

STOCHASTIC CONVERGENCE OF INCOME IN TURKIYE: A METHODOLOGICAL REINVESTIGATION OF PROVINCES

Altan BOZDOĞAN* 

Abstract

This study revisits income convergence among Turkish provinces for 1992-2019 and differs from most empirical literature due to its unique structural and methodological framework. Stochastic convergence is tested by employing a battery of panel stationarity tests that allow cross-sectional dependence and structural breaks. Breaks are further analyzed with respect to the nature of breaks as sharp and smooth. Sharp breaks are identified endogenously, while smooth breaks are accounted for using the Fournier approximation. Although σ -convergence is detected, there are no shreds of evidence of stochastic convergence at the panel level. Univariate test statistics demonstrate that at the provincial level, there is no single case that applies to all provinces. As additional dimensions of the data-generating process are evaluated in the testing procedure, outcomes about stochastic convergence slightly shift for provinces. However, findings at the panel level remain consistent and do not produce stochastic convergence. At the provincial level, mixed results are obtained.

Keywords: Stochastic Convergence, Fourier Approximation, Panel Unit Root, Regional Economics, Stationarity

JEL Classification: C23, O47, R10

1. Introduction

The existence of regional disparities and their patterns are quite crucial not just from an academic intellectual curiosity viewpoint but also because they have the power to govern the agenda of policy-makers. In that respect, this study tries to revisit some old yet still relevant issues in Turkiye using a province level. The first and foremost aim is to explore convergence structure employing a solid methodological approach quite different from the common practice in the literature.

The idea of convergence, in the contemporaneous understanding, was introduced by Solow (1956) under the framework of the neoclassical growth theory, which is inevitable under the diminishing return to physical or human capital assumptions because that tenet forces each economy¹ to approach its own steady state in the long run. Relative distance to their steady states

* Marmara University, Department of Economics, E-mail: altan.bozdogan@marmara.edu.tr, ORCID: 0000-0002-4976-9043

1 Hereinafter, instead of economies or countries, “regions” are used in order to be aligned with the content of this study.

governs their growth rate, producing *conditional* convergence. However, once the model posits identical preferences and homogenous technologies for all regions, they share the same unique steady state irrespective of initial conditions. Regions far away from the long-run equilibrium grow faster than regions closer, and eventually; poorer regions become as rich as the initially rich regions. That sort of catch-up is called *absolute* convergence. The neoclassical growth theory does not predict absolute convergence but occurs as a particular case.

On the other side, endogenous growth theories initiated by Romer (1986) and Lucas (1988) criticized the critical building blocks of the neoclassical growth theory and incorporated positive externalities or spillovers into the setup through increasing returns into the production function in the form of intentional human capital accumulation and R&D activities. The long-run growth is determined within the model endogenously rather than by taking it as an exogenous factor. This strand of growth literature predicts no convergence or even *divergence* as the initial condition of a region is determined by endogenous drivers. Besides absolute and conditional convergence, Galor (1996) proposes a third alternative: club convergence – regions with similar structural features (e.g., initial conditions) or heterogeneity in factor endowments form clusters with distinct steady-states even in the neoclassical growth model.

Empirical testing of convergence can be classified broadly into four different methodologies²: i) cross-section approach, ii) panel approach, iii) times series approach, and iv) distribution approach (Islam, 2003). The distribution approach fundamentally differs from the rest because it deals with the entire income distribution instead of directly working with regression analysis. Markov chain analysis is one way to account for such distribution dynamics (Quah, 1993a). The other tool is σ -convergence, a convergence that seeks a decline in income dispersion and is quantified by either standard deviation or coefficient of variation (Dowrick and Nguyen, 1989; Friedman, 1992; Boyle and McCarty, 1997). The cross-section approach (Barro, 1991; Mankiw, Romer, and Weil, 1992; Barro and Sala-i Martin, 1992) searches for a negative relationship between initial income level and growth rate of per capita income. This method is called β -convergence. Absolute or conditional β -convergence division is based on whether other structural characteristics beyond the initial income level are controlled. However, such initial level regression (i.e., Barro-type regressions) is criticized by Quah (1993b) for being an example of Galton's fallacy. Additionally, differences in initial technology levels are seen in the error term of the regression, and besides the capital deepening as a source of income convergence, technology diffusion as the other source disappears due to the assumption of homogenous technologies across regions (Islam, 2003). The panel approach (Islam, 1995; Caselli, Esquivel, and Lefort, 1996; Barro, 1996) is viewed as a potential candidate for solving this problem. Explicit control of technology terms has a dual advantage. First, the technology term captures more than technology (e.g., other aspects of the economic structure), and second, omitted variable bias stemming from unobserved heterogeneity is solved with the individual³ (regional) effect in the regression equation (Temple, 1999; Islam,

2 For an extensive literature review on different conceptualizations of convergence phenomenon, please see Temple (1999), Islam (2003) and Durlauf, Johnson and Temple (2005).

3 Caselli, Esquivel and Lefort (1996) stress that for regions that share similar technologies, bias from individual effect

2003). The time series approach to convergence underpins this study. Thus, I discuss it in-depth in the Section 3.

The plan of the paper is as follows. Section 2 presents the related empirical literature on Türkiye. Section 3 provides the theoretical foundations of the stochastic convergence. Section 4 introduces the data and some descriptive analysis. Section 5 outlines the econometrics methodology and Section 6 concludes.

2. Related Literature Review

Several studies were conducted to reveal convergence dynamics in Türkiye's regions. Filiztekin (2018) proved the existence of conditional β -convergence among 65 provinces between 1975 and 1995, yet divergence was detected via σ -convergence, particularly after the late 1970s to late 1980s. Tansel and Güngör (1999) studied the same period for 67 provinces using productivity instead of regional GDP. They found that absolute β -convergence and the speed of convergence accelerated after 1980, which is attributed to the liberalization practices. However, σ -convergence exhibited different patterns for western and eastern provinces. Although, Doğruel and Doğruel (2003) documented β -convergence for 1987-1999 for all, high-income and low-income provinces, failure to find σ -convergence in low-income provinces was tied to decreased public investments in those regions. Karaca (2004) investigated the 1975-2000 period, but could not find evidence of β -convergence, and divergence was explored as income dispersion widened. Önder, Deliktaş, and Karadağ (2010) conducted a series of panel techniques and observed conditional convergence for NUTS-2 regions during 1980-2001; however, the transportation component of public capital stock was found as a factor that exacerbated regional disparities. Gömleksiz, Şahbaz, and Mercan (2017) also supported the role of government in stimulating convergence for 2004-2014 at NUTS2 level.

Using the panel approach, Bolkol (2019) found shreds of evidence on both unconditional and conditional convergence for different regional units, including provinces, from 2005 to 2017. Despite the fall in the variation, strong arguments about σ -convergence were unavailable, but the 2008-09 crisis period contributed to the convergence experience. Later, Bolkol (2023) added an endogenous growth perspective and stressed that policies based on R&D personnel would not lead to convergence but rather a divergence.

Aksoy, Taştan, and Kama (2019) observed convergence clubs rather than absolute or conditional convergence for the 1987-2001 and 2004-2017 periods. Similar convergence clubs were obtained in Sakarya, Baran, and İpek (2024) for 2004-2022. However, two subsets, 2004-2016 and 2017-2022, exhibited different patterns. The tendency of convergence turned into divergence for 81 provinces. There are also some studies (Gezici and Hewings, 2004; Aldan and Gaygısız, 2006) mainly concentrating on spatial links and some studies (Aldan and Gaygısız, 2006; Karahasan,

may be trivial. Time-specific effect captures world growth and commons shocks (Durlauf et al. 2005).

2017 and 2020) with Markov chain analysis; yet all of them demonstrated the continuity in the regional income variation. Beside the β -convergence, a strand of literature was flourished after Carlino and Mills (1993), Quah (1993a), Bernard and Durlauf (1995).

