Introduction

Electrical signal disturbance detection is of significant importance in electrical networks. The disturbances in signal quality may threat electrical devices which are vulnerable to these disturbances such as harmonics, sags, high frequency discharges [1] etc. There are various studies have been conducted for detection of these disturbances recently [2-5].

Most of the disturbances exhibit non-linear signal waveforms and oscillations. For this purpose, time-series signal analysis and frequency spectrum investigations are required for proper analysis. To improve the performance of the proposed techniques, time series decomposition has been investigated for detecting the oscillations [6-7]. Investigated signals (especially proposed HV discharges) may display time varying characteristics where adaptive time series decomposition is quite effective tool and required. In this study, a comparative research of empirical mode decomposition (EMD) and variational mode decomposition (VMD) is conducted.

As an efficient decomposition method, EMD has been introduced for non-linear signals (especially signals with non-stationary characteristics) [8]. EMD is used for effective decomposing of original time-series signal into signal components, which are called intrinsic mode functions (IMFs). Most of the EMD applications contain non-linear time series signal denoising approaches [9-13]. Due to the limitations of EMD, the improper (mode mixing error) IMFs or redundant IMFs can be obtained where the reconstruction of original signal could be challenging task [8].

Variational mode decomposition is a robust and recently introduced non-recursive adaptive time series decomposition method [14]. Unlike EMD, the VMD is computed in frequency domain by updating center frequencies for each mode, which is namely variational computation. In VMD, investigated signal is transformed into a few modes, in which these modes are updated by using Wiener filter. By the aid of the filter, VMD is not prone to background noise [14]. VMD has numerous
applications in literature such as biomedical signals, power signals, speech signals and mechanical systems signals [15-19] etc.

In order to investigate highly oscillated HV discharge signals a single-phase transformer with 0.4kV/40kV, 50Hz, 8kVA rated values is employed. During the tests, voltage signals and current signals are obtained and analyzed. Voltage signals are measured via capacitive HV probe (2000:1) and current readings are measured high frequency sensitive current probe. All the signal waveforms recorded via high-speed oscilloscope. After adaptive time series decomposition of voltage and current signals, the IMFs are obtained and analyzed for the comparison. Each IMFs cross correlation coefficients with investigated decomposed time series signal are computed.

This paper is organized as follows. In section 2, the test setup and employed equipment is introduced. In section 3 and section 4 the mathematical background of EMD and VMD methods are presented respectively. In section 5, the test results are provided. Finally, in section 6, the conclusion is given.

**Test Setup**

High voltage discharge signals are collected by using test setup, which is given in Figure 1. The discharge signal is generated approximately at 5.5kV. In the setup, transformer’s secondary winding is open circuited and secondary voltage and discharge currents are recorded. In order to utilize all the aspects of the discharge current signal (evaluate all the frequency components) the current probe is employed. The current probe is capable of measuring frequency components up to 30MHz. Capacitive HV probe (2000:1 turn ratio) is used for measuring voltage of the transformer.

The block diagram of the test setup is given in Figure 2. In order to eliminate time delay between voltage and current recordings an equal length data cables are used to connect high-speed oscilloscope and probes.

**Empirical Mode Decomposition (EMD)**

Empirical mode decomposition method is a novel method for decomposing nonlinear, multicomponent time series signals iteratively [8]. The decomposed time series functions (modes) are called IMFs, which exhibit instantaneous frequency and amplitude characteristics processed by Hilbert transform. The EMD algorithm is computed by experimental concept (empirical) rather than analytical calculations [20,21].

According to the EMD criterions, the IMFs should satisfy two limitations: (a) the number of extrema (over-shoot and under-shoot) and the number of zero crossing of the signal must be equal or they might be different at most by one; and (b) the average of the upper and lower envelope which is defined by the local extrema points should be zero everywhere. The local average is zero for each point [8].

