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Theory and Applications of the Double Laplace Transform for Local Derivatives with the Mittag-Leffler Kernel

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Mittag Leffler Çekirdeği ile Lokal Türevler için Çift Katlı Laplace Dönüşümü Teorisi ve Uygulamaları

Burak OZKUCUK*, Erdal BAS

Fırat University, Faculty of Science, Department of Mathematics, Elazığ , Türkiye

Abstract
This article aims to define the M-derivative double Laplace transform, which is the general form of the six-parameter conformable derivative involving the Mittag-Leffler function. It is expressed with several theorems and will give us a useful and dependable method for solving fractional M-derivative partial differenial equations. Furthermore, the application of these given definitions and theorems to fractional partial differential equations is shown. Finding solutions to partial differential equations containing M-derivatives that can match mathematical, engineering, and physical mode ls may be accomplished with the use of this transformation.

***Keywords** M-derivative, Laplace transform, Double M-Derivative, Truncated Mittag-Leffler function.*

Öz
Bu makale, Mittag-Leffler fonksiyonunu içeren altı parametreli uyumlu türevin genel formu olan M-türevi çift katlı Laplace dönüşümünü tanımlamayı amaçlamaktadır. Birkaç teoremle ifade edilir ve bize M-türevi kısmi diferansiyel denklemleri çözmek için kullanışlı ve güvenilir bir yöntem verecektir. Ayrıca, verilen bu tanım ve teoremlerin kesirli kısmi diferansiyel denklemlere uygulanması gösterilmiştir. Matematiksel, mühendislik ve fiziksel modellerle eşleşebilecek M-türevlerini içeren kısmi diferansiyel denklemlere çözümler bulmak, bu dönüşümün kullanılmasıyla gerçekleştirilebilir.

***Anahtar Kelimeler** M-türev, Laplace dönüşümü, İki katlı M- Laplace dönüşümü, Mittag Leffler fonksiyonu

1. Introduction

One of the most significant studies conducted by humanity to better comprehend nature is described in a letter L'Hospital wrote to Leibniz in 1695 and which establishes the terms derivative and integral, the building blocks of fractional calculations. Following this letter, it caught the interest of other scientists, leading to the substantial contributions of numerous mathematicians including Euler, Lagrange, Laplace, Lacroix, Fourier, Liouville, Riemann, Greer, Holmgren, Grünwald, Letnikov, Sonin, Laurent, Nekrassov, Krug, and Weyl [Kurt 2018, Özkan and Kurt 2018]. Their contributions have helped physics, engineering, and other scientific fields advance. In recent years, there have been numerous advancements in the field of fractional calculus, as well as numerous definitions of fractional derivatives. Riemann-Liouville, Caputo, Grüwald-Letnikov, Hadamard, Marchaud, Riesz, Wely, and Erdely-Kober are a few of the authors who defined fractional derivative definitions [Kilbas 2006]. The Riemann-Liouville fractional nonlocal derivative had certain drawbacks, thus Caputo defined a new non local fractional derivative in 1967 that was superior to many other fractional non local derivatives. But since the derivative of the resultant function, product, and quotient of two functions is not defined by these non-local fractional derivative definitions, Khail and his colleagues defined the local conformable derivative [Khalil, R., Horani, M. Al., Yousef, A., Sababheh, M. 2014] and the local conformable integral [J.V.D.C Sousa, E.C. de Oliveira, 2017], which are close to the classical derivative.

Additionally, Abdeljawad developed the chain rule, exponential functions, Gronwall's inequality, partial integration, Taylor series expansion, which are some important features of the harmonic derivative, and defined the Laplace transform in terms of the harmonic derivative [Abdeljawad, T., 2015]. Another type of local derivative and integral was defined by Katugampola [Katugampola, U.N., 2011]. In 2018, Ozkan and Kurt expressed and proved certain fundamental features of the conformable Laplace transform, and then they used these properties to get the solutions of conformable fractional integral and integro-differential equations. In a different work, Ozkan and Kurt defined the double conformable Laplace transform and discussed some of its

*Sorumlu Yazar/Corresponding Author: Burak OZKUCUK
e-posta/e-mail:burakozkucukidil@gmail.com

characteristics. Using these characteristics, they were able to fully solve the conformable fractional-order heat and telegraph problem [Kurt 2018, Özkan and Kurt 2018]. Jarad F., Uğurlu E., Abdeljawad T. and Baleanu, D., investigated definitions and theorems in conformable fractional partial derivatives in 2017 [Jarad, F., Uğurlu, E., Abdeljawad, T. and Baleanu, D., 2017]. The M –derivative, a brand-new local derivative using the Mittag-Leffler function, was published in 2017 by Sousa and Oliveira [J.V.D.C Sousa, E.C. de Oliveira, 2017]. The characteristics of integer-order computations are satisfied by this newly found M-derivative. [Katugampola, U.N., 2011]. In 2020, Jarad F. and Abdeljawad T. examined the convolution theorem and Laplace transform in comformable derivatives [Jarad, F., Abdeljawad, T., 2020]. The spectrum solutions of differential equations with fractional M-derivatives under starting circumstances were studied in 2020 by Bas E. and Acay B. [Bas, E., Acay, B., 2020]. Bas, Acay, and Abdeljawad published this work to scientific resources in 2020 after obtaining the corresponding values of the M-derivative in Laplace transforms [Bas, E., Acay, B., and T. Abdeljawad, 2020].

2. Description of the truncated M-derivative and some fundamental tools

Definition 2.1. The fractional M-derivative for $0 < \beta \leq 1$ is;

$$ {}_iD_M{}^{\beta\varphi} g(t) = \lim_{\epsilon \to 0} \frac{g\left(t_i E_\varphi(\varepsilon t^{-\beta})\right) - g(t)}{\varepsilon} \quad (1)$$

it's described as [Jarad, F., Uğurlu, E., Abdeljawad, T. and Baleanu, D., 2017].

Definition 2.2. The left M-integral of an integrable function $f, (a,t]$ with $a \geq 0, t \geq a,$ and $0 < \rho \leq 1$ is defined as [Bas, E., Acay, B., and T. Abdeljawad, 2020]:

$$ {}_M\mathcal{J}_a^{\rho\gamma} f(t) = \int_a^t f(x) d_\rho(a,x)$$

$$= \Gamma(\gamma+1) \int_a^t f(x)(x-a)^{\rho-1} dx. \quad (2)$$

where $d_\rho(x,a) = \Gamma(\gamma+1)(x-a)^{\rho-1}\ d_\rho x$ can be written, and if the M-integral from the right at the point $a = 0$ is defined as follows [Bas, E., Acay, B., and T. Abdeljawad, 2020]:

$$ {}^{\rho\gamma}_M\mathcal{J}_a f(t) = \int_a^t f(x) d_\rho(b,x)$$

$$= \Gamma(\gamma+1) \int_a^t f(x)(b-x)^{\rho-1} dx. \quad (3)$$

Definition 2.3. Let $f, g\colon [a,b] \to \mathbb{R}$ and f, g differentiable functions and $0 < \rho \leq 1$. Then the partial integration of the M-derivative from the left and right respectively is as follows [Bas, E., Acay, B., and T. Abdeljawad, 2020]:

$$\Gamma(\gamma+1) \int_a^b (t-a)^{\rho-1} f(t)\ {}_iD_M{}^{\rho\Upsilon} g(t) dt$$

$$= f(t).g(t) l_a^b - \Gamma(\gamma+1) \int_a^b (t-a)^{\rho-1} g(\mathrm{t})$$

$$\mathrm{x}\ {}_iD_M{}^{\rho\Upsilon} f(t) dt. \quad (4)$$

and

$$\Gamma(\gamma+1) \int_a^b (t-a)^{\rho-1} f(t)\ {}_iD_M{}^{\rho\Upsilon} g(t) dt = f(t) g(t) l_a^b$$

$$+ \Gamma(\gamma+1) \int_a^b (b-t)^{\rho-1} g(\mathrm{t})\ {}_iD_M{}^{\rho\Upsilon} f(t) dt. \quad (5)$$