Erlat and Özkan (2006) used CADF panel unit root and tested the time series approach to convergence in Turkish provinces. They found that different regions involved different patterns signaling some sort of club formations but failed to get clear evidence on absolute convergence for 1975-2000. Aslan and Kula (2011) analyzed 67 provinces from 1975 to 2001 with a univariate LM unit root test that enabled the endogenous determination of structural breaks. Allowing two structural breaks resulted in stochastic convergence for all provinces except Bitlis, Erzurum, and Hakkari so that shocks to relative income had only transitory impact. Durusu-Çiftçi and Nazlıoğlu (2019) applied a series of univariate unit root tests to 73 provinces from 1992 to 2013, allowing for sharp shifts and smooth shifts. However, they took the presence of stochastic convergence as a necessary but not sufficient condition and checked β -convergence for each province following Tomljanovich and Vogelsang (2002). The clear divergence between eastern and western provinces was reached. Akkay (2022) employed similar univariate unit root tests as Durusu-Çiftçi and Nazlıoğlu (2019) and extended the terminal year to 2019. All provinces experienced stochastic convergence, and this result remained consistent regardless of whether structural breaks, primarily in 2002 and 2008, were taken into account.

The literature on regional stochastic convergence in various countries is extensive. Notable studies include Tomljanovich and Vogelsang (2002) on regions in the United States, DeJuan and Tomljanovich (2005) on Canadian provinces, Constantini and Arbia (2006) on Italian regions, Carrion-i-Silvestre and German-Soto (2009) on Mexican regions, and Misra, Kar, Nazlıoğlu, and Karul (2024) on Indian states.

3. Theoretical Foundations of Stochastic Convergence

Quah (1992) encapsulates the convergence phenomenon using several approaches and defines one approach as the absence of unit root or deterministic time trend in income disparities between countries that is intrinsically and fundamentally different from initial level regression analysis. Bernard and Durlauf (1995; 1996) also express that regions ⁴ i and j convergence between time t and $t + T$, when the per capita output difference is expected to fall. $\mathcal{Y}_{i,t}$ corresponds to natural logarithm of real per capita output and if $\mathcal{Y}_{i,t} > \mathcal{Y}_{j,t}$ then the previous statement can be demonstrated as $E(\mathcal{Y}_{i,t+T} - \mathcal{Y}_{j,t+T} | I_t) < \mathcal{Y}_{i,t} - \mathcal{Y}_{j,t}$ in the time series context. This structure is later elaborated to capture variant aspects of the convergence such that two regions are said to converge if the long-term forecasts of per capita output for both regions are equal to a fixed time t , conditional on some information set at t , including time, current and deeper lags of $\mathcal{Y}_{i,t}$ (see

4 Definitions are based on countries but since this study explores regional convergence, from now on “region” replaces “country” in such definitions.

Bernard and Durlauf, 1995 and 1996). The benchmark unit appears as a problem to be solved, and there are two paths of practice: choosing a reference country or taking a sample average ⁵.

According to Evans and Karras (1996) i regions are said to convergence if, and only if, a common trend a_t , which is unobservable by nature and equivalent to technology ⁶, and finite parameters $\mu_1, \mu_2, \dots, \mu_i$ exist such that

$$\lim_{T \rightarrow \infty} E(y_{i,t+T} - a_{t+T} | I_t) = \mu_i \quad (1)$$

μ_i is a parameter governing the balanced growth path of the region i . Common trend is obtained by averaging over i regions so that

$$\lim_{T \rightarrow \infty} E(\bar{y}_{t+T} - a_{t+T} | I_t) = \frac{1}{i} \sum_{i=1}^i \mu_i \quad (2)$$

where $\bar{y}_t = \sum_{i=1}^i \frac{y_{i,t}}{i}$. The level of common trend is defined as $\lim_{T \rightarrow \infty} E(\bar{y}_{t+T} - a_{t+T} | I_t) = 0$, so common trend equals to average behavior of i economies. To eliminate it, we subtract (2) from (1) and generate

$$\lim_{T \rightarrow \infty} E(y_{i,t+T} - \bar{y}_{t+T} | I_t) = \mu_i \quad (3)^7$$

(3) is isolated from common trend and is left with deviations of per capita income from average behavior. Namely, long run forecasts of relative per capita incomes approach to a constant as the forecasting horizon tends to infinity and this can be directly tested by checking the stationarity of the deviation of output, $y_{i,t+T} - \bar{y}_{t+T}$ (Evans and Karras, 1996; Bernard and Durlauf, 1995 and 1996) ⁸.

Using a similar rationale, Carlino and Mills (1993) first suggest (*stochastic convergence*) to test whether shocks to relative income are temporary or not. In case of stationarity, idiosyncratic regional specific factors are also immune to long-run economic growth and shocks have only transitory impacts (Carrion-i Silvestre and Soto, 2009). On the other hand, non-stationarity triggers a shock of permanent deviations in relative per capita income and hampers any tendency of stochastic convergence. Thus, future trajectories of such behaviors cannot be projected. Temple (1999) also emphasizes the link between convergence and stationarity testing but is aware of how hard to get precise interpretations.

A body of empirical literature on this issue emerges in the context of whether or not there is a time trend ⁹. Trend stationarity case is named as *stochastic convergence* (Carlino and Mills, 1993;

5 Latter strategy is adopted to bring into alignment with regional convergence literature. See Islam (2003) for possible problems of taking deviations from either reference economy or sample average.

6 "Not just technology but resource endowments, climate, institutions and so on; it may therefore differ across countries" Mankiw, Romer and Weil (1992: 5-6).

7 Bernard and Durlauf (1995; 1996) used $\lim_{T \rightarrow \infty} E(y_{i,t+T} - \bar{y}_{t+T} | I_t) = 0$ version of the formula.

8 For the bivariate case, incomes have to be cointegrated. See Bernard and Durlauf (1995); Stengos and Yazgan (2014) for details.

9 See Islam (2003) for discussion of stochastic and deterministic trends.

Strazicich, Lee and Day, 2004) or *catching-up* (Oxley and Greasley, 1995; Cunada and Garcia, 2006) while level stationarity as either *deterministic convergence*¹⁰ (Li and Papell, 1999; Cunada and Garcia, 2006) or *long-run convergence* (Oxley and Greasley, 1995). However, Li and Papell (1999) remark a caveat about a time trend as it can cause permanent per capita income differences making it vulnerable to criticism. Zero mean stationarity, without a constant and time trend case, is also discussed (Bernard and Durlauf, 1995; Cunada and Garcia, 2006). A generic explanation of divergence, in our case, is that per capita income gap between a region and country average consistently widens and requires non-stationarity.

However, it is worth noting that the time series approach, to a large extent, is inherently statistical and not linked explicitly to growth theories because initial conditions have no role in the long-run trajectories (Oxley and Greasley, 1995; and Durlauf, Johnson, and Temple, 2005). On the other hand, the impacts of initial cross-country differences in physical and human capital on the long-run patterns construct the backbone of neoclassical and endogenous growth theories (see Durlauf, *et al.*, 2005). Evans and Karras (1996) and Evans (1998) put some effort into reconciling the time series approach with growth theories, aiming at strengthening the weak ties. Evans (1998) argues that $y_{i,t+T}$ reverts to common trend, measured by \bar{y}_{t+T} , lends some support to exogenous growth theory, while the case of non-reverting $y_{i,t+T}$ to common trend provides what the endogenous growth models require¹¹. The former case corresponds to stationarity, whereas non-stationarity leads to the latter case. Relevant models need to be tested appropriately to get more definitive and concrete outcomes (Oxley and Greasley, 1995), so this study avoids such certain claims. A further taxonomy is also possible rooted in Evans and Karras (1996) by modifying equation (3) as follows: i) *absolute convergence* takes place when $\mu_i = 0$ for all i s, or ii) *conditional convergence* if $\mu_i \neq 0$ for some i . To be clearer based on the distinction made above, zero mean stationarity implies the same steady-state for all regions (King and Ramlogan-Dobson, 2014) and is analogous to *absolute convergence* (Cunada and Garcia, 2006). It is also proposed that a constant term (Strazicich *et al.* 2004) or α_{t+T} can capture some time-invariant differences giving rise to *conditional convergence* (Islam, 2003). As a matter of fact, most of the earlier literature is based upon Dickey-Fuller type equation estimation (Carlino and Mills, 1993; Oxley and Greasley, 1995; Bernard and Durlauf, 1995; Li and Papell, 1999). Using a well-behaved neoclassical production function, the following equation¹² can be written to test for convergence

$$y_{it} = \mu_i - \beta g t + (1 + \beta)y_{i,t-1} + \varepsilon_{it} \quad (4)$$

If region subscripts are removed, it becomes the Dickey-Fuller equation¹³ with constant and time trend. To achieve (stochastic) convergence, $(1 + \beta)$ has to be less than one, that is to say β should be negative or it should not contain unit root (Islam, 2003). Although technology (A_t) specification

10 Li and Papell (1999) label Bernard and Durlauf (1995; 1996) case as *deterministic convergence*.

11 For a more straightforward interpretation, cross-section specific intercepts should be checked as well. For more, see Evans and Karras (1996), and Evans (1998).

