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# Fixed point results with $c_A$ distance in cone A-metric spaces

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**Abstract**

In this paper, the concept of a $c_A$-distance in a cone A-metric space with some illustrative examples is presented. Then, some common fixed point theorems for weakly compatible self mappings are gotten by using this new distance. Also, some common fixed point results in cone A-metric spaces for weakly compatible self mappings are held without assumption of the normality for cones.

## 1 Introduction and Preliminaries

The easily dealing of Banach contraction mapping principle (Banach, 1922), stimulated many authors to introduce new spaces and give various generalizations of this principle. In 1989, Bakhtin (Bakhtin, 1989). presented the concept of b-metric space as a generalization of metric spaces, in which many researchers considered the fixed point theory. In 1993, Czeru (Czerwik, 1993; 1998), enlarged many results related to the b-metric spaces. In 1994. Matthews (Matthew's, 1994) gave the subject of partial metric space in which the self distance of any point of space may not be zero. In 2013, Shukla (Shukla, 2013) extended both the concept of b-metric and partial metric spaces by introducing the partial b-metric spaces. In 1984, Dhage in his Ph.D. thesis (Dhage, 1984) introduced the concept of a D-metric space. Then Dhage (Dhage, 2000) studied topological properties of D-metric space. Naidu et al. (Naidu et al., 2004) proved that the concepts of convergent sequences and sequerrtial continuity are not well defmed in D-metric spaces. Naidu et al. (Naidu et al., 2005) pointed out some drawbacks in the idea of open balls in D-metric space. In 2003, Mustafa and Sims (2003) identified condition third as a weakness in Dhage's theory of D-metric space. The tetrahedral inequality in D-metric has been replaced with the prototypical rectangular inequality adopted by Mustafa and Sims (Mustafa & Sims, 2006) in 2006 and introduced the notion of $G$-metric space and suggested an important generalization of metric space. Aghajani et al. (Aghajani et al., 2014) introduced $G_b$-metric space and established common fixed point of generalized weak contractive mapping in partially ordered $G_b$-metric spaces.

Sedghi et al. (Sedghi, Rao, et al., 2007) have introduced $D^*$-metric spaces which is a probable modification of the definition of D- metric spaces introduced by Dhage (Dhage, 1984) and proved some basic properties in $D^*$-metric spaces, (see (Sedghi, Shobe, et al., 2007)). Every $G$-metric space is a $D^*$-metric space. The converse, however, is false in general: a $D^*$-metric space is not necessarily a $G$-metric space. Sedghi et al. (Sedghi et al., 2012) identified condition third of the $G$-metric space as a peculiar limitation but classified the symmetry condition as a common weakness of both $G$- and $D^*$-metric spaces. To overcome these difficulties. Sedghi et al. (Sedghi et al., 2012) introduced a new generalized metric space called an $S$-metric space.The $S$-metric space is a space with three dimensions. Sedghi et a1. (Sedghi et al., 2012) asserted that every $G$-metric is an $S$-metric, see [(Sedghi et al., 2012), Remarks 1.3] and [(Sedghi et al., 2012), Remarks 2.2]. The Example 2.1 and Example 2.2 of Dung et al. (Dung et al., 2014). shows that this assertion is not correct. Moreover, the class of all $S$-metrics and the class of all $G$-metrics are distinct. Souayah et al. (Souayak & Mlaiki, 2016) have introduced $S_b$-metric space and established some fixed point theorems. Recently, Abbas et al. (Abbas et al., 2015) introduced the concept of an n-tuple metric space and studied its topological properties. This new structure is named as A-metric space.

On the other hand, Huang and Zhang (2007). introduced cone metric spaces and proved some fixed point theorems of contractive mappings on cone metric spaces in 2007. Then, in 2008, Rezapour and Hamlbarani (2008). obtained generalizations of some results in (Huang Zhang, 2007). by omitting the assumption of normality. Then many researchers are obtained fixed point theorems on cone metric spaces.(see (Abbas Jungck, 2008), (Abbas Rhoades, 2009), (Ilić Rakočević, 2008), (Janković et al., 2011), (Turkoglu Abuloha, 2010), (Vetro, 2007))

In 2017, Dhamodharan and Krishnakumar (Dhamodharan & Krishnakumar, 2017) extended and generalized the notion of S-metric spaces to cone S-metric spaces with some properties and proved some fixed point results under the condition of normality for cones. Then, Saluja (Saluja, 2020), proved some fixed point theorems on cone S-metric spaces using implicit relation under the condition of normality for cones. In 2018, Singh and Singh (Singh & Singh, 2018) have introduced the concept of a cone $S_b$-metric space which is a generalization of the S-metric space, the $S_b$-metric space and the cone S-metric space and they proved some fixed point results (see [(Singh & Singh, 2020), (Saluja, 2021).). In 2022 Fadail et al. (Fadail et al., 2022) introduced the concept of a $c_s$-distance in a cone S-metric space with some properties and give some examples. And they proved some common fixed point and fixed point theorems for weakly compatible self mappings with this new distance. After that, as a direct application of this new distance, they obtained some common fixed point and fixed point results in the setting of cone S-metric spaces for weakly compatible self mappings with out assumption of the normality for cones.

In this paper, the concept of distance $c_A$ in cone A-metric space is presented with some illustrative examples inspired by (Abbas et al., 2015) and (Fadail et al., 2022). Then, some common fixed point theorems for weakly compatible self mappings are gotten by using this new distance. Also, some common fixed point results in cone A-metric spaces for weakly compatible self mappings are held without assumption of the normality for cones.