Definition 2.4. Let $f\colon [a,\infty) \to \mathbb{R}$ be a real-valued function, $a \in R$, $\gamma > 0$ and $0 < \rho \leq 1$. In this case, the Laplace transform of the M-derivative of the function f is;

$$\mathcal{L}_{\rho,\gamma}^a \{f(t)\}(s) = F_{\rho,\gamma}^a(s)$$

$$= \Gamma(\rho+1) \int_b^\infty e^{-s\frac{\Gamma(\gamma+1)(t-a)^\rho}{\rho}} f(t)(t-a)^{\rho-1} dt \quad (6)$$

it is defined as [Bas, E., Acay, B., and T. Abdeljawad 2020].

Theorem 2.1. Let f:$[a,\infty) \to \mathbb{R}$ be a defined function. $\mathcal{L}_{\rho,\gamma}^a \{f(t)\}(s) = \mathrm{F}_{\rho,\gamma}^a(s)$ and from the classical Laplace transform

$$\mathcal{L}_{\rho,\gamma}^a \{f(t)\}(s) = F_{\rho,\gamma}^a(s) = \mathcal{L}\left\{ f\left(a + \left(\frac{\rho t}{\Gamma(\gamma+1)}\right)^{\frac{1}{\rho}} \right) \right\} \quad (7)$$

the expression is obtained by [Bas, E., Acay, B., and T. Abdeljawad, 2020].

Definition 2.4. Let the functions $f(t)$ and $g(t)$ have a piecewise continuous and exponential order, the convolution integral of the functions f and g on the M-derivative is

$$(f * g)(t) = \Gamma(\gamma+1)$$

$$x \int_a^t f(\tau) g(a + ((t-a)^\rho - (\tau-a)^\rho) \frac{1}{\rho} (t-a)^{\rho-1} dt.$$

(8)

it is defined as [Bas, E., Acay, B., and T. Abdeljawad, 2020].

Theorem 2.2. $g : [a,\infty) \to \mathbb{R}$, $a \in \mathbb{R}$, $0 < \rho \leq 1$ $\Upsilon > 0$, $s > 0$ and let $f(t)$ and $g(t)$ be functions such that

$\mathcal{L}_{\rho,\gamma}^a[f(t)](s) = \mathcal{F}_{\rho,\gamma}^a(s)$ and $\mathcal{L}_{\rho,\gamma}^a[g(t)](s) = \mathcal{G}_{\rho,\gamma}^a(s)$

To ensure the conditions

$$\mathcal{L}_{\rho,\gamma}^a \{f * g\}(\mathrm{t}) = \mathcal{F}_{\rho,\gamma}^a(s).\ \mathcal{G}_{\rho,\gamma}^a(s),$$

the operation { $f * g$ } defined by its equality, is called the convolution of functions f and g [Bas, E., Acay, B., and T. Abdeljawad, 2020].

We can express the double Laplace transformations of some functions by means of the M-derivative in the following table [Bas, E., Acay, B., and T. Abdeljawad, 2020]:

- $\mathcal{L}^{a}_{\rho,\gamma}\{1\}(s) = \frac{1}{s}, s > 0.$
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\left(\Gamma(\gamma+1)\frac{(t-a)^{\rho}}{\rho}\right)^{\beta}\right\}(s) = \frac{\Gamma(\beta+1)}{s^{1+\beta}}$, $Re(\beta) > 0, s > 0.$
- $\mathcal{L}^{a}_{\rho,\gamma}\{t\}(s) = \frac{\Gamma\left(\frac{1}{\rho}+1\right)\left(\frac{\rho}{\Gamma(\gamma+1)}\right)^{\frac{1}{\rho}}}{s^{\frac{\rho+1}{\rho}}}, s > 0.$
- $\mathcal{L}^{a}_{\rho,\gamma}\{t^{k}\}(s) = \frac{\Gamma\left(\frac{k}{\rho}+1\right)\left(\frac{\rho}{\Gamma(\gamma+1)}\right)^{\frac{k}{\rho}}}{s^{\frac{\rho+k}{\rho}}}$, $s > 0$ and k is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{e^{c\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}}\right\}(s) = \frac{1}{s-c}$, $s > c$ and c is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}e^{c\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}}\right\}(s)=\frac{1}{(s-c)^{2}}$, c is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\sin(b\,\Gamma(\gamma+1)\frac{t^{\rho}}{\rho})\right\}(s)=\frac{b}{b^{2}+s^{2}}$, b is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\cos(b\,\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}\right\}(s)=\frac{s}{b^{2}+s^{2}}$, b is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{e^{-c\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}}\sin(b\;\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}\right\}(s)$ $= \frac{b}{b^{2}+(s+c)^{2}}$, b and c is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{e^{-c\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}}\cos(b\,\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}\right\}(s)$ $=\frac{s+c}{b^{2}+(s+c)^{2}}$, b and c is any constant.

3. Main Theorical Results and Applications

Definition 3.1. Let $u(x,t)$ be a piecewise continuous function on the interval $[0,\infty)\times[0,\infty)$ of exponential order. Consider for some $\gamma,\sigma > 0$ $sup_{x>0,t>0}\frac{|u(x,t)|}{e^{\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}} < \infty$. Under these conditions M −derivative double Laplace transform is defined by

$${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{u(x,t)\}= U(p,s)$$

$$= \int_{0}^{\infty}\int_{0}^{\infty} \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}$$

$$x\, u(x,t)d_{\alpha}t d_{\beta}x \quad (9)$$

where $p,s \in ₵, 0 < \alpha, \beta \leq 1$ and the integrals are by means of conformable fractional integral with respect to t and x respectively.

Theorem 3.1. Let $u(x,t), v(x,t)$ be two functions which have the M-Derivative double Laplace transform and $c_1, c_2 \in \mathbb{R}$

$${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{c_1 u(x,t) + c_2 v(x,t)\}$$

$$= c_1{}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{u(x,t)\}+ c_2{}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{v(x,t)\} \quad (10)$$

equality is ensured, that is ${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}$ the operator is linear.

