Black Sea Journal of Engineering and Science
doi: 10.34248/bsengineering.1575776

Open Access Journal
e-ISSN: 2619 – 8991

Research Article
Volume 8 - Issue 1: xx-xx / January 2025

EXACT SOLUTIONS OF TIME-FRACTIONAL THIN-FILM FERROELECTRIC MATERIAL EQUATION WITH CONFORMABLE FRACTIONAL DERIVATIVE

Mustafa EKICI[1]*

[1]*Çanakkale Onsekiz Mart University, Faculty of Education, Department of Mathematics and Science, 17100, Çanakkale, Türkiye*

Abstract: This study employs the unified method, a powerful approach, to address the intricate challenges posed by fractional differential equations in mathematical physics. The principal objective of this study is to derive novel exact solutions for the time-fractional thin-film ferroelectric material equation. Fractional derivatives in this study are defined using the conformable fractional derivative, ensuring a robust mathematical foundation. Through the unified method, we derive solitary wave solutions for the governing equation, which models wave dynamics in these materials and holds significance in various fields of physics and hydrodynamics. The behavior of these solutions is analyzed using the conformable derivative, shedding light on their dynamic properties. Analytical solutions, formulated in hyperbolic, periodic, and trigonometric forms, illustrating the impact of fractional derivatives on these physical phenomena. This paper highlights the capability of the unified method in tackling complex issues associated with fractional differential equations, expanding both mathematical techniques and our understanding of nonlinear physical phenomena.

Keywords: The thin-film ferroelectric material equation, the unified method, conformable fractional derivative

***Corresponding author:** Canakkale Onsekiz Mart University, Faculty of Education, Department of Mathematics and Science, 17100, Canakkale, Türkiye
E mail: mustafa.ekici@comu.edu.tr (M. EKICI)
Mustafa EKICI https://orcid.org/ 0000-0003-2494-8229
Received: October 29, 2024
Accepted: December 16, 2024
Published: January 15, 2025

Cite as: Ekici M. 2025. Exact solutions of time-fractional thin-film ferroelectric material equation with conformable fractional derivative. BSJ Eng Sci, 8(1): xx-xx.

Introduction

Nonlinear partial differential equations (NLPDEs) are fundamental in the fields of science and engineering, providing critical frameworks for modeling and analyzing real-world phenomena (Sun et al., 2018). PDEs are widely applied across diverse disciplines, from physics and chemistry to engineering and finance, where they enable a more nuanced understanding of complex physical and dynamical systems. In recent years, fractional differential equations (FDEs) have attracted substantial attention due to their generalization of classical integer-order models, offering enhanced flexibility and accuracy in representation (Ray et al., 2014). The introduction of fractional order derivatives introduces additional degrees of freedom, allowing these models to more precisely capture the intricacies of physical processes and yielding improved results in practical applications when compared to traditional integer-order models.

Fractional partial differential equations (FPDEs) have thus become instrumental across multiple domains, such as physics, chemistry, control theory, acoustics, viscoelasticity, electrochemistry, fluid dynamics, and engineering. These equations support a more accurate modeling of real-world phenomena, especially those with memory and hereditary properties, by reflecting the temporal and spatial dependencies in a system more comprehensively. Nonlinear FDEs, in particular, allow for exact solutions that describe a variety of complex nonlinear behaviors, offering essential insights into the dynamics of sophisticated systems. The expanded capability of FPDEs to capture and interpret complex behaviors highlights their growing importance in advancing scientific understanding across these diverse fields (Mainardi, 2018; Wang et al., 2023).

The quest to find reliable and exact solutions for nonlinear FDEs has spurred significant research efforts, as such solutions are essential for a complete understanding of the physical implications of these equations. Researchers have developed various methods to tackle nonlinear FDEs, providing a rich set of tools for analyzing and modeling intricate phenomena. Solitary wave solutions, in particular, have garnered attention for their effectiveness in elucidating the fundamental physics behind a wide range of phenomena, contributing to fields as varied as hydrodynamics, optics, and materials science. This pursuit has not only enriched the repertoire of mathematical techniques available but has also deepened our understanding of nonlinear physical phenomena, marking nonlinear FDEs as a vital element in modern scientific and engineering research (Wang et al., 2023).