Let $G$ be a real Banach space, $\theta_G$ denote the zero element in $G$ and $M$ a subset of $G$. We called $M$ a cone if satisfies the following three conditions:

i. $M$ is a nonempty closed set and $M \neq \{\theta_G\}$

ii. If $a, b$ are nonnegative real numbers and $k, l \in M$ then $ak + bl \in M$;

iii. $k \in M$ and $-k \in M$ imply $k = \theta_G$

For any cone $M \subset G$ , the partial ordering $\preccurlyeq$ with respect to $M$ by $k \preccurlyeq l$ if and only if $l - k \in M$. The notation of $<$ stand for $k \preccurlyeq l$ but $k \neq l$. Also, we used $k \ll l$ to indicate that $l - k \in intM$ ,where $intM$

denotes the interior of $M$. A cone $M$ is called normal if there exists a number $\lambda$ such that

$$\theta_G \preccurlyeq k \preccurlyeq l \quad implies \quad \| k \| \leq \lambda \| l \| \tag{1}$$

for all $k, l \in G$. Equivalently, the cone $M$ is normal if

$$\text{For all } n, k_n \preccurlyeq l_n \preccurlyeq m_n \text{ and } \lim_{n\to\infty} k_n = \lim_{n\to\infty} m_n = k \quad \text{imply} \quad \lim_{n\to\infty} l_n = k \tag{2}$$

The least positive number $\lambda$ satisfying condition is called the normal constant of $M$.( for more details about cones see (Aleksić et al., 2018) ) In this paper, the cone $M$ is assumed to be solid, namely $intM \neq \theta_G$. The next lemma is helpful in our work.

**Lemma 1.1.** *(Jungck et al., 2009) Let $G$ be a real Banach space with a solid cone $M$. Suppose that $k_n$ is a sequence in $M$. Then we have :*

*i.* *If $k \preccurlyeq \nu k$ where $k \in M$ and $0 \leq \nu < 1$, then $k = \theta_G$*

*ii.* *If $\varepsilon \in intM, \theta_G \preccurlyeq k_n$ and $k_n \longrightarrow \theta_G$, then there exists a positive integer $N$ such that $k_n \ll \varepsilon$ for all $n \geq N$*

*iii.* *If $k \preccurlyeq l$ and $l \ll \varepsilon$, then $k \ll \varepsilon$*

*iv.* *If $\theta_G \preccurlyeq k \ll \varepsilon$ for each $\theta_G \ll \varepsilon$, then $k = \theta_G$*

**Definition 1.2.** (Abbas et al., 2015) Let $\Delta$ be a non-empty set. A function $A : \Delta^n \to [0, \infty)$ is called an $A$-metric on $\Delta$ if $\forall k_i, k \in \Delta$, $i = 1, 2, 3, \ldots, n$, the following conditions are satisfied:

i. $A(k_1, k_2, \ldots, k_{n-1}, k_n) \geq 0$,

ii. $A(k_1, k_2, \ldots, k_{n-1}, k_n) = 0$ if and only if $k_1 = k_2 = \ldots = k_{n-1} = k_n$ ,

iii. $A(k_1, k_2, \ldots, k_{n-1}, k_n) \leq A(k_1, k_1, .., (k_1)_{n-1}, k) + A(k_2, k_2, ..., (k_2)_{n-1}, k)$
$+ A(k_3, k_3, ...., (k_3)_{n-1}, k) + .... + A(k_{n-1}, k_{n-1}, \ldots, (k_{n-1})_{n-1}, k)$
$+ A(k_n, k_n, \ldots, (k_n)_{n-1}, k)$

The pair $(\Delta, A)$ is called an $A$-metric space.

**Definition 1.3.** (Fernandez et al., 2017) Suppose that $G$ is a real Banach space, $M$ is a cone in $G$ with $intM \neq \emptyset$ and $\preccurlyeq$ is partial ordering in $G$ with respect to $M$. Let $\Delta$ be a non-empty set, $\forall k_i, k \in \Delta$, $i = 1, 2, 3, \ldots, n$ and let the function $A : \Delta^n \to G$ satisfy the following conditions

i. $\theta_G \preccurlyeq A(k_1, k_2, \ldots, k_{n-1}, k_n)$,

ii. $A(k_1, k_2, \ldots, k_{n-1}, k_n) = \theta_G$ if and only if $k_1 = k_2 = \ldots = k_{n-1} = k_n$ ,

iii. $A(k_1, k_2, \ldots, k_{n-1}, k_n) \preccurlyeq A(k_1, k_1, .., (k_1)_{n-1}, k) + A(k_2, k_2, ..., (k_2)_{n-1}, k) + A(k_3, k_3, ...., (k_3)_{n-1}, k)$
$+ ... + A(k_{n-1}, k_{n-1}, \ldots, (k_{n-1})_{n-1}, k) + A(k_n, k_n, \ldots, (k_n)_{n-1}, k)$

Then, the function A is called a cone $A$-metric on $\Delta$ and the pair $(\Delta, A)$ is called a cone $A$-metric space.

**Definition 1.4.** Let $(\Delta, A)$ be a cone A-metric space, $\{k_n\}$ be a sequence in $\Delta$ and $k \in \Delta$ .

i. For all $\varepsilon \in G$ with $\theta_G \ll \varepsilon$, if there exists a positive integer N such that $A(k_n, k_n, ..., k_n, k) \ll \varepsilon$ for all $n > N$ , then $\{k_n\}$ is said to be convergent and k is the limit of $\{k_n\}$. We denote this by $k_n \longrightarrow k$.

ii. For all $\varepsilon \in G$ with $\theta_G \ll \varepsilon$, if there exists a positive integer N such that $A(k_n, k_n, ..., k_n, k_m) \ll \varepsilon$ for all $n, m > N$, then $\{k_n\}$ is called a Cauchy sequence in $\Delta$.

iii. A cone A-metric space $(\Delta, A)$ is called a complete if every Cauchy sequence in $\Delta$ is convergent.