12 The proof of this equation can be found in Islam (1995 and 2003).

13 Dickey and Fuller (1979) model (c) is $y_t = \mu - \beta t + \rho y_{t-1} + \varepsilon_t$

plays a role in the type¹⁴ of convergence, this study quests for only stochastic convergence under different sets of assumptions of the data-generating process (DGP).

Bernard and Durlauf (1995 and 1996) put forward a prominent remark about the inappropriateness of such time-series¹⁵ testing for economies that are far from their steady-states, pointing out that unit root null hypothesis can be spuriously accepted because, in this case, the data may be generated by a transitional law of motion rather than by an invariant random process. Thus, the sample moments of the data are not representative to the population moments. This research acknowledges the aforementioned empirical concerns. Even though the true DGP for provincial per capita income in Türkiye may be difficult to have or even unattainable fully because provinces may not be close to their steady-states, the true DGP can be approximated considering all probable and relevant peculiarities of the data.

4. Data and Descriptive Analysis

The income per capita relative to a benchmark, mostly the average of the regions, is needed to test the stochastic convergence. Nevertheless, the fact that per capita income is not reported regularly at the provincial level prevents the use of official statistics retrieved from Turkstat. The official series covers 1987-2001 (with the *base* year 1987) and 2004-2022 (with the *reference*¹⁶ year 2009). Methodological change to the chain-volume index¹⁷ from the constant-price approach in the calculation of GDP and the missing period of 2002-2003 do not make it feasible (Düşündere, 2019; Akkay, 2022). Thus, in unreliable¹⁸ or unavailable subnational data, luminosity can be used as a proxy for economic performance (Chen and Nordhaus, 2011; Henderson, Storeygard, and Weil, 2012). For this purpose, Düşündere (2019 and 2020) estimates luminosity-based income per capita at the provincial level for 1992-2019¹⁹ using satellite nighttime light data. This study utilizes that new dataset and converts provincial GDP (chain-volume index) in Turkish Lira into GDP per capita for 81 provinces using population data. Then, for each province and each year, per capita incomes are divided by average of provinces for the corresponding year to generate relative incomes, which are later taken their natural logarithms.

14 This study does not follow stochastic and deterministic convergence definitions based on the deterministic or stochastic trend discussed in Islam (2003) because they may create confusion with the stochastic and deterministic convergence I define here.

15 For an assessment of cross-section and time-series approaches to convergence see Bernard and Durlauf (1995 and 1996).

16 See Bakış (2018) for details.

17 Income per capita was first announced in 2016 and revised in 2020 for 2009-2019. Chain volume index was adopted in 2016.

18 Chen and Nordhaus (2011) grade countries from A to E in terms of output and luminosity compliance where A is the highest grade while E is the lowest. Türkiye has the grade C and luminosity has small value added in A, B, and C due to high measurement error. Therefore, the extended income per capita series by Düşündere (2019 and 2020) may have no significant information additions to the subnational income per capita series. There are strong evidences to support such that for all years and provinces, correlation between the predicted and official income per capita ranges between 99.38% and 99.9% (Düşündere, 2019). Besides, official data in 2020, 2021 and 2022 are not used in this study owing to different sources.

19 This dataset is constructed on behalf of The Economic Policy Research Foundation of Türkiye (TEPAV).

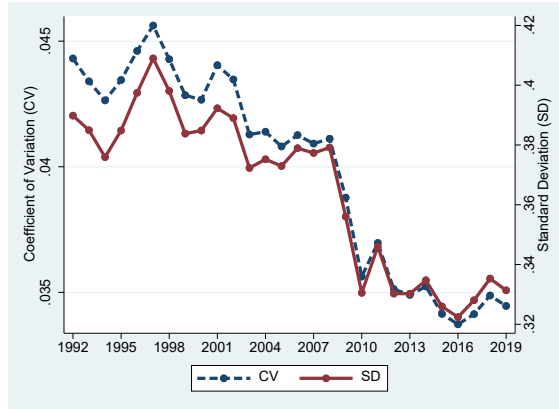


Figure 1: σ - convergence

Figure 1 presents σ -convergence using standard deviation and coefficient of variation. The 1990s were characterized by relatively higher income dispersion among provinces. After 2000, a radical fall in statistics can be seen that is equivalent to an improvement in income distribution. The decline in income dispersion intensified during the 2008-2010 period, which can be attributed to the global financial crisis. Thus, it may signal convergence towards low – income provinces. Indeed, σ -convergence does not tell where the provinces heading to low income or high-income. Figure 2 and 3 represent choropleth maps about average real income levels and real GDP growth rates from 1992 to 2019. East and West distinction is explicitly monitored. Eastern Anatolia and the South-east Anatolia stay at the lowest quartile, whereas Western provinces are located at the highest quartile. There is a smooth transition from high-income to low-income provinces. Tunceli, Erzincan, Trabzon, Rize, and Artvin disturb this smooth transition. Differences among provinces are eroded during that time period in favor of the North-west Anatolia, according to Figure 3. There are some individual units as well in which growth rates belong to the highest quartile and no significant pattern is observed.

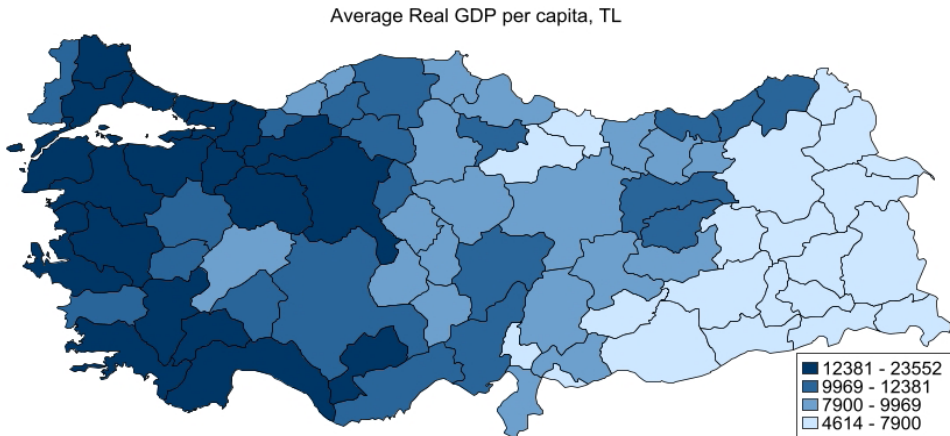


Figure 2: Average Real GDP per capita, TL

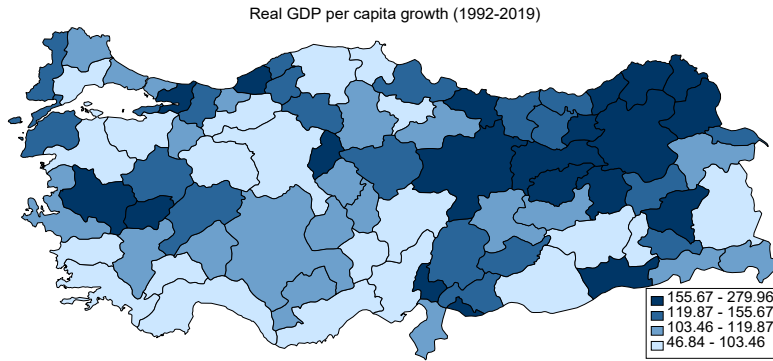


Figure 3: Real GDP per capita Growth Rate (1992-2019)