Proof. The theorem is easily proved easily by using the linearity property of the double Laplace transform with the definition of double M-Laplace.

Theorem 3.2. $u(x,t)$ is a function that provides a double M-Laplace transform, and the ${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}$ transform, $c, d \in R$, provides the translation property as follows.

$${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\left\{e^{-c\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}-d\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}u(x,t)\right\}$$

$=U(p+c, s+d).$

Proof. If the function $e^{-c\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}-d\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}u(x,t)$ is written instead of the function $u(x,t)$ expressed in the equation (9) in the definition of the M-derivative Laplace transform,

$$= \Gamma(\gamma+1)\,\Gamma(\sigma+1)$$

$$x\int_{0}^{\infty}\int_{0}^{\infty} e^{-p\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}e^{-s\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}e^{-c\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}-d\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}u(x,t)d_{\alpha}t d_{\beta}x$$

$$=\int_{0}^{\infty}\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}-d\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}$$

$$\text{X}\left[\int_{0}^{\infty}\Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}-c\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}u(x,t)d_{\beta}x\right]d_{\alpha}t \quad (11)$$

is obtained. If we consider the definition of M-Laplace transform

$$= \int_{0}^{\infty}\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}-d\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}[U(p+c,t)]\,d_{\alpha}t$$

$= U(p+c, s+d)$

it is found, which completes the proof.

Theorem 3.3. The function $u(x,t)$ is β with respect to x. according to order and t, α. the double M −derivative Laplace transforms of the fractional partial derivatives of order, ${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}[u(x,t)]= U(p,s)$, β and α. of the function $u(x,t)$ the double M-derivative Laplace transformations of order fractional partial derivatives are respectively

$$(i)\ {}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{{}_{M}D_{x}{}^{\beta}u(x,t)\} = pU(p,s) - U(0,s) \quad (10)$$

$$(ii)\ {}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{{}_{M}D_{t}{}^{\alpha}u(x,t)\} = sU(p,s) - U(p,0) \quad (11)$$

$$(iii)\ {}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{{}_{M}D_{x}{}^{\beta}{}_{M}D_{t}{}^{\alpha}u(x,t)\}$$

$$= psU(p,s) - pU(p,0) - s\,U(0,s) + U(0,0) \quad (12)$$

it is expressed in the form.

Proof.

(i) (9) if $u(x,t)$ is written instead of the function ${}_{M}D_x{}^{\beta}u(x,t)$ in the double M-Laplace transform given by the definition

$${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{\beta}u(x,t)\}$$

$$=\int_0^\infty\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}$$

$$\text{x}\,\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}[{}_{M}D_x{}^{\beta}u(x,t)]d_\beta x d_\alpha t$$

$$=\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}$$

$$x\left[\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\,[{}_{M}D_x{}^{\beta}u(x,t)]d_\beta x\right]d_\alpha t$$

$$=\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}$$

$$x\left[\Gamma(\sigma+1)\int_0^\infty e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\,[{}_{M}D_x{}^{\beta}u(x,t)]d_\beta x\right]d_\alpha t$$

$$=\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}\left(u(x,t)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\Big|_0^\infty\right.$$

$$\left.+p\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\,(u(x,t))d_\beta x\right)d_\alpha t$$

$$=\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}[pU(p,t)-U(0,t)]\,d_\alpha t$$

$$=p\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}\,U(p,t)d_\alpha t$$

$$-\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}\,U(0,t)d_\alpha t$$

$$=pU(p,s)-U(0,s)$$

is acquired.

(ii) (9) the proof is made if ${}_{M}D_t{}^{\alpha}u(x,t)$ is written instead of the function $u(x,t)$ in the double M-Laplace transform given by the definition and the operations of Theorem 3.3 (i) are repeated.

(iii) (9) if the double M-Laplace transform given by the equation ${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{\beta}{}_{M}D_t{}^{\alpha}u(x,t)\}$ is taken instead of the function $u(x,t)$ in the definition,

$$=\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}$$

$$\text{x}\left[\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}\,[{}_{M}D_x{}^{\beta}{}_{M}D_t{}^{\alpha}u(x,t)]d_\alpha t\right]d_\beta x$$

$$=\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}$$

$$x\left[\Gamma(\gamma+1)\int_0^\infty e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}\,[{}_{M}D_x{}^{\beta}{}_{M}D_t{}^{\alpha}u(x,t)]d_\alpha t\right]d_\beta x$$

$$=\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}$$

$$x\left(e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}{}_{M}D_x{}^{\beta}u(x,t)\Big|_0^\infty\right.$$

$$\left.+s\int_0^\infty e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}\,{}_{M}D_x{}^{\beta}\Gamma(\gamma+1)u(x,t)d_\alpha t\right)d_\beta x$$

$$=\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}$$

$$x[s\,{}_{M}D_x{}^{\beta}U(x,s)-{}_{M}D_x{}^{\beta}U(x,0)]\,d_\beta x$$

$$=s\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\,[{}_{M}D_x{}^{\beta}U(x,s)]d_\beta x$$

$$-\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\,{}_{M}D_x{}^{\beta}\,[U(x,0)]d_\beta x$$

$$=s\,(pU(p,s)-U(0,s))-\left(e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}U(x,0)\Big|_0^\infty\right.$$

$$\left.-\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}u(x,0)\,d_\beta x\right)$$

$$=psU(p,s)-pU(p,0)-s\,U(0,s)+U(0,0)$$

The proof is completed.

Theorem 3.4. Let $0<\alpha,\beta\leq 1$ and be $u(x,t)\in \mathbb{C}^l(\mathbb{R}^+\times\mathbb{R}^+)$ and $l=max(m,n)$ with $m,n\in\mathbb{N}$. In addition, Let there be double M-Laplace transformations of the functions $u(x,t)$, ${}_{M}D_x{}^{(\mathrm{i})\beta}\,u(x,t)$ and ${}_{M}D_t{}^{(\mathrm{j})\alpha}\,u(x,t)$, $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$. In this case

(i) $${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{(\mathrm{m})\beta}u(x,t)\}=p^mU(p,s)$$

$$-p^{m-1}U(0,s)-\sum_{i=1}^{m-1}p^{m-1-i}\,{}^{M}\mathcal{L}_t^{\alpha}[{}_{M}D_x{}^{(\mathrm{i})\beta}u(0,t)]$$

(ii) $${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_t{}^{(\mathrm{n})\alpha}u(x,t)\}=s^nU(p,s)$$

$$-s^{n-1}U(p,0)-\sum_{j=1}^{n-1}s^{n-1-j}\,{}^{M}\mathcal{L}_x^{\beta}[{}_{M}D_t{}^{(\mathrm{j})\alpha}u(x,0)]$$

(iii) $${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{(\mathrm{m})\beta}{}_{M}D_t{}^{(\mathrm{n})\alpha}(u(x,t))\}$$

$$=p^ms^n(U(p,s)-s^{-1}U(p,0)-p^{-1}U(0,s)$$

$$-\sum_{j=1}^{n-1}s^{-1-j}\,{}^{M}\mathcal{L}_x^{\beta}[{}_{M}D_t{}^{(\mathrm{j})\alpha}U(p,0)]$$

$$-\sum_{i=1}^{m-1}p^{-1-i}\,{}^{M}\mathcal{L}_t^{\alpha}[{}_{M}D_x{}^{(\mathrm{i})\beta}U(0,s)]$$

$$+\sum_{j=1}^{n-1}s^{-1-j}p^{-1}\left({}_{M}D_t{}^{(\mathrm{j})\alpha}U(0,0)\right)$$

$$+\sum_{i=1}^{m-1}s^{-1}\,p^{-1-i}\,{}_{M}D_x{}^{(\mathrm{i})\beta}U(0,0)$$

$$+\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}s^{-1-j}p^{-1-i}\,{}_{M}D_t{}^{(\mathrm{j})\alpha}\,U(0,0)$$

$$+p^{-1}s^{-1}U(0,0))$$

equality is ensured.