Thin-film ferroelectric materials are characterized by unique dielectric properties, notably their ability to retain polarization even after an external electric field is removed. The mathematical modeling of these materials often involves a complex set of equations to describe the relationship between the electric displacement field, polarization, and electric field within the thin film structure. The governing equation, typically derived from the Landau-Ginzburg-Devonshire theory, incorporates a nonlinear polarization term, accounting for ferroelectric hysteresis, as well as gradient energy terms that describe domain wall behavior. This equation is crucial for understanding the dynamic response and stability of polarization in thin films, which are widely used in memory storage devices, sensors, and actuators. Furthermore, solving these equations provides insights into optimizing ferroelectric material properties at nanoscale dimensions, where size-dependent effects play a significant role. The thin-film ferroelectric equation, therefore, serves not only as a theoretical framework but also as a practical tool in engineering next-generation electronic devices (Martin and Rappe, 2016).

Further the time-fractional thin-film ferroelectric material equation (TFFEME) holds a significant role across various branches of physics and thermodynamics, providing critical insights into the behavior and optimization of thin-film materials. Thin films, characterized by material layers with thicknesses typically in the micrometer range, are profoundly influenced by the deposition process, which largely determines their properties. Recent advancements in optimizing the performance of these materials have greatly expanded their applications, enhancing their relevance in contemporary technology (Setter et al., 2006). The TFFEME finds practical utility across multiple domains, with applications ranging from memory devices and actuators to sensors, each with unique performance requirements tailored to specific contexts. For instance, ferroelectric sensors prioritize high spontaneous polarization, while ferroelectric memory devices demand enhanced fatigue resistance (Gruverman et al., 1997).

In response to the increasing demand for miniaturization within microelectronics, TFFEME-based devices have been scaled down in feature size, now approaching the nanoscale (Qin et al., 2008). Such advancements underscore the adaptability of TFFEME across diverse applications, reinforcing its importance within modern technological landscapes. The one-dimensional time-fractional form of the TFFEME is given by (Zahran et al., 2022) (equation 1).

$$\frac{n_c}{q_d^2}\frac{\partial^\beta G(x,t)}{\partial t^\beta}+[(g_2-2\alpha)G+g_4G^3+g_6G^5]-w\Delta G=0, \qquad 1$$

where n_c and q_d are density of mass and particles. g_2, g_4, g_6 are indicating both pressure and temperature. w is the space non-uniformity coefficient, and α is the reciprocal of the electric susceptibility.

These phenomena can be effectively modeled using NLPDEs. Consequently, obtaining traveling wave solutions of NLPDEs is of significant importance. In order to gain insight into the underlying mechanisms of these physical phenomena, it is imperative to investigate their solutions. Solutions to NLPDEs not only address specific problems but also provide profound insights into the fundamental physical aspects within related fields. As a result, numerous powerful methodologies have been developed to obtain exact solutions for nonlinear equations. These methods, including the tanh-function expansion method (Fan, 2000), Jacobi elliptic function expansion method (Liu et al., 2001), homogeneous balance method (Wang et al., 1996), exponential function method (He and Wu, 2006; Ekici and Unal 2020), (G'/G)-expansion method (Zhang et al., 2008; Ekici and Unal, 2022), Adomian decomposition method (El-Sayed and Gaber, 2006), homotopy analysis method (Arafa et al., 2011), differential transformation method (Odibat and Momani, 2008; Ekici and Ayaz 2017), unified method (Akcagil and Aydemir, 2018), and Kudryashov's method (Kaplan et al., 2016; Ekici, 2023), among others, have been widely employed in exploring nonlinear phenomena across various scientific disciplines.

