917	0.0429	0.0101	0.0515	0.0254	0.0702	0.0515	0.0722	0.0632	0.0504	0.0501	0.0473	0.0585	0.0489
	100	Mielke	0.1910	0.0482	0.0072	0.0673	0.0283	0.1329	0.1463	0.0590	0.0423	0.0207	0.0210	0.0164	0.0374	0.0200
		Hubert	0.1394	0.0482	0.0072	0.0641	0.0280	0.1096	0.0935	0.0696	0.0423	0.0207	0.0218	0.0159	0.0358	0.0196
		Light	0.1394	0.0482	0.0072	0.0642	0.0280	0.1096	0.0935	0.0698	0.0425	0.0209	0.0219	0.0161	0.0360	0.0198
	500	Mielke	0.1892	0.0458	0.0046	0.0637	0.0244	0.1192	0.1025	0.0563	0.0412	0.0198	0.0211	0.0155	0.0354	0.0188
		Hubert	0.1377	0.0458	0.0046	0.0619	0.0242	0.1077	0.0873	0.0652	0.0412	0.0198	0.0214	0.0154	0.0347	0.0187
		Light	0.1377	0.0459	0.0046	0.0619	0.0242	0.1077	0.0873	0.0652	0.0412	0.0199	0.0214	0.0154	0.0347	0.0188
	1000	Mielke	0.1891	0.0457	0.0043	0.0632	0.0241	0.1165	0.0945	0.0554	0.0405	0.0194	0.0205	0.0149	0.0344	0.0182
		Hubert	0.1376	0.0457	0.0043	0.0618	0.0240	0.1085	0.0871	0.0641	0.0405	0.0194	0.0207	0.0148	0.0339	0.0181
		Light	0.1376	0.0457	0.0043	0.0618	0.0240	0.1086	0.0871	0.0641	0.0406	0.0194	0.0207	0.0148	0.0340	0.0181
	5000	Mielke	0.1888	0.0454	0.0041	0.0622	0.0237	0.1127	0.0886	0.0551	0.0401	0.0188	0.0202	0.0143	0.0337	0.0176
		Hubert	0.1372	0.0454	0.0041	0.0616	0.0237	0.1091	0.0870	0.0635	0.0401	0.0188	0.0202	0.0143	0.0334	0.0176
		Light	0.1372	0.0454	0.0041	0.0616	0.0237	0.1091	0.0870	0.0635	0.0401	0.0188	0.0203	0.0143	0.0334	0.0176
5	100	Mielke	0.2340	0.0493	0.0051	0.0748	0.0289	0.1815	0.2113	0.0592	0.0312	0.0098	0.0099	0.0058	0.0268	0.0096
		Hubert	0.1730	0.0493	0.0051	0.0709	0.0287	0.1456	0.1321	0.0689	0.0312	0.0098	0.0103	0.0057	0.0251	0.0093
		Light	0.1730	0.0493	0.0052	0.0710	0.0288	0.1456	0.1321	0.0690	0.0313	0.0098	0.0104	0.0057	0.0252	0.0093
	500	Mielke	0.2330	0.0469	0.0026	0.0710	0.0247	0.1612	0.1554	0.0559	0.0309	0.0084	0.0096	0.0041	0.0249	0.0078
		Hubert	0.1715	0.0469	0.0026	0.0687	0.0246	0.1388	0.1210	0.0651	0.0309	0.0084	0.0097	0.0040	0.0243	0.0078
		Light	0.1715	0.0469	0.0026	0.0687	0.0247	0.1388	0.1210	0.0651	0.0309	0.0085	0.0097	0.0040	0.0243	0.0078
	1000	Mielke	0.2326	0.0467	0.0023	0.0703	0.0244	0.1564	0.1404	0.0553	0.0305	0.0081	0.0094	0.0037	0.0244	0.0075
		Hubert	0.1712	0.0467	0.0023	0.0686	0.0244	0.1400	0.1208	0.0643	0.0305	0.0081	0.0095	0.0037	0.0239	0.0074
		Light	0.1712	0.0467	0.0023	0.0686	0.0244	0.1400	0.1208	0.0643	0.0305	0.0081	0.0095	0.0037	0.0239	0.0074
	5000	Mielke	0.2327	0.0465	0.0021	0.0693	0.0240	0.1510	0.1270	0.0552	0.0302	0.0076	0.0092	0.0034	0.0239	0.0071
		Hubert	0.1711	0.0465	0.0021	0.0685	0.0240	0.1431	0.1222	0.0641	0.0302	0.0076	0.0092	0.0034	0.0236	0.0071
		Light	0.1711	0.0465	0.0021	0.0685	0.0240	0.1431	0.1222	0.0641	0.0302	0.0076	0.0092	0.0034	0.0236	0.0071

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table 7. MSE values computed for high agreement tables with three raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Mielke	0.1423	0.0559	0.0177	0.0672	0.0396	0.0959	0.1051	0.1008	0.0918	0.0739	0.0737	0.0725	0.0883	0.0742
		Hubert	0.1050	0.0559	0.0177	0.0656	0.0386	0.0867	0.0716	0.1047	0.0918	0.0739	0.0753	0.0713	0.0864	0.0736
		Light	0.1050	0.0560	0.0178	0.0656	0.0386	0.0867	0.0716	0.1053	0.0924	0.0744	0.0760	0.0719	0.0870	0.0741
	500	Mielke	0.1381	0.0531	0.0157	0.0637	0.0359	0.0875	0.0736	0.0955	0.0927	0.0851	0.0847	0.0841	0.0909	0.0848
		Hubert	0.1013	0.0531	0.0157	0.0627	0.0357	0.0834	0.0673	0.0970	0.0927	0.0851	0.0853	0.0838	0.0901	0.0846
		Light	0.1013	0.0531	0.0157	0.0627	0.0357	0.0834	0.0673	0.0972	0.0928	0.0852	0.0855	0.0840	0.0902	0.0848
	1000	Mielke	0.1370	0.0524	0.0153	0.0627	0.0352	0.0852	0.0691	0.0934	0.0914	0.0854	0.0851	0.0844	0.0898	0.0850
		Hubert	0.1003	0.0524	0.0153	0.0620	0.0351	0.0825	0.0663	0.0947	0.0914	0.0854	0.0854	0.0843	0.0892	0.0849
		Light	0.1003	0.0524	0.0153	0.0620	0.0351	0.0825	0.0663	0.0948	0.0915	0.0855	0.0855	0.0843	0.0893	0.0850
	5000	Mielke	0.1368	0.0522	0.0151	0.0622	0.0350	0.0836	0.0667	0.0922	0.0904	0.0850	0.0848	0.0839	0.0886	0.0844
		Hubert	0.1001	0.0522	0.0151	0.0619	0.0350	0.0824	0.0661	0.0934	0.0904	0.0850	0.0849	0.0839	0.0884	0.0844
		Light	0.1001	0.0522	0.0151	0.0619	0.0350	0.0824	0.0661	0.0934	0.0905	0.0850	0.0849	0.0839	0.0884	0.0844
	100	Mielke	0.2320	0.0570	0.0079	0.0794	0.0353	0.1643	0.2398	0.0855	0.0576	0.0266	0.0292	0.0225	0.0499	0.0265
		Hubert	0.1595	0.0570	0.0079	0.0761	0.0347	0.1306	0.1147	0.0883	0.0576	0.0266	0.0296	0.0219	0.0480	0.0260
		Light	0.1594	0.0570	0.0079	0.0761	0.0347	0.1306	0.1147	0.0884	0.0577	0.0268	0.0297	0.0220	0.0482	0.0262
	500	Mielke	0.2306	0.0549	0.0064	0.0763	0.0320	0.1482	0.1470	0.0787	0.0565	0.0310	0.0323	0.0258	0.0488	0.0295
		Hubert	0.1576	0.0549	0.0064	0.0743	0.0319	0.1309	0.1116	0.0793	0.0565	0.0310	0.0325	0.0257	0.0480	0.0294
		Light	0.1576	0.0549	0.0064	0.0743	0.0319	0.1309	0.1116	0.0794	0.0566	0.0311	0.0326	0.0257	0.0480	0.0294
	1000	Mielke	0.2313	0.0550	0.0063	0.0759	0.0318	0.1450	0.1304	0.0774	0.0559	0.0312	0.0332	0.0267	0.0484	0.0300
		Hubert	0.1581	0.0550	0.0063	0.0745	0.0317	0.1325	0.1125	0.0777	0.0559	0.0312	0.0333	0.0266	0.0479	0.0299
		Light	0.1581	0.0550	0.0063	0.0745	0.0318	0.1325	0.1125	0.0777	0.0559	0.0313	0.0333	0.0266	0.0479	0.0300
	5000	Mielke	0.2309	0.0547	0.0061	0.0749	0.0315	0.1393	0.1169	0.0768	0.0553	0.0310	0.0335	0.0270	0.0479	0.0300
		Hubert	0.1578	0.0547	0.0061	0.0742	0.0315	0.1335	0.1131	0.0768	0.0553	0.0310	0.0336	0.0270	0.0477	0.0300
		Light	0.1578	0.0547	0.0061	0.0742	0.0315	0.1335	0.1131	0.0768	0.0553	0.0310	0.0336	0.0270	0.0477	0.0300
4	100	Mielke	0.3118	0.0580	0.0045	0.0894	0.0340	0.2503	0.3883	0.0536	0.0234	0.0056	0.0059	0.0012	0.0525	0.0525
		Hubert	0.2123	0.0580	0.0045	0.0849	0.0337	0.1832	0.1693	0.0525	0.0234	0.0056	0.0060	0.0012	0.0525	0.0525
		Light	0.2123	0.0580	0.0046	0.0849	0.0338	0.1832	0.1692	0.0524	0.0232	0.0054	0.0059	0.0012	0.0524	0.0524
	500	Mielke	0.3097	0.0558	0.0032	0.0852	0.0302	0.2206	0.2643	0.1289	0.0903	0.0636	0.0675	0.0593	0.0958	0.0754
		Hubert	0.2098	0.0558	0.0032	0.0825	0.0301	0.1760	0.1578	0.1261	0.0903	0.0636	0.0674	0.0592	0.0950	0.0753
		Light	0.2098	0.0558	0.0032	0.0826	0.0301	0.1760	0.1578	0.1262	0.0903	0.0636	0.0674	0.0592	0.0950	0.0753
	1000	Mielke	0.3091	0.0555	0.0031	0.0842	0.0297	0.2119	0.2214	0.0873	0.0440	0.0139	0.0188	0.0098	0.0369	0.0141
		Hubert	0.2094	0.0555	0.0031	0.0822	0.0297	0.1793	0.1603	0.0842	0.0440	0.0139	0.0188	0.0097	0.0362	0.0140
		Light	0.2094	0.0555	0.0031	0.0822	0.0297	0.1793	0.1603	0.0842	0.0440	0.0139	0.0188	0.0098	0.0362	0.0140
	5000	Mielke	0.3093	0.0554	0.0030	0.0832	0.0295	0.2019	0.1824	0.0863	0.0433	0.0135	0.0187	0.0097	0.0360	0.0138
		Hubert	0.2095	0.0554	0.0030	0.0822	0.0295	0.1854	0.1660	0.0831	0.0433	0.0135	0.0187	0.0097	0.0357	0.0137
		Light	0.2095	0.0554	0.0030	0.0822	0.0295	0.1854	0.1660	0.0831	0.0433	0.0135	0.0187	0.0097	0.0357	0.0137

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table 8. MSE values computed for low agreement tables with four raters