Proof.

(i) According to the total method, we must first show its accuracy for $m=1$. we proved its correctness for $m=1$ in Theorem 3.3 (i). Now we will accept its correctness for $m-1$ and show that this equality is correct for m.

$${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{(m)\beta}u(x,t)\}$$

$$=p^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{(m-1)\beta}u(x,t)\}-{}^{M}\mathcal{L}_t^{\alpha}\{{}_{M}D_x{}^{(m-1)\beta}u(0,t)\}$$

it can be written. since equality is considered correct for $m-1$

$${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{(m)\beta}u(x,t)\}$$

$$=p(p^{m-1}U(p,s)-p^{m-2}U(0,s)$$

$$-\sum_{i=1}^{m-2}p^{m-2-i}\,{}^{M}\mathcal{L}_t^{\alpha}\left[{}_{M}D_x{}^{(i)\beta}u(0,t)\right]$$

$$-{}^{M}\mathcal{L}_t^{\alpha}\{{}_{M}D_x{}^{(m-1)\beta}u(0,t)\}$$

$$=p^{m}U(p,s)-p^{m-1}U(0,s)$$

$$-\sum_{i=1}^{m-2}p^{m-1-i}\,{}^{M}\mathcal{L}_t^{\alpha}\left[{}_{M}D_x{}^{(i)\beta}u(0,t)\right]$$

$$-{}^{M}\mathcal{L}_t^{\alpha}\{{}_{M}D_x{}^{(m-1)\beta}u(0,t)\}$$

$$=p^{m}U(p,s)-\;p^{m-1}U(0,s)$$

$$-\sum_{i=1}^{m-1}p^{m-1-i}\,{}^{M}\mathcal{L}_t^{\alpha}\{{}_{M}D_x{}^{(i)\beta}u(0,t)\}$$

is obtained.

(ii) If the necessary operations are performed in a manner similar to Theorem 3.4 (i), the proof will be made.

(iii) We will use the equations (i) and (ii) to prove this theorem. (i) from equality

$${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{(m)\beta}{}_{M}D_t{}^{(n)\alpha}(u(x,t)\}$$

$$=\;{}^{M}\mathcal{L}_t^{\alpha}\,\{p^{m}{}_{M}D_t{}^{(n)\alpha}U(p,t)-\;p^{m-1}{}_{M}D_t{}^{(n)\alpha}U(0,t)$$

$$-\sum_{i=1}^{m-1}p^{m-1-i}({}_{M}D_t{}^{(n)\alpha}{}_{M}D_x{}^{(i)\beta}u(0,t))\}$$

it can be arranged as. Due to the linearity property of the Laplace transform

$$=p^{m}\,{}^{M}\mathcal{L}_t^{\alpha}\left\{{}_{M}D_t{}^{(n)\alpha}U(p,t)\right\}-p^{m-1}{}^{M}\mathcal{L}_t^{\alpha}\left\{{}_{M}D_t{}^{(n)\alpha}U(0,t)\right\}$$

$$-{}^{M}\mathcal{L}_t^{\alpha}\left\{\sum_{i=1}^{m-1}p^{m-1-i}({}_{M}D_t{}^{(n)\alpha}{}_{M}D_x{}^{(i)\beta}u(0,t))\right\}$$

it can be written. (ii) in accordance with the equality contained in the expression

$${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{{}_{M}D_x{}^{(m)\beta}{}_{M}D_t{}^{(n)\alpha}(u(x,t))\}$$

$$=p^{m}(s^{n}U(p,s)-s^{n-1}U(p,0)$$

$$-\sum_{j=1}^{n-1}s^{n-1-j}\left({}_{M}D_t{}^{(j)\alpha}U(p,0)\right))$$

$$-p^{m-1}(s^{n}U(0,s)-s^{n-1}U(0,0)$$

$$\left.-\sum_{j=1}^{n-1}s^{n-1-j}\left({}_{M}D_t{}^{(j)\alpha}U(0,0)\right)\right)$$

$$-s^{n}\textstyle\sum_{i=1}^{m-1}p^{m-1-i}\left({}_{M}D_x{}^{(i)\beta}U(0,s)\right)$$

$$+s^{n-1}\textstyle\sum_{i=1}^{m-1}p^{m-1-i}\left({}_{M}D_x{}^{(i)\beta}U(0,0)\right)$$

$$-\textstyle\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}p^{m-1-i}\,s^{n-1-j}{}_{M}D_t{}^{(j)\alpha}{}_{M}D_x{}^{(i)\beta}U(0,0)$$

$${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\left\{{}_{M}D_x{}^{(m)\beta}{}_{M}D_t{}^{(n)\alpha}\big(u(x,t)\big)\right\}$$

$$=p^{m}s^{n}(U(p,s)-s^{-1}U(p,0)-p^{-1}U(0,s)$$

$$-\textstyle\sum_{j=1}^{n-1}s^{-1-j}{}^{M}\mathcal{L}_x^{\beta}[{}_{M}D_t{}^{(j)\alpha}U(p,0)]$$

$$-\;\textstyle\sum_{i=1}^{m-1}p^{-1-i}{}^{M}\mathcal{L}_t^{\alpha}[{}_{M}D_x{}^{(i)\beta}U(0,s)]$$

$$+\textstyle\sum_{j=1}^{n-1}s^{-1-j}p^{-1}\left({}_{M}D_t{}^{(j)\alpha}U(0,0)\right)$$

$$+\textstyle\sum_{i=1}^{m-1}s^{-1}\,p^{-1-i}{}_{M}D_x{}^{(i)\beta}U(0,0)$$

$$+\textstyle\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}s^{-1-j}p^{-1-i}\;{}_{M}D_t{}^{(j)\alpha}{}_{M}D_x{}^{(i)\beta}\;U(0,0)$$

$$+p^{-1}s^{-1}U(0,0))$$

the result is reached. Thus, the proof is completed

Theorem 3.5. We can express the double Laplace transformations of some functions by means of the M-derivative in the following way:

- ${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{1\}=\frac{1}{ps},\;p>0,s>0$
- ${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{x^{m}t^{n}\}=\frac{1}{ps},$
  x$\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\left(\frac{\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}\,\Gamma\left(1+\frac{n}{\alpha}\right)\;\Gamma\left(1+\frac{m}{\beta}\right),$
  $p>0,s>0$ m and n are any constant.