This study can be summarized as follows: Section 2 provides an overview of the conformable fractional derivative and its fundamental properties, along with a step-by-step explanation of the unified method. In Section 3, the unified method, a key approach in solving fractional partial differential equations and the central focus of this work, is given. Additionally, exact solutions for fractional partial differential equations are obtained using the unified method in Section 3. Finally, a comprehensive discussion of the findings is presented, along with suggestions for future research directions.

2. Materials and Methods

We give with a brief overview of the conformable fractional derivative, emphasizing its fundamental properties.

Definition; Let $\beta \in (0,1]$ and $\varpi: \mathbb{R}^{+} \cup \{0\} \to R$ are be given. The conformable fractional derivative of ϖ of order β is defined as follows (equation 2):

$$(T_\beta \varpi)(t)=\lim_{\varepsilon\to 0}\frac{\varpi(t+\varepsilon t^{1-\beta})-\varpi(t)}{\varepsilon}, (t>0). \qquad 2$$

Theorem; Let $\beta \in (0,1], t>0$ and ϖ, ψ be $\beta-$differentiable. Then we can write the following properties.

- $T_\beta(k\varpi+s\psi)=k(T_\beta\varpi)+s(T_\beta\psi)$, for all $k,s \in \mathbb{R}$.
- $T_\beta(t^m)=mt^{m-\beta}$ for all $m \in \mathbb{R}$.
- $T_\beta(\lambda)=0$, for all constant functions $\varpi(t)=\lambda$.
- $T_\beta(\varpi\psi)=\varpi(T_\beta\psi)+\psi(T_\beta\varpi)$.
- $T_\beta\left(\frac{\varpi}{\psi}\right)=\frac{\psi(T_\beta\varpi)-\varpi(T_\beta\psi)}{\psi^2}$.
- If, in addition, ϖ is differentiable, then (equation 3);

$$\left(T_\beta \varpi\right)(t)=t^{1-\beta}\left(\frac{d\varpi}{dt}\right). \quad 3$$

The derivative of order β for a constant is zero. Derivatives can be applied to both differentiable and non-differentiable functions (Abdeljawad, 2015; Li and Peng, 2023).

The unified method is an advanced analytical approach that has gained considerable recognition in mathematical physics for its effectiveness in solving nonlinear fractional differential equations. The method has gained renown for its capacity to produce precise analytical solutions to complex and nonlinear equations, thus providing researchers with a robust instrument with which to address a wide range of challenging problems. A key strength of the unified method lies in its flexibility and adaptability, making it especially valuable for cases involving fractional derivative equations and complex boundary conditions.

In this study, we apply the unified method to derive stable and explicit soliton solutions for FDEs. This method accommodates a general form of nonlinear evolution equations, offering a structured approach for exploring the intricate dynamics inherent in fractional models, as outlined below. By employing the unified method, we aim to advance our understanding of these equations and highlight its potential as a powerful framework for future research in nonlinear fractional systems.

In this section we illustrate the unified method for solving NPDEs. Suppose that a NPDEs are in the following form (equation 4):

$$P(u, D_t^\beta u, u_x, D_t^\beta u_x, D_t^{2\beta} u, \dots)=0, \quad 4$$

where β denotes the conformable fractional derivative, while P denotes a polynomial involving u and its various partial derivatives, encompassing the highest order derivative and nonlinear terms. The unified method will be elucidated to derive typical and broad-spectrum soliton solutions for NFDEs. The fundamental phases of the unified method are outlined as follows:

Step 1: Assign a compound variable ξ with the real variables x and t by the following transformation(equation 5):

$$u(x,t)=U(\xi), \xi=x-\frac{k}{\Gamma(1+\beta)}t^\beta, \quad 5$$

where k is wave velocity. The wave variable assigned in equation 5 transforms equation 4 into the following ordinary differential equation (ODE);

$$Q(U,-kU',U',-kU'',k^2U'',\dots)=O. \quad 6$$

Here, Q represents a polynomial involving U and its derivatives with respect to ξ. We integrate equation 6 as many times as feasible, and for the sake of simplicity, we set the constant(s) of integration to zero.