R	n	Coef.	Balanced							Unbalanced							
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ	
3	100	Hubert	0.0500	0.0244	0.0090	0.0281	0.0154	0.0367	0.0271	0.0334	0.0247	0.0174	0.0166	0.0145	0.0220	0.0162	
		Light	0.0500	0.0245	0.0090	0.0282	0.0154	0.0367	0.0272	0.0337	0.0251	0.0177	0.0169	0.0148	0.0223	0.0165	
	500	Hubert	0.0482	0.0223	0.0064	0.0262	0.0128	0.0348	0.0241	0.0306	0.0218	0.0134	0.0134	0.0113	0.0190	0.0125	
		Light	0.0482	0.0223	0.0064	0.0262	0.0129	0.0348	0.0241	0.0307	0.0219	0.0135	0.0135	0.0113	0.0190	0.0126	
	1000	Hubert	0.0482	0.0222	0.0062	0.0261	0.0127	0.0347	0.0239	0.0302	0.0214	0.0129	0.0129	0.0108	0.0185	0.0120	
		Light	0.0482	0.0222	0.0062	0.0261	0.0127	0.0347	0.0239	0.0302	0.0214	0.0129	0.0130	0.0108	0.0186	0.0121	
	5000	Hubert	0.0480	0.0219	0.0059	0.0258	0.0123	0.0345	0.0235	0.0302	0.0213	0.0127	0.0128	0.0106	0.0184	0.0118	
		Light	0.0480	0.0219	0.0059	0.0258	0.0123	0.0345	0.0235	0.0302	0.0213	0.0127	0.0128	0.0106	0.0184	0.0119	
	4	100	Hubert	0.0705	0.0258	0.0059	0.0328	0.0149	0.0531	0.0435	0.0301	0.0156	0.0072	0.0067	0.0049	0.0126	0.0063
			Light	0.0705	0.0259	0.0059	0.0328	0.0149	0.0531	0.0435	0.0303	0.0158	0.0074	0.0068	0.0050	0.0127	0.0064
500		Hubert	0.0690	0.0237	0.0033	0.0310	0.0121	0.0511	0.0394	0.0283	0.0139	0.0044	0.0044	0.0023	0.0107	0.0038	
		Light	0.0690	0.0237	0.0033	0.0310	0.0121	0.0511	0.0394	0.0283	0.0139	0.0045	0.0044	0.0023	0.0107	0.0038	
1000		Hubert	0.0689	0.0235	0.0030	0.0309	0.0118	0.0513	0.0390	0.0280	0.0135	0.0040	0.0040	0.0018	0.0104	0.0034	
		Light	0.0689	0.0235	0.0030	0.0309	0.0119	0.0513	0.0390	0.0280	0.0136	0.0040	0.0040	0.0019	0.0104	0.0034	
5000		Hubert	0.0689	0.0233	0.0027	0.0307	0.0116	0.0515	0.0387	0.0278	0.0134	0.0037	0.0037	0.0015	0.0101	0.0031	
		Light	0.0689	0.0233	0.0027	0.0307	0.0116	0.0515	0.0387	0.0278	0.0134	0.0037	0.0037	0.0015	0.0101	0.0031	
5		100	Hubert	0.0852	0.0266	0.0047	0.0364	0.0155	0.0690	0.0607	0.0291	0.0110	0.0040	0.0036	0.0038	0.0084	0.0034
			Light	0.0852	0.0266	0.0047	0.0365	0.0155	0.0690	0.0607	0.0292	0.0111	0.0041	0.0036	0.0038	0.0085	0.0035
	500	Hubert	0.0839	0.0246	0.0022	0.0348	0.0126	0.0651	0.0546	0.0279	0.0100	0.0015	0.0014	0.0008	0.0071	0.0012	
		Light	0.0839	0.0246	0.0022	0.0348	0.0126	0.0651	0.0546	0.0279	0.0100	0.0015	0.0014	0.0008	0.0071	0.0012	
	1000	Hubert	0.0840	0.0244	0.0019	0.0346	0.0123	0.0653	0.0540	0.0277	0.0097	0.0011	0.0011	0.0004	0.0068	0.0009	
		Light	0.0840	0.0245	0.0019	0.0346	0.0123	0.0653	0.0540	0.0277	0.0097	0.0011	0.0011	0.0004	0.0068	0.0009	
	5000	Hubert	0.0839	0.0241	0.0016	0.0344	0.0120	0.0658	0.0536	0.0275	0.0096	0.0009	0.0008	0.0001	0.0066	0.0007	
		Light	0.0839	0.0242	0.0016	0.0344	0.0120	0.0658	0.0536	0.0275	0.0096	0.0009	0.0008	0.0001	0.0066	0.0007	

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table 9. MSE values computed for medium agreement tables with four raters

R	n	Coef.	Balanced							Unbalanced							
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ	
3	100	Hubert	0.0747	0.0628	0.0484	0.0483	0.0448	0.0579	0.0471	0.0962	0.0472	0.0140	0.0556	0.0298	0.0745	0.0578	
		Light	0.0755	0.0635	0.0492	0.0489	0.0455	0.0586	0.0478	0.0962	0.0473	0.0140	0.0557	0.0299	0.0745	0.0578	
	500	Hubert	0.0731	0.0639	0.0512	0.0508	0.0480	0.0593	0.0497	0.0923	0.0436	0.0108	0.0521	0.0261	0.0708	0.0526	
		Light	0.0732	0.0640	0.0513	0.0510	0.0482	0.0594	0.0498	0.0923	0.0436	0.0108	0.0522	0.0262	0.0708	0.0526	
	1000	Hubert	0.0725	0.0633	0.0504	0.0503	0.0474	0.0586	0.0490	0.0919	0.0432	0.0104	0.0517	0.0257	0.0704	0.0519	
		Light	0.0725	0.0633	0.0505	0.0503	0.0475	0.0587	0.0490	0.0919	0.0432	0.0105	0.0517	0.0257	0.0704	0.0519	
	5000	Hubert	0.0723	0.0632	0.0504	0.0502	0.0474	0.0585	0.0489	0.0916	0.0428	0.0101	0.0514	0.0253	0.0701	0.0514	
		Light	0.0723	0.0632	0.0504	0.0502	0.0474	0.0586	0.0489	0.0916	0.0428	0.0101	0.0514	0.0253	0.0701	0.0514	
	4	100	Hubert	0.0656	0.0397	0.0184	0.0187	0.0130	0.0328	0.0169	0.1403	0.0486	0.0069	0.0643	0.0276	0.1100	0.0934
			Light	0.0659	0.0400	0.0186	0.0189	0.0131	0.0330	0.0172	0.1403	0.0486	0.0069	0.0644	0.0277	0.1100	0.0934
500		Hubert	0.0638	0.0402	0.0192	0.0205	0.0146	0.0336	0.0179	0.1376	0.0459	0.0046	0.0620	0.0244	0.1077	0.0875	
		Light	0.0639	0.0403	0.0192	0.0205	0.0147	0.0337	0.0180	0.1376	0.0459	0.0046	0.0621	0.0244	0.1078	0.0875	
1000		Hubert	0.0638	0.0402	0.0190	0.0205	0.0146	0.0337	0.0178	0.1374	0.0458	0.0044	0.0618	0.0240	0.1084	0.0872	
		Light	0.0638	0.0403	0.0190	0.0205	0.0146	0.0337	0.0179	0.1374	0.0458	0.0044	0.0618	0.0240	0.1084	0.0872	
5000		Hubert	0.0636	0.0401	0.0189	0.0203	0.0143	0.0335	0.0176	0.1369	0.0453	0.0040	0.0614	0.0236	0.1088	0.0868	
		Light	0.0636	0.0401	0.0189	0.0203	0.0144	0.0335	0.0176	0.1369	0.0453	0.0041	0.0614	0.0236	0.1088	0.0868	
5		100	Hubert	0.0669	0.0300	0.0082	0.0090	0.0042	0.0237	0.0078	0.1736	0.0498	0.0048	0.0715	0.0287	0.1459	0.1325
			Light	0.0670	0.0302	0.0083	0.0090	0.0042	0.0238	0.0079	0.1736	0.0499	0.0048	0.0715	0.0288	0.1459	0.1325
	500	Hubert	0.0646	0.0304	0.0080	0.0094	0.0037	0.0239	0.0074	0.1712	0.0470	0.0026	0.0689	0.0248	0.1388	0.1211	
		Light	0.0646	0.0305	0.0081	0.0094	0.0037	0.0239	0.0074	0.1712	0.0470	0.0026	0.0689	0.0249	0.1388	0.1211	
	1000	Hubert	0.0644	0.0305	0.0080	0.0095	0.0037	0.0239	0.0074	0.1714	0.0468	0.0023	0.0687	0.0243	0.1400	0.1210	
		Light	0.0645	0.0305	0.0080	0.0095	0.0037	0.0239	0.0074	0.1714	0.0468	0.0023	0.0687	0.0243	0.1400	0.1210	
	5000	Hubert	0.0641	0.0302	0.0077	0.0093	0.0034	0.0237	0.0071	0.1712	0.0465	0.0021	0.0684	0.0239	0.1430	0.1221	
		Light	0.0641	0.0302	0.0077	0.0093	0.0034	0.0237	0.0071	0.1712	0.0465	0.0021	0.0684	0.0239	0.1430	0.1221	

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table 10. MSE values computed for high agreement tables with four raters

R	n	Coef.	Balanced							Unbalanced							
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ	
3	100	Hubert	0.1040	0.0558	0.0178	0.0653	0.0386	0.0861	0.0713	0.1009	0.0881	0.0699	0.0719	0.0679	0.0826	0.0697	
		Light	0.1040	0.0558	0.0178	0.0653	0.0386	0.0861	0.0713	0.1019	0.0889	0.0706	0.0727	0.0688	0.0835	0.0704	
	500	Hubert	0.1004	0.0526	0.0155	0.0622	0.0355	0.0827	0.0668	0.0939	0.0899	0.0826	0.0826	0.0812	0.0872	0.0819	
		Light	0.1004	0.0526	0.0155	0.0622	0.0355	0.0827	0.0668	0.0942	0.0901	0.0828	0.0828	0.0814	0.0874	0.0822	
	1000	Hubert	0.1002	0.0524	0.0153	0.0619	0.0351	0.0825	0.0663	0.0942	0.0910	0.0850	0.0850	0.0839	0.0888	0.0844	
		Light	0.1002	0.0524	0.0153	0.0619	0.0351	0.0825	0.0663	0.0943	0.0911	0.0851	0.0851	0.0840	0.0889	0.0845	
	5000	Hubert	0.1002	0.0522	0.0152	0.0619	0.0350	0.0825	0.0662	0.0937	0.0907	0.0851	0.0851	0.0841	0.0886	0.0845	
		Light	0.1002	0.0522	0.0152	0.0619	0.0350	0.0825	0.0662	0.0937	0.0907	0.0852	0.0851	0.0841	0.0886	0.0845	
	4	100	Hubert	0.1628	0.0586	0.0082	0.0786	0.0363	0.1330	0.1169	0.0738	0.0495	0.0258	0.0218	0.0107	0.0552	0.0459
			Light	0.1628	0.0586	0.0082	0.0786	0.0363	0.1329	0.1168	0.0738	0.0495	0.0260	0.0218	0.0107	0.0552	0.0460
500		Hubert	0.1584	0.0554	0.0065	0.0751	0.0324	0.1316	0.1125	0.0783	0.0547	0.0287	0.0307	0.0236	0.0467	0.0272	
		Light	0.1584	0.0554	0.0065	0.0751	0.0324	0.1316	0.1125	0.0784	0.0547	0.0287	0.0307	0.0237	0.0467	0.0272	
1000		Hubert	0.1578	0.0550	0.0063	0.0745	0.0318	0.1324	0.1126	0.0790	0.0567	0.0315	0.0344	0.0275	0.0489	0.0306	
		Light	0.1578	0.0550	0.0063	0.0745	0.0318	0.1324	0.1126	0.0790	0.0568	0.0315	0.0344	0.0276	0.0489	0.0307	
5000		Hubert	0.1574	0.0546	0.0061	0.0740	0.0314	0.1332	0.1129	0.0764	0.0551	0.0309	0.0333	0.0268	0.0474	0.0298	
		Light	0.1574	0.0546	0.0061	0.0740	0.0314	0.1332	0.1129	0.0764	0.0551	0.0309	0.0333	0.0268	0.0474	0.0298	
5		100	Hubert	0.2131	0.0586	0.0045	0.0858	0.0342	0.1835	0.1694	0.1579	0.0916	0.0364	0.1034	0.1130	0.0341	0.0126
			Light	0.2131	0.0586	0.0045	0.0858	0.0342	0.1835	0.1694	0.1360	0.0768	0.0286	0.0587	0.0477	0.0341	0.0127
	500	Hubert	0.2098	0.0559	0.0032	0.0828	0.0303	0.1763	0.1582	0.1214	0.0677	0.0251	0.0590	0.0583	0.0316	0.0110	
		Light	0.2097	0.0559	0.0032	0.0828	0.0303	0.1763	0.1582	0.1109	0.0609	0.0216	0.0374	0.0266	0.0316	0.0110	
	1000	Hubert	0.2095	0.0557	0.0031	0.0824	0.0298	0.1792	0.1604	0.1579	0.0916	0.0364	0.1034	0.1130	0.0341	0.0126	
		Light	0.2095	0.0557	0.0031	0.0825	0.0298	0.1792	0.1604	0.1360	0.0768	0.0286	0.0587	0.0477	0.0341	0.0127	
	5000	Hubert	0.2093	0.0554	0.0029	0.0820	0.0293	0.1853	0.1659	0.0834	0.0427	0.0129	0.0185	0.0095	0.0353	0.0133	
		Light	0.2093	0.0554	0.0030	0.0820	0.0293	0.1853	0.1659	0.0834	0.0427	0.0129	0.0185	0.0095	0.0353	0.0133	

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table 11. MSE values computed for low agreement tables with five raters