Proof.

If $u(x,t)=x^{m}t^{n}$is taken instead of the function $u(x,t)$ in the equation (8) in the definition of the fractional M-derivative Laplace transform, the double M-Laplace transform

$${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{x^{m}t^{n}\}=\int_0^{\infty}\int_0^{\infty}\Gamma(\sigma+1)\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}$$

$$\text{x}\;\Gamma(\gamma+1)\mathrm{e}^{-s\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}}x^{m}t^{n}d_{\alpha}t d_{\beta}x$$

$$=\int_0^{\infty}\Gamma(\sigma+1)\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}x^{m}$$

$$x\left[\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}t^n d_\alpha t\right] d_\beta x$$

it is expressed in the form. Let's solve the integral

$I = \int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}t^n d_\alpha t$ here

if $u = s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}$ is transformed and written instead of in the integral

$$du = s\Gamma(\gamma+1)\frac{\alpha t^{\alpha-\alpha}}{\alpha}d_\alpha t,\quad \frac{du}{s} = \Gamma(\gamma+1)d_\alpha t$$

$t^n = \left(\frac{u\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}$ *I, then*

$$I = \int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}t^n d_\alpha t$$

$$= \int_0^\infty e^{-u}\left(\frac{u\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\frac{du}{s} = \frac{1}{s}\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\int_0^\infty e^{-u}u^{\frac{n}{\alpha}}du$$

it is obtained that, by looking at how the gamma function is defined in the $\int_0^\infty e^{-u}u^{\frac{n}{\alpha}}du$ integral

$$\int_0^\infty e^{-u}u^{\frac{n}{\alpha}}du = \Gamma\left(1+\frac{n}{\alpha}\right),$$

$$=\frac{1}{s}\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\int_0^\infty e^{-u}u^{\frac{n}{\alpha}}du = \frac{1}{s}\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\Gamma\left(1+\frac{n}{\alpha}\right).$$

$II = \int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}x^m d_\beta x$ let's solve the integral.

Where $v = p\Gamma(\sigma+1)\frac{x^\beta}{\beta}$ transformed and

$$dv = p\Gamma(\sigma+1)\frac{\beta x^{\beta-\beta}}{\beta}d_\beta x,\quad \frac{dv}{p} = \Gamma(\sigma+1)d_\beta x$$

If $x^m = \left(\frac{v\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}$ is written instead in the II integral,

$$II = \int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}x^m d_\beta x$$

$$= \int_0^\infty e^{-v}\left(\frac{v\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}\frac{dv}{p} = \frac{1}{p}\left(\frac{v\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}\int_0^\infty e^{-v}v^{\frac{m}{\beta}}dv$$

It is discovered that, In addition, considering the definition of the gamma function $\int_0^\infty e^{-v}v^{\frac{m}{\beta}}dv$ integral

$$\int_0^\infty e^{-v}v^{\frac{m}{\beta}}dv = \Gamma\left(1+\frac{m}{\beta}\right),$$

$$=\frac{1}{p}\left(\frac{v\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}\int_0^\infty e^{-v}v^{\frac{m}{\beta}}dv = \frac{1}{p}\left(\frac{v\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}\Gamma\left(1+\frac{m}{\beta}\right).$$

$$= \int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}x^m$$

$$x\left[\int_0^\infty \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}t^n d_\alpha t\right] d_\beta x$$

$$= \int_0^\infty \Gamma(\sigma+1)\, e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}x^m$$

$$x\left[\frac{1}{s}\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\Gamma\left(1+\frac{n}{\alpha}\right)\right]d_\beta x$$

$$= \frac{1}{s}\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\Gamma\left(1+\frac{n}{\alpha}\right)\int_0^\infty \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}x^m d_\beta x$$

$$= \frac{1}{s}\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\Gamma\left(1+\frac{n}{\alpha}\right)\frac{1}{p}\left(\frac{\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}\Gamma\left(1+\frac{m}{\beta}\right)$$

$$= \frac{1}{ps}\left(\frac{\alpha}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\alpha}}\left(\frac{\beta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\beta}}\Gamma\left(1+\frac{n}{\alpha}\right)\Gamma\left(1+\frac{m}{\beta}\right)$$

the result is reached.

- ${}^M\mathcal{L}_t^\alpha {}^M\mathcal{L}_x^\beta\left\{e^{c\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}+d\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\right\}$
  $= \frac{1}{(p-d)(s-c)}, p > c, s > d,$ c and d are any constant.
- ${}^M\mathcal{L}_t^\alpha {}^M\mathcal{L}_x^\beta\left\{\sin\left(a\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\right.$
  $\left. x\sin\left(b\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\} = \frac{ab}{(a^2+s^2)(b^2+p^2)},$ a and b are any constant.
- ${}^M\mathcal{L}_t^\alpha {}^M\mathcal{L}_x^\beta\left\{\cos\left(a\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\right.$
  $\left. x\cos\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\} = \frac{ps}{(a^2+s^2)(b^2+p^2)},$ a and b are any constant.
- ${}^M\mathcal{L}_t^\alpha {}^M\mathcal{L}_x^\beta\left\{\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right.$
  $\left. x\, e^{c\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}+d\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\right\}$
  $= \frac{1}{(p-d)^2(s-c)^2},$
  $p > c, s > d, c$ and d are any constant.
- ${}^M\mathcal{L}_t^\alpha {}^M\mathcal{L}_x^\beta\left\{\sinh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\right.$
  $\left. x\sinh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\}$
  $= \frac{1}{(s^2-1)(p^2-1)}.$

Proof.

(9) instead of the function $u(x,t)$ in the double M-Laplace transform given by the definition

if $\sinh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\sinh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)$ is written and

$$\sinh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right) = \frac{e^{\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}} - e^{-\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}}{2}$$

$$ve\ \sinh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right) = \frac{e^{\Gamma(\sigma+1)\frac{x^\beta}{\beta}} - e^{-\Gamma(\sigma+1)\frac{x^\beta}{\beta}}}{2}$$

expansions are taken into account,

$${}^M\mathcal{L}_t^\alpha {}^M\mathcal{L}_x^\beta\left\{\sinh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\sinh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\}$$

$= \int_0^\infty \int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} \Gamma(\gamma+1)$

x$\left[sinh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right) sinh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right] e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}$

$x\, d_\alpha t d_\beta x$

$= \int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} \left[\frac{e^{\Gamma(\sigma+1)\frac{x^\beta}{\beta}} - e^{-\Gamma(\sigma+1)\frac{x^\beta}{\beta}}}{2}\right]$

x$\left(\int_0^\infty \Gamma(\gamma+1) e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}} \left[\frac{e^{\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}} - e^{-\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}}{2}\right] d_\alpha t \right)$

x$d_\beta x$

$= \int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}}$

$x \left[\frac{e^{\Gamma(\sigma+1)\frac{x^\beta}{\beta}} - e^{-\Gamma(\sigma+1)\frac{x^\beta}{\beta}}}{2}\right] \left(\frac{1}{s^2-1}\right) d_\beta x$

