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A nondominated sorting ant colony optimization algorithm for complex assembly line balancing problem incorporating incompatible task sets

Uyumsuz iş setlerini içeren karmaşık montaj hattı dengeleme problemi için bastırılmamış sınıflandırmalı karınca koloni optimizasyonu algoritması

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Abstract

Two-sided assembly lines are heavily used in automotive industry for producing large-sized products such as buses, trucks and automobiles. Mixed-model lines help manufacturers satisfy customized demands at a reasonable cost with desired quality. This paper addresses to mixed-model two-sided lines incorporating incompatible task groups and proposes a new method for minimizing two conflicting objectives, namely cycle time and the number of workstations, to maximize line efficiency. While such an approach yields to a so-called type-E problem in the line balancing domain, the proposed nondominated sorting ant colony optimization (NSACO) approach provides a set of solutions dominating others in terms of both objectives (pareto front solutions). The solution which has the highest line efficiency among pareto front solutions is then determined as the best solution. An additional performance criterion is also applied when two different solutions have the same values for both objectives. The solution which has the smoother workload distribution is favoured when both criteria are the same. NSACO is described and a numerical example is provided to exhibit its running mechanism. The performance of the algorithm is tested through test problems in two conditions, i.e. incompatible task sets are considered and not considered, and computational results are presented for the first time. The results indicate that NSACO has a promising solution capacity.

Keywords: Assembly line balancing, Mixed-model, Two-sided lines, Incompatible task set constraints, Nondominated sorting ant colony optimization

Öz

İki-taraflı montaj hatları otomotiv endüstrisinde otobüs, kamyon ve otomobil gibi geniş hacimli ürünlerin üretiminde yoğunlukla kullanılmaktadır. Karışık modelli hatlar ise üreticilere müşterilerin kişiselleştirilmiş talebini uygun maliyetle ve istenen kalitede ulaştırmak için yardımcı olmaktadır. Bu çalışma uyumsuz iş gruplarını içeren karışık-modelli iki taraflı montaj hatlarını konu almaktadır ve hat etkinliğini maksimize etmek için birbiriyle çelişen iki amacı (çevrim zamanı ve istasyon sayısı) minimize eden yeni bir yöntem önermektedir. Böyle bir yaklaşım montaj hattı alanında tip-E olarak adlandırılan probleme işaret etse de önerilen bastırılmamış sınıflandırmalı karınca koloni algoritması (NSACO) diğer çözümleri her iki amaç açısından da bastıran çözüm seti sunmaktadır (pareto yüzey çözümler). Çözümler arasından en yüksek hat etkinliğine sahip olanı en iyi çözüm olarak belirlenmektedir. İki farklı çözüm her iki amaç açısından da aynı değerlere sahip olduğu zaman, ilave bir performans kriteri uygulanmaktadır. Her iki amaç da aynı değerlere sahip olduğu zaman daha düzgün iş yükü dağılımına sahip olan çözüm tercih edilmektedir. NSACO tanımlanmıştır ve çalışma prensibi bir sayısal örnek üzerinden anlatılmıştır. Algoritmanın performansı, uyumsuz iş seti kısıtlarını dikkate alarak ve almayarak iki durum altında test edilmiştir ve araştırma sonuçları ilk defa sunulmuştur. Sonuçlar göstermektedir ki NSACO ümit verici çözüm kapasitesine sahiptir.

Anahtar kelimeler: Montaj hattı dengeleme, Karışık-model, İki taraflı hatlar, Uyumsuz iş seti kısıtları, Bastırılmamış sınıflandırmalı karınca koloni optimizasyonu

1 Introduction

An assembly line is a sequential order of workstations linked to each other via a conveyor or moving belt. Each workstation performs a set of job pieces, called tasks, assigned to it within a predetermined time, called cycle time [1],[2]. Assembly line balancing problem is determining the configuration for assignment of tasks to workstations in such a way that a performance criterion (or sometimes more than one criterion) is optimised. There are essential constraints which must be ensured during the balancing process, i.e. assignment constraint, capacity constraint and precedence relationship constraint [3]. Assembly line balancing problems can be classified as type-I, type-II and type-E based on the performance criterion, or objective, sought. The number of workstations is minimised given cycle time in type-I problems whereas cycle time is minimised given the number of workstations in type-II problems. The two conflicting objectives, namely cycle time and the number of workstations, are minimised concurrently in type-E problems [4]. Assembly line balancing problems are also divided into two groups based on the configuration of workstations across the line, (i) one-sided lines and, (ii) two-sided lines. In one-sided lines, workstations are located in only one side of the line, i.e. left or right. On the other hand, workstations are located on both left and right sides of the line in two-sided lines. Therefore, another constraint, operation side constraint, needs to be considered in two-sided lines which makes problem even harder to solve in compare with one-sided lines. In two-sided lines, workstations facing each other are called mated-stations. The minimisation of the number of mated-stations, which corresponds to the length of the line, is also considered as a performance criterion in two-sided lines [5].