EMD process is the iterative method, which is computed systematically. Initially the local extrema points are computed for a given time series signal \( x(t) \). The envelope signals of local minima \( (e_{\text{min}}(t)) \) and local maxima\( (e_{\text{max}}(t)) \) are obtained. The next step is to calculate local average [9-10]:

\[
m(t) = \left[ e_{\text{min}}(t) + e_{\text{max}}(t) \right] / 2 \quad (1)
\]

The IMFs are calculated recursively (by the \( i \) parameter) by using local average.

\[
c_i(t) = x(t) - m(t) \quad (2)
\]

The IMF function is checked for whether calculated IMF is valid according to the criterions mentioned previously. If IMF is not valid, the iterative process is repeated [8,18]. If IMF is valid then IMF is set to \( z_i(t) = c_i(t) \).
\[ x_i(t) = x(t) - z_i(t) \]  

This iteration is repeated for \( i = 1 \ldots n \) where signal decomposition is completed and IMFs are generated.

### Variational Mode Decomposition (VMD)

Variational Mode Decomposition is a robust process, which decomposes the given signal into different signal waveforms (modes). These modes are characterized by their center frequencies [14]. As a variational approach, the modes (\( c_k \)) are evaluated for \( k \) values where sum of each mode (IMF) is equal to the given time series signal \( x(t) \).

By Hilbert transform is conducted for obtaining frequency spectrum for each mode [14].

\[ \sigma(t) + \frac{j}{\pi t} c_k(t) \]  

By using computed center frequency, spectrum shifting is fulfilled for each mode where \( w_k \) is the center frequency.

\[ \sigma(t) + \frac{j}{\pi t} c_k(t) \sum \left[ \left( \sigma(t) + \frac{j}{\pi t} c_k(t) \right) e^{-jw_k t} \right] \]  

As an initial step, VMD process is employed for calculation of \( c_1, w_1, \lambda_1 \) which are updated (\( c_k^{n+1}, w_k^{n+1}, \lambda^{n+1} \)) for further orders (\( n \)) in upcoming steps. The further calculations are conducted in frequency domain where parameters in frequency domain are given by \( \hat{\cdot} \) [18].

\[ w_k^{n+1} = \frac{1}{2\pi} \int_0^\infty \left| \hat{c}_k(w) \right|^2 dw \]  

The analyzed signal is decomposed into modes (\( c_k \)) by employing center frequencies (\( w_k \)) [14].

\[ w_k^{n+1} = \int c_k(w) \hat{c}_k(w) dw \]  

According to the VMD algorithm, each mode is updated in the frequency domain and the center frequencies are re-calculated for each iteration.

### Test Results

HV discharges are recorded and investigated via high-speed oscilloscope. High frequency discharges are observed on voltage and current waveforms. The voltage signal of the transformer secondary is given in Figure 3.

The corresponding current signal of the transformer secondary is given in Figure 4.

HV discharges tend to produce harmonic components with high frequencies. In order to analyze frequency spectrum fast Fourier transform (FFT) of discharge signals are obtained. The frequency spectrum of voltage signal is given in Figure 5.

The frequency spectrum of current signal is given in Figure 6. Both voltage and current signals exhibit higher harmonics.
The time series current and voltage signals are analyzed by using EMD and VMD algorithms and IMFs are obtained. Decomposed modes are interpreted for comparison purposes.

**EMD analysis**

The non-linear signals contain higher harmonics where signal decomposition may reveal signal characteristics. In this study, EMD method is employed for time series analysis. The decomposed IMFs of EMD algorithm for voltage signal are given in Figure 7.

![Frequency spectrum of voltage signal](image1)

**Figure 7.** Decompositions of EMD on voltage signal

All the modes (IMFs) are expected to exhibit investigated signal characteristics in small scale. The decomposed IMFs of EMD algorithm for current signal are given in Figure 8.

![Frequency spectrum of current signal](image2)

**Figure 8.** Decompositions of EMD on current signal

Each IMF (mode) is given sequentially where IMFs are lined up according to their frequencies from high to low frequency modes. By definition, summation of all IMFs should generate original time series signal. In order to compare the produced IMFs and the correlation of the original signal, the cross correlation coefficients are calculated. To facilitate comparison between IMFs, cross-correlation coefficient analysis is quite effective tool [22]. The cross correlation coefficients of IMFs and original signals for EMD algorithm is given in Table 1.