**Lemma 1.5.** *(Fernandez et al., 2017) Let* $(\Delta, A)$ *be a cone A-metric space. Then,* $A(k,k,...,k,l)=A(l,l,...,l,k)$ *for all* $k, l \in \Delta$.

**Definition 1.6.** Let $(\Delta, A)$ be a cone A-metric space, $t : \Delta \to \Delta$ and $s : \Delta \to \Delta$ be two mappings and let $l, m \in \Delta$. Recall that, if $m = sl = tl$ then we called $l$ is a coincidence point of mappings and $m$ is a point of coincidence. If $l = sl = tl$, then we called $l$ is a common fixed point of $t$ and $s$. The mappings t and s are called weakly compatible if $stl = tsl$ whenever $sl = tl$.

## 2 $c_A$-distance

Now, the notion of a $c_A$-distance on a cone A-metric space and some of its properties are given.

**Definition 2.1.** Let $(\Delta, A)$ be a cone A-metric space. A function $c_A : \Delta^n \longrightarrow G$ is called $c_A$-distance on $\Delta$ if the following conditions hold

$(c_A1)$ $\theta_G \preccurlyeq c_A(k_1, k_2, ..., k_n)$ for all $k_1, k_2, .., k_n \in \Delta$

$(c_A2)$ $c_A(k_1, k_2, ..., k_n) \preccurlyeq c_A(k_1, k_1, ..., k) + c_A(k_2, k_2, .., k) + .... + c_A(k_{n-1}, k_{n-1}, .., k)c_A(k, k, ..., k_n)$ for all $k_1, k_2, ..k_n, k \in \Delta$

$(c_A3)$ for each $k \in \Delta$ and $n \geq 1$ , if $c_A(k, k, ..., k, l_n) \preccurlyeq m$ for some $m \in M$, then $c_A(k, k, .., k, l) \preccurlyeq m$ whenever $\{l_n\}$ is a sequence in $\Delta$ converging to a point $l \in \Delta$

$(c_A4)$ for all $\varepsilon \in G$ with $\theta_G \ll \varepsilon$, there exists $m \in G$ with $\theta_G \ll m$ such that $k_1, k_2, ..k_n, k \in \Delta$, $c_A(k, k, ..., k, k_1) \ll m$ ,$c_A(k, k, ..., k, k_2) \ll m$ ,..., $c_A(k, k, ..., k, k_{n-1}) \ll m$ and $c_A(k, k, ..., k, k_n) \ll m$ imply $A(k_1, k_2, ..., k_{n-1}, k_n) \ll \varepsilon$.

**Example 2.2.** Let $G = C^1_{\mathbb{R}}([0, 1])$ and $M = \{m \in E | m(t) \geq 0 \text{ on } t \in [0, 1]\}$. Let $\Delta = [0, 1]$ and define a mapping $A : \Delta^n \to G$ by

$$A(k_1, k_2, \ldots, k_n)(s) = \sum_{i=1}^{n} \sum_{i<j} \mid k_i - k_j \mid^2 e^s \text{ for all } k_1, k_2, ..., k_{n-1}, k_n \in \Delta$$

Then $(\Delta, A)$ is a complete cone A-metric space. Define a

$$c_A : \Delta^n \to G \text{ by } \quad c_A(k_1, k_2, ..k_{n-1}, k_n) = \sum_{i=1}^{n} \sum_{i<j} \mid k_i - k_j \mid^2 e^s \text{ for all } k_1, k_2, ..., k_{n-1}, k_n \in \Delta$$

It is clear that, $c_A$ satisfies $(c_A1)$,$(c_A2)$ and $(c_A3)$. Let $\varepsilon \in G$ with $\theta_G \ll \varepsilon$ be given and put $m = \frac{\varepsilon}{2n}$. Let $k \in \Delta$ and $c_A(k, k, .., k, k_1) \ll m$ ,$c_A(k, k, ..., k, k_2) \ll m$ ,..., $c_A(k, k, ..., k, k_{n-1}) \ll m$ and $c_A(k, k, ..., k, k_n) \ll m$. Then we have

$$\begin{aligned} c_A(k, k, .., k, k_1) &= (n-1)|k - k_1|^2 e^s \ll m \\ c_A(k, k, .., k, k_2) &= (n-1)|k - k_2|^2 e^s \ll m \\ &\cdots \\ c_A(k, k, .., k, k_n) &= (n-1)|k - k_n|^2 e^s \ll m \end{aligned}$$

for all $k_i, k \in \Delta$ , i=1,2,3,..,n.