R	n	Coef.	Balanced							Unbalanced							
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ	
3	100	Hubert	0.0494	0.0238	0.0084	0.0277	0.0149	0.0360	0.0265	0.0334	0.0247	0.0174	0.0166	0.0145	0.0220	0.0162	
		Light	0.0494	0.0238	0.0084	0.0278	0.0150	0.0360	0.0265	0.0337	0.0251	0.0177	0.0169	0.0148	0.0223	0.0165	
	500	Hubert	0.0484	0.0223	0.0063	0.0262	0.0128	0.0347	0.0240	0.0306	0.0218	0.0134	0.0134	0.0113	0.0190	0.0125	
		Light	0.0484	0.0223	0.0064	0.0262	0.0128	0.0348	0.0241	0.0307	0.0219	0.0135	0.0135	0.0113	0.0190	0.0126	
	1000	Hubert	0.0482	0.0221	0.0061	0.0260	0.0126	0.0346	0.0238	0.0302	0.0214	0.0129	0.0129	0.0108	0.0185	0.0120	
		Light	0.0482	0.0221	0.0061	0.0260	0.0126	0.0346	0.0238	0.0302	0.0214	0.0129	0.0130	0.0108	0.0186	0.0121	
	5000	Hubert	0.0480	0.0219	0.0059	0.0259	0.0123	0.0345	0.0235	0.0302	0.0213	0.0127	0.0128	0.0106	0.0184	0.0118	
		Light	0.0480	0.0219	0.0059	0.0259	0.0123	0.0345	0.0235	0.0302	0.0213	0.0127	0.0128	0.0106	0.0184	0.0119	
	4	100	Hubert	0.0702	0.0250	0.0051	0.0324	0.0142	0.0526	0.0428	0.0301	0.0156	0.0072	0.0067	0.0049	0.0126	0.0063
			Light	0.0702	0.0251	0.0051	0.0324	0.0142	0.0526	0.0430	0.0303	0.0158	0.0074	0.0068	0.0050	0.0127	0.0064
500		Hubert	0.0691	0.0236	0.0032	0.0310	0.0121	0.0511	0.0393	0.0283	0.0139	0.0044	0.0044	0.0023	0.0107	0.0038	
		Light	0.0691	0.0237	0.0032	0.0310	0.0121	0.0511	0.0394	0.0283	0.0139	0.0045	0.0044	0.0023	0.0107	0.0038	
1000		Hubert	0.0691	0.0235	0.0029	0.0309	0.0118	0.0513	0.0390	0.0280	0.0135	0.0040	0.0040	0.0018	0.0104	0.0034	
		Light	0.0691	0.0235	0.0029	0.0309	0.0118	0.0513	0.0391	0.0280	0.0136	0.0040	0.0040	0.0019	0.0104	0.0034	
5000		Hubert	0.0690	0.0234	0.0027	0.0307	0.0116	0.0515	0.0387	0.0278	0.0134	0.0037	0.0037	0.0015	0.0101	0.0031	
		Light	0.0690	0.0234	0.0027	0.0307	0.0116	0.0515	0.0388	0.0278	0.0134	0.0037	0.0037	0.0015	0.0101	0.0031	
5		100	Hubert	0.0846	0.0257	0.0040	0.0359	0.0148	0.0680	0.0595	0.0291	0.0110	0.0040	0.0036	0.0038	0.0084	0.0024
			Light	0.0846	0.0257	0.0040	0.0360	0.0148	0.0680	0.0596	0.0292	0.0111	0.0041	0.0036	0.0038	0.0085	0.0025
	500	Hubert	0.0840	0.0244	0.0020	0.0347	0.0125	0.0650	0.0543	0.0279	0.0100	0.0015	0.0014	0.0008	0.0071	0.0012	
		Light	0.0840	0.0244	0.0020	0.0347	0.0125	0.0650	0.0544	0.0279	0.0100	0.0015	0.0014	0.0008	0.0071	0.0012	
	1000	Hubert	0.0840	0.0243	0.0018	0.0346	0.0122	0.0652	0.0539	0.0277	0.0097	0.0011	0.0011	0.0004	0.0068	0.0009	
		Light	0.0840	0.0243	0.0018	0.0346	0.0123	0.0652	0.0539	0.0277	0.0097	0.0011	0.0011	0.0004	0.0068	0.0009	
	5000	Hubert	0.0839	0.0241	0.0016	0.0344	0.0120	0.0658	0.0535	0.0275	0.0096	0.0009	0.0008	0.0001	0.0066	0.0007	
		Light	0.0839	0.0241	0.0016	0.0344	0.0120	0.0658	0.0535	0.0275	0.0096	0.0009	0.0008	0.0001	0.0066	0.0007	

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table 12. MSE values computed for medium agreement tables with five raters

R	n	Coef.	Balanced							Unbalanced							
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ	
3	100	Hubert	0.0938	0.0452	0.0126	0.0539	0.0285	0.0720	0.0553	0.0755	0.0643	0.0504	0.0498	0.0464	0.0593	0.0488	
		Light	0.0938	0.0453	0.0126	0.0540	0.0286	0.0720	0.0553	0.0764	0.0651	0.0512	0.0507	0.0473	0.0602	0.0496	
	500	Hubert	0.0919	0.0432	0.0105	0.0517	0.0258	0.0702	0.0518	0.0732	0.0641	0.0515	0.0511	0.0483	0.0595	0.0499	
		Light	0.0919	0.0432	0.0106	0.0517	0.0258	0.0702	0.0520	0.0734	0.0643	0.0517	0.0513	0.0485	0.0597	0.0502	
	1000	Hubert	0.0918	0.0430	0.0103	0.0516	0.0255	0.0701	0.0516	0.0732	0.0641	0.0513	0.0510	0.0482	0.0594	0.0498	
		Light	0.0918	0.0430	0.0103	0.0516	0.0256	0.0701	0.0516	0.0733	0.0642	0.0514	0.0511	0.0483	0.0595	0.0499	
	5000	Hubert	0.0916	0.0428	0.0101	0.0514	0.0253	0.0700	0.0513	0.0723	0.0633	0.0505	0.0502	0.0474	0.0586	0.0489	
		Light	0.0916	0.0428	0.0101	0.0514	0.0253	0.0700	0.0513	0.0724	0.0633	0.0505	0.0502	0.0474	0.0586	0.0489	
	4	100	Hubert	0.1391	0.0479	0.0065	0.0640	0.0273	0.1087	0.0921	0.0653	0.0389	0.0173	0.0182	0.0124	0.0321	0.0162
			Light	0.1391	0.0480	0.0065	0.0641	0.0274	0.1087	0.0923	0.0657	0.0392	0.0176	0.0184	0.0126	0.0324	0.0166
500		Hubert	0.1374	0.0458	0.0045	0.0619	0.0243	0.1072	0.0868	0.0639	0.0404	0.0193	0.0207	0.0148	0.0338	0.0181	
		Light	0.1374	0.0458	0.0045	0.0619	0.0243	0.1073	0.0869	0.0640	0.0405	0.0194	0.0207	0.0148	0.0339	0.0182	
1000		Hubert	0.1372	0.0456	0.0043	0.0616	0.0239	0.1081	0.0867	0.0641	0.0406	0.0194	0.0208	0.0149	0.0340	0.0182	
		Light	0.1372	0.0456	0.0043	0.0616	0.0239	0.1081	0.0868	0.0641	0.0407	0.0194	0.0209	0.0149	0.0341	0.0182	
5000		Hubert	0.1369	0.0453	0.0040	0.0613	0.0235	0.1086	0.0865	0.0635	0.0401	0.0189	0.0203	0.0144	0.0335	0.0176	
		Light	0.1369	0.0453	0.0040	0.0613	0.0235	0.1086	0.0865	0.0635	0.0401	0.0189	0.0203	0.0144	0.0335	0.0176	
5		100	Hubert	0.1731	0.0488	0.0042	0.0712	0.0282	0.1453	0.1318	0.0656	0.0282	0.0066	0.0077	0.0032	0.0220	0.0064
			Light	0.1731	0.0489	0.0042	0.0713	0.0283	0.1454	0.1318	0.0658	0.0284	0.0067	0.0078	0.0032	0.0222	0.0066
	500	Hubert	0.1715	0.0469	0.0025	0.0688	0.0247	0.1388	0.1207	0.0646	0.0303	0.0079	0.0093	0.0035	0.0238	0.0073	
		Light	0.1715	0.0469	0.0025	0.0689	0.0247	0.1388	0.1208	0.0646	0.0304	0.0079	0.0093	0.0036	0.0238	0.0073	
	1000	Hubert	0.1714	0.0467	0.0023	0.0687	0.0244	0.1399	0.1208	0.0643	0.0303	0.0078	0.0094	0.0035	0.0238	0.0073	
		Light	0.1714	0.0467	0.0023	0.0687	0.0244	0.1399	0.1208	0.0643	0.0303	0.0078	0.0094	0.0036	0.0238	0.0073	
	5000	Hubert	0.1712	0.0464	0.0021	0.0683	0.0239	0.1430	0.1220	0.0641	0.0302	0.0077	0.0092	0.0034	0.0237	0.0071	
		Light	0.1712	0.0464	0.0021	0.0683	0.0239	0.1430	0.1220	0.0641	0.0302	0.0077	0.0092	0.0034	0.0237	0.0071	

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table 13. MSE values computed for high agreement tables with five raters

R	n	Coef.	Balanced							Unbalanced							
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ	
3	100	Hubert	0.1030	0.0548	0.0171	0.0646	0.0381	0.0846	0.0697	0.0944	0.0824	0.0650	0.0659	0.0620	0.0767	0.0642	
		Light	0.1030	0.0548	0.0172	0.0647	0.0382	0.0846	0.0698	0.0956	0.0834	0.0657	0.0669	0.0630	0.0777	0.0659	
	500	Hubert	0.1002	0.0524	0.0155	0.0620	0.0353	0.0823	0.0662	0.0945	0.0904	0.0830	0.0831	0.0816	0.0877	0.0824	
		Light	0.1002	0.0524	0.0155	0.0620	0.0353	0.0823	0.0664	0.0948	0.0906	0.0833	0.0833	0.0819	0.0880	0.0827	
	1000	Hubert	0.1002	0.0524	0.0153	0.0620	0.0353	0.0825	0.0662	0.0946	0.0913	0.0854	0.0854	0.0843	0.0892	0.0848	
		Light	0.1002	0.0524	0.0154	0.0620	0.0353	0.0825	0.0663	0.0947	0.0914	0.0855	0.0855	0.0844	0.0893	0.0850	
	5000	Hubert	0.1000	0.0521	0.0151	0.0618	0.0349	0.0823	0.0659	0.0938	0.0907	0.0852	0.0852	0.0842	0.0887	0.0846	
		Light	0.1000	0.0521	0.0151	0.0618	0.0349	0.0823	0.0660	0.0938	0.0908	0.0852	0.0852	0.0842	0.0887	0.0846	
	4	100	Hubert	0.1620	0.0580	0.0078	0.0783	0.0360	0.1323	0.1161	0.0753	0.0533	0.0281	0.0322	0.0257	0.0460	0.0283
			Light	0.1620	0.0580	0.0079	0.0784	0.0361	0.1323	0.1159	0.0753	0.0533	0.0282	0.0322	0.0257	0.0460	0.0280
500		Hubert	0.1581	0.0552	0.0064	0.0746	0.0321	0.1309	0.1114	0.0753	0.0533	0.0281	0.0322	0.0257	0.0460	0.0283	
		Light	0.1581	0.0552	0.0064	0.0746	0.0321	0.1309	0.1115	0.0753	0.0533	0.0282	0.0322	0.0257	0.0460	0.0280	
1000		Hubert	0.1579	0.0549	0.0063	0.0744	0.0318	0.1322	0.1122	0.0816	0.0584	0.0323	0.0369	0.0311	0.0478	0.0296	
		Light	0.1579	0.0549	0.0063	0.0744	0.0318	0.1322	0.1122	0.0802	0.0572	0.0315	0.0347	0.0278	0.0479	0.0297	
5000		Hubert	0.1573	0.0545	0.0061	0.0739	0.0313	0.1330	0.1126	0.0763	0.0551	0.0310	0.0336	0.0271	0.0475	0.0301	
		Light	0.1573	0.0545	0.0061	0.0739	0.0313	0.1330	0.1126	0.0763	0.0551	0.0310	0.0336	0.0271	0.0475	0.0301	
5		100	Hubert	0.2126	0.0579	0.0042	0.0855	0.0339	0.1826	0.1683	0.0809	0.0405	0.0118	0.0091	0.0016	0.0809	0.0809
			Light	0.2126	0.0580	0.0042	0.0855	0.0339	0.1826	0.1688	0.0810	0.0406	0.0120	0.0092	0.0016	0.0810	0.0818
	500	Hubert	0.2097	0.0558	0.0032	0.0825	0.0301	0.1759	0.1574	0.0836	0.0428	0.0129	0.0192	0.0085	0.0825	0.0814	
		Light	0.2097	0.0558	0.0032	0.0826	0.0301	0.1759	0.1576	0.0837	0.0429	0.0130	0.0192	0.0085	0.0823	0.0810	
	1000	Hubert	0.2094	0.0555	0.0030	0.0823	0.0298	0.1789	0.1599	0.1128	0.0621	0.0226	0.0489	0.0515	0.0325	0.0111	
		Light	0.2094	0.0555	0.0030	0.0823	0.0298	0.1789	0.1600	0.1034	0.0554	0.0187	0.0322	0.0222	0.0325	0.0108	
	5000	Hubert	0.2094	0.0554	0.0029	0.0820	0.0293	0.1852	0.1656	0.1120	0.0625	0.0234	0.0498	0.0536	0.0350	0.0134	
		Light	0.2094	0.0554	0.0029	0.0820	0.0293	0.1852	0.1656	0.1015	0.0551	0.0191	0.0319	0.0230	0.0350	0.0134	