The IMFs (intrinsic mode functions from first to fourteenth) are decomposed sequentially from high frequency based mode to low frequency based mode as mentioned. Considering current signal, high center frequency modes have higher correlation coefficients. However, voltage signal exhibit distributed characteristics in terms of mode coefficients.
Unlike EMD, VMD have a few modes to reconstruct original non-linear signal. In this study, only the decomposed mode (IMF) is considered. The decomposed IMFs of VMD algorithm for voltage signal are given in Figure 9.

VMD is more efficient method than the EMD algorithm, since it is capable of decomposing given signal into lesser modes with non-recursive iteration. The decomposed IMFs of VMD algorithm for current signal are given in Figure 10.

The cross correlation coefficients are calculated by employing VMD algorithm for comparison purposes. The cross correlation coefficients of IMFs (intrinsic mode functions from first to third) and original signals for VMD algorithm is given in Table 2.

The correlation coefficients especially current signal coefficients are significantly displaying signal characteristic since they have higher coefficients. Additional modes have similar coefficients with the first three modes and hence proposed three VMD modes are assumed to be adequate. VMD algorithm can analyze characteristics of discharges with a few modes in contrast to EMD algorithm. Besides results have revealed that current characteristics of discharge signals are distinctive in terms of mode decomposition.

Conclusions

It is a quite challenging task to quantify non-linearity of discharge signals in electrical networks. Decomposing non-linear signals based on their distributed signal frequency components is an efficient technique to quantify non-linearity of a discharge signal. For this purpose, EMD and recently introduced VMD algorithms are employed to investigate HV discharge signals. A comparative study of EMD and VMD on non-linear HV discharge signals is conducted and modes (IMFs) are decomposed in time domain. In order to analyze obtained modes, cross correlation coefficients are computed and investigated. According to the

<table>
<thead>
<tr>
<th>IMF</th>
<th>The correlation coefficients for current signal</th>
<th>The correlation coefficients for voltage signal</th>
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<tbody>
<tr>
<td>IMF1</td>
<td>0.470</td>
<td>0.195</td>
</tr>
<tr>
<td>IMF2</td>
<td>0.280</td>
<td>0.142</td>
</tr>
<tr>
<td>IMF3</td>
<td>0.208</td>
<td>0.174</td>
</tr>
<tr>
<td>IMF4</td>
<td>0.188</td>
<td>0.075</td>
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<tr>
<td>IMF5</td>
<td>0.153</td>
<td>0.116</td>
</tr>
<tr>
<td>IMF6</td>
<td>0.062</td>
<td>0.164</td>
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<tr>
<td>IMF7</td>
<td>0.085</td>
<td>0.233</td>
</tr>
<tr>
<td>IMF8</td>
<td>0.130</td>
<td>0.302</td>
</tr>
<tr>
<td>IMF9</td>
<td>0.144</td>
<td>0.209</td>
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<tr>
<td>IMF10</td>
<td>0.135</td>
<td>0.223</td>
</tr>
<tr>
<td>IMF11</td>
<td>0.191</td>
<td>0.303</td>
</tr>
<tr>
<td>IMF12</td>
<td>0.007</td>
<td>0.423</td>
</tr>
<tr>
<td>IMF13</td>
<td>0.017</td>
<td>0.025</td>
</tr>
<tr>
<td>IMF14</td>
<td>0.027</td>
<td>0.006</td>
</tr>
</tbody>
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<th>IMF</th>
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<th>The correlation coefficients for voltage signal</th>
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</thead>
<tbody>
<tr>
<td>IMF1</td>
<td>0.420</td>
<td>0.195</td>
</tr>
<tr>
<td>IMF2</td>
<td>0.458</td>
<td>0.142</td>
</tr>
<tr>
<td>IMF3</td>
<td>0.481</td>
<td>0.174</td>
</tr>
</tbody>
</table>
results, distinctive coefficients are observed on EMD and VMD algorithms for noisy signals. Especially current modes have higher correlation with the given noisy current signals. Besides VMD can reveal signal modes with a few decompositions rather than EMD algorithm. The proposed algorithms are capable of on-line monitoring of non-linear signal modes in time domain. By using these algorithms (especially VMD algorithm) an efficient detection of system signals with discharges is plausible.

Acknowledgement

I would like to show my gratitude and thank to the TÜBİTAK National Metrology Institute for the HV discharge tests which are conducted in Power and Energy Laboratory.

6. References