Step 2: We express the exact solution of equation 6 in the following form (equation 7):

$$U(\xi)=a_0+\sum_{i=1}^{M}\left[a_i\varphi^i+b_i\varphi^{-i}\right], \quad 7$$

where M is positive integers, a_0, a_i, b_i $(i=1,2,3,\dots,M)$ are constants to be determined and $\varphi=\varphi(\xi)$ satisfies following the Riccati differential equation 8.

$$\varphi'(\xi)=\varphi^2(\xi)+\lambda, \quad 8$$

where $\varphi'=\frac{d\varphi}{d\xi}$ and λ is a constant. The general solution of equation 8 as follows:

Set 1: When $\lambda<0$, the solutions of equation 8

$$\varphi(\xi)=\begin{cases}\dfrac{\sqrt{-(A^2+B^2)\lambda}-A\sqrt{-\lambda}cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)}{Asinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)+B},\\ \dfrac{-\sqrt{-(A^2+B^2)\lambda}-A\sqrt{-\lambda}cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)}{Asinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)+B},\\ \sqrt{-\lambda}-\dfrac{2A\sqrt{-\lambda}}{A+cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)-sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)},\\ -\sqrt{-\lambda}+\dfrac{2A\sqrt{-\lambda}}{A+cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)+sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)},\end{cases}$$

where A, B and ξ_0 are arbitrary constants.

Set 2: When $\lambda>0$, the solutions of equation 8

$$\varphi(\xi)=\begin{cases}\dfrac{\sqrt{(A^2-B^2)\lambda}-A\sqrt{\lambda}cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)}{Asin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+B},\\ \dfrac{-\sqrt{(A^2-B^2)\lambda}-A\sqrt{\lambda}cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)}{Asin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+B},\\ i\sqrt{\lambda}-\dfrac{2Ai\sqrt{\lambda}}{A+cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)-i\,sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)},\\ -i\sqrt{\lambda}+\dfrac{2A\sqrt{\lambda}}{A+cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+i\,sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)},\end{cases}$$

where A, B and ξ_0 are arbitrary constants.

Set 3: When $\lambda=0$, the solutions of equation 8

$$\varphi(\xi)=-\frac{1}{\xi+\xi_0},$$

where ξ_0 arbitrary constant (Akter et al., 2020)

Step 3: Employing the homogeneous balance method outlined in equation 6 enables us to determine the positive integer values of M corresponding to the solution described in equation 7. By substituting the solution from equation 7 into equation 6 and incorporating the Riccati equation depicted in equation 8, we obtain a polynomial expression in terms of $U(\xi)$. This polynomial, upon equating coefficients of similar powers of U(ξ) to zero, yields specific sets of algebraic equations.

Step 4: Upon substituting equation 7 into equation 6 alongside equation 8, a polynomial expression in terms of $U(\xi)$ is derived. Equating all coefficients of $U(\xi)$ to zero leads to a system of algebraic equations. By employing the Maple program, we can effectively solve this system to determine the values of parameters such as a_0, a_i, b_i $(i=1,2,3,\dots,M)$, and λ. Subsequently, upon substituting these values and equation 8 into equation 7, exact solutions for the reduced equation 4 can be obtained.

3. Application

Now we apply the unifed method to obtain for analytic solution of the time-fractional thin-film ferroelectric material equation. This equation can be written as (equation 9);

$$\frac{n_c}{q_d^2}\frac{\partial^\beta G(x,t)}{\partial t^\beta}+[(g_2-2\alpha)G+g_4G^3+g_6G^5]-w\frac{\partial^2 G(x,t)}{\partial x^2}=0. \quad 9$$