$= \frac{1}{s^2-1} \int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} \left[\frac{e^{\Gamma(\sigma+1)\frac{x^\beta}{\beta}} - e^{-\Gamma(\sigma+1)\frac{x^\beta}{\beta}}}{2}\right] d_\beta x$

$= \left(\frac{1}{s^2-1}\right) \left[\frac{1}{2}\int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} e^{\Gamma(\sigma+1)\frac{x^\beta}{\beta}} d_\beta x \right.$

$\left. -\frac{1}{2}\int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} e^{-\Gamma(\sigma+1)\frac{x^\beta}{\beta}} d_\beta x \right]$

$= \left(\frac{1}{s^2-1}\right) \left[\frac{1}{2(p-1)} - \frac{1}{2(p+1)}\right] = \frac{1}{(s^2-1)(p^2-1)}.$

- ${}^{M}\mathcal{L}_t^{\alpha}\,{}^{M}\mathcal{L}_x^{\beta} \left\{ cosh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\right.$

  $\left. x\ cosh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\} = \frac{ps}{(s^2-1)(p^2-1)}$

Proof.

(9) instead of the function $u(x,t)$ in the double M-Laplace transform given by the definition

$cosh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right) cosh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)$ is written, the proof is completed if the

$cosh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right) = \frac{e^{\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}} + e^{-\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}}{2}$

$, cosh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right) = \frac{e^{\Gamma(\sigma+1)\frac{x^\beta}{\beta}} + e^{-\Gamma(\sigma+1)\frac{x^\beta}{\beta}}}{2}$ expansions are taken into account and similar operations are performed as in the (vii) form of the theorem.

- ${}^{M}\mathcal{L}_t^{\alpha}\,{}^{M}\mathcal{L}_x^{\beta} \left\{ e^{c\Gamma(\gamma+1)\frac{t^\alpha}{\alpha} + d\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\right.$

  $\left. x\cos\left(a\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\cos\left(b\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\}$

  $= \frac{(p-d)(s-c)}{(a^2+(s-c)^2)(b^2+(p-d)^2)}$, a, b, c and d are any constant.

- ${}^{M}\mathcal{L}_t^{\alpha}\,{}^{M}\mathcal{L}_x^{\beta} \left\{ e^{c\Gamma(\gamma+1)\frac{t^\alpha}{\alpha} + d\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\right.$

  $\left. x\sin\left(a\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)\sin\left(b\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\}$

  $= \frac{ab}{(a^2+(s+c)^2)(b^2+(p+d)^2)}$, a, b, c and d are any constant.

- ${}^{M}\mathcal{L}_t^{\alpha}\,{}^{M}\mathcal{L}_x^{\beta} \left\{ e^{c\Gamma(\gamma+1)\frac{t^\alpha}{\alpha} + d\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\right.$

  $\left. x sinh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right) sinh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\}$

  $= \frac{1}{((s-c)^2-1)((p-d)^2-1)}$, c and d are any constant.

- ${}^{M}\mathcal{L}_t^{\alpha}\,{}^{M}\mathcal{L}_x^{\beta} \left\{ e^{c\Gamma(\gamma+1)\frac{t^\alpha}{\alpha} + d\Gamma(\sigma+1)\frac{x^\beta}{\beta}}\right.$

  $\left. x cosh\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right) cosh\left(\Gamma(\sigma+1)\frac{x^\beta}{\beta}\right)\right\}$

  $= \frac{(p-d)(s-c)}{((s-c)^2-1)((p-d)^2-1)}$, c and d are any constant.

- ${}^{M}\mathcal{L}_t^{\alpha}\,{}^{M}\mathcal{L}_x^{\beta} \left\{ \left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)^b \left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^c \right\}$

  $= \frac{\Gamma(1+b)}{s^{1+b}} \frac{\Gamma(1+c)}{p^{1+c}}$,

  $Re\,(b) > 0, Re\,(c) > 0, s > 0$ and $p > 0$.

Proof.

(9) instead of the function $u(x,t)$ in the double M-Laplace transform given by the definition

$\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)^b \left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^c$ is written and similar operations are performed

${}^{M}\mathcal{L}_t^{\alpha}\,{}^{M}\mathcal{L}_x^{\beta} \left\{ \left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)^b \left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^c \right\}$

$= \int_0^\infty \int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}} \Gamma(\gamma+1)$

x$\left[\left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)^b \left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^c\right] d_\alpha t d_\beta x$

$= \int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} \left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^c$

$x \left(\int_0^\infty \Gamma(\gamma+1) e^{-s\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}} \left(\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}\right)^b d_\alpha t\right) d_\beta x$

$= \frac{\Gamma(1+b)}{s^{1+b}} \int_0^\infty \Gamma(\sigma+1) e^{-p\Gamma(\sigma+1)\frac{x^\beta}{\beta}} \left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^c d_\beta x$

$= \frac{\Gamma(1+b)}{s^{1+b}}\, \mathcal{L}^{a}_{\beta,\sigma} \left\{ \left(\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta} \right)^{c} \right\}$

$= \frac{\Gamma(1+b)}{s^{1+b}} \frac{\Gamma(1+c)}{p^{1+c}}$.

Definition 3.2. Let $f(x,t)$ and $g(x,t)$ be continuous functions. M-derivative double convolution in the sense of derivative

$(f ** g)(x,t)$

$= \Gamma(\gamma+1)\,\Gamma(\sigma+1)\int_0^x\int_0^t f(x-v,t-\mu)g(v,\mu)$
$x d_\beta v d_\alpha \mu$

it is defined by its integral.

Theorem 3.6. (Convolutions Theorem) Let $U(x,t)$ and $v(x,t)$ functions $U(p,s) = {}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{u(x,t)\}$ and $V(p,s) = {}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{v(x,t)\}$ have a double M-Laplace transform for $s>0$ and $p>0$. In this case, to express the double convolutions of the functions $u(x,t) ** v(x,t)$ to

${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{u(x,t) ** v(x,t)\} = U(p,s)\,V(p,s)$

is in the form.

Proof.

${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{u(x,t) ** v(x,t)\}$

$= {}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}[u\left(\left(\frac{\beta x}{\Gamma(\sigma+1)}\right)^{\beta}, \left(\frac{\alpha t}{\Gamma(\gamma+1)}\right)^{\alpha}\right)$

$** \, v\left(\left(\frac{\beta x}{\Gamma(\sigma+1)}\right)^{\beta}, \left(\frac{\alpha t}{\Gamma(\gamma+1)}\right)^{\alpha}\right)]$

It can be written. Since the double M-derivative Laplace transform on the right side of the above equation is known to provide the convolution property

${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{u\left(\left(\frac{\beta x}{\Gamma(\sigma+1)}\right)^{\beta}, \left(\frac{\alpha t}{\Gamma(\gamma+1)}\right)^{\alpha}\right)$

$** \, v\left(\left(\frac{\beta x}{\Gamma(\sigma+1)}\right)^{\beta}, \left(\frac{\alpha t}{\Gamma(\gamma+1)}\right)^{\alpha}\right)\}$

$= {}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\left\{u\left(\left(\frac{\beta x}{\Gamma(\sigma+1)}\right)^{\beta}, \left(\frac{\alpha t}{\Gamma(\gamma+1)}\right)^{\alpha}\right)\right\}$

x $\mathcal{L}^{\alpha}_{t}\mathcal{L}^{\beta}_{x}\{v\left(\left(\frac{\beta x}{\Gamma(\sigma+1)}\right)^{\beta}, \left(\frac{\alpha t}{\Gamma(\gamma+1)}\right)^{\alpha}\right)\}$

$= U(p,s)\,V(p,s)$,

this proves the theorem.