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

5.2.1 Comparison of the weighted kappa coefficients

The coefficients are compared across weighting schemes, levels of true agreement and table structure. The results in Tables 5-12 and Tables A1-A8 show that:

- In the balanced tables with three raters, the unweighted versions of three kappa coefficients are compared, Hubert's and Light's coefficients performing better than Mielke's. For example, for the tables with three categories where $n = 100$ and the agreement level is low, the MSE values of the unweighted version of Hubert's and Light's coefficients were around 0.050, whereas Mielke's was 0.064. A similar outcome is observed when exponential linear and exponential quadratic weights are employed (Table 5).
- In the balanced tables with three raters where the agreement level is low or medium, all the MAE/MSE values of three coefficients are similar when linear and quadratic, ridit linear and ridit quadratic weights are used. For instance, when linear weights were employed for tables with three categories where $n = 100$, the MSE values of the three coefficients were found around 0.025 for low agreement and 0.046 for medium agreement (Table 5 and Table 6). When ridit quadratic weights were employed for the tables comprising four categories, where $n = 1000$, the MSE values of the three coefficients were around 0.012 for low agreement and 0.024 for medium agreement (Table 5 and Table 6).
- In the balanced tables with three raters where there is high agreement, all the MAE/MSE values of the three coefficients are similar when linear and quadratic weights are used. For example, when linear and quadratic weights were used for the tables with three categories where $n = 100$, the MSE values of all three coefficients were around 0.060 and 0.017, respectively (Table 7).
- In the balanced tables with three raters, all the MAE/MSE values of Hubert's and Light's coefficients are similar.
- In the balanced tables with three raters, when using the unweighted version of kappas, exponential linear and exponential quadratic weights, the difference between the coefficients becomes more noticeable. When using the unweighted version of kappas, exponential linear or exponential quadratic weights, Hubert's and Light's weighted kappa coefficients perform similarly and better than Mielke's. For example, when exponential quadratic weights were used for the tables with five categories where $n = 5000$ and the agreement level is medium, the MSE values of Hubert's and Light's coefficients were around 0.209 where Mielke's was 0.309 (Table 6).
- In the unbalanced tables with three raters, three coefficients perform similarly. The difference occurs when the unweighted version of the kappa coefficients is used. The unweighted version of

Mielke's kappa coefficient performs better than the other coefficients. For example, for the tables with three categories where $n = 100$ and the agreement level is low, the MSE values of unweighted version of Hubert's and Light's coefficients were around 0.035 where Mielke's was 0.028 (Table 5).

- In the balanced tables with four or five raters, the MAE/MSE values of Hubert's and Light's weighted kappa coefficients are similar.
- In the unbalanced tables with four or five raters, the MAE/MSE values of these two coefficients are also mostly similar. The performance of Hubert's and Light's weighted kappa coefficients is similar when there is low or medium agreement. When there is high agreement among four raters' classifications, Light's kappa, calculated using linear, quadratic, ridit linear and ridit quadratic weights, shows slightly better performance than Hubert's kappa when there are five categories and $n = 100, 500, 1000$. For instance, in the unbalanced, high agreement tables with four raters where $R = 5$ and $n = 100$, the MSE value of Light's ridit linear weighted coefficient is around 0.058, whereas Hubert's is 0.103 (Table 10).
- When there is high agreement among five raters' classifications, Light's kappa, calculated using linear, quadratic, ridit linear and ridit quadratic weights, shows slightly better performance than Hubert's kappa when there are five categories and $n = 1000, 5000$. For instance, in the unbalanced, high agreement tables with five raters where $R = 5$ and $n = 1000$, the MSE value of Light's ridit quadratic weighted coefficient is around 0.022, whereas Hubert's is 0.051 (Table 13).

The coefficients in relation to their best performance when there are three raters are summarized in Table 14.

5.2.2. Comparison of the weighting schemes

The comparative analysis of weighting schemes is performed based on the weighted kappa coefficients, level of true agreement, and table structure (Tables 5-12 and Tables A1-A8).

The accuracy of the coefficients depends on the weights used, the number of categories, and the structure of the table. In the balanced tables, the kappa coefficients with quadratic weights, followed by the ridit quadratic weights, perform better than the other weights in every scenario. Unweighted kappa coefficients generate poorer results than the other weighting schemes. For instance, in the balanced, low agreement tables with three raters where $R = 3$ and $n = 100$, the MSE values of Hubert's unweighted, linearly, quadratically, ridit linearly, ridit quadratically, exponential linearly, and exponential quadratically weighted coefficients were found to be 0.050, 0.025, 0.009, 0.029, 0.016, 0.037, and 0.028, respectively (Table 5). This result also indicates that the ordinal structure of the categories should not be ignored and that the coefficients for ordinal tables should be used in applications.

Table 14. The summary of coefficients related to MSE in three rater studies

Structure	<i>n</i>	UW	L	Q	RL	RQ	EL	EQ
Balanced	100, 500, 1000	Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Hubert, Light	Hubert, Light
	5000	Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light
Unbalanced	All	Mielke	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

In the case of unbalanced tables, the results vary depending on the kappa coefficients, the number of categories, and the degree of true agreement:

- When three raters are involved, the coefficients employing ridit quadratic weights perform better than other weighting schemes. The performance of exponential quadratic weights follows that of ridit quadratic weights in most scenarios. However, it fails to estimate true agreement only in scenarios where $R = 5$ and $n = 100$, which result in very sparse tables. In contrast, the ridit quadratic and quadratic weights perform better than the others even in such sparse tables. For instance, in the unbalanced, medium agreement tables with three raters where $R = 5$ and $n = 100$, the MSE values of Mielke's unweighted, linearly, quadratically, ridit linearly, ridit quadratically, exponential linearly, and exponential quadratically weighted coefficients were found to be 0.0592, 0.0312, 0.0098, 0.0099, 0.0058, 0.0268, and 0.0096, respectively (Table 6).
- In scenarios where four raters categorise subjects into three or four categories and the agreement levels are low or high, the Hubert's and Light's kappa coefficients with ridit quadratic weights demonstrate better performance in comparison to alternative weighting schemes. For instance, in the unbalanced, low agreement tables with four raters where $R = 4$ and $n = 500$, the MSE values of Hubert's unweighted, linearly, quadratically, ridit linearly, ridit quadratically, exponential linearly, and exponential quadratically weighted coefficients were found to be 0.028, 0.013, 0.004, 0.004, 0.023, 0.010, and 0.003, respectively (Table 8). In the same conditions but the agreement level is medium, the Hubert's and Light's kappa coefficients with quadratic, followed by ridit quadratic weights demonstrate better performance. For instance, in the unbalanced, medium agreement tables with four raters where $R = 4$ and $n = 500$, the MSE values of Hubert's unweighted, linearly, quadratically, ridit linearly, ridit quadratically, exponential linearly, and exponential quadratically weighted coefficients were found to be 0.137, 0.045, 0.004, 0.062, 0.024, 0.107, and 0.087, respectively (Table 9).
- In scenarios where four raters categorise subjects into five categories, Hubert's and Light's kappa coefficients with exponential quadratic weights perform slightly better than the other weighting schemes in the low agreement tables where $n = 100$ and in the high agreement tables where $n = 100, 500, 1000$. For instance, in the unbalanced, high agreement tables with four raters where $R = 5$ and $n = 100$, the MSE values of Hubert's unweighted, linearly, quadratically, ridit linearly, ridit quadratically, exponential linearly, and exponential quadratically weighted coefficients were found to be 0.157, 0.091, 0.036, 0.103, 0.113, 0.034, and 0.012, respectively (Table 10). However, if $R = 5$ and $n = 5000$, the MSE values of Hubert's unweighted, linearly, quadratically, ridit linearly, ridit quadratically, exponential linearly, and exponential quadratically weighted coefficient were found to be 0.083, 0.042, 0.012, 0.018, 0.009, 0.035, and 0.013, respectively (Table 10). In this case, ridit quadratic weights result in lower MSE values than other weighting schemes.
- When five raters are present, Hubert's and Light's kappa coefficients using ridit quadratic weights, followed by exponential quadratic weights, tend to outperform other weight schemes. In cases of low agreement scenarios with $n = 100$ and $R = 5$, Hubert's and Light's kappa coefficients with exponential quadratic weights are performed slightly better than the other weights. The MSE values of Hubert's unweighted, linearly, quadratically, ridit linearly, ridit quadratically, exponential linearly, and exponential quadratically weighted coefficients were found to be 0.0291, 0.0110, 0.0040, 0.0036, 0.0038, 0.0084, and 0.0024, respectively (Table 11). Besides, exponential quadratic weights are performed better than the other weights in high agreement scenarios with $n = 1000, 5000$.

5.2.3. Effect of the number of categories on weighting schemes

In this section, we aim to examine the impact of the number of categories on weighting schemes (Tables 5-12 and Tables A1-A8).

Depending on the weighting scheme and distribution, the effect of number of categories on the MAE/MSE values differ:

- In the balanced tables, it is observed that the MSE values of the unweighted kappas and the weighted kappas with linear, ridit linear, and exponential linear weights increase as the number of categories increases. Conversely, the MSE values of the kappas with quadratic, ridit quadratic, and exponential quadratic weights decrease as the number of categories increases. For instance, in the balanced, low agreement tables with three raters where $n = 100$, the MSE values of unweighted Mielke's coefficient were found as 0.064, 0.092, and 0.109 where $R = 3$, $R = 4$, and $R = 5$, respectively (Table 5). The MSE values of quadratically weighted Mielke's coefficient were found as 0.009, 0.006, and 0.005 where $R = 3$, $R = 4$, and $R = 5$, respectively (Table 5).
- In the unbalanced tables, it is observed that the MSE values of the Mielke's unweighted kappa and the weighted kappa with linear, quadratic, ridit linear, and ridit quadratic weights decrease as the number of categories increases. For instance, in the unbalanced, medium agreement tables with three raters where $n = 500$, the MSE values of Mielke's coefficient with

ridit linear weights were found as 0.050, 0.021, and 0.009 where $R = 3$, $R = 4$, and $R = 5$, respectively (Table 6).

- In the unbalanced tables with low or medium agreement, the MSE values of the Mielke's kappa with exponential linear and exponential quadratic weights also decrease as the number of categories increases. For instance, in the unbalanced, medium agreement tables with three raters where $n = 1000$, the MSE values of Mielke's coefficient with exponential quadratic weights were found as 0.049, 0.018, and 0.007 where $R = 3$, $R = 4$, and $R = 5$, respectively (Table 6).
- When there is low agreement in the unbalanced tables, the increase in the number of categories has a minimal effect on MSE. However, as the level of agreement rises, so does the impact of MSE. For instance, in the unbalanced, low agreement tables with three raters where $n = 100$, the MSE values of Light's coefficient with quadratic weights were found as 0.021, 0.010, and 0.005 where $R = 3$, $R = 4$, and $R = 5$, respectively (Table 5). However, in high agreement tables, these values were found as 0.074, 0.026, 0.005, respectively (Table 7).

6. CONCLUSIONS

This study can guide researchers in choosing a weighting scheme and the weighted kappa coefficient when more than two raters rate the ordinal categories. The best combination of weights and weighted kappa coefficients is summarized in Table 15.