5. Econometric Methodology

Bai and Ng (2005) underline the importance of the null hypothesis of stationarity, which is more natural than the null hypothesis of a unit root for many economic problems. It can be argued that if convergence is rejected for the stationary null, this would provide stronger evidence against the convergence hypothesis than simply failing to reject the unit root null hypothesis (Bai and Ng, 2005). Becker, Enders, and Lee (2006) also shed more light on this debate by pointing out that tests with the null hypothesis of a unit root have low power in stationarity when a theory has to be tested under the null of stationarity. Therefore, I follow in their footsteps, and a battery of stationarity tests has been implemented to check the regional income convergence dynamics. In addition, instead of merely univariate tests, a common practice in the literature, panel tests that utilize more information are used as the provinces are adjacent to each other and likely to be affected to varying degrees by the same shocks. Besides panel outcomes, a dual perspective is adopted due to the possibility of interpretations of individual series in terms of stationarity. Univariate time series stationarity tests suffer from low power, while panel counterparts can enhance the power due to a higher number of observations but can be difficult to interpret (Maddala, 1999; Smith and Fuertas, 2010). First of all, the information is always obtained from univariate tests; thus, as Maddala (1999) proposed, movement to panel tests may not solve the varying conclusions, but more powerful tests can be a natural remedy. Therefore, this study challenges the recent empirical literature of (stochastic) convergence in Türkiye on the grounds of a series of tests considering potential maladies that can harm the power of the tests.

5.1. No-shift: Hadri (2000) and Cross-Sectional Dependence

Hadri (2000) extends the residual-based Lagrange multiplier (LM) univariate stationary test of Kwiatkowski, Phillips, Schmidt, and Shin (1992)²⁰ and introduces panel data stationarity test with

²⁰ Smith and Fuertas (2010) emphasize that Kwiatkowski, Phillips, Schmidt and Shin (1992), hereafter, KPSS is sensitive to the bandwidth selection. Unless, it is reported, all bandwidths for spectral window are set to $4(T/100)^{2/9}$.

the null hypothesis of series are stationary around a deterministic trend against the alternative hypothesis of unit root. The model can be written as follows:

$$y_{it} = z_t' \delta_i + r_{it} + \epsilon_{it} \quad (5)$$

$$r_{it} = r_{i,t-1} + u_{it} \quad (6)$$

where $\delta_i = [\alpha_i, \beta_i]'$ and $z_t = [1, t]'$ with the trend model. r_{it} is a random walk. $u_{it} \sim IIN(0, \sigma_u^2)$ and $\epsilon_{it} \sim IIN(0, \sigma_\epsilon^2)$ are mutually independent normal, and independent and identically distributed across i and over t . The stationarity null hypothesis is $\sigma_u^2 = 0$ against the alternative of $\sigma_u^2 > 0$. The initial values of r_{i0} are heterogenous fixed unknowns and the trend model can be written as

$$y_{it} = r_{i0} + \beta_i t + \sum_{t=1}^t u_{it} + \epsilon_{it} \quad (7)$$

Partial sum of residuals (S_{it}) is obtained from equation (7) using OLS. The LM test that is the average of the Kwiatkowski *et al.* (1992) test statistic across i , allowing heteroskedasticity, and estimated using the below formula

$$LM = \frac{1}{N} \left(\sum_{i=1}^N \left(\frac{1}{T^2} \frac{\sum_{t=1}^T S_{it}^2}{\sigma_{\epsilon_i}^2} \right) \right) \quad (8)$$

The benchmark panel test statistic, which is the normalized version of (8), is computed as Z . The above test statistic is normalized to obtain the benchmark panel test statistics

$$Z = \frac{\sqrt{N}(LM - \xi)}{\zeta} \sim N(0,1) \quad (9)$$

where ξ is the mean and ζ^2 is the variance with 1/15 and 11/6300, respectively (Hadri, 2000). Z is standard normal; thus, there is no need to compute a new set of critical values. Such stationary or unit root tests with presumed cross-sectional independence are first-generation tests. Hadri (2000) panel stationarity test is deliberately preferred in this work because new features are added into the same structure in each stage.

In contrast to spatial economics, where cross-correlation is related to geographic factors such as distance, location, and space, this study treats contemporaneous correlation stemming from unobserved global shocks, local interactions, or pure idiosyncratic correlation among individuals (Moscone and Tosetti, 2009).

The existence of common shocks and unobserved common components pave the way for interdependencies across cross-sectional units (De Hoyos and Sarafidis, 2006). Cross-sectional dependence and potential structural breaks can result in inconsistent and biased inferences. Besides, such issues will also determine what kind of panel unit root or stationary tests have to be adopted. The recently flourishing literature suggests two approaches to identifying cross-sectional dependence (Ditzen, 2021): direct testing for the CD (Pesaran, 2015) and estimating the strength of the dependence (Bailey, Kapetanios, and Pesaran, 2016). Both methods detect the

cross-sectional dependence in relative GDP per capita. First, Pesaran (2015, 2021) test statistic is estimated using the following equation:

$$CD = \left[\frac{TN(N-1)}{2} \right]^{1/2} \left(\frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \widehat{\rho}_{ij} \right) \tag{10}$$

where $\widehat{\rho}_{ij}$ is the pair-wise correlation coefficient. However, Pesaran (2015) offers to shift the null hypothesis of cross section independence of Pesaran (2004) with weak²¹ cross-sectional dependence for panels with large N. The null hypothesis can be shown as $0 \leq \alpha < (2 - d)/4$, and α measures the degree of cross-sectional dependence (Pesaran, 2015). In other words, CD test examines for $\alpha < 0.5$ ²³ (Ditzen, 2021). So, with 81 provinces, I can safely use the null hypothesis of weak cross-sectional dependence against strong cross-sectional dependence. According to Table 1, weak convergence cannot be rejected as the p-value is greater than 0.10. As an alternative way to gauge the cross correlation, CD* test of Pesaran and Xie (2023) which a bias corrected version of Pesaran (2015) is estimated using the following equation:

$$CD^* = \frac{CD + \sqrt{\frac{T}{2}\theta_n}}{1 - \theta_n} \tag{11}$$

where θ_n is the bias-corrected term. The result of CD* ends up with strong cross-sectional dependence. Although, outcomes of CD and CD* are enough to justify the utilization of panel tests capturing cross-correlations, as a final attempt to settle the degree of cross-correlation, the exponent of cross-sectional dependence is estimated using Bailey *et al.* (2016), which has quite decent small sample property. This approach tries to determine the value of α from the range of [0,1]. The range of [0.5,1] corresponds to different degrees of strong cross-sectional dependence, while the range of [0,0.5] corresponds to different degrees of weak cross-sectional dependence. It would be more appropriate to verify the degree of cross-sectional dependence is sufficiently large, that is to say, $\alpha > 1/2$, to justify the use of Bailey *et al.* (2016) method. Here CD* test can be referred to better interpret $\hat{\alpha}$ in Table 1. $\hat{\alpha}$ is the bias-adjusted estimator²² of α , which is close to 1, implying strong cross-sectional dependence.

Table 1: Testing Cross-Sectional Dependence

CD	CD*	$\hat{\alpha}_{0.05}$	$\hat{\alpha}$	$\hat{\alpha}_{0.95}$
1.01 (0.315)	-1.68 (0.093)	0.851	0.918 [0.041]	0.986

Notes: Numbers in parentheses are p-values. The number in brackets is the standard error. The first 4 principal components are used in the estimation of CD*. $\hat{\alpha}_{0.05}$ and $\hat{\alpha}_{0.95}$ give the 90% confidence interval bands.