Using the wave variable, substituting equation 5 into equation 9 reduces to the nonlinear ODE;

$$\left(\frac{k^2n_c}{q_d^2}-w\right)U''+[(g_2-2\alpha)U+g_4U^3+g_6U^5]=0, \quad 10$$

where $k, w, \alpha, g_2, g_4, g_6, n_c$ and q_d are constants (Zahran et al., 2022). By comparing the term U'' with U^5, utilizing the homogeneous balance principle (Wang et al., 2023), $N+2=5N$ is generated. Hence, $N=\frac{1}{2}$. When the transformation is $U(\xi)=\sqrt{P(\xi)}$ used, equation 10 can be decreased, as:

$$\left(\frac{k^2n_c}{q_d^2}-w\right)\left[\frac{1}{2}PP''-\frac{1}{4}(P')^2\right]-[(g_2-2\alpha)P^2+g_4P^3+g_6P^4]=0, \quad 11$$

where $P'=\frac{dP}{d\xi}$. Balancing the highest order term PP'' and P^4 in equation 11 we have $N=1$.

$$P(\xi)=a_0+a_1\varphi+b_1\varphi^{-1}. \quad 12$$

We substitute equation 12 into equation 11 and collect all the terms with the same power of $P^i(\xi)$ $(i=0,1,2,\ldots,8)$, and equating each coefficient to zero, yields a set of algebraic equations. Solving these equations with the aid of the mathematical software Maple, yields the following solutions for k, s, b_1, a_0, a_1 :

Case 1:

$$a_0=\frac{-3g_4}{8g_6},\ b_1=\frac{9g_4^2}{256a_1g_6^2},\ \lambda=-\frac{9g_4^2}{256a_1g_6^2},$$
$$w=-\frac{4}{3}a_1^2g_6+k^2\frac{n_c}{q_d^2},\ \alpha=\frac{g_2}{2}-\frac{3}{2}\frac{g_4^2}{g_6}.$$

Substituting these results into equation 12, we reach the results:

(a) Hyperbolic function solutions (when $\lambda<0$):

$$P_{11}(\xi)=\frac{-3g_4}{8g_6}+\frac{9g_4^2\left\{A\sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)+B\right\}}{256a_1g_6^2\left\{A\sqrt{-(A^2+B^2)\lambda}-A\sqrt{-\lambda}\cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)\right\}},$$

$$P_{12}(\xi)=\frac{-3g_4}{8g_6}-\frac{9g_4^2\left\{A\sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)+B\right\}}{256a_1g_6^2\left\{\sqrt{-(A^2+B^2)\lambda}+A\sqrt{-\lambda}\cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)\right\}},$$

$$P_{13}(\xi)=\frac{-3g_4}{8g_6}+\frac{9g_4^2\left\{A+\cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)-\sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)\right\}}{256a_1g_6^2\sqrt{-\lambda}\left\{\cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)-\sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)-A\right\}},$$

$$P_{14}(\xi)=\frac{-3g_4}{8g_6}+\frac{9g_4^2\left\{A+\cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)\right\}}{256a_1g_6^2\sqrt{-\lambda}\left\{A-\cosh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)-\sinh\left(2\sqrt{-\lambda}(\xi+\xi_0)\right)\right\}}.$$

(b) Trigonometric function solutions (when $\lambda>0$):

$$P_{15}(\xi)=\frac{-3g_4}{8g_6}+\frac{9g_4^2\left\{A\sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+B\right\}}{256a_1g_6^2\left\{\sqrt{(A^2-B^2)\lambda}-A\sqrt{\lambda}\cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)\right\}},$$

$$P_{16}(\xi)=\frac{-3g_4}{8g_6}-\frac{9g_4^2\left\{A\sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+B\right\}}{256a_1g_6^2\left\{\sqrt{(A^2-B^2)\lambda}+A\sqrt{\lambda}\cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)\right\}},$$

$$P_{17}(\xi)=\frac{-3g_4}{8g_6}+\frac{9g_4^2\left\{A+\cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)-i\sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)\right\}}{256a_1g_6^2\sqrt{\lambda}\left\{i\cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+\sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)-iA\right\}},$$

$$P_{18}(\xi)=\frac{-3g_4}{8g_6}+\frac{9g_4^2\left\{A+\cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)\right\}}{256a_1g_6^2\sqrt{\lambda}\left\{-i\cos\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+\sin\left(2\sqrt{\lambda}(\xi+\xi_0)\right)+iA\right\}}.$$

(c) Rational function solutions (when $\lambda=0$)

$$P_{19}(\xi)=\frac{-3g_4}{8g_6}-\frac{9g_4^2}{256a_1g_6^2(\xi+\xi_0)},$$

where $\xi=x-\frac{k}{\Gamma(1+\beta)}t^\beta$.

