

Ms Excel Macro Applications of Finite Difference and Integration Method for Simply Supported Rectangular Plates Under Sinusoidal Load

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Abstract

In this study, the finite differences and the method of integration were compared to find displacements in an elastic plate under sinusoidal loading. This question type, which is one of the common examples in the literature, is available with various methods. Application engineers mostly use Microsoft Excel excel macro program, VBA, has written program codes for both finite element method and integration method. The difference between the results of these programs and the CPU time is calculated.

Keywords: Finite Difference Method, Integration Method, Elastic Plate, Microsoft Excel Macro

Sinüzoidal Yükleme Altında Basit Mesnetlenmiş Dikdörtgen Plaklarda Sonlu Farklar ve İntegrasyon Metodunun Ms Excel Makro Uygulaması

Özet

Bu çalışmada sinüzoidal yükleme altındaki bir elastik plaktaki deplasmanları bulmak için sonlu farklar ve integrasyon yöntemi karşılaştırılmıştır. Bu soru tipi literatürde yaygın olup, çeşitli metotlarla çözümü yapılmıştır. Bu soru tipinin uygulama mühendislerinin çoğunlukla kullandığı Microsoft Excel' in makro programında her iki metod için çözümü sunulmuştur. Yapılan çözüm sonuçları ve çözüm zamanı arasındaki farklar elde edilmiştir.

Anahtar Kelimeler: Sonlu farklar metodu, integrasyon metodu, elastik plak, Microsoft Excel Makro

1. Introduction

Numerous calculations have been made up to now on the solution of elastic plates. These studies were carried out by varying the dimensions of the elastic plaques from time to time and changing the loading conditions applied on the bases, changing the boundary conditions at the bases. In this study, four edges under sinusoidal loading resolve a simple supported elastic plate problem [1]. These solutions were compared with manual solutions and also within themselves.

In this analysis, VBA programming language in Microsoft Excel is used. The reason for choosing this applet is that it is easy to access and is easy to implement by every engineer. In addition, the ability to read data from the file and write data to the file is easier, as well as the wide selection of graphics. However, the CPU time is

not the performance seen in other programming languages .

2. Sinusoidal Load Under Elastic Plate

2.a. System geometry and loading condition

In the coordinate system given 2.1, the rectangular plate with long side a, short side b is simply supported. We assumed that the sinusoidal load distributed over surface of rectangular plate is given by the expression

$$q = q_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{a}\right) \quad (1)$$

in which q_0 represents the intensity of the load at the center of the plate.

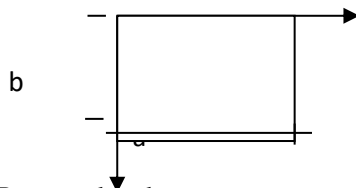


Figure 1. Rectangular plate

3. Integration Method

Generally, numerical approximation methods require reaching in several steps of solving by dividing time or space region into specific intervals, or in most cases require successive iteration solutions [2,3,4]. Numerical methods developed for this reason. They have various advantages or disadvantages in terms of the computational needs they require, the time spent on the solution or the CPU (Central Processing Unit) time, the stability of the equations, the number of node points used, the preprocessing for problem solving, and most importantly the time spent on this solution.

In this method, Elastic curve equation for (1) equation loading condition ;

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad (2)$$

The differential equation (2) for the deflection surface in this case becomes;

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_0}{D} \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{a}\right) \quad (3)$$

in which the quantity D, taking the place of the quantity EI in the case of beams, is called the flexural rigidity of the plate. The boundary conditions for simply supported edges are

$$w=0 \quad M_x=0 \quad \text{for } x=0 \text{ and } x=a$$

$$w=0 \quad M_y=0 \quad \text{for } y=0 \text{ and } y=b$$

we can represent the boundary conditions in the following form:

$$w=0 \quad \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{for } x=0 \text{ and } x=a$$

$$w=0 \quad \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{for } y=0 \text{ and } y=b$$

All boundary conditions are satisfied if we take for deflections the expression;

$$w = C \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{a}\right) \quad (4)$$

in which the constant C must be chosen so as to satisfy Eq. (3). Substituting expression (4) into Eq. (3), we find

$$\pi^4 \left[\frac{1}{a^2} + \frac{1}{b^2} \right]^2 C = \frac{q_0}{D} \quad (5)$$

and we conclude that the deflection surface satisfying Eq. (3) and boundary conditions is

$$w = \frac{q_0}{\pi^4 \left[\frac{1}{a^2} + \frac{1}{b^2} \right]^2 D} \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{a}\right) \quad (6)$$

In the light of these equations, we have applied this equation to find the displacements along the entire plate surface with the aid of the program we have done in excel macro.