Now,

$$\begin{aligned}
A(k_1,k_2,\dots,k_n) &= \textstyle\sum_{i=1}^{n}\sum_{i<j} \mid k_i-k_j \mid^2 e^s \\
&= \textstyle\sum_{i=1}^{n}\sum_{i<j} \mid (k_i-k) - (k_j-k) \mid^2 e^s \\
&\leq \{2(n-1)|k_1-k|^2+2|k_2-k|^2+2|k_3-k|^2+....+2|k_n-k|^2\}e^s \\
&\quad +\{2(n-2)|k_2-k|^2+2|k_3-k|^2+....+2|k_n-k|^2\}e^s \\
&\quad +\{2(n-3)|k_3-k|^2+2|k_4-k|^2+....+2|k_n-k|^2\}e^s+... \\
&\quad +\{2(2)|k_{n-2}-k|^2+2|k_{n-1}-k|^2+2|k_n-k|^2\}e^s \\
&\quad +2|k_{n-1}-k|^2e^s+|k_n-a|^2e^s \\
&= 2(n-1)|k_1-k|^2e^s+2(n-1)|k_2-k|^2e^s+.... \\
&\quad +2(n-1)|k_{n-1}-k|^2e^s+2(n-1)|k_n-a|^2e^s \\
&= 2[c_A(k,k,...k,k_1)+c_A(k,k,..,k,k_2)+...+c_A(k,k,...,k,k_{n-1})+c_A(k,k,..,k,k_n)] \\
&\ll [\tfrac{\varepsilon}{2n}+\tfrac{\varepsilon}{2n}+...+\tfrac{\varepsilon}{2n}+\tfrac{\varepsilon}{2n}] \\
&= \varepsilon
\end{aligned}$$

Hence $c_A$ satisfies $(c_A4)$.Thus, $c_A$ is a $c_A$-distance on $\Delta$.

**Example 2.3.** Let $\Delta=[0,1]$ and $G=C^1_{\mathbb{R}}([0,1])$ with $\parallel m \parallel = \parallel m \parallel_\infty + \parallel m' \parallel_\infty$, $m\in G$ and let $M=\{m\in G|m(t)\geq 0 \text{ on } t\in[0,1]\}$. This cone is solid but it is not normal. Define a cone A-metric $A:\Delta^n\to G$ by,

$$A(k_1,k_2,..,k_{n-1},k_n)(s) \quad = (|k_1-k_n|+|k_2-k_n|+...+|k_{n-1}-k_n|)e^s$$

for all $k_1,k_2,...,k_n\in\Delta$. Then $(\Delta,A)$ is a complete cone A-metric space, Define a mapping $c_A:\Delta^n\to G$ by $c_A(k_1,k_2,...,k_n)=(k_n+k_{n-1})e^s$ for all $k_1,k_2,...,k_n\in\Delta$. It is clear that $c_A$ satisfy $(c_A1),(c_A2)$ and $(c_A3)$. Let $\varepsilon\in G$ with $\theta_G\ll\varepsilon$ be given put $m=\frac{\varepsilon}{2n-2}$. Let $k\in\Delta$ and suppose that $c_A(k,k,...,k_1)\ll m$, $c_A(k,k,...,k_2)\ll m$ ,...,$c_A(k,k,...,k_n)\ll m$. Then we have

$$\begin{aligned}
A(k_1,k_2,..,k_{n-1},k_n)(s) &= \quad (|k_1-k_n|+|k_2-k_n|+,,,+|k_{n-1}-k_n|)e^s \\
&\leq \quad (k_1+k_2+k_1+k_3+....+k_1+k_n)e^s \\
&= \quad (n-1)k_1e^s+k_2e^s+...+k_ne^s \\
&\leq \quad (n-1)(k_1+k)e^s+(k_2+k)e^s+...+(k_n+k)e^s \\
&= \quad (n-1)c_A(k,k,...,k_1)(s)+c_A(k,k,...k_2)(s)+....+c_A(k,k,...,k_n)(s) \\
&\ll \quad (n-1)\tfrac{\varepsilon}{2n-2}+\tfrac{\varepsilon}{2n-2}+....+\tfrac{\varepsilon}{2n-2} \\
&= \quad \epsilon.
\end{aligned}$$

Hence satisfies $(c_A4)$.Thus, is a $c_A$-distance on $\Delta$.

**Lemma 2.4.** *The cone A-metric function is a $c_A$-distance on $\Delta$ where $(\Delta,A)$ is a cone A-metric space.*

**Proof.** Let $(\Delta,A)$ be a cone A-metric space and define the function $c_A:\Delta^n\to G$ by $c_A(k_1,k_2,...,k_{n-1},k_n)=A(k_1,k_2,...,k_{n-1},k_n)$ for all $k_1,k_2,...,k_{n-1},k_n\in\Delta$. . It is clear that $c_A$ satisfy $(c_A1),(c_A2)$ and $(c_A3)$. Let $\varepsilon\in G$ with $\theta\ll\varepsilon$ be given and put $m=\frac{\varepsilon}{n}$. Suppose that $c_A(k,k,..,k_1)\ll m$ ,$c_A(k,k,...,k,k_2)\ll m$

,..., $c_A(k, k, ..., k, k_{n-1}) \ll m$ and $c_A(k, k, ..., k, k_n) \ll m$ imply $c_A(k_1, k_2, ..., k_{n-1}, k_n) \ll m$.Then we have

$$\begin{aligned} A(k_1, k_2, ..., k_{n-1}, k_n) &\leq A(k_1, k_1, ..., k_1, k) + A(k_2, k_2, .., k_2, k) + ... + A(k_n, k_n, ..., k_n, k) \\ &= A(k, k, .., k, k_1) + A(k, k, ..., k, k_2) + ... + A(k, k, .., k, k_n) \\ &= c_A(k, k, .., k, k_1) + c_A(k, k, ..., k, k_2) + ... + c_A(k, k, .., k, k_n) \\ &\ll \tfrac{\varepsilon}{n} + \tfrac{\varepsilon}{n} + .... + \tfrac{\varepsilon}{n} \\ &= \varepsilon. \end{aligned}$$