Other cases of solutions can be obtained in a similar manner to the above case; however, these are omitted here for simplicity.

Case 2:

$$a_0=\frac{-3g_4}{8g_6},\ b_1=\frac{9g_4^2}{256a_1g_6^2},\ \lambda=\frac{9g_4^2}{256a_1^2g_6^2},$$
$$w=-\frac{4}{3}a_1^2g_6+k^2\frac{n_c}{q_d^2},\ \alpha=\frac{g_2}{2}-\frac{15}{128}\frac{g_4^2}{g_6}.$$

Case 3:

$$a_0=\frac{-3g_4}{8g_6},\ b_1=0,\ \lambda=-\frac{9}{64}\frac{g_4^2}{a_1^2g_6^2},$$
$$w=-\frac{4}{3}a_1^2g_6+k^2\frac{n_c}{q_d^2},\ \alpha=\frac{g_2}{2}-\frac{3}{32}\frac{g_4^2}{g_6}.$$

Case 4:

$$a_0=\frac{-3g_4}{8g_6},\ a_1=0,\ \lambda=-\frac{64}{9}\frac{b_1^2g_6^2}{g_4^2},$$
$$w=k^2\frac{n_c}{q_d^2}-\frac{27}{1024}\frac{g_4^4}{b_1^2g_6^3},\ \alpha=\frac{g_2}{2}-\frac{3}{32}\frac{g_4^2}{g_6}.$$

4. Conclusion

In this study, the time-fractional TFFEME problem was tackled using the unified method, which provided exact solutions. Through the unified method, intricate solutions for rational, trigonometric and hyperbolic function types were derived, allowing for the identification of periodic, w-shaped, dark, and bright soliton structures in closedform solutions that characterize the governing model. The findings confirm that the unified method serves as an efficient and adaptable mathematical approach for deriving a range of solitary wave solutions under the influence of temporal fractional operators. All computations were conducted using Maple, demonstrating the method's capability in extracting complex wave profiles with high accuracy and consistency. These solutions offer valuable insights into the intricate mechanisms underlying nonlinear physical phenomena, particularly in the context of wave collaboration. Our findings underscore the directness and efficiency of the unified method, highlighting its applicability to a wide array of nonlinear PDEs in mathematical physics.

Author Contributions

The percentages of the author contributions are presented below. The author reviewed and approved the final version of the manuscript.

	M.E.
C	100
D	100
S	100
DCP	100
DAI	100
L	100
W	100
CR	100
SR	100
PM	100
FA	100

C=Concept, D= design, S= supervision, DCP= data collection and/or processing, DAI= data analysis and/or interpretation, L= literature search, W= writing, CR= critical review, SR= submission and revision, PM= project management, FA= funding acquisition.

Conflict of Interest

The author declared that there is no conflict of interest.

Ethical Consideration

Ethics committee approval was not required for this study because of there was no study on animals or humans.

Acknowledgements

This study is not funded by any institution. The author expresses their sincere thanks for the reviewer's suggestions and comments.

References

Abdeljawad T. 2015. On conformable fractional calculus. J Comput Appl Math, 279: 57-66.

Akcagil S, Aydemir T. 2018. A new application of the unified method. New Trends Mathl Sci, 2018: 6(1).

Akter S, Sen RK, Roshid HO. 2020. Dynamics of interaction between solitary and rogue wave of the space-time fractional Broer–Kaup models arising in shallow water of harbor and coastal zone. SN Appl Sci, 2: 1-12.

Arafa AAM, Rida SZ, Mohamed H. 2011. Homotopy analysis method for solving biological population model. Commun Theor Phys, 56(5): 797.