Table 15. The summary of **weights** and **coefficients**

M	Structure	R = 3			R = 4			R = 5		
		Low	Medium	High	Low	Medium	High	Low	Medium	High
3	Balanced	Q	Q	Q	Q	Q	Q	Q	Q	Q
		Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light
	Unbalanced	RQ	RQ	RQ	RQ	RQ	RQ	RQ	RQ	RQ
		Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke, Hubert, Light	Mielke
4	Balanced	Q	Q	Q	Q	Q	Q	Q	Q	Q
		Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Mielke, Hubert, Light
	Unbalanced	RQ	RQ	RQ	RQ	RQ	RQ	RQ, EQ*	RQ	RQ, EQ*
		Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light
5	Balanced	Q	Q	Q	Q	Q	Q	Q	Q	Q
		Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Mielke, Hubert, Light
	Unbalanced	RQ	RQ	RQ	RQ	RQ	RQ	RQ, EQ*	RQ	RQ, EQ*
		Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Hubert, Light	Mielke

Abbr.: Quadratic (Q), Ridit (R), Exponential (E). *: EQ can be used when $n = 100$ while RQ is appropriate when $n = 500, 1000, 5000$. +: EQ can be used when $n = 100, 500, 1000$ while RQ is appropriate when $n = 5000$. #: RQ can be used when $n = 100, 500$ while RQ is appropriate when $n = 1000, 5000$.

The main results of the study are summarized as follows:

- The simulation study results show that the coefficients can show different behavior when different weights are used. It can be said that the different weights used have more influence on the coefficients. In terms of estimating the true agreement, the study has shown that all three coefficients give consistent results when the correct weight is used.
- The results indicate that the structure of the table, the level of agreement, and the number of categories also have an effect on the coefficients. The MSE of low agreement is found to be smaller than the MSE of medium agreement and the MSE of medium agreement is found to be smaller than the MSE of high agreement, except for the situations with Mielke's weighted kappa with linear, quadratic, ridit linear, and ridit quadratic weights in the unbalanced tables when there are five categories. The impact of the level of agreement on MSE and MAE values is less than the other weighing schemes for quadratic and ridit quadratic weights.
- Hubert's and Light's weighted kappas have similar results in most situations.
- The unweighted kappa coefficients yield poorer results compared to other weighting schemes. This result shows the importance of using weighted kappa coefficients when dealing with ordinal categories in a table.
- In the balanced tables, any of these three coefficients can be used with the quadratic weighting scheme.
- In the case of tables with three or four categories, the ridit quadratic weighting scheme can be implemented, using any of these three coefficients. For tables with three raters who assess subjects in five categories, Mielke's kappa coefficients can be used with the ridit quadratic weighting scheme in the unbalanced and high agreement tables. In the low or high agreement tables with four or five raters rating subjects in five categories, ridit quadratic or exponential quadratic weights can be utilized depending on the sample size.

The illustrative example in Section 4 is classified by three raters into three ordered categories when $n = 765$, with an imbalanced sample size design. Based on the results from the simulation study, the inter-rater agreement is summarized by the ridit quadratic weighting scheme. The agreement among the raters ranges from 0.342 to 0.418, depending on the coefficient used.

DECLARATION OF ETHICAL STANDARDS

The author of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

AUTHORS' CONTRIBUTIONS

Ayfer Ezgi YILMAZ: Design of the work; implementation of simulations; analysis of results; interpretation of results; the creation of software used in work; have drafted the work and revised it.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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APPENDIX

The MAE results are summarized in Tables A1-A9 by the number of raters, number of categories, sample size, true level of agreement, coefficient, and weighting scheme.

Tables A1-A3 show the results for the three raters, Tables A4-A6 for the four raters, and Tables A7-A9 for the five raters.

Table A1. MAE values computed for low agreement tables with three raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Mielke	0.2495	0.1491	0.0838	0.1643	0.1149	0.1960	0.1780	0.1561	0.1501	0.1246	0.1188	0.1152	0.1443	0.1214
		Hubert	0.2202	0.1491	0.0838	0.1612	0.1134	0.1868	0.1582	0.1761	0.1501	0.1246	0.1218	0.1138	0.1413	0.1206
		Light	0.2202	0.1492	0.0840	0.1613	0.1135	0.1869	0.1583	0.1768	0.1510	0.1258	0.1225	0.1146	0.1421	0.1215
	500	Mielke	0.2481	0.1479	0.0763	0.1625	0.1113	0.1898	0.1589	0.1526	0.1460	0.1127	0.1116	0.1029	0.1373	0.1093
		Hubert	0.2189	0.1479	0.0763	0.1606	0.1110	0.1856	0.1542	0.1737	0.1460	0.1127	0.1128	0.1025	0.1359	0.1091
		Light	0.2189	0.1479	0.0763	0.1606	0.1110	0.1856	0.1542	0.1739	0.1462	0.1130	0.1130	0.1027	0.1361	0.1093
	1000	Mielke	0.2482	0.1481	0.0766	0.1623	0.1114	0.1885	0.1561	0.1523	0.1455	0.1122	0.1115	0.1021	0.1363	0.1084
		Hubert	0.2191	0.1481	0.0766	0.1609	0.1113	0.1858	0.1538	0.1733	0.1455	0.1122	0.1122	0.1019	0.1353	0.1083
		Light	0.2191	0.1481	0.0766	0.1609	0.1113	0.1858	0.1539	0.1734	0.1456	0.1123	0.1123	0.1020	0.1353	0.1084
	5000	Mielke	0.2481	0.1482	0.0766	0.1616	0.1113	0.1871	0.1540	0.1522	0.1454	0.1118	0.1121	0.1022	0.1356	0.1081
		Hubert	0.2192	0.1482	0.0766	0.1610	0.1113	0.1858	0.1536	0.1733	0.1454	0.1118	0.1124	0.1021	0.1352	0.1081
		Light	0.2192	0.1482	0.0766	0.1610	0.1113	0.1858	0.1536	0.1733	0.1454	0.1119	0.1124	0.1022	0.1352	0.1081
4	100	Mielke	0.3013	0.1541	0.0675	0.1803	0.1118	0.2459	0.2403	0.1407	0.1190	0.0821	0.0749	0.0686	0.1096	0.0765
		Hubert	0.2636	0.1541	0.0675	0.1758	0.1108	0.2280	0.2049	0.1694	0.1190	0.0821	0.0771	0.0680	0.1062	0.0759
		Light	0.2636	0.1543	0.0676	0.1759	0.1110	0.2280	0.2050	0.1697	0.1195	0.0828	0.0776	0.0683	0.1067	0.0764
	500	Mielke	0.3004	0.1527	0.0518	0.1777	0.1074	0.2347	0.2083	0.1383	0.1160	0.0614	0.0606	0.0432	0.1029	0.0570
		Hubert	0.2628	0.1527	0.0518	0.1750	0.1072	0.2260	0.1979	0.1678	0.1160	0.0614	0.0617	0.0429	0.1013	0.0567
		Light	0.2628	0.1527	0.0518	0.1751	0.1072	0.2260	0.1979	0.1678	0.1161	0.0616	0.0618	0.0430	0.1014	0.0569
	1000	Mielke	0.3008	0.1527	0.0513	0.1773	0.1075	0.2326	0.2024	0.1375	0.1154	0.0597	0.0597	0.0390	0.1016	0.0551
		Hubert	0.2629	0.1527	0.0513	0.1753	0.1074	0.2267	0.1973	0.1670	0.1154	0.0597	0.0605	0.0388	0.1005	0.0550
		Light	0.2629	0.1527	0.0513	0.1753	0.1074	0.2267	0.1973	0.1670	0.1154	0.0599	0.0605	0.0388	0.1005	0.0551
	5000	Mielke	0.3006	0.1527	0.0515	0.1763	0.1076	0.2298	0.1981	0.1377	0.1153	0.0599	0.0603	0.0381	0.1009	0.0549
		Hubert	0.2627	0.1527	0.0515	0.1754	0.1076	0.2270	0.1969	0.1667	0.1153	0.0599	0.0605	0.0380	0.1004	0.0549
		Light	0.2627	0.1527	0.0515	0.1754	0.1076	0.2270	0.1969	0.1667	0.1153	0.0599	0.0605	0.0380	0.1004	0.0549
5	100	Mielke	0.3297	0.1556	0.0594	0.1896	0.1125	0.2817	0.2844	0.1354	0.0976	0.0596	0.0560	0.0538	0.0883	0.0545
		Hubert	0.2889	0.1556	0.0594	0.1844	0.1120	0.2586	0.2412	0.1678	0.0976	0.0596	0.0549	0.0563	0.0847	0.0541
		Light	0.2889	0.1558	0.0596	0.1845	0.1121	0.2587	0.2413	0.1681	0.0979	0.0598	0.0550	0.0564	0.0850	0.0543
	500	Mielke	0.3299	0.1550	0.0406	0.1878	0.1091	0.2678	0.2498	0.1333	0.0981	0.0360	0.0321	0.0262	0.0835	0.0317
		Hubert	0.2894	0.1550	0.0406	0.1847	0.1089	0.2543	0.2324	0.1663	0.0981	0.0360	0.0327	0.0263	0.0816	0.0316
		Light	0.2894	0.1551	0.0406	0.1848	0.1089	0.2543	0.2324	0.1663	0.0982	0.0360	0.0328	0.0263	0.0817	0.0316
	1000	Mielke	0.3298	0.1552	0.0397	0.1874	0.1093	0.2645	0.2411	0.1329	0.0974	0.0309	0.0281	0.0180	0.0821	0.0268
		Hubert	0.2894	0.1552	0.0397	0.1852	0.1092	0.2550	0.2317	0.1655	0.0974	0.0309	0.0286	0.0181	0.0808	0.0266
		Light	0.2894	0.1552	0.0397	0.1852	0.1092	0.2550	0.2317	0.1656	0.0975	0.0310	0.0286	0.0180	0.0809	0.0267
	5000	Mielke	0.3299	0.1554	0.0394	0.1866	0.1095	0.2611	0.2337	0.1333	0.0973	0.0280	0.0266	0.0096	0.0813	0.0240
		Hubert	0.2896	0.1554	0.0394	0.1855	0.1095	0.2566	0.2315	0.1656	0.0973	0.0280	0.0268	0.0097	0.0808	0.0240
		Light	0.2896	0.1554	0.0394	0.1855	0.1095	0.2566	0.2315	0.1656	0.0973	0.0280	0.0268	0.0097	0.0808	0.0240

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Redit (R), Exponential (E).