Hence $c_A$ satisfies $(c_A4)$.Thus, is a $c_A$-distance on $\Delta$. □

**Lemma 2.5.** *Let $(\Delta, A)$ be a cone A-metric space and $c_A$ is a $c_A$-distance on $\Delta$. Let $\{k_s\}$ and $\{l_s\}$ be two sequences in $\Delta$ and $k, l \in \Delta$ . Suppose that $\mu_s$ is a sequence in $M$ converging to $\theta_G$ .Then the following hold*

i. *If $c_A(k_s, k_s, ..., k_s, k) \leq \mu_s$ and $c_A(k_s, k_s, ..., k_s, l) \leq \mu_s$, then $k = l$*

ii. *If $c_A(k_s, k_s, ..., k_s, l_s) \leq \mu_s$ and $c_A(k_s, k_s, ..., k_s, l) \leq \mu_s$, then $\{l_s\}$ converges to $l$.*

iii. *If $c_A(k_s, k_s, ..., k_s, k_t) \leq \mu_s$ for $t > s$, then $\{k_s\}$ is a Cauchy sequence in $\Delta$.*

iv. *If $c_A(k, k, ..., k, k_s) \leq \mu_s$ ,then $\{k_s\}$ is a Cauchy sequence in $\Delta$.*

**Proof.** i. Since $\mu_s$ is a sequence in $M$ converging to $\theta_G$ , then there exist a positive integer N and

$$\theta_G \ll \varepsilon \in intM,$$

such that $\mu_s \ll \varepsilon$ for all $s \geq N$. Hence $c_A(k_s, k_s, ..., k_s, k) \ll \varepsilon$ and $c_A(k_s, k_s, ..., k_s, l) \ll \varepsilon$. By $(c_A4)$ with $m = \varepsilon$, it follows that $A(k, k, ..., k, l) \ll \varepsilon$. By Lemma1(4), it follows that $A(k, k, ..., k, l) = \theta_G$. Hence $k = l$

ii. As in the proof of (1), there exist a positive integer N and $\theta_G \ll \varepsilon \in intM$ such that $\mu_s \ll \varepsilon$ for all $s \in N$. Hence $c_A(k_s, k_s, ..., k_s, l_s) \ll \varepsilon$ and $c_A(k_s, k_s, ..., k_s, l) \ll \varepsilon$. By $(c_A4)$ with $m = \varepsilon$, it follows that $A(l_s, l_s, ..., l_s, l) \ll \varepsilon$. Definition4 (1) shows that $\{l_s\}$ converges to $l$ .

iii. As in the proof of (1) and (2), there exist a positive integer $s_0$ and $\theta_G \ll \varepsilon \in intM$ such that $\mu_m \ll \varepsilon$ for all $s \geq s_0$. Hence, $c_A(k_s, k_s, ..., k_s, k_t) \ll \varepsilon$ for all $t > s \geq s_0$. Clearly that $c_A(k_s, k_s, ..., k_s, k_{s+1}) \ll \varepsilon$. Now, we have $c_A(k_s, k_s, ..., k_s, k_{s+1}) \ll \varepsilon$ and $c_A(k_s, k_s, ..., k_s, k_t)) \ll \varepsilon$, by $(c_A4)$ with $m = \varepsilon$ , it follows that $c_A(k_{s+1}, k_{s+1}, ..., k_{s+1}, k_t) \ll \varepsilon$. Definition 4 (2) shows that $\{k_s\}$ is a Cauchy sequence in $\Delta$.

iv. The proof is similar to (3).

□

**Remark.** i. $c_A(k, k, ..., k, l) = d(l, l, ..., l, k)$ does not necessarily for all $k, l \in \Delta$.

ii. $c_A(k_1, k_2, .., k_{n-1}, k_n) = \theta_G$ is not necessarily equivalent to
$k_1 = k_2 = ... = k_{n-1} = k_n$ for all $k_1, k_2, ..., k_{n-1}, k_n \in \Delta$.

## 3 Common fixed point and fixed point results with $c_a$ distance in cone A-metric spaces

In this section, we will study the problems of the common fixed point and the fixed point for weakly compatible self mappings in cone A-metric spaces with a $c_a$-distance.

**Theorem 3.1.** *Suppose that $(X, A)$ be a cone A-metric space and d is a $c_a$-distance on X. Let $f, g : X \to X$ be two self mappings satisfy the following contractive condition*

$$\begin{aligned} d(fx, fx, ..., fx, fy) &\leq \alpha_1 d(gx, gx, .., gx, gy) + \alpha_2 d(gx, gx, ..., gx, fx) \\ &\quad + \alpha_3 d(gy, gy, ..., gy, fy) + \alpha_4 d(gx, gx, .., gx, fy), \end{aligned}$$

*for all $x, y \in X$ where $\alpha_i \in (0, 1), i = 1, 2, 3, 4$ such that $\alpha_1 + \alpha_2 + \alpha_3 + 3\alpha_4 < 1$. If f(X) is a subset of g(X) and g(X) is a complete subspace of X, then f and g have a coincidence point $x^*$ in X. In addition, if $z = gx^* = fx^*$ then $d(z, z, ..., z, z) = \theta$. Furthermore, if f and g are weakly compatible, then f and g have a unique common fixed point.*