Applications 3.1.

${}_{M}D_{t}{}^{(2)\alpha}u(x,t) - {}_{M}D_{x}{}^{(2)\beta}u(x,t) + u(x,t)$

$+\int_0^x\int_0^t \Gamma(\sigma+1)\;\Gamma(\gamma+1)$

x $e^{\frac{\Gamma(\gamma+1)(t-\mu)^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{(x-v)^{\beta}}{\beta}}\; u(v,\mu)d_\beta v d_\alpha \mu$

$= e^{\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}\left(1+ e^{\Gamma(\gamma+1)\frac{t^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}\right)$ (13)

about to be

$u(0,t) = e^{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}}$

$u(x,0) = e^{\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}$ (14)

${}_{M}D_{x}{}^{\beta}u\,(0,t) = e^{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}}$

${}_{M}D_{t}{}^{\alpha}u(x,0) = e^{\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}$

solve the M-derivative partial integro-differential equation given by the initial conditions.

Solution.

(13) if the double M-Laplace transform is applied to both sides of the M-derivative integro-differential equation given by the equation and the convolution theorem is applied

${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{{}_{M}D_{t}{}^{(2)\alpha}u(x,t)\}$

$=s^2\,U(p,s) - sU(p,0) - {}_{M}D_{t}{}^{\alpha}u(p,0)$ (15)

${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\{{}_{M}D_{x}{}^{(2)\beta}u(x,t)\}$

$=p^2\,U(p,s) - pU(0,s) - {}_{M}D_{x}{}^{\beta}u(0,s)$ (16)

$\int_0^x\int_0^t \Gamma(\sigma+1)\;\Gamma(\gamma+1)e^{\frac{\Gamma(\gamma+1)(t-\mu)^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{(x-v)^{\beta}}{\beta}}$

$x u(v,\mu)d_\beta v d_\alpha \mu = \frac{1}{(s-1)(p-1)}\,U(p,s)$ (17)

is acquired.

${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\left\{e^{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}\right\} = \frac{1}{(s-1)(p-1)}$ (18)

${}^{M}\mathcal{L}^{\alpha}_{t}{}^{M}\mathcal{L}^{\beta}_{x}\left\{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}e^{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}\right\}$

$= \frac{1}{(s-1)^2(p-1)^2}$ (19)

equations (15), (16), (17), (18), and (19). If the provided M-derivative Integro-differential equation is substituted with equation (13)

$s^2\,U(p,s) - sU(p,0) - {}_{M}D_{t}{}^{\alpha}u(p,0) - p^2\,U(p,s)$

$+pU(0,s) + {}_{M}D_{x}{}^{\beta}u(0,s) + U(p,s)$

$+\frac{1}{(s-1)(p-1)}\,U(p,s) = \frac{1}{(s-1)(p-1)} + \frac{1}{(s-1)^2(p-1)^2}$ (20)

the equation is found. If the double M-Laplace transform is applied to the initial conditions given by (14), then

$U(0,s)={}^{M}\mathcal{L}_t^{\alpha}\left\{e^{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}}\right\}=\frac{1}{(s-1)}$

$U(p,0)={}^{M}\mathcal{L}_x^{\beta}\left\{e^{\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}\right\}=\frac{1}{(p-1)}$

$_{M}D_t{}^{\alpha}u(p,0)={}^{M}\mathcal{L}_x^{\beta}\left\{e^{\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}\right\}=\frac{1}{(p-1)}$ (21)

$_{M}D_x{}^{\beta}u(0,s)={}^{M}\mathcal{L}_t^{\alpha}\left\{e^{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}}\right\}=\frac{1}{(s-1)}.$

(21) equations are written in place in equation (20) and

$$s^2\,U(p,s)-\frac{s}{(p-1)}-\frac{1}{(p-1)}-p^2\,U(p,s)+\frac{p}{(s-1)}+\frac{1}{(s-1)}$$

$$+\frac{1}{(s-1)(p-1)}U(p,s)+\,U(p,s)$$

$$=\frac{1}{(s-1)(p-1)}+\frac{1}{(s-1)^2(p-1)^2}$$

if the necessary actions are taken

$U(p,s)=\frac{1}{(s-1)(p-1)}.$ (22)

is obtained. From here, similar operations to the examples given above are performed by

$u(x,t)=e^{\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}+\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}}.$ (23)

Applications 3.2.

$_{M}D_t{}^{\alpha}u(x,t)+{}_{M}D_t{}^{(3)\alpha}u(x,t)$

$-\int_0^t\Gamma(\gamma+1)sinh\left(\frac{\Gamma(\gamma+1)(t-y)^{\alpha}}{\alpha}\right){}_{M}D_x{}^{(3)\beta}u(x,y)d_{\alpha}y=0$ (24)

about to be

$u(0,t)=0$

$_{M}D_x{}^{\beta}u\,(0,t)=sin\left(\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}\right)$

$_{M}D_x{}^{(2)\beta}u\,(0,t)=0$ (25)

$u(x,0)=0$

$_{M}D_t{}^{\alpha}u(x,0)=\,\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}$

$_{M}D_t{}^{(2)\alpha}u(x,0)=0$

solve the M-derivative partial integro-differential equation given by the initial conditions.

Solution.

If the double M-Laplace transform is applied to both sides of the M-derivative integro-differential equation given by (24), then,

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{_{M}D_t{}^{\alpha}u(x,t)\}=sU(p,s)-U(p,0)$ (26)

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{_{M}D_t{}^{(3)\alpha}u(x,t)\}=s^3\,U(p,s)-s^2U(p,0)$

$-s\,{}_{M}D_t{}^{\alpha}u(p,0)-{}_{M}D_t{}^{(2)\alpha}u(p,0)$ (27)

Definition 3.2. and Theorem 3.5 in the integral on the right side of the M-derivative integro-differential equation given by (24). If applied,

$$\int_0^t\Gamma(\gamma+1)sinh\left(\frac{\Gamma(\gamma+1)(t-y)^{\alpha}}{\alpha}\right)$$

$$x\,{}_{M}D_x{}^{(3)\beta}u(x,y)d_{\alpha}y$$

$$={}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\left\{sinh\left(\frac{\Gamma(\gamma+1)(t)^{\alpha}}{\alpha}\right){}_{M}D_x{}^{(3)\beta}u(x,t)\right\}$$