21 Weak cross-sectional dependence means that the correlation between units at each point in time converges to zero as the number of cross sections goes to infinity. Under strong dependence the correlation converges to a constant.

22 The details can be found in Bailey *et al.* (2016).

5.2. No-shift: Hadri and Kurozumi (2011)

Hadri and Kurozumi (2011 and 2012) modify the data-generating process of Hadri (2000) and incorporate cross-sectional dependence in the form of a common factor. Error component ϵ_{it} is redefined as following

$$\epsilon_{it} = f_t \gamma_i + v_{it} \quad (12)$$

f_t is a one-dimensional latent common factor, and each individual is very likely to be affected by the common factor with the loading factor γ_i . To eliminate cross-sectional dependence, Pesaran (2007) methodology is followed. Cross-sectional average of the model, composed of (5), (6), and (12), is taken to remove the common factor²³. New partial sum of residuals (S_{it}^w) is constructed from the cross-sectional average model using OLS. Then, using Hadri (2000) procedure, same statistics are obtained as in (8) and (9) but to differentiate the notation, A subscripts are added as LM_A and Z_A . Individual test statistics are seen in the innermost parenthesis in (8), and that term is divided by a consistent long-run variance estimator to correct for serial correlation, so the innermost parenthesis is replaced by $\frac{1}{\sigma_f^2 T^2} \sum_{t=1}^T S_{it}^{w^2}$. As suggested in Hadri and Kurozumi (2012), that estimator is chosen following Sul, Phillips, and Choi (2005) to enhance the power of the test, especially for the trend case. This study applies Sul *et al.* (2005) with quadratic spectral specification.

Beyond cross-sectional dependence, another problem potentially undermining the power of the test, is well documented in Perron (1989) and Lee, Huang, and Shin (1997), may arise due to erroneous omission of structural breaks. Lee *et al.* (1997) depict that stationarity tests ignoring the potential structural break(s) are biased towards rejecting the stationarity null hypothesis and create a size distortion problem. Alongside this, mis-specified placing and numbering of the breaks can severely distort the power of the test; thus, to refrain from such complications, a stationarity test, Carrion-i-Silvestre, Barrio-Castro, and Lopez-Bazo. (2005), that can endogenously determine both number and location of breaks. This test also addresses cross-sectional dependence through the nonparametric bootstrapping of Maddala and Wu (1999).

5.3. Sharp-shift: Carrion-i-Silvestre, Barrio-Castro, and Lopez-Bazo (2005)

Carrion-i-Silvestre *et al.* (2005) attach two new components to the random walk process of equation (2) in the form of dummy variables as the changes in the level and slope to capture the date of the break(s). Equations (5) and (6) are adjusted in line with $\delta_i = [\alpha_i, \beta_i]'$ and $z_t = [1, t, DU_{i,1,t}, \dots, DU_{i,m_i,t}, DT_{i,1,t}^*, \dots, DT_{i,m_i,t}^*]'$. For reasons of parsimony, under the null hypothesis the data generating process of the model with shifts in the mean and time trend is assumed to be

$$y_{it} = r_{it} + \beta_i t + \sum_{s=1}^{m_i} \theta_{i,s} DU_{i,s,t} + \sum_{s=1}^{m_i} \gamma_{i,s} DT_{i,s,t}^* + \epsilon_{it} \quad (13)$$

23 In order to save space, averaged model is not added but can be seen in Hadri and Kurozumi (2011 and 2012).

The dummy variable $DT_{i,s,t}^* = t - T_{b,s}^i$ for $t > T_{b,s}^i$ and 0 otherwise, where $s = 1, \dots, m_i$ and m_i is the maximum number of structural breaks imposed, and $T_{b,s}^i$ is the s th date of the break for the individual i . The dummy variable $DU_{i,s,t} = 1$ for $t > T_{b,s}^i$ and 0 otherwise. The null hypothesis of stationarity is slightly modified compared to Hadri (2000) and Hadri and Kurozumi (2011) to $\sigma_{u,i}^2 = 0$ against the nonstationary alternative of $\sigma_{u,i}^2 > 0$. Partial sum of residuals is obtained from equation (13) again using OLS. As it is built upon the framework of the Hadri (2000), equation (8) is estimated for $LM(\lambda)$ where λ stands for the dependence of the test on the break dates. Finally, $Z(\lambda)$ is assessed by rewriting $LM(\lambda)$ for LM in equation (8) for the panel test statistics. $Z(\lambda)$ can also be calculated by assuming that long-run variance is homogeneous across individuals. The number of breaks is estimated using LWZ criterion as suggested by Carrion-i-Silvestre *et al.* (2005) when trending regressors are included. Long run variance estimator in our analysis is Sul, Phillips, and Choi (2005) with quadratic spectral quadratic spectral and m is set to 5.

5.4. Smooth-shift: Nazlıoğlu and Karul (2017)

Tests directly identifying the number of breaks, location of breaks, or even their functional form examine the phenomenon of *sharp breaks* with the help of time dummies. However, such time dummy practices may not be enough to fully comprehend and transmit the true nature of breaks. The trend is considered to consist of sections that are linear between breaks, while discontinuity is in the realm of possibility (Enders and Lee, 2004). Thus, false specifications of breaks can be as detrimental as their total ignorance. As has been a common topic of debate recently (Enders and Lee, 2004; Becker *et al.* 2006), many macroeconomic time series are characterized by rather *smooth breaks* or *gradual breaks*, corresponding to structural breaks with an unknown number of breaks, dates, duration, and functional form. The Fourier approximation can mimic various forms of structural breaks or nonlinearities in the deterministic term²⁴ (Becker *et al.*, 2006). Nazlıoğlu and Karul (2017) borrow the univariate framework of Becker *et al.*, (2006), extend it, and build their novel panel stationarity test. They insert a deterministic term as a function of time as $z_i(t)$ instead of $z_t^i \delta_i$ into the DGP in equation (5). The model below is slightly different from Becker *et al.* (2006) as it includes the common factor.

$$y_{it} = z_i(t) + r_{it} + f_t \gamma_i + v_{it} \quad (14)$$

A Fourier expansion with a single frequency component, as in equation (15), is capable of constructing a level and trend shift model.

$$z_i(t) = \alpha_i + \beta_i t + \gamma_{1i} \sin\left(\frac{2\pi kt}{T}\right) + \gamma_{2i} \cos\left(\frac{2\pi kt}{T}\right) \quad (15)$$

k is the Fourier frequency component, and $r_{i0} = 0$ for all i . γ_{1i} measures the amplitude and displacement of shifts is captured by γ_{2i} . As opposed to sharp breaks, smooth breaks using the

²⁴ A strictly linear trend is just a special case.

Fourier approximation has a weakness arising out of unknown form, numbers, and dates of breaks is that it is not possible to analyze the changes of the values of the constant and time trend before and after the structural changes (Tsong, Lee, Tsai and Hu, 2016), which has a vast empirical literature on it starting with Tomljanovich and Vogelsang (2002). Individual test statistics are computed using the following equation

$$\tau_{\tau_i}(k) = \frac{1}{T^2} \frac{\sum_{t=1}^T \tilde{\delta}_{it}(k)^2}{\tilde{\sigma}_{vi}^2} \quad (16)$$

where $\tilde{\delta}_{it}(k)$ is the sum of OLS residuals from equation (14) and $\tilde{\sigma}_{vi}^2$ is the long run variance²⁵. The average of individual statistics (τ_{τ}) is taken to obtain the below panel test statistic.

$$FP(k) = \frac{1}{N} \left(\sum_{i=1}^N (\tau_{\tau_i}(k)) \right) \quad (17)$$

The null hypothesis of stationarity converges to the standard normal distribution. Thus, the final version of panel test statistic is defined as

$$FZ(k) = \frac{\sqrt{N}(FP(k) - \xi(k))}{\zeta(k)} \sim N(0,1) \quad (18)$$

Values of $\xi(k)$ and $\zeta^2(k)$ for constant, and constant and trend models can be found in Table 1 in Nazlioglu and Karul (2017). The long-run variance is estimated with the Bartlett kernel with Kurozumi (2002), as suggested by Nazlioglu and Karul (2017), due to their superior performance over the rule of Sul et al. (2005).