Table A2. MAE values computed for medium agreement tables with three raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Mielke	0.3511	0.2083	0.1032	0.2311	0.1620	0.2803	0.2668	0.2565	0.2492	0.2184	0.2149	0.2117	0.2426	0.2172
		Hubert	0.3039	0.2083	0.1032	0.2271	0.1598	0.2658	0.2317	0.2708	0.2492	0.2184	0.2181	0.2097	0.2395	0.2161
		Light	0.3039	0.2084	0.1034	0.2272	0.1600	0.2658	0.2318	0.2714	0.2498	0.2191	0.2188	0.2105	0.2401	0.2168
	500	Mielke	0.3505	0.2073	0.1007	0.2293	0.1597	0.2717	0.2357	0.2557	0.2506	0.2235	0.2219	0.2171	0.2429	0.2209
		Hubert	0.3030	0.2073	0.1007	0.2269	0.1593	0.2650	0.2277	0.2683	0.2506	0.2235	0.2232	0.2166	0.2414	0.2206
		Light	0.3030	0.2073	0.1007	0.2269	0.1593	0.2650	0.2278	0.2686	0.2509	0.2238	0.2234	0.2169	0.2416	0.2209
	1000	Mielke	0.3504	0.2070	0.1005	0.2286	0.1594	0.2693	0.2308	0.2560	0.2510	0.2239	0.2226	0.2172	0.2426	0.2208
		Hubert	0.3028	0.2070	0.1005	0.2268	0.1592	0.2648	0.2270	0.2686	0.2510	0.2239	0.2234	0.2169	0.2415	0.2206
		Light	0.3028	0.2071	0.1005	0.2268	0.1592	0.2648	0.2270	0.2687	0.2511	0.2241	0.2235	0.2170	0.2416	0.2208
	5000	Mielke	0.3502	0.2070	0.1004	0.2276	0.1591	0.2668	0.2275	0.2558	0.2511	0.2241	0.2233	0.2173	0.2420	0.2207
		Hubert	0.3027	0.2070	0.1004	0.2268	0.1591	0.2648	0.2267	0.2685	0.2511	0.2241	0.2236	0.2172	0.2416	0.2207
		Light	0.3027	0.2070	0.1004	0.2268	0.1591	0.2648	0.2267	0.2685	0.2511	0.2242	0.2236	0.2172	0.2416	0.2207
	100	Mielke	0.4344	0.2138	0.0710	0.2538	0.1566	0.3601	0.3736	0.2361	0.1961	0.1250	0.1287	0.1102	0.1825	0.1247
		Hubert	0.3703	0.2138	0.0710	0.2474	0.1552	0.3269	0.3002	0.2568	0.1961	0.1250	0.1313	0.1081	0.1780	0.1234
		Light	0.3703	0.2139	0.0711	0.2475	0.1554	0.3269	0.3002	0.2571	0.1966	0.1257	0.1317	0.1085	0.1785	0.1239
	500	Mielke	0.4344	0.2130	0.0633	0.2514	0.1537	0.3445	0.3183	0.2357	0.2009	0.1367	0.1414	0.1196	0.1857	0.1332
		Hubert	0.3705	0.2130	0.0633	0.2476	0.1533	0.3274	0.2945	0.2537	0.2009	0.1367	0.1422	0.1191	0.1837	0.1328
		Light	0.3705	0.2130	0.0634	0.2477	0.1534	0.3274	0.2945	0.2538	0.2010	0.1369	0.1423	0.1192	0.1838	0.1330
	1000	Mielke	0.4346	0.2132	0.0634	0.2508	0.1539	0.3409	0.3067	0.2347	0.2004	0.1372	0.1415	0.1195	0.1844	0.1328
		Hubert	0.3707	0.2132	0.0634	0.2481	0.1537	0.3291	0.2947	0.2524	0.2004	0.1372	0.1421	0.1193	0.1831	0.1327
		Light	0.3707	0.2132	0.0634	0.2481	0.1538	0.3291	0.2947	0.2525	0.2004	0.1373	0.1421	0.1193	0.1831	0.1327
	5000	Mielke	0.4345	0.2130	0.0634	0.2493	0.1537	0.3357	0.2975	0.2347	0.1999	0.1368	0.1418	0.1193	0.1832	0.1323
		Hubert	0.3704	0.2130	0.0634	0.2480	0.1537	0.3302	0.2948	0.2519	0.1999	0.1368	0.1419	0.1192	0.1826	0.1322
		Light	0.3704	0.2130	0.0634	0.2480	0.1537	0.3302	0.2948	0.2519	0.1999	0.1369	0.1419	0.1193	0.1827	0.1322
4	100	Mielke	0.4822	0.2169	0.0584	0.2687	0.1582	0.4228	0.4534	0.2380	0.1681	0.0811	0.0828	0.0603	0.1535	0.0803
		Hubert	0.4138	0.2169	0.0584	0.2614	0.1576	0.3786	0.3595	0.2574	0.1681	0.0811	0.0848	0.0595	0.1481	0.0789
		Light	0.4138	0.2170	0.0586	0.2615	0.1578	0.3786	0.3596	0.2576	0.1684	0.0815	0.0851	0.0597	0.1483	0.0792
	500	Mielke	0.4824	0.2156	0.0456	0.2656	0.1549	0.4009	0.3923	0.2354	0.1739	0.0866	0.0935	0.0562	0.1558	0.0836
		Hubert	0.4137	0.2156	0.0456	0.2612	0.1547	0.3720	0.3471	0.2540	0.1739	0.0866	0.0940	0.0558	0.1535	0.0833
		Light	0.4137	0.2157	0.0457	0.2612	0.1547	0.3721	0.3472	0.2540	0.1740	0.0867	0.0940	0.0559	0.1536	0.0834
	1000	Mielke	0.4822	0.2156	0.0454	0.2647	0.1551	0.3952	0.3738	0.2346	0.1737	0.0872	0.0948	0.0570	0.1551	0.0840
		Hubert	0.4136	0.2156	0.0454	0.2615	0.1549	0.3739	0.3473	0.2531	0.1737	0.0872	0.0951	0.0568	0.1535	0.0838
		Light	0.4136	0.2156	0.0455	0.2615	0.1550	0.3739	0.3473	0.2531	0.1737	0.0873	0.0951	0.0568	0.1535	0.0839
	5000	Mielke	0.4823	0.2155	0.0452	0.2631	0.1548	0.3885	0.3562	0.2349	0.1735	0.0869	0.0956	0.0575	0.1542	0.0839
		Hubert	0.4136	0.2155	0.0452	0.2616	0.1548	0.3782	0.3495	0.2530	0.1735	0.0869	0.0957	0.0574	0.1536	0.0839
		Light	0.4136	0.2155	0.0452	0.2616	0.1548	0.3782	0.3495	0.2530	0.1735	0.0870	0.0957	0.0574	0.1536	0.0839

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table A3. MAE values computed for high agreement tables with three raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Mielke	0.3724	0.2314	0.1262	0.2533	0.1906	0.3041	0.3125	0.3055	0.2913	0.2586	0.2567	0.2540	0.2844	0.2586
		Hubert	0.3193	0.2314	0.1262	0.2501	0.1880	0.2892	0.2613	0.3116	0.2913	0.2586	0.2597	0.2517	0.2812	0.2573
		Light	0.3193	0.2314	0.1263	0.2502	0.1881	0.2892	0.2613	0.3127	0.2922	0.2594	0.2608	0.2528	0.2822	0.2582
	500	Mielke	0.3708	0.2294	0.1240	0.2512	0.1879	0.2948	0.2698	0.3062	0.3017	0.2889	0.2881	0.2870	0.2986	0.2883
		Hubert	0.3173	0.2294	0.1240	0.2492	0.1874	0.2878	0.2583	0.3088	0.3017	0.2889	0.2892	0.2865	0.2974	0.2880
		Light	0.3173	0.2294	0.1240	0.2492	0.1874	0.2878	0.2583	0.3090	0.3020	0.2891	0.2894	0.2868	0.2976	0.2882
	1000	Mielke	0.3697	0.2283	0.1230	0.2498	0.1868	0.2914	0.2622	0.3042	0.3009	0.2908	0.2901	0.2890	0.2981	0.2900
		Hubert	0.3163	0.2283	0.1230	0.2484	0.1866	0.2868	0.2569	0.3063	0.3009	0.2908	0.2907	0.2887	0.2973	0.2898
		Light	0.3163	0.2283	0.1230	0.2484	0.1866	0.2868	0.2569	0.3064	0.3010	0.2909	0.2909	0.2889	0.2974	0.2899
	5000	Mielke	0.3697	0.2283	0.1229	0.2493	0.1869	0.2891	0.2581	0.3034	0.3005	0.2912	0.2909	0.2894	0.2974	0.2902
		Hubert	0.3163	0.2283	0.1229	0.2486	0.1869	0.2870	0.2570	0.3054	0.3005	0.2912	0.2911	0.2893	0.2971	0.2902
		Light	0.3163	0.2283	0.1229	0.2486	0.1869	0.2870	0.2570	0.3054	0.3005	0.2912	0.2912	0.2894	0.2971	0.2902
	100	Mielke	0.4785	0.2346	0.0804	0.2771	0.1795	0.4005	0.4740	0.2861	0.2333	0.1529	0.1579	0.1343	0.2151	0.1518
		Hubert	0.3962	0.2346	0.0804	0.2712	0.1779	0.3575	0.3338	0.2908	0.2333	0.1529	0.1594	0.1323	0.2109	0.1503
		Light	0.3962	0.2346	0.0805	0.2712	0.1779	0.3575	0.3338	0.2909	0.2335	0.1534	0.1598	0.1327	0.2112	0.1507
	500	Mielke	0.4796	0.2336	0.0783	0.2753	0.1774	0.3841	0.3801	0.2792	0.2364	0.1741	0.1775	0.1580	0.2193	0.1696
		Hubert	0.3964	0.2336	0.0783	0.2717	0.1769	0.3610	0.3332	0.2802	0.2364	0.1741	0.1780	0.1574	0.2174	0.1693
		Light	0.3964	0.2336	0.0783	0.2718	0.1769	0.3610	0.3332	0.2804	0.2365	0.1743	0.1781	0.1575	0.2175	0.1694
	1000	Mielke	0.4806	0.2341	0.0785	0.2751	0.1775	0.3804	0.3599	0.2775	0.2357	0.1756	0.1809	0.1619	0.2191	0.1720
		Hubert	0.3973	0.2341	0.0785	0.2724	0.1773	0.3636	0.3349	0.2780	0.2357	0.1756	0.1812	0.1617	0.2179	0.1718
		Light	0.3973	0.2341	0.0785	0.2724	0.1773	0.3636	0.3349	0.2780	0.2357	0.1757	0.1812	0.1617	0.2179	0.1719
	5000	Mielke	0.4804	0.2339	0.0782	0.2736	0.1773	0.3731	0.3417	0.2769	0.2350	0.1759	0.1829	0.1640	0.2187	0.1731
		Hubert	0.3971	0.2339	0.0782	0.2724	0.1773	0.3653	0.3362	0.2769	0.2350	0.1759	0.1830	0.1639	0.2181	0.1730
		Light	0.3971	0.2339	0.0782	0.2724	0.1773	0.3653	0.3362	0.2769	0.2350	0.1759	0.1830	0.1640	0.2181	0.1730
4	100	Mielke	0.5562	0.2370	0.0576	0.2947	0.1757	0.4962	0.6123	0.2275	0.1450	0.0577	0.0662	0.0267	0.2236	0.2236
		Hubert	0.4584	0.2370	0.0576	0.2871	0.1747	0.4247	0.4071	0.2236	0.1450	0.0577	0.0662	0.0272	0.2236	0.2236
		Light	0.4584	0.2371	0.0578	0.2872	0.1747	0.4247	0.4071	0.2233	0.1449	0.0575	0.0662	0.0270	0.2233	0.2233
	500	Mielke	0.5561	0.2355	0.0544	0.2911	0.1721	0.4688	0.5091	0.3304	0.2525	0.1688	0.1845	0.1474	0.2494	0.1821
		Hubert	0.4575	0.2355	0.0544	0.2865	0.1717	0.4189	0.3965	0.3255	0.2525	0.1688	0.1843	0.1470	0.2473	0.1818
		Light	0.4575	0.2355	0.0544	0.2865	0.1717	0.4189	0.3965	0.3255	0.2525	0.1689	0.1844	0.1470	0.2473	0.1818
	1000	Mielke	0.5557	0.2353	0.0542	0.2897	0.1716	0.4599	0.4678	0.2950	0.2092	0.1168	0.1361	0.0972	0.1913	0.1174
		Hubert	0.4574	0.2353	0.0542	0.2863	0.1714	0.4231	0.4000	0.2897	0.2092	0.1168	0.1360	0.0970	0.1897	0.1172
		Light	0.4574	0.2353	0.0542	0.2863	0.1714	0.4231	0.4000	0.2897	0.2092	0.1168	0.1360	0.0970	0.1897	0.1173
	5000	Mielke	0.5561	0.2354	0.0542	0.2883	0.1716	0.4492	0.4268	0.2937	0.2080	0.1161	0.1366	0.0983	0.1895	0.1171
		Hubert	0.4576	0.2354	0.0542	0.2867	0.1716	0.4305	0.4074	0.2882	0.2080	0.1161	0.1366	0.0982	0.1888	0.1170
		Light	0.4576	0.2354	0.0542	0.2867	0.1716	0.4305	0.4074	0.2882	0.2080	0.1161	0.1366	0.0982	0.1888	0.1170