**Proof.** Let $x_0$ be an arbitrary point in X. Choose a point $x_1$ in X such that $gx_1 = fx_0$. This can be done for $f(X) \subseteq g(X)$. Continuing this process we obtain a sequence $\{x_n\}$ in X such that $gx_{n+1} = fx_n$. Then we have

$$\begin{aligned}
d(gx_n, gx_n, ..., gx_n, gx_{n+1}) &= d(fx_{n-1}, fx_{n-1}, ..., fx_{n-1}, fx_n)\\
&\leq \alpha_1 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\quad +\alpha_2 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, fx_{n-1})\\
&\quad +\alpha_3 d(gx_n, gx_n, ..., gx_n, fx_n)\\
&\quad +\alpha_4 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, fx_n)\\
&= \alpha_1 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\quad +\alpha_2 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\quad +\alpha_3 d(gx_n, gx_n, ..., gx_n, gx_{n+1})\\
&\quad +\alpha_4 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_{n+1})\\
&\leq \alpha_1 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\quad +\alpha_2 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\quad +\alpha_3 d(gx_n, gx_n, ..., gx_n, gx_{n+1})\\
&\quad +\alpha_4 d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\quad +d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n) + d(gx_n, gx_n, ..., gx_n, gx_{n+1})\\
&= (\alpha_1 + \alpha_2 + 2\alpha_4) d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\quad +(\alpha_3 + \alpha_4) d(gx_n, gx_n, ..., gx_n, gx_{n+1}).
\end{aligned}$$

So,

$$\begin{aligned}
d(gx_n, gx_n, ..., gx_n, gx_{n+1}) &\leq \tfrac{\alpha_1+\alpha_2+2\alpha_4}{1-\alpha_3-\alpha_4} d(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&= hd(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n)\\
&\leq h^2 d(gx_{n-2}, gx_{n-2}, ..., gx_{n-2}, gx_{n-1})\\
&\vdots\\
&\vdots\\
&\leq h^n d(gx_0, gx_0, ..., gx_0, gx_1),
\end{aligned}$$

where $h = \frac{\alpha_1+\alpha_2+2\alpha_4}{1-\alpha_3-\alpha_4} < 1$.
Let $m > n \geq 1$. Then we get

$$\begin{aligned}
d(gx_n, gx_n, ..., gx_n, gx_m) &\leq 2d(gx_n, gx_n, ..., gx_n, gx_{n+1}) + 2d(gx_{n+1}, gx_{n+1}, ..., gx_{n+1}, gx_{n+2})\\
&\quad +... + 2d(gx_{m-1}, gx_{m-1}, ..., gx_{m-1}, gx_m)\\
&\leq 2[d(gx_n, gx_n, ..., gx_n, gx_{n+1}) + d(gx_{n+1}, gx_{n+1}, ..., gx_{n+1}, gx_{n+2})\\
&\quad +... + d(gx_{m-1}, gx_{m-1}, ..., gx_{m-1}, gx_m)]\\
&\leq 2(h^n + h^{n+1} + ... + h^{m-1}) d(gx_0 + gx_0 + ... + gx_0 + gx_1)\\
&\leq 2\tfrac{h^n}{1-h} d(gx_0 + gx_0 + ... + gx_0 + gx_1) \to \theta \text{ as } n \to +\infty
\end{aligned}$$

Consequently, Lemma 11 (3) explicates that $\{gx_n\}$ is a Cauchy sequence in X. Since g(X) is complete, there exists $x^* \in X$ such that $gx_n \to g^*$ as $n \to \infty$ .Therefore,we have

$$d(gx_n, gx_n, ..., gx_n, gx^*) \leq 2\frac{h^n}{1-h} d(gx_0 + gx_0 + ... + gx_0 + gx_1) \to \theta \text{ as } n \to +\infty \tag{3}$$

Note that

$$d(fx_{n-1}, fx_{n-1}, ..., fx_{n-1}, fx_n) = d(gx_n, gx_n, ..., gx_n, gx_{n+1}) \leq hd(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx_n). \tag{4}$$

By utilization (3.2),we get

$$\begin{aligned}
d(gx_n, gx_n, ..., gx_n, fx^*) &= d(fx_{n-1}, fx_{n-1}, ..., fx_{n-1}, fx^*)\\
&\leq hd(gx_{n-1}, gx_{n-1}, ..., gx_{n-1}, gx^*)\\
&\leq 2h\tfrac{h^n}{1-h} d(gx_0, hx_0, .., gx_0, gx_1)\\
&= 2h\tfrac{h^n}{1-h} d(gx_0, hx_0, .., gx_0, gx_1) \to \theta \text{ as } n \to \infty
\end{aligned}$$

Consequently ,Lemma 11 (1), (3.1) and (3.3) explicate that $gx^* = fx^*$. Therefore, $x^*$ is a coincidence point of f and g and z is a point of coincidence of f and g where $z = gx^* = fx^*$ for some $x^*$ in X.

Suppose that $z = gx^* = fx^*$. Then we have

$$\begin{aligned} d(z,z,..,z,) &= d(fx^*,fx^*,..,fx^*) \\ &\leq \alpha_1 d(gx^*,gx^*,...,gx^*) + \alpha_2 d(gx^*,gx^*,...,fx^*) \\ &\quad + \alpha_3 d(gx^*,gx^*,...,fx^*) + \alpha_4 d(gx^*,gx^*,...,fx^*) \\ &= \alpha_1 d(z,z,..,z) + \alpha_2 d(z,z,..,z) + \alpha_3 d(z,z,..,z) + \alpha_4 d(z,z,..,z) \\ &= (\alpha_1+\alpha_2+\alpha_3+\alpha_4) d(z,z,..,z). \end{aligned}$$