Which frequency has to be preferred needs great attention and depends upon the sort of data. As argued by Becker, Enders, and Hurn (2004), highly persistent macroeconomic data requires the value of k as 1 or 2 to control for breaks²⁶ and test for the stationarity versus non-stationarity, where the higher frequencies are not associated with structural breaks but stochastic parameter variability. Nazlioglu and Karul (2017) assume homogenous²⁷ frequency across cross-sections in order to obtain the asymptotic distribution of panel statistics. According to Lee, Wu, and Yang (2016) homogenous frequency does not mean identical breaks across cross-sections.

25 For the details of the long run variance see Becker *et al.* (2006).

26 "It is difficult to distinguish between a structural break and certain types of nonlinearities. Clearly, a series with a break can be viewed as a special case of a process that is nonlinear in its parameters. As such, our approach can be viewed as an attempt to provide a general procedure to approximate unknown nonlinear components (Becker *et al.* 2006: p.2)"

27 Proper frequency selection especially in time series is possible through grid-search by minimizing sum of squared residuals (Becker *et al.* 2006). To the best my knowledge, similar procedure is not available for panel case.

6. Results

According to Hadri (2000) test statistics, stochastic convergence is not observed in 38 provinces, while shocks have only a transitory impact on 41 provinces across the country but are mostly located in the Mediterranean and Eastern Black Sea regions. There is no clear-cut East-West distinction in terms of convergence. The number of provinces converging to the country average slightly increases to 45 provinces. Although the outcome of 32 provinces does not change when cross-sectional dependence is controlled, the bias that may arise due to erroneous omission of this facet is eliminated. The discrepancy between stochastically convergent provinces according to no-shift models is quite obvious. However, the novelty of this study is the merging of information obtained from univariate test statistics and panel test statistics simultaneously. Panel B parts of Tables 2, 3, and 4 depict panel tests. Panel A parts of Tables 2, 3, and 4 reflect univariate test statistics for each province. Yet, univariate tests in Panel A indicate that only concentrating on the panel level may hide the inner dynamics; thus, this also validates the approach adopted in the study. The null hypothesis of stationarity is rejected in Table 2, leading to, to some extent, divergence at the panel level.

On the other hand, the time span is 28 years, which is quite long for a developing country such as Türkiye. Many significant economic crises (1994 and 2001) stemmed from inner sources or (2007-2008) transmitted from the world during that period may disrupt the estimation of tests that ignore structural breaks. To avoid this and mitigate potential issues, it is necessary to capture the underlying dynamics by allowing the test to account for structural breaks. The groundbreaking feature of Carrion-i Silvestre et al. (2005) is the endogenous determination of structural breaks, and the restriction in front of the number of breaks is removed. Table 3 shows that incorporating structural breaks improved the number of provinces with stochastic convergence to 54. Once again, metropolitan cities such as Istanbul, Ankara, İzmir, and Bursa appear as consistently convergent irrespective of the panel unit root test. In 13 provinces, there is no structural break and most structural breaks take place in Gümüşhane with 5 breaks. Dates of structural breaks vary across the country, but mostly, they correspond to economic crisis periods. The effect of the 1994 crisis may be detected in 1995 and later years; the 2000-2001 crisis is less visible as a break, but as a caveat, it should be noted that 1999 is the catastrophic earthquake year and subsequent years can capture this. Besides, the following years can experience this demolition on economy as a prolonged shock. 2007-08 global financial crises also spilled over and appeared as a break for some provinces. Local, national, and presidential election years should be monitored carefully as a potential source of breaks in this respect.

The difference in structural break dates may also signal out that the same shock may have different impacts on the regions. Panel tests are estimated for two separate cases in Table 3, cross-sectional independence and cross-sectional dependence. Assuming cross-sectional independence results in no stochastic convergence overall, and this conclusion is robust to both homogenous and heterogenous long-run variances. To take into consideration the cross-sectional dependence, bootstrap critical values are obtained, and according to Panel C, homogeneity in the long-run

variance ends up with a rejection of stationarity while heterogeneity leads to stationarity in the panel. Therefore, strong interpretations are not possible here.

Smooth-shift models are displayed in Table 4 for k stands for Fourier frequencies. Following Becker, et al. (2004), higher frequencies are not suitable for usage. Once again, at the panel level, FzK statistics reject the stochastic convergence. On the other hand, for 24 provinces, individual test statistics are in the rejection region of the 10% critical value, leading to stochastic convergence. The remaining 54 provinces do not follow a convergence path. When the panel stationarity test is conducted at $k=2$, the number of convergent provinces falls to only 18. As a result, when cross-sectional dependence is controlled and instead of sharp shift, smooth shifts are allowed in the model, stochastic convergence at the provincial level weakens significantly. Additionally, findings from univariate cases approach to the panel findings, which are robust to the model selection. Unlike the country basis analysis of which outcomes of the panel tests are sensitive to the selection of the panel members such as missing data, membership of an organization, or interest of researchers in particular countries (Ford, Jackson and Kline, 2006), working with a single country and its regions, to some degree, help us to avoid such a problem. However, this study admits that socio-cultural and socio-economic factors may have great importance in settling this convergence issue.

Table 2: No Shift Model

Panel A: province-by-province tests							
		Hadri (2000)		Hadri and Kurozumi (2012)		Hadri (2000)	Hadri and Kurozumi (2012)
Nuts3	Province	KPSS	KPSS	Nuts3	Province	KPSS	KPSS
TR100	İstanbul	0.068	0.072	TR811	Zonguldak	0.070	0.045
TR211	Tekirdağ	0.104	0.101	TR812	Karabük	0.117	0.126
TR212	Edirne	0.182	0.172	TR813	Bartın	0.074	0.088
TR213	Kırklareli	0.071	0.070	TR821	Kastamonu	0.058	0.076
TR221	Balıkesir	0.136	0.123	TR822	Çankırı	0.142	0.140
TR222	Çanakkale	0.163	0.148	TR823	Sinop	0.138	0.150
TR310	İzmir	0.076	0.113	TR831	Samsun	0.128	0.135
TR321	Aydın	0.094	0.104	TR832	Tokat	0.127	0.162
TR322	Denizli	0.102	0.108	TR833	Çorum	0.060	0.077
TR323	Muğla	0.147	0.132	TR834	Amasya	0.088	0.092
TR331	Manisa	0.089	0.147	TR901	Trabzon	0.077	0.055
TR332	Afyonkarahisar	0.117	0.113	TR902	Ordu	0.073	0.066
TR333	Kütahya	0.112	0.109	TR903	Giresun	0.182	0.180
TR334	Uşak	0.098	0.091	TR904	Rize	0.126	0.098
TR411	Bursa	0.115	0.151	TR905	Artvin	0.162	0.156
TR412	Eskişehir	0.043	0.043	TR906	Gümüşhane	0.148	0.147
TR413	Bilecik	0.128	0.118	TRA11	Erzurum	0.098	0.053
TR421	Kocaeli	0.062	0.092	TRA12	Erzincan	0.135	0.134
TR422	Sakarya	0.076	0.110	TRA13	Bayburt	0.104	0.137
TR423	Düzce	0.092	0.074	TRA21	Ağrı	0.126	0.103
TR424	Bolu	0.082	0.092	TRA22	Kars	0.074	0.055
TR425	Yalova	0.154	0.169	TRA23	Iğdır	0.122	0.095
TR510	Ankara	0.103	0.100	TRA24	Ardahan	0.041	0.093
TR521	Konya	0.167	0.157	TRB11	Malatya	0.127	0.146
TR522	Karaman	0.064	0.061	TRB12	Elazığ	0.095	0.078
TR611	Antalya	0.166	0.154	TRB13	Bingöl	0.165	0.153
TR612	Isparta	0.160	0.133	TRB14	Tunceli	0.173	0.168
TR613	Burdur	0.173	0.160	TRB21	Van	0.161	0.145
TR621	Adana	0.159	0.132	TRB22	Muş	0.086	0.097
TR622	Mersin	0.154	0.154	TRB23	Bitlis	0.092	0.065
TR631	Hatay	0.139	0.153	TRB24	Hakkari	0.189	0.174
TR632	Kahramanmaraş	0.043	0.068	TRC11	Gaziantep	0.146	0.158
TR633	Osmaniye	0.171	0.155	TRC12	Adıyaman	0.165	0.160
TR711	Kırıkkale	0.069	0.071	TRC13	Kilis	0.127	0.122
TR712	Aksaray	0.191	0.188	TRC21	Şanlıurfa	0.085	0.086
TR713	Niğde	0.153	0.155	TRC22	Diyarbakır	0.083	0.057
TR714	Nevşehir	0.127	0.154	TRC31	Mardin	0.169	0.150
TR715	Kırşehir	0.061	0.074	TRC32	Batman	0.082	0.069
TR721	Kayseri	0.105	0.101	TRC33	Şırnak	0.205	0.192
TR722	Sivas	0.070	0.119	TRC34	Siirt	0.134	0.103
TR723	Yozgat	0.063	0.096				