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table A4. MAE values computed for low agreement tables with four raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Hubert	0.2202	0.1497	0.0818	0.1615	0.1129	0.1867	0.1579	0.1754	0.1474	0.1164	0.1142	0.1047	0.1372	0.1122
		Light	0.2202	0.1498	0.0819	0.1616	0.1131	0.1867	0.1580	0.1765	0.1485	0.1177	0.1155	0.1059	0.1385	0.1135
	500	Hubert	0.2189	0.1479	0.0763	0.1604	0.1107	0.1854	0.1538	0.1734	0.1454	0.1117	0.1124	0.1020	0.1353	0.1082
		Light	0.2189	0.1479	0.0764	0.1604	0.1108	0.1854	0.1538	0.1736	0.1456	0.1121	0.1126	0.1023	0.1355	0.1085
	1000	Hubert	0.2192	0.1482	0.0767	0.1609	0.1112	0.1858	0.1538	0.1730	0.1452	0.1117	0.1120	0.1018	0.1349	0.1079
		Light	0.2192	0.1483	0.0767	0.1609	0.1112	0.1858	0.1539	0.1731	0.1453	0.1119	0.1122	0.1019	0.1351	0.1081
	5000	Hubert	0.2191	0.1480	0.0762	0.1606	0.1108	0.1856	0.1533	0.1735	0.1458	0.1124	0.1127	0.1025	0.1355	0.1085
		Light	0.2191	0.1480	0.0762	0.1606	0.1108	0.1856	0.1533	0.1735	0.1458	0.1124	0.1127	0.1025	0.1355	0.1085
	100	Hubert	0.2634	0.1551	0.0636	0.1759	0.1104	0.2274	0.2041	0.1680	0.1149	0.0693	0.0674	0.0563	0.1008	0.0649
		Light	0.2634	0.1552	0.0637	0.1760	0.1106	0.2274	0.2041	0.1686	0.1158	0.0703	0.0680	0.0567	0.1015	0.0657
4	500	Hubert	0.2622	0.1526	0.0519	0.1748	0.1073	0.2255	0.1974	0.1670	0.1155	0.0604	0.0607	0.0405	0.1006	0.0557
		Light	0.2622	0.1526	0.0519	0.1749	0.1073	0.2255	0.1975	0.1671	0.1157	0.0607	0.0609	0.0406	0.1008	0.0559
	1000	Hubert	0.2624	0.1528	0.0518	0.1751	0.1075	0.2262	0.1971	0.1668	0.1153	0.0598	0.0605	0.0383	0.1004	0.0549
		Light	0.2624	0.1528	0.0518	0.1751	0.1075	0.2262	0.1971	0.1669	0.1154	0.0599	0.0606	0.0384	0.1005	0.0550
	5000	Hubert	0.2624	0.1527	0.0516	0.1751	0.1072	0.2268	0.1967	0.1667	0.1154	0.0601	0.0606	0.0381	0.1004	0.0550
		Light	0.2624	0.1527	0.0516	0.1751	0.1072	0.2268	0.1967	0.1667	0.1154	0.0601	0.0606	0.0381	0.1004	0.0550
	100	Hubert	0.2904	0.1579	0.0556	0.1865	0.1130	0.2605	0.2435	0.1661	0.0946	0.0503	0.0480	0.0494	0.0803	0.0469
		Light	0.2904	0.1581	0.0558	0.1867	0.1133	0.2606	0.2435	0.1665	0.0952	0.0508	0.0482	0.0494	0.0807	0.0472
	500	Hubert	0.2894	0.1557	0.0409	0.1855	0.1097	0.2547	0.2331	0.1661	0.0975	0.0323	0.0306	0.0221	0.0812	0.0290
		Light	0.2894	0.1557	0.0410	0.1855	0.1098	0.2547	0.2331	0.1661	0.0977	0.0324	0.0307	0.0221	0.0813	0.0291
5	1000	Hubert	0.2897	0.1558	0.0401	0.1856	0.1097	0.2553	0.2321	0.1660	0.0975	0.0288	0.0282	0.0162	0.0811	0.0256
		Light	0.2897	0.1558	0.0401	0.1856	0.1097	0.2553	0.2321	0.1660	0.0975	0.0289	0.0282	0.0161	0.0812	0.0257
	5000	Hubert	0.2896	0.1553	0.0392	0.1853	0.1091	0.2566	0.2314	0.1658	0.0976	0.0283	0.0272	0.0085	0.0811	0.0243
		Light	0.2896	0.1553	0.0393	0.1853	0.1091	0.2566	0.2314	0.1658	0.0977	0.0284	0.0272	0.0085	0.0811	0.0243

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table A5. MAE values computed for medium agreement tables with four raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Hubert	0.3067	0.2119	0.1064	0.2304	0.1634	0.2685	0.2344	0.2666	0.2432	0.2095	0.2095	0.2005	0.2324	0.2068
		Light	0.3067	0.2120	0.1066	0.2305	0.1636	0.2686	0.2345	0.2681	0.2445	0.2112	0.2110	0.2021	0.2338	0.2083
	500	Hubert	0.3031	0.2076	0.1013	0.2272	0.1597	0.2652	0.2280	0.2687	0.2509	0.2237	0.2231	0.2166	0.2415	0.2205
		Light	0.3031	0.2076	0.1013	0.2272	0.1597	0.2652	0.2280	0.2690	0.2512	0.2241	0.2235	0.2169	0.2418	0.2208
	1000	Hubert	0.3028	0.2073	0.1009	0.2269	0.1593	0.2649	0.2273	0.2684	0.2507	0.2233	0.2230	0.2165	0.2412	0.2201
		Light	0.3028	0.2073	0.1009	0.2269	0.1594	0.2649	0.2273	0.2686	0.2508	0.2235	0.2232	0.2167	0.2413	0.2203
	5000	Hubert	0.3026	0.2069	0.1003	0.2266	0.1589	0.2647	0.2266	0.2688	0.2513	0.2243	0.2238	0.2174	0.2418	0.2208
		Light	0.3026	0.2069	0.1003	0.2266	0.1589	0.2647	0.2266	0.2688	0.2513	0.2243	0.2238	0.2175	0.2418	0.2209
	100	Hubert	0.3726	0.2160	0.0703	0.2494	0.1566	0.3288	0.3018	0.2520	0.1934	0.1231	0.1254	0.0997	0.1739	0.1187
		Light	0.3726	0.2162	0.0705	0.2495	0.1568	0.3289	0.3018	0.2525	0.1941	0.1243	0.1261	0.1005	0.1746	0.1196
4	500	Hubert	0.3705	0.2134	0.0640	0.2482	0.1542	0.3277	0.2949	0.2516	0.1991	0.1354	0.1404	0.1173	0.1817	0.1310
		Light	0.3705	0.2134	0.0641	0.2482	0.1542	0.3277	0.2949	0.2518	0.1993	0.1356	0.1405	0.1175	0.1819	0.1312
	1000	Hubert	0.3704	0.2134	0.0640	0.2481	0.1539	0.3290	0.2949	0.2520	0.1998	0.1364	0.1418	0.1190	0.1826	0.1321
		Light	0.3704	0.2134	0.0641	0.2481	0.1539	0.3290	0.2949	0.2521	0.1999	0.1365	0.1419	0.1191	0.1827	0.1322
	5000	Hubert	0.3700	0.2127	0.0632	0.2476	0.1533	0.3298	0.2945	0.2520	0.2002	0.1371	0.1421	0.1194	0.1828	0.1324
		Light	0.3700	0.2127	0.0633	0.2476	0.1533	0.3298	0.2945	0.2520	0.2002	0.1372	0.1421	0.1194	0.1828	0.1325
	100	Hubert	0.4153	0.2193	0.0569	0.2635	0.1597	0.3799	0.3611	0.2558	0.1680	0.0778	0.0821	0.0517	0.1474	0.0758
		Light	0.4153	0.2194	0.0572	0.2636	0.1599	0.3799	0.3611	0.2561	0.1684	0.0784	0.0825	0.0519	0.1478	0.0763
	500	Hubert	0.4135	0.2159	0.0461	0.2617	0.1555	0.3721	0.3474	0.2535	0.1733	0.0862	0.0937	0.0549	0.1530	0.0829
		Light	0.4135	0.2159	0.0462	0.2617	0.1555	0.3721	0.3474	0.2536	0.1734	0.0865	0.0938	0.0550	0.1531	0.0831
5	1000	Hubert	0.4139	0.2158	0.0455	0.2617	0.1550	0.3739	0.3476	0.2535	0.1739	0.0874	0.0958	0.0575	0.1539	0.0843
		Light	0.4139	0.2158	0.0455	0.2617	0.1550	0.3739	0.3476	0.2535	0.1740	0.0875	0.0958	0.0575	0.1540	0.0844
	5000	Hubert	0.4137	0.2155	0.0451	0.2614	0.1544	0.3781	0.3494	0.2532	0.1738	0.0874	0.0959	0.0577	0.1538	0.0842
		Light	0.4137	0.2155	0.0451	0.2614	0.1544	0.3781	0.3494	0.2532	0.1738	0.0874	0.0959	0.0577	0.1538	0.0842

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table A6. MAE values computed for high agreement tables with four raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Hubert	0.3189	0.2318	0.1272	0.2507	0.1894	0.2893	0.2619	0.3097	0.2889	0.2554	0.2579	0.2500	0.2788	0.2543
		Light	0.3188	0.2319	0.1274	0.2508	0.1895	0.2892	0.2619	0.3113	0.2903	0.2566	0.2595	0.2516	0.2803	0.2557
	500	Hubert	0.3161	0.2284	0.1233	0.2484	0.1869	0.2867	0.2573	0.3047	0.2979	0.2855	0.2854	0.2829	0.2935	0.2843
		Light	0.3161	0.2284	0.1233	0.2484	0.1869	0.2867	0.2573	0.3050	0.2983	0.2859	0.2857	0.2832	0.2938	0.2846
	1000	Hubert	0.3162	0.2284	0.1232	0.2483	0.1865	0.2868	0.2570	0.3061	0.3007	0.2906	0.2906	0.2886	0.2970	0.2896
		Light	0.3162	0.2284	0.1232	0.2483	0.1866	0.2868	0.2570	0.3062	0.3009	0.2908	0.2908	0.2888	0.2972	0.2898
	5000	Hubert	0.3164	0.2284	0.1231	0.2487	0.1869	0.2871	0.2572	0.3059	0.3009	0.2916	0.2915	0.2897	0.2975	0.2905
		Light	0.3164	0.2284	0.1231	0.2487	0.1869	0.2871	0.2572	0.3059	0.3010	0.2916	0.2916	0.2898	0.2975	0.2905
	100	Hubert	0.4013	0.2387	0.0831	0.2763	0.1828	0.3617	0.3380	0.2647	0.2149	0.1490	0.1368	0.0865	0.2249	0.1959
		Light	0.4012	0.2388	0.0833	0.2763	0.1829	0.3616	0.3380	0.2648	0.2150	0.1496	0.1370	0.0866	0.2251	0.1962
4	500	Hubert	0.3976	0.2347	0.0791	0.2732	0.1785	0.3622	0.3345	0.2787	0.2326	0.1676	0.1732	0.1515	0.2145	0.1631
		Light	0.3976	0.2347	0.0792	0.2732	0.1786	0.3622	0.3345	0.2788	0.2327	0.1678	0.1733	0.1516	0.2146	0.1632
	1000	Hubert	0.3971	0.2342	0.0787	0.2725	0.1776	0.3636	0.3351	0.2804	0.2376	0.1766	0.1843	0.1646	0.2204	0.1741
		Light	0.3971	0.2342	0.0787	0.2725	0.1776	0.3636	0.3351	0.2805	0.2377	0.1768	0.1844	0.1647	0.2205	0.1742
	5000	Hubert	0.3967	0.2336	0.0780	0.2720	0.1769	0.3649	0.3360	0.2763	0.2345	0.1756	0.1824	0.1634	0.2175	0.1725
		Light	0.3967	0.2336	0.0780	0.2720	0.1769	0.3649	0.3360	0.2763	0.2345	0.1756	0.1824	0.1634	0.2176	0.1726
	100	Hubert	0.4601	0.2391	0.0589	0.2895	0.1772	0.4260	0.4084	0.3691	0.2763	0.1668	0.2479	0.2281	0.1846	0.1125
		Light	0.4600	0.2392	0.0590	0.2896	0.1773	0.4260	0.4084	0.3514	0.2610	0.1547	0.2078	0.1720	0.1847	0.1126
	500	Hubert	0.4577	0.2358	0.0548	0.2871	0.1726	0.4195	0.3971	0.3291	0.2432	0.1439	0.1843	0.1484	0.1774	0.1044
		Light	0.4577	0.2358	0.0548	0.2871	0.1726	0.4195	0.3971	0.3207	0.2361	0.1384	0.1650	0.1212	0.1776	0.1045
5	1000	Hubert	0.4576	0.2357	0.0547	0.2868	0.1720	0.4231	0.4001	0.3691	0.2763	0.1668	0.2479	0.2281	0.1846	0.1125
		Light	0.4576	0.2358	0.0547	0.2868	0.1720	0.4231	0.4001	0.3514	0.2610	0.1547	0.2078	0.1720	0.1847	0.1126
	5000	Hubert	0.4575	0.2353	0.0541	0.2863	0.1711	0.4304	0.4072	0.2886	0.2066	0.1134	0.1359	0.0972	0.1879	0.1154
		Light	0.4575	0.2353	0.0541	0.2863	0.1711	0.4304	0.4072	0.2886	0.2066	0.1134	0.1359	0.0972	0.1879	0.1154

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Redit (R), Exponential (E).