$={}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\left\{sinh\left(\frac{\Gamma(\gamma+1)(t)^{\alpha}}{\alpha}\right)\right\}{}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{_{M}D_x{}^{(3)\beta}u(x,t)\}$

$=\frac{1}{s^2-1}\left(p^3\,U(p,s)-p^2U(0,s)-p\,{}_{M}D_x{}^{\beta}u(0,s)\right.$

$\left.-{}_{M}D_x{}^{(2)\beta}u(0,s)\right)$ (28)

is acquired. If we apply the double M-Laplace transform to the initial conditions given by (25)

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{u(0,t)\}=0$

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{_{M}D_x{}^{\beta}u(0,t)\}=\frac{1}{s^2+1}$

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{u(x,0)\}=0$

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{_{M}D_x{}^{(2)\beta}u(0,t)\}=0$ (29)

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{_{M}D_t{}^{\alpha}u(x,0)\}=\frac{1}{p^2}$

${}^{M}\mathcal{L}_t^{\alpha}{}^{M}\mathcal{L}_x^{\beta}\{_{M}D_t{}^{(2)\alpha}u(x,0)\}=0.$

If (26), (27), (28) and (29) are considered in the M-derivative integro-differential equation given by their equation (24), then

$$sU(p,s)-U(p,0)+s^3\,U(p,s)-s^2U(p,0)$$

$$-s\,{}_{M}D_t{}^{\alpha}u(p,0)-{}_{M}D_t{}^{(2)\alpha}u(p,0)$$

$$-\frac{1}{s^2-1}\left(p^3\,U(p,s)-p^2U(0,s)-p\,{}_{M}D_x{}^{\beta}u(0,s)\right.$$

$$\left.-{}_{M}D_x{}^{(2)\beta}u(0,s)\right)=0$$

If the necessary actions are taken here

$U(p,s)=\frac{1}{(s^2+1)p^2}$ (30)

can be found. If a double inverse M-Laplace transform is performed on both sides of the resulting $U(p,s)$, then

$u(x,t)=\,\Gamma(\sigma+1)\frac{x^{\beta}}{\beta}sin\left(\frac{\Gamma(\gamma+1)t^{\alpha}}{\alpha}\right)$ (31)

the solution is reached.

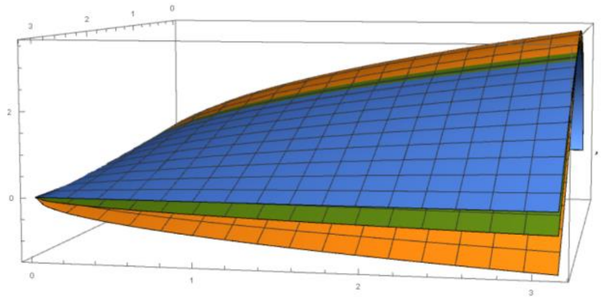

Figure 1. The image of the solution of the equation (31) for different x and t values with the values α= β=0.5, γ= σ =0, ,γ= σ =0.5 and γ = σ= 0.8

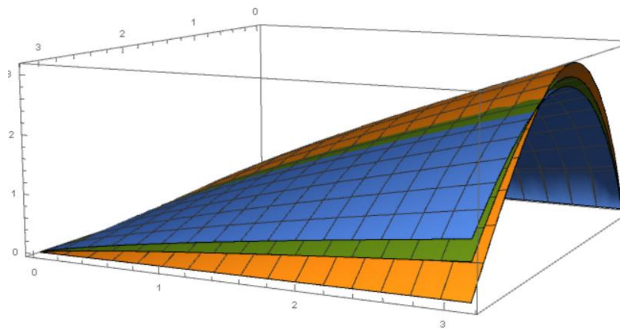

Figure 2. The image of the solution of the equation (31) for different x and t values with the values α= β=0.8, γ= σ =0, ,γ= σ =0.5 and γ = σ= 0.8

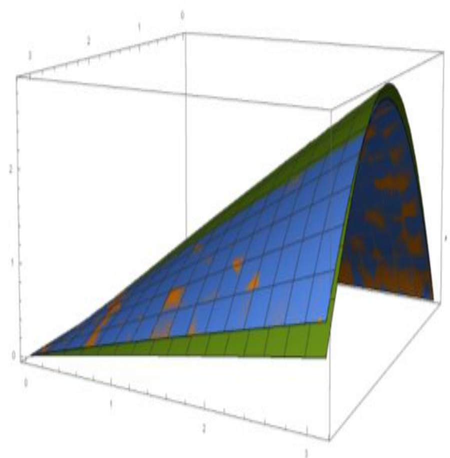

Figure 3. The image of the solution of the equation (31) for different x and t values with the values α= β=0.95, γ= σ =0, ,γ= σ =0.5 and γ = σ= 0.8

4. Visural result and scientific discusison

The double conformable derivative Laplace transform has been converted to the double M derivative Laplace form by taking advantage of increasing the parameters. (9) when we consider the equation, there are variables γ, σ, unlike the conformable fractional equation, there are variables 0 < α, $\beta \leq 1$, γ,σ. The parameters of the conformable derivative are α, β, t, x, while the M-derivative also varies depending on the parameters $\alpha, \beta, t, x, \gamma, \sigma$ This change is shown in Figure 1. Figure 2 and Figure 3. with different values γ=σ=0, γ=σ=0.5 and γ=σ=0.8 for $\alpha = \beta = 0.5$. γ=σ=0, γ=σ=0.5 and γ=σ=0.8 for $\alpha = \beta = 0.8$, γ=σ=0, γ=σ=0.5 and γ=σ=0.8 for $\alpha = \beta = 0.95$ the solution curves of the equation for different values γ=σ=0, γ= σ=0.5 and γ= σ=0.8 (13) The solution curves of the equation are shown. When we evaluate these solution curves, α and β tend to be the same as the conformable derivative for the value γ=σ=0, approaching the derivative in the classical sense when approaching 1. while the M-derivative comparison is made with the classical derivative by changing the α and β parameters, the derivative comparison compatible with the M-derivative is made within the values of γ, σ.

5. Conclusions

In this study, the definition of M-derivative double Laplace transforms, the theorems and values corresponding to these double M-Laplace transformations were expressed and proved. With these theorems, partial differential equations containing M −derivatives were solved using values corresponding to double M-Laplace transformations, and comparisons were made between double comformable Laplace transformations and double M-Laplace transformations by showing these solutions with graphs. It is thought that this transformation will be an effective technique for finding solutions to partial differential equations containing M-derivatives and that it will be used in modeling in mathematical, physical and engineering sciences using this technique.

Declaration of Ethical Standards

The authors declare that they comply with all ethical standards.

Credit Authorship Contribution Statement

Author-1: Conceptualization, investigation, methodology and software, visualization and writing – original draft.

Author-2: Conceptualization, investigation, methodology and software, supervision and writing – review and editing.

Declaration of Competing Interest

The authors have no conflicts of interest to declare regarding the content of this article

Data Availability

All data generated or analyzed during this study are included in this published article.

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