Ekici M, Ünal M. 2020. Application of the exponential rational function method to some fractional soliton equations. IGI Global, Newyork, USA, pp: 13-32.

Ekici M, Ayaz F. 2017. Solution of model equation of completely passive natural convection by improved differential transform method. Res Eng Struct Mat, 3(1): 1-10.

Ekici M, Ünal M. 2022. Application of the rational (G'/G)-expansion method for solving some coupled and combined wave equations. Commun Fac Sci Univ Ank Ser A1 Math Stat, 71(1): 116-132.

Ekici M. 2023. Exact solutions to some nonlinear time-fractional evolution equations using the generalized Kudryashov method in mathematical physics. Symmetry, 15(10): 1961.

El-Sayed AMA, Gaber M. 2006. The Adomian decomposition method for solving partial differential equations of fractal order in finite domains. Phys Lett A, 359(3): 175-182.

Fan E. 2000. Extended tanh-function method and its applications to nonlinear equations. Phys Lett A, 277(4): 212-218.

Gruverman A, Tokumoto H, Prakash AS, Aggarwal S, Yang B, Wuttig M, Venkatesan T. 1997. Nanoscale imaging of domain dynamics and retention in ferroelectric thin films. Appl Phys Lett, 71(24): 3492-3494.

He JH, Wu XH. 2006. Exp-function method for nonlinear wave equations. Chaos Solitons Fract, 30(3): 700-708.

Kaplan M, Bekir A, Akbulut A. 2016. A generalized Kudryashov method to some nonlinear evolution equations in mathematical physics. Nonlinear Dyn, 85(4): 2843-2850.

Li Z, Peng C. 2023. Bifurcation, phase portrait and traveling wave solution of time-fractional thin-film ferroelectric material equation with beta fractional derivative. Phys Lett A, 484: 129080.

Liu S, Fu Z, Liu S, Zhao Q. 2001. Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations. Phys Lett A, 289(1): 69-74.

Mainardi F. 2018. Fractional calculus: Theory and applications. Mathemat, 6(9): 145.

Martin LW, Rappe AM. 2016. Thin-film ferroelectric materials and their applications. Nat Rev Mater, 2(2): 1-14.

Odibat Z, Momani S. 2008. A generalized differential transform method for linear partial differential equations of fractional order. Appl Math Lett, 21(2): 194-199.

Qin M, Yao K, Liang YC. 2008. High efficient photovoltaics in nanoscaled ferroelectric thin films. Appl Phys Lett, 2008: 93(12).

Ray SS, Atangana A, Noutchie SC, Kurulay M, Bildik N, Kilicman A. 2014. Fractional calculus and its applications in applied mathematics and other sciences. Math Probl Eng, 2014(2): 849395.

Setter N, Damjanovic D, Eng L, Fox G, Gevorgian S, Hong S, Streiffer S. 2006. Ferroelectric thin films: Review of materials, properties, and applications. J Appl Phys, 2006: 100(5).

Sun H, Zhang Y, Baleanu D, Chen W, Chen Y. 2018. A new collection of real world applications of fractional calculus in science and engineering. Commun Nonlinear Sci Numer Simul, 64: 213-231.

Wang X, Ehsan H, Abbas M, Akram G, Sadaf M, Abdeljawad T.

2023. Analytical solitary wave solutions of a time-fractional thin-film ferroelectric material equation involving beta-derivative using modified auxiliary equation method. Results Phys, 48: 106411.

Wang M, Zhou Y, Li Z. 1996. Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics. Phys Lett A, 216(1-5): 67-75.

Zahran E H, Mirhosseini-Alizamini SM, Shehata MS, Rezazadeh H, Ahmad H. 2022. Study on abundant explicit wave solutions of the thin-film Ferro-electric materials equation. Opt Quantum Electron, 54(1): 48.

Zhang S, Tong J L, Wang W. 2008. A generalized (G'/G)-expansion method for the mKdV equation with variable coefficients. Phys Lett A, 372(13): 2254-2257.