Panel B: panel tests

	Stat.	p-value
	10.634	0.000
Hadri (2000)	10.691	0.000

Hadri and Kurozumi (2011)

Notes: The critical values are 0.119, 0.146, and 0.216 for 10%, 5%, and 1%, respectively. The bold numbers show the rejection of the null hypothesis of stationarity.

Table 3: Sharp Shift Model

Panel A: province-by-province tests		Carrion-i Silvestre <i>et al.</i> (2005)							Bootstrap Critical Values		
Nuts3	Province	KPSS	m	T ₁	T ₂	T ₃	T ₄	T ₅	0.90	0.95	0.99
TR100	İstanbul	0.063	2	2000	2009				0.076	0.173	0.331
TR211	Tekirdağ	0.093	2	1999	2009	2010	2010		0.586	0.774	1.173
TR212	Edirne	0.157	3	1999	2005	2015	2015		0.162	0.180	0.218
TR213	Kırklareli	0.036	2	1999	2008	2014	2014		0.299	0.435	0.665
TR221	Balıkesir	0.406	3	1999	2008	2006			0.125	0.153	0.313
TR222	Çanakkale	0.107	2	1996	2012	2010			0.170	0.209	0.358
TR310	İzmir	0.065	0	1997	2010	2009			0.262	0.379	0.724
TR321	Aydın	0.036	2	1996	2008	2013			0.308	0.375	0.519
TR322	Denizli	0.037	2	1996	2007	2006			0.190	0.259	0.368
TR323	Muğla	0.076	1	2009	2001	2007			0.354	0.434	0.572
TR331	Manisa	0.561	1	1999	2008	2007			0.116	0.164	0.362
TR332	Afyonkarahisar	0.041	1	2007	2009				0.154	0.193	0.323
TR333	Kütahya	0.043	1	1996	2013				0.112	0.132	0.250
TR334	Uşak	0.070	3	1995	2007				0.167	0.207	0.265
TR411	Bursa	0.085	4	2005	2009				0.563	0.615	0.725
TR412	Eskişehir	0.171	0	1998	2012				0.260	0.355	0.601
TR413	Bilecik	0.069	1	2005	2013				0.159	0.180	0.240
TR421	Kocaeli	0.158	2	2013	2015				0.126	0.263	0.503
TR422	Sakarya	0.038	2	1996	2007				0.103	0.169	0.284
TR423	Düzce	0.073	1	1997	2005				0.219	0.327	0.564
TR424	Bolu	0.257	2	1999	2001				0.193	0.229	0.311
TR425	Yalova	0.043	2	1999	2008				0.135	0.182	0.267
TR510	Ankara	0.072	2	2009	2007				0.073	0.108	0.275
TR521	Konya	0.764	2	1996	2009				0.160	0.181	0.299
TR522	Karaman	0.050	2	2000	2002				0.086	0.103	0.182
TR611	Antalya	0.064	2	1996	2011				0.189	0.219	0.382
TR612	Isparta	0.055	1	2003	2014				0.245	0.311	0.463
TR613	Burdur	0.092	2	2000	2002				0.198	0.252	0.360
TR621	Adana	0.469	1	1999	2002				0.148	0.194	0.342
TR622	Mersin	0.218	1	1997					0.143	0.164	0.264
TR631	Hatay	0.216	1	1995					0.168	0.218	0.356
TR632	Kahramanmaraş	0.402	3	1999					0.078	0.128	0.165
TR633	Osmaniye	0.388	4	1998					0.126	0.144	0.172
TR711	Kırıkkale	0.052	1	1997					0.239	0.331	0.518
TR712	Aksaray	0.086	3	1997					0.166	0.177	0.208
TR713	Niğde	0.049	1	1997					0.280	0.334	0.480
TR714	Nevşehir	0.211	0	1999					0.252	0.346	0.597
TR715	Kırşehir	0.118	0	1995					0.247	0.352	0.593
TR721	Kayseri	0.081	2	1999					0.217	0.283	0.406
TR722	Sivas	0.093	2	2004					0.141	0.279	0.500
TR723	Yozgat	0.175	1	2002					0.184	0.275	0.421
TR811	Zonguldak	0.105	0	1995					0.261	0.356	0.598
TR812	Karabük	0.184	0	1995					0.266	0.368	0.628
TR813	Bartın	0.225	0						0.265	0.383	0.641
TR821	Kastamonu	0.162	3						0.399	0.452	0.578
TR822	Çankırı	0.100	1						0.172	0.200	0.324
TR823	Sinop	0.082	2						0.132	0.224	0.424
TR831	Samsun	0.170	2						0.081	0.116	0.195
TR832	Tokat	1.698	4						0.074	0.098	0.138
TR833	Çorum	0.174	3						0.418	0.485	0.635
TR834	Amasya	0.146	0						0.245	0.330	0.557

Notes: The bold numbers show the rejection of the null hypothesis of stationarity. The number of breaks is selected using the LWZ criteria. Bootstrap critical values are obtained with 4000 replications.

Table 3: Sharp Shift Model (continued)

Panel A: province-by-province tests											
Nuts3	Province	KPSS	Carrion-i Silvestre <i>et al.</i> (2005)					Bootstrap Critical Values			
			m	T _{b1}	T _{b2}	T _{b3}	T _{b4}	T _{b5}	0.90	0.95	0.99
TR901	Trabzon	0.104	0						0.249	0.351	0.589
TR902	Ordu	0.170	2	2004	2014	2005	2011	2015	0.100	0.119	0.188
TR903	Giresun	0.296	1	2004	1999	2011	2015		0.146	0.165	0.212
TR904	Rize	0.087	0	2010	2005	2008			0.250	0.357	0.623
TR905	Artvin	0.298	1	1995	2001	2012			0.159	0.177	0.248
TR906	Gümüşhane	0.962	2	1995	2015	2014			0.523	0.572	0.661
TRA11	Erzurum	0.040	1	1999	2008	2013			0.369	0.428	0.544
TRA12	Erzincan	0.917	4	1995	2011				0.247	0.325	0.582
TRA13	Bayburt	0.097	0	1995	2009				0.246	0.357	0.643
TRA21	Ağrı	0.041	1	2002	2012				0.332	0.397	0.552
TRA22	Kars	0.038	1	1996	2010				0.372	0.431	0.552
TRA23	Iğdır	0.046	1	2005	2007				0.184	0.246	0.395
TRA24	Ardahan	0.028	0	1999	2012				0.259	0.366	0.637
TRB11	Malatya	0.099	0	2000	2009				0.255	0.355	0.635
TRB12	Elazığ	0.085	3	1995	2015				0.073	0.081	0.107
TRB13	Bingöl	0.271	2	2000	2009				0.228	0.326	0.527
TRB14	Tunceli	0.086	2	2000	2009				0.138	0.173	0.351
TRB21	Van	0.168	2	1995	2008				0.411	0.517	0.708
TRB22	Muş	0.035	2	2000					0.204	0.236	0.302
TRB23	Bitlis	0.165	2	2010					0.190	0.248	0.394
TRB24	Hakkari	0.613	2	2002					0.113	0.167	0.325
TRC11	Gaziantep	0.429	3	2007					0.414	0.468	0.567
TRC12	Adıyaman	0.183	2	1999					0.101	0.110	0.160
TRC13	Kilis	0.051	1	2000					0.164	0.187	0.251
TRC21	Şanlıurfa	0.291	2	2008					0.087	0.115	0.216
TRC22	Diyarbakır	0.625	2	2000					0.375	0.492	0.719
TRC31	Mardin	0.353	2						0.108	0.120	0.207
TRC32	Batman	0.044	3						0.111	0.170	0.267
TRC33	Şırnak	0.144	1						0.180	0.233	0.361
TRC34	Siirt	0.222	3						0.069	0.111	0.395