Table A7. MAE values computed for low agreement tables with five raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Hubert	0.2195	0.1488	0.0796	0.1612	0.1127	0.1857	0.1568	0.1754	0.1474	0.1164	0.1142	0.1047	0.1372	0.1122
		Light	0.2195	0.1489	0.0797	0.1613	0.1129	0.1858	0.1568	0.1765	0.1485	0.1177	0.1155	0.1059	0.1385	0.1135
	500	Hubert	0.2193	0.1483	0.0767	0.1608	0.1112	0.1856	0.1538	0.1734	0.1454	0.1117	0.1124	0.1020	0.1353	0.1082
		Light	0.2193	0.1483	0.0767	0.1608	0.1112	0.1856	0.1540	0.1736	0.1456	0.1121	0.1126	0.1023	0.1355	0.1085
	1000	Hubert	0.2193	0.1482	0.0765	0.1609	0.1112	0.1857	0.1536	0.1730	0.1452	0.1117	0.1120	0.1018	0.1349	0.1079
		Light	0.2193	0.1483	0.0766	0.1609	0.1112	0.1857	0.1537	0.1731	0.1453	0.1119	0.1122	0.1019	0.1351	0.1081
	5000	Hubert	0.2191	0.1480	0.0763	0.1607	0.1109	0.1856	0.1532	0.1735	0.1458	0.1124	0.1127	0.1025	0.1355	0.1085
		Light	0.2191	0.1480	0.0763	0.1607	0.1109	0.1856	0.1532	0.1735	0.1458	0.1124	0.1127	0.1025	0.1355	0.1085
	100	Hubert	0.2635	0.1537	0.0593	0.1761	0.1100	0.2271	0.2035	0.1680	0.1149	0.0693	0.0674	0.0563	0.1008	0.0649
		Light	0.2635	0.1539	0.0595	0.1763	0.1103	0.2271	0.2039	0.1686	0.1158	0.0703	0.0680	0.0567	0.1015	0.0657
4	500	Hubert	0.2626	0.1528	0.0520	0.1752	0.1077	0.2256	0.1975	0.1670	0.1155	0.0604	0.0607	0.0405	0.1006	0.0557
		Light	0.2626	0.1528	0.0520	0.1753	0.1078	0.2256	0.1977	0.1671	0.1157	0.0607	0.0609	0.0406	0.1008	0.0559
	1000	Hubert	0.2627	0.1528	0.0517	0.1754	0.1077	0.2263	0.1971	0.1668	0.1153	0.0598	0.0605	0.0383	0.1004	0.0549
		Light	0.2627	0.1528	0.0518	0.1754	0.1077	0.2263	0.1973	0.1669	0.1154	0.0599	0.0606	0.0384	0.1005	0.0550
	5000	Hubert	0.2626	0.1528	0.0516	0.1753	0.1074	0.2269	0.1968	0.1667	0.1154	0.0601	0.0606	0.0381	0.1004	0.0550
		Light	0.2626	0.1528	0.0516	0.1753	0.1074	0.2269	0.1968	0.1667	0.1154	0.0601	0.0606	0.0381	0.1004	0.0550
	100	Hubert	0.2899	0.1561	0.0512	0.1860	0.1126	0.2591	0.2417	0.1661	0.0946	0.0503	0.0480	0.0494	0.0803	0.0469
		Light	0.2898	0.1562	0.0514	0.1862	0.1129	0.2592	0.2418	0.1665	0.0952	0.0508	0.0482	0.0494	0.0807	0.0472
	500	Hubert	0.2896	0.1554	0.0404	0.1855	0.1098	0.2545	0.2326	0.1661	0.0975	0.0323	0.0306	0.0221	0.0812	0.0290
		Light	0.2896	0.1555	0.0404	0.1855	0.1099	0.2545	0.2327	0.1661	0.0977	0.0324	0.0307	0.0221	0.0813	0.0291
5	1000	Hubert	0.2898	0.1555	0.0395	0.1856	0.1097	0.2552	0.2318	0.1660	0.0975	0.0288	0.0282	0.0162	0.0811	0.0256
		Light	0.2898	0.1555	0.0396	0.1856	0.1097	0.2552	0.2318	0.1660	0.0975	0.0289	0.0282	0.0161	0.0812	0.0257
	5000	Hubert	0.2896	0.1553	0.0392	0.1853	0.1091	0.2565	0.2313	0.1658	0.0976	0.0283	0.0272	0.0085	0.0811	0.0243
		Light	0.2896	0.1553	0.0392	0.1853	0.1091	0.2565	0.2313	0.1658	0.0977	0.0284	0.0272	0.0085	0.0811	0.0243

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Redit (R), Exponential (E).

Table A8. MAE values computed for medium agreement tables with five raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Hubert	0.3038	0.2086	0.1029	0.2279	0.1613	0.2649	0.2304	0.2695	0.2477	0.2160	0.2151	0.2067	0.2370	0.2126
		Light	0.3038	0.2087	0.1031	0.2280	0.1615	0.2650	0.2302	0.2712	0.2492	0.2178	0.2171	0.2087	0.2387	0.2141
	500	Hubert	0.3026	0.2071	0.1007	0.2265	0.1591	0.2642	0.2267	0.2694	0.2520	0.2251	0.2244	0.2180	0.2425	0.2217
		Light	0.3026	0.2071	0.1007	0.2265	0.1591	0.2643	0.2270	0.2698	0.2523	0.2256	0.2248	0.2185	0.2429	0.2221
	1000	Hubert	0.3027	0.2070	0.1004	0.2266	0.1591	0.2645	0.2266	0.2700	0.2525	0.2255	0.2251	0.2187	0.2431	0.2222
		Light	0.3027	0.2070	0.1005	0.2267	0.1591	0.2645	0.2267	0.2702	0.2527	0.2258	0.2253	0.2189	0.2433	0.2224
	5000	Hubert	0.3026	0.2068	0.1002	0.2266	0.1588	0.2646	0.2264	0.2688	0.2514	0.2244	0.2238	0.2175	0.2418	0.2209
		Light	0.3026	0.2068	0.1002	0.2266	0.1588	0.2646	0.2264	0.2689	0.2514	0.2245	0.2239	0.2175	0.2419	0.2210
	100	Hubert	0.3714	0.2150	0.0691	0.2494	0.1573	0.3272	0.3001	0.2522	0.1918	0.1198	0.1243	0.0979	0.1727	0.1162
		Light	0.3714	0.2152	0.0694	0.2496	0.1576	0.3273	0.3004	0.2528	0.1926	0.1211	0.1251	0.0987	0.1735	0.1170
4	500	Hubert	0.3703	0.2133	0.0640	0.2480	0.1542	0.3270	0.2940	0.2521	0.1999	0.1366	0.1417	0.1188	0.1827	0.1322
		Light	0.3703	0.2134	0.0641	0.2480	0.1542	0.3270	0.2942	0.2523	0.2002	0.1370	0.1419	0.1191	0.1829	0.1324
	1000	Hubert	0.3702	0.2131	0.0637	0.2479	0.1538	0.3285	0.2941	0.2528	0.2011	0.1382	0.1434	0.1208	0.1839	0.1338
		Light	0.3702	0.2131	0.0637	0.2479	0.1539	0.3285	0.2943	0.2529	0.2012	0.1384	0.1435	0.1209	0.1840	0.1339
	5000	Hubert	0.3699	0.2127	0.0632	0.2474	0.1531	0.3296	0.2941	0.2520	0.2002	0.1372	0.1422	0.1196	0.1829	0.1325
		Light	0.3699	0.2127	0.0632	0.2474	0.1531	0.3296	0.2941	0.2520	0.2002	0.1372	0.1423	0.1196	0.1829	0.1326
	100	Hubert	0.4150	0.2176	0.0534	0.2638	0.1603	0.3796	0.3607	0.2540	0.1638	0.0703	0.0774	0.0453	0.1432	0.0700
		Light	0.4150	0.2178	0.0537	0.2640	0.1605	0.3796	0.3606	0.2542	0.1643	0.0711	0.0777	0.0456	0.1436	0.0710
	500	Hubert	0.4139	0.2157	0.0456	0.2617	0.1554	0.3722	0.3470	0.2536	0.1732	0.0860	0.0940	0.0550	0.1531	0.0829
		Light	0.4139	0.2158	0.0457	0.2618	0.1555	0.3722	0.3470	0.2537	0.1734	0.0862	0.0941	0.0552	0.1532	0.0830
5	1000	Hubert	0.4139	0.2158	0.0455	0.2618	0.1553	0.3739	0.3473	0.2533	0.1736	0.0870	0.0956	0.0572	0.1537	0.0840
		Light	0.4139	0.2158	0.0455	0.2619	0.1553	0.3739	0.3473	0.2533	0.1737	0.0871	0.0957	0.0573	0.1538	0.0840
	5000	Hubert	0.4137	0.2154	0.0450	0.2614	0.1543	0.3781	0.3493	0.2531	0.1736	0.0872	0.0959	0.0577	0.1537	0.0841
		Light	0.4137	0.2155	0.0450	0.2614	0.1543	0.3781	0.3493	0.2531	0.1736	0.0872	0.0959	0.0577	0.1537	0.0840

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).

Table A9. MAE values computed for high agreement tables with five raters

R	n	Coef.	Balanced							Unbalanced						
			UW	L	Q	RL	RQ	EL	EQ	UW	L	Q	RL	RQ	EL	EQ
3	100	Hubert	0.3181	0.2307	0.1259	0.2504	0.1895	0.2877	0.2598	0.3008	0.2808	0.2482	0.2486	0.2406	0.2700	0.2457
		Light	0.3181	0.2308	0.1261	0.2505	0.1897	0.2876	0.2599	0.3028	0.2824	0.2495	0.2505	0.2425	0.2717	0.2489
	500	Hubert	0.3159	0.2283	0.1233	0.2481	0.1867	0.2862	0.2564	0.3060	0.2992	0.2866	0.2866	0.2841	0.2947	0.2854
		Light	0.3159	0.2283	0.1234	0.2482	0.1867	0.2862	0.2566	0.3064	0.2995	0.2870	0.2870	0.2845	0.2951	0.2858
	1000	Hubert	0.3163	0.2285	0.1233	0.2487	0.1871	0.2868	0.2569	0.3068	0.3014	0.2913	0.2914	0.2894	0.2978	0.2903
		Light	0.3163	0.2285	0.1234	0.2487	0.1872	0.2868	0.2570	0.3070	0.3016	0.2916	0.2916	0.2897	0.2980	0.2906
	5000	Hubert	0.3161	0.2282	0.1228	0.2485	0.1868	0.2868	0.2567	0.3060	0.3011	0.2917	0.2917	0.2899	0.2976	0.2907
		Light	0.3161	0.2282	0.1228	0.2485	0.1868	0.2868	0.2567	0.3061	0.3011	0.2918	0.2917	0.2900	0.2977	0.2907
	100	Hubert	0.4006	0.2378	0.0822	0.2764	0.1832	0.3611	0.3372	0.2737	0.2300	0.1664	0.1773	0.1576	0.2133	0.1663
		Light	0.4006	0.2379	0.0824	0.2765	0.1835	0.3611	0.3370	0.2737	0.2300	0.1667	0.1774	0.1577	0.2133	0.1657
4	500	Hubert	0.3972	0.2343	0.0788	0.2725	0.1778	0.3613	0.3331	0.2737	0.2300	0.1664	0.1773	0.1576	0.2133	0.1663
		Light	0.3972	0.2343	0.0788	0.2725	0.1778	0.3613	0.3332	0.2737	0.2300	0.1667	0.1774	0.1577	0.2133	0.1657
	1000	Hubert	0.3971	0.2341	0.0785	0.2725	0.1777	0.3633	0.3346	0.2829	0.2391	0.1770	0.1859	0.1668	0.2182	0.1712
		Light	0.3971	0.2341	0.0785	0.2725	0.1777	0.3633	0.3347	0.2817	0.2379	0.1760	0.1839	0.1638	0.2182	0.1715
	5000	Hubert	0.3966	0.2334	0.0779	0.2718	0.1767	0.3646	0.3355	0.2762	0.2346	0.1759	0.1833	0.1644	0.2179	0.1732
		Light	0.3966	0.2335	0.0779	0.2718	0.1767	0.3646	0.3355	0.2762	0.2346	0.1759	0.1833	0.1644	0.2179	0.1733
	100	Hubert	0.4599	0.2384	0.0578	0.2898	0.1781	0.4255	0.4078	0.2830	0.1989	0.1039	0.0871	0.0343	0.2830	0.2830
		Light	0.4599	0.2385	0.0580	0.2899	0.1782	0.4255	0.4083	0.2832	0.1992	0.1044	0.0875	0.0342	0.2832	0.2845
	500	Hubert	0.4577	0.2357	0.0546	0.2867	0.1722	0.4190	0.3963	0.2890	0.2066	0.1124	0.1375	0.0902	0.2788	0.2702
		Light	0.4577	0.2357	0.0547	0.2868	0.1722	0.4190	0.3965	0.2891	0.2068	0.1129	0.1376	0.0903	0.2710	0.2698
5	1000	Hubert	0.4575	0.2354	0.0543	0.2866	0.1719	0.4228	0.3997	0.3218	0.2348	0.1347	0.1804	0.1517	0.1802	0.1054
		Light	0.4575	0.2354	0.0544	0.2866	0.1719	0.4228	0.3997	0.3135	0.2272	0.1282	0.1628	0.1232	0.1802	0.1039
	5000	Hubert	0.4576	0.2352	0.0540	0.2863	0.1710	0.4303	0.4069	0.3195	0.2347	0.1368	0.1796	0.1546	0.1871	0.1155
		Light	0.4576	0.2352	0.0540	0.2863	0.1710	0.4303	0.4069	0.3104	0.2265	0.1298	0.1611	0.1255	0.1871	0.1156

Abbr.: Unweighted (UW), Linear (L), Quadratic (Q), Ridit (R), Exponential (E).