Since $(\alpha_1+\alpha_2+\alpha_3+\alpha_4) < 1$, Lemma 1.1(1) explicates that $d(z,z,...,z) = \theta$.
Finally, impose there is another point of coincidence $\mu$ of f and g such that $\mu = fy^* = gy^*$ for some $y^*$ in X. Then we have

$$\begin{aligned} d(z,z,...,\mu) &= d(fx^*,fx^*,...,fy^*) \\ &\leq \alpha_1 d(gx^*,gx^*,...,gy^*) + \alpha_2 d(gx^*,gx^*,...,fx^*) \\ &\quad + \alpha_3 d(gy^*,gy^*,...,fy^*) + \alpha_4 d(gx^*,gx^*,...,fy^*) \\ &= \alpha_1 d(z,z,..,\mu) + \alpha_2 d(z,z,..,z) + \alpha_3 d(\mu,\mu,..,\mu) + \alpha_4 d(z,z,..,\mu) \\ &= (\alpha_1+\alpha_4) d(z,z,..,\mu) \end{aligned}$$

Since $(\alpha_1+\alpha_4) < 1$, Lemma 1 (1) explicates that $d(z,z,...,\mu) = \theta$. Also, we have $d(z,z,...,z) = \theta$, Thus, Lemma 11 (1) explicates that $z = \mu$. Therefore, z is the unique point of coincidence. Now, let $z = gx^* = fx^*$.Since f and g are weakly compatible , we have

$$gz = ggx^* = gfx^* = fgx^* = fgx^* = fz$$

Hence, gz is a point of coincidence. The uniqueness of the point of coincidence implies that $gz = gx^*$ . Therefore,$z = gz = fz$ .Hence, z is the unique common fixed point of f and g. ☐

As a consequence of Theorem 13, we have the following common fixed point result under the concept of a $c_a$–distance in cone A-metric spaces.

**Result 3.2.** *Suppose that (X,A) be a cone A-metric space and d is a $c_a$-distance on X. Let $f,g : X \to X$ be two self mappings satisfy one of the following contractive conditions for all $x,y \in X$:*

*i.*

$$d(fx,fx,..,fy) \leq \alpha d(gx,gx,..,gy),$$

*where $\alpha \in [0,1)$ is a constant.*

*ii.*

$$d(fx,fx,..,fy) \leq \alpha_1 d(gx,gx,..,fx) + \alpha_2 d(gy,gy,...,fy),$$

*where $\alpha_1,\alpha_2 \in [0,1)$ are constants such that $\alpha_1+\alpha_2 < 1$.*

*iii.*

$$d(fx,fx,..,fy) \leq \alpha_1 d(gx,gx,..,gy) + \alpha_2 d(gx,gx,...,fx) + \alpha_3 d(gy,gy,..,fy),$$

*where $\alpha_1,\alpha_2,\alpha_3 \in [0,1)$ are constants such that $\alpha_1+\alpha_2+\alpha_3 < 1$.*

*If f(X) is a subset of g(X) and g(X) is a complete subspace of X, then f and g have a coincidence point $x^*$ in X. In addition, if $z = gx^* = fx^*$ then $d(z,z,...,z) = \theta_G$. Furthermore, if f and g are weakly compatible, then f and g have a unique common fixed point.*

**Example 3.3.** Consider Example 9. Define the mappings $f : X \to X$ by $fx = \frac{x^2}{4}$ and $g : X \to X$ by

$g = \frac{x}{2}$ for all X . It is clear that $f(X) \subseteq g(X)$ and g(X) is a complete subset of X. Let $x, y \in X$ , we have

$$\begin{aligned} d(fx, fx, ..., fy)(t) &= d(\tfrac{x^2}{4}, \tfrac{x^2}{4}, ..., \tfrac{y^2}{4})(t) \\ &= (\tfrac{y^2}{4} + \tfrac{X^2}{4})e^t \\ &\leq \tfrac{1}{2}(\tfrac{y^2}{2} + \tfrac{X^2}{2})e^t \\ &= \tfrac{1}{2}d(gx, gx, .., gy)(t) \end{aligned}$$

with $\alpha = \frac{1}{2} < 1$. Also, f and g are weakly compatible at $x = 0$. Therefore, all conditions of Corollary 14 are satisfied. Hence, f and g have a unique common fixed point $y = 0$ and $f(0) = g(0) = 0$ with $d(0, 0, ..., 0) = \theta_G$.

In the following theorem ,we prove the fixed point theorem for self mappings in a complete cone A-metric space with a $c_a$-distance.

**Theorem 3.4.** *Let (X,A) be a complete cone A-metric space and d is a $c_a$-distance on X.Let $f : X \to X$ be a self mapping satisfies the following contractive condition*

$$d(fx, fx, .., fy) \leq \alpha_1 d(x, x, .., y) + \alpha_2 d(x, x, ..., fx) + \alpha_3 d(y, y, .., fy) + \alpha_4 d(x, x, ..., fy),$$

*for all $x, y \in X$ where $\alpha_i \in (01, ), i = 1, 2, 3, 4$ such that $\alpha_1 + \alpha_2 + \alpha_3 + 3\alpha_4 < 1$. Then f has a fixed point $x^* \in X$ and f or any $x \in X$ , iterative sequence $\{f^n x\}$ converges to the fixed point. If $\mu = f\mu$ then $d(\mu, \mu, .., \mu) = \theta$.The fixed point is unique.*

**Proof.** In Theorem 13, put $gx = x$ .The proof is complete. □

As a consequence of Theorem 16, we get the fixed point theorem of Banach contraction type ,Kannan contraction type and Reich contraction type under the concept of a $c_a$-distance in a cone A-metric space respectively.