Step of program:

- 1- Once plates surface is defined.
- 2- The plate lengths are divided into equal parts
- 3- Cyclic increments are made in the x and y directions
- 4- These values are placed in the equation
- 5- Program is stopped

1. Finite Difference Method

Boundary value problems do not have many definite and closed solutions. Approximate solutions are available for this. Approximate methods are divided into characters. Some agrees with the provision of boundary conditions, but requires that the differential equation be precisely met. In the second group of methods, there is approximation in the presence of the differential equation, even if the boundary conditions are precise. The finite difference method is a method

that can be applied to almost every other form entering this second group[1].

It is possible to approximate the derivative of a function $y = f(x)$ at a certain set of discrete points, using known values. Similarly, approximate partial derivative formulas can be given for multivariable functions of type $z = f(x, y)$. Considering the definition of derivative, advanced difference approach, backward difference approach and central difference approach are emphasized.

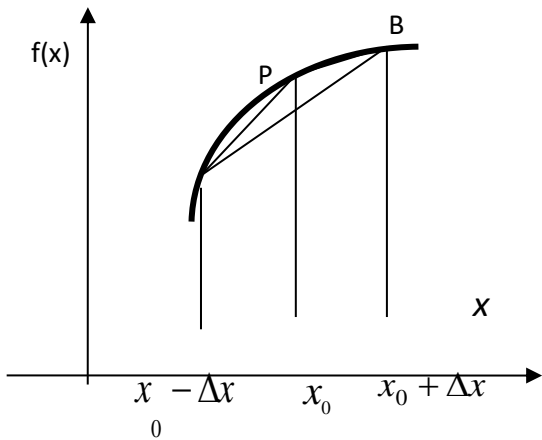


Figure 2. The derivation of $f(x)$ at point P using forward and backward and central differences

Advanced difference formula:

$$f'(x_0) = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \quad (7)$$

Back difference formula:

$$f'(x_0) = \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} \quad (8)$$

Central difference formula:

$$f'(x_0) = \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x} \quad (9)$$

The theory of elasticity is a branch of the theoretical physics, and the strain and shape changes that occur in elastic bodies under the influence of the objective external loads are calculated precisely by moving from a certain set of differential equations. In this case, the vehicle

used for the solution is the differential equation theory[3].

The principal equation of plane elasticity theory;

$$\Delta(\sigma_x + \sigma_y) = \Delta\Delta F = \frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial y^2 \partial x^2} + \frac{\partial^4 F}{\partial y^4} = 0 \quad (10)$$

From the boundary conditions:

$$F_s = M(s) \text{ and } \left(\frac{dF}{dn}\right) = -Q_t(s) \quad (11)$$

Here;

F_s : Stress function,

$M(s)$: Static moment,

Q_t : It is the trace of the boundary value forces on the tangent.

The way to solve the problem of boundary value:

- Equation (11) calculates the values of the den function at the boundary precisely,
- The coefficients of the finite difference are calculated
- We extend the function to the outside of the region to be calculated,
- Divide the calculated shape into the desired number of node points to form the finite difference equations,
- Since the gauss-elimination method is used, conversion of the results to square matrix
- By solving these results into a matrix format.

5. Summary and Conclusion

In this study, an elastic plate finite difference method and a generalized computer program were used to obtain the elastic plate deflection graph for a complex loading situation of the geometry. The results were compared and the solutions were made. In the comparison made, 0.01 percent error was observed between the integration method and the final difference method. Displacement graphics have topped. The reason is that the number of segment is excessive. But the solution time is ten times more in the finite difference method.

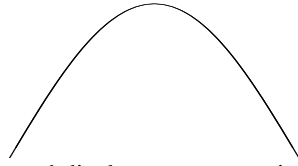


Figure 3. Exaggerated displacement curve in the long side

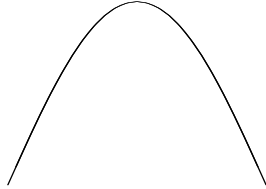


Figure 4. Exaggerated displacement curve in the short side

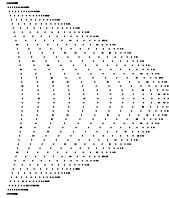


Figure 5. Plate displacement view

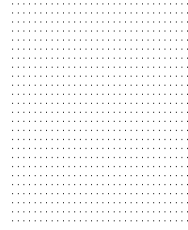


Figure 6. Plate top view

6. References

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