Panel B: panel tests assuming cross-section independence		
	Stat.	p-value
LM(λ)-homogenous	70.187	0.000
LM(λ)-heterogenous	19.163	0.000

Panel C: panel tests assuming cross-section dependence (bootstrap distribution)			
	0.90	0.95	0.99
LM(λ)-homogenous	25.016	26.076	28.101
LM(λ)-heterogenous	42.749	44.874	49.616

Notes: The bold numbers show the rejection of the null hypothesis of stationarity. The number of breaks is selected using the LWZ criteria. Bootstrap critical values are obtained with 4000 replications.

Table 4: Smooth Shift Model

Panel A: province-by-province tests							
Nazlioglu and Karul (2017)				Nazlioglu and Karul (2017)			
Nuts3	Province	k=1 KPSS	k=2 KPSS	Nuts3	Province	k=1 KPSS	k=2 KPSS
TR100	İstanbul	0.072	0.092	TR811	Zonguldak	0.036	0.038
TR211	Tekirdağ	0.059	0.126	TR812	Karabük	0.044	0.126
TR212	Edirne	0.074	0.147	TR813	Bartın	0.056	0.142
TR213	Kırklareli	0.072	0.092	TR821	Kastamonu	0.058	0.110
TR221	Balıkesir	0.054	0.145	TR822	Çankırı	0.052	0.145
TR222	Çanakkale	0.049	0.152	TR823	Sinop	0.063	0.142
TR310	İzmir	0.040	0.134	TR831	Samsun	0.040	0.142
TR321	Aydın	0.051	0.114	TR832	Tokat	0.042	0.142
TR322	Denizli	0.038	0.121	TR833	Çorum	0.063	0.106
TR323	Muğla	0.082	0.170	TR834	Amasya	0.036	0.066
TR331	Manisa	0.035	0.147	TR901	Trabzon	0.062	0.080
TR332	Afyonkarahisar	0.044	0.142	TR902	Ordu	0.053	0.080
TR333	Kütahya	0.042	0.132	TR903	Giresun	0.036	0.191
TR334	Uşak	0.069	0.153	TR904	Rize	0.041	0.100
TR411	Bursa	0.065	0.143	TR905	Artvin	0.063	0.148
TR412	Eskişehir	0.065	0.055	TR906	Gümüşhane	0.059	0.139
TR413	Bilecik	0.029	0.129	TRA11	Erzurum	0.063	0.033
TR421	Kocaeli	0.062	0.107	TRA12	Erzincan	0.059	0.139
TR422	Sakarya	0.056	0.129	TRA13	Bayburt	0.052	0.170
TR423	Düzce	0.048	0.096	TRA21	Ağrı	0.064	0.135
TR424	Bolu	0.052	0.109	TRA22	Kars	0.055	0.038
TR425	Yalova	0.031	0.146	TRA23	İğdir	0.062	0.159
TR510	Ankara	0.081	0.140	TRA24	Ardahan	0.034	0.091
TR521	Konya	0.055	0.163	TRB11	Malatya	0.044	0.180
TR522	Karaman	0.054	0.036	TRB12	Elazığ	0.055	0.120
TR611	Antalya	0.056	0.158	TRB13	Bingöl	0.048	0.136
TR612	Isparta	0.065	0.168	TRB14	Tunceli	0.054	0.169
TR613	Burdur	0.077	0.151	TRB21	Van	0.043	0.169
TR621	Adana	0.055	0.140	TRB22	Muş	0.064	0.107
TR622	Mersin	0.049	0.139	TRB23	Bitlis	0.079	0.119
TR631	Hatay	0.035	0.190	TRB24	Hakkari	0.071	0.141
TR632	Kahramanmaraş	0.035	0.055	TRC11	Gaziantep	0.068	0.141
TR633	Osmaniye	0.056	0.147	TRC12	Adıyaman	0.057	0.142
TR711	Kırıkkale	0.065	0.080	TRC13	Kilis	0.040	0.138
TR712	Aksaray	0.048	0.156	TRC21	Şanlıurfa	0.038	0.064
TR713	Niğde	0.061	0.222	TRC22	Diyarbakır	0.052	0.058
TR714	Nevşehir	0.036	0.174	TRC31	Mardin	0.062	0.143
TR715	Kırşehir	0.039	0.074	TRC32	Batman	0.057	0.114
TR721	Kayseri	0.083	0.143	TRC33	Şırnak	0.024	0.145
TR722	Sivas	0.051	0.132	TRC34	Siirt	0.074	0.164
TR723	Yozgat	0.059	0.133				

Panel B: panel tests		
	Stat.	p-value
FzK (k=1)	16.912	0.000
FzK (k=2)	17.381	0.000

Notes: The bold numbers show the rejection of the null hypothesis of stationarity. k represents the Fourier frequency. Critical values are 0.0471 (10%), 0.0546 (5%), and 0.0716 (1%) for $k=1$; 0.1034, 0.1321, and 0.2022 for $k=2$.

7. Conclusion

Turkiye shows up as a very suitable candidate for analyzing the convergence phenomenon due to flagrant regional disparities manifested in the West and East. Despite the voluminous literature, there is no consensus on the presence or type of convergence. This research adheres to the findings of the aforementioned empirical literature on Turkiye, and partially documents some supportive outcomes using the choropleth maps and σ -convergence. Economic growth spreads unevenly at the provincial level, making catch-up challenging for initially low-income areas. For example, the Northeastern part overperformed compared to the rest in terms of real per capita growth. On the one hand, generally, low-income areas experienced relatively low progress. On the other hand, income dispersion gets narrower, especially after the 2000s. The speed of σ -convergence soared up around 2008. As a matter of fact, this can provide some evidence for approaching the level of lower-income regions as a country.

This study follows an alternative formulation and perspective to shed more light on convergence in Turkiye between 1992 and 2019. It adopts a more statistical attitude and adds new flavors to estimation mechanics in pursuit of the best data-generating process of income per capita. As a result, this study focuses on stochastic convergence. However, some limitations arise in this approach as other economic factors and initial income levels cannot be controlled, but dealing with the pure data itself and its relative ratio to the province averages contributes to understanding of convergence from a different angle. Incorporating data and region-specific elements like cross-sectional dependence, endogenously determined structural breaks, and smooth breaks requires implementing a set of panel stationarity tests. Four different panel stationarity tests are employed, constructed on the same structure. Therefore, no methodological probable inconsistency exists, and results can be directly comparable. The panel framework permits the study to examine the univariate cases as well. Thus, empirical results are interpreted in two layers. The results of the panel stationarity tests partially track the literature, and Turkiye, with its provinces, does not have stochastic convergence. Controlling different potential features of the data also does not alter that conclusion. However, stochastic convergence or stochastic divergence is not omnipresent at the provincial level, and there is no regional pattern. Provinces should be discussed in-depth to reveal the reasoning behind this absence. There is also a tendency towards obtaining fewer provinces as stochastically convergent. The results, in particular, from univariate cases, demand a lot of care, and perhaps further technical carving out.

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