**Result 3.5.** *Let $(X, A)$ be a complete cone A-metric space and d is a $c_a$-distance on X. Let $f : X \to X$ be a self mapping satisfies one of the following contractive conditions for all $x, y \in X$*

*i.*

$$d(fx, fx, .., fy) \leq \alpha d(x, x, .., y),$$

*where $\alpha \in [0, 1)$ is a constant.*

*ii.*

$$d(fx, fx, .., fy) \leq \alpha_1 d(x, x, .., fx) + \alpha_2 d(y, y, ..., fy),$$

*where $\alpha_1, \alpha_2 \in [0, 1)$ are constants such that $\alpha_1 + \alpha_2 < 1$.*

*iii.*

$$d(fx, fx, .., fy) \leq \alpha_1 d(x, x, .., y) + \alpha_2 d(x, x, ..., fx) + \alpha_3 d(y, y, .., fy),$$

*where $\alpha_1, \alpha_2, \alpha_3 \in [0, 1)$ are constants such that $\alpha_1 + \alpha_2 + \alpha_3 < 1$.*

*Then f has a fixed point $x^* \in X$ and f or any $x \in X$ , iterative sequence $\{f^n x\}$ converges to the fixed point. If $\mu = f\mu$ then $d(\mu, \mu, .., \mu) = \theta$.The fixed point is unique.*

## 4 Some applications

Using Lemma 11, we prove some common fixed point and fixed point theorems in cone A-metric spaces with out assumption of normality for cones. Our results extend and generalize the fixed point results of

Dhamodharan and Krishnakumar (Dhamodharan & Krishnakumar, 2017) and Saluja (Saluja, 2020).

**Theorem 4.1.** *Let (X,A) be a complete cone A-metric space and d is a $c_a$-distance on X.Let $f : X \to X$ be a self mapping satisfies the following contractive condition*

$$A(fx, fx, ..., fy) \leq \alpha_1 A(gx, gx, .., gy) + \alpha_2 A(gx, gx, ..., fx) + \alpha_3 A(gy, gy, ..., fy) + \alpha_4 A(gx, gx, .., fy),$$

*for all $x, y \in X$ where $\alpha_i \in (0, 1), i = 1, 2, 3, 4$ such that $\alpha_1 + \alpha_2 + \alpha_3 + 3\alpha_4 < 1$. If f(X) is a subset of g(X) and g(X) is a complete subspace of X, then f and g have a coincidence point $x^*$ in X. Furthermore, if f and g are weakly compatible, then f and g have a unique common fixed point.*

**Proof.** Since the cone A-metric function is a $c_a$–distance on X by Lemma 10. Put $d(x_1, x_2, ..., x_n) = A(x_1, x_2, ..., x_n)$ in Theorem 13. The proof is complete. In the following theorem, we prove the fixed point theorem for self mappings in a complete cone A-metric space. ☐

**Theorem 4.2.** *Let (X,A) be a complete cone A-metric space and d is a $c_a$-distance on X.Let $f : X \to X$ be a self mapping satisfies the following contractive condition*

$$A(fx, fx, ..., fy) \leq \alpha_1 A(x, x, .., y) + \alpha_2 A(x, x, ..., fx) + \alpha_3 A(y, y, ..., fy) + \alpha_4 A(x, x, .., fy),$$

*for all $x, y \in X$ where $\alpha_i \in (0, 1), i = 1, 2, 3, 4$ such that $\alpha_1 + \alpha_2 + \alpha_3 + 3\alpha_4 < 1$. Then f has a fixed point $x^* \in X$ and for any $x \in X$ , iterative sequence $\{f^n x\}$ converges to the fixed point. The fixed point is unique*

**Proof.** In Theorem 4.1, put gx =x . The proof is complete. ☐

As a consequence of Theorem 4.2, we get the fixed point theorem of Banach contraction type, Kannan contraction type and Reich contraction type in a cone A-metric space respectively.

**Result 4.3.** *Let (X,A) be a complete cone A-metric space. Let $f : X \to X$ be a self mapping satisfies one of the following contractive conditions for all $x, y \in X$*

*i.*

$$A(fx, fx, .., fy) \leq \alpha A(x, x, .., y),$$

*where $\alpha \in [0, 1)$ is a constant.*

*ii.*

$$A(fx, fx, .., fy) \leq \alpha_1 A(x, x, .., fx) + \alpha_2 A(y, y, ..., fy),$$

*where $\alpha_1, \alpha_2 \in [0, 1)$ are constants such that $\alpha_1 + \alpha_2 < 1$.*

*iii.*

$$A(fx, fx, .., fy) \leq \alpha_1 A(x, x, .., y) + \alpha_2 A(x, x, ..., fx) + \alpha_3 A(y, y, .., fy),$$

*where $\alpha_1, \alpha_2, \alpha_3 \in [0, 1)$ are constants such that $\alpha_1 + \alpha_2 + \alpha_3 < 1$.*

*Then f has a fixed point $x^* \in X$ and f or any $x \in X$ , iterative sequence $\{f^n x\}$ converges to the fixed point.The fixed point is unique.*

## 5 Conclusion

In 2017, Fernandez, Saelee, Saxena, Malvia and Kumam presented the concept of an $A$-cone metric space over Banach algebra as a generalization of A-metric spaces and cone metric spaces over Banach algebra. They also defined generalized Lipschitz and expansive maps in such maps and establish some fixed point theorems for such maps in the setting of the new space. We present the concept of a $c_A$-distance in a cone A-metric space with some illustrative examples inspired from their article. Then, some common fixed point theorems for weakly compatible self mappings are gotten by using this new distance. Also, some common fixed point results in cone A-metric spaces for weakly compatible self mappings are held without assumption of the normality for cones. Our results extend and generalize the fixed point results of Dhamodharan and Krishnakumar (Dhamodharan & Krishnakumar, 2017) and Saluja (Saluja, 2020).

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