Mixed-model lines have emerged as a response to the effort of meeting customized demands at a reasonable cost. The main advantage of a mixed-model line over a single-model line is that more than one model of a product can be assembled on the same line in an inter-mixed sequence [6],[7]. No setup is needed (or it is negligible) between model changes as models are similar to each other. Mixed-model line balancing problem was introduced by Thomopoulos [8] and attracted many researchers to this domain. Several exact and approximate (heuristic/metaheuristic) solution techniques have been proposed to deal with it considering various objectives and constraints. One can refer to Boysen, Fliedner [9], Battaïa and Dolgui [1] for a comprehensive classification scheme for assembly line balancing problems and solution methods presented. Specifically, Emde, Boysen [10] provided a computational evaluation of objectives to smoothen workload in mixed-model lines.

Two-sided lines are frequently used in producing homogeneous large-sized products, such as buses, trucks and automobiles, in mass quantities [11]. The two-sided line balancing problem was introduced by Bartholdi [12]. This was followed by many researchers and the problem has been dealt in various aspects. Abdullah Make, Ab. Rashid [13] presented a review of optimization methods, objective functions, and specific constraints used in solving two-sided assembly line balancing problems. The majority of the researches on two-sided assembly line balancing problem focused on single-model production, see for example Kim, Kim [14], Lee, Kim [15], Kim, Song [16], Ozcan and Toklu [17], Ozbakir and Tapkan [18], Purnomo, Wee [19] and Li, Tang [20]. However, mixed-model lines are also utilised frequently in industry though they have been received less attention.

The mixed-model two-sided assembly line balancing problem (MTALBP) was introduced by Simaria and Vilarinho [21]. Ozcan and Toklu [22] presented a mathematical model and a simulated annealing algorithm for the solution of the problem. Chutima and Chimklai [23] developed a particle swarm optimisation algorithm with negative knowledge. Rabbani, Moghaddam [24] dealt with a mixed-model two-sided line configured as a multiple U-shaped layout. Kucukkoc and Zhang [7] introduced the problem of balancing and sequencing mixed-model parallel two-sided lines, modelled the problem mathematically [25] and proposed a new hybrid genetic - ant colony algorithm approach [26] for solving the problem.

The incompatible task set (ITS) concept was introduced by Zhang, Kucukkoc [27] through a case study for rebalancing (i.e. minimisation of the cycle time). Though, no comprehensive research results were presented. Kucukkoc [28] proposed an ant colony algorithm approach for solving the MTALBP multi-objectively. ITS constraint was not incorporated in that research. Building on the work of Kucukkoc [28], Kucukkoc [29] handled the MTALBP and proposed a nondominated sorting approach for solving the problem multi-objectively again with no consideration of ITS.

This research differs from the studies existing in the literature by presenting the first computational test results for mixed-model two-sided assembly line balancing incorporating incompatible task sets. Incompatible task set constraint is different from negative zoning constraints as will be explained in Section 2. The main contribution of this paper is the newly proposed running mechanism of a competitive ant colony optimisation algorithm for multi-objectively solving the MTALBP under the ITS constraints.

The next section briefly describes the problem studied, followed by the detailed description of the proposed NSACO algorithm in Section 3. A numerical example is provided in Section 4, in which the steps of the NSACO is explained. The results of the computational tests are reported in Section 5 and finally, the conclusions are drawn in Section 6.

2 Problem statement

MTALBP is to find the best assignment configuration of tasks in mixed-model two-sided lines in such a way that one or more performance criterion is optimised. The performance criteria to be optimised within the scope of this paper are cycle time, the number of mated stations/workstation and smoothness index. As the major contribution of this paper is on the methodology side, the problem definition part will be given very briefly.

A mixed-model two-sided line has workstations located on both of its sides (left and right) to build similar-models ($m = 1,2, \dots, M$) of a product in an intermixed sequence. There is no setup between model changes and the models can be produced as low as single lots. Each workstation ($k = 1,2, \dots, NS$) is responsible for completing a set of tasks ($i = 1,2, \dots, I$) assigned to it within certain amount of time, called cycle time (C). Cycle time is determined dividing the planning horizon by the total demand for models required by the customers within this horizon. Each task requires a deterministic operation (or processing) time (t_{im}), which may vary from one model to another. There are precedence relationships between tasks caused by the technological or organisational constraints. P_i denotes the set of predecessors of task i. For example, $P_9 = \{2,6\}$ means that tasks 2 and 6 must be completed to initialise task 9. The capacity and operation side constraints also exist in the problem. The capacity constraint ensures that there is no workstation filled by tasks of which the sum of total processing times for any model exceed the cycle time. The operation side constraint limits the assignment of tasks, in such a way that some tasks can only be assigned to left side (L) while some on the right (R). There also are some tasks that can be assigned to either side (E). Figure 1 presents the precedence relationship of a simple 9-task problem. The operation sides and processing times of tasks are given over nodes in the format "(X,Y,Z)", where X, Y and Z denote operation side, processing time for model A and processing time for model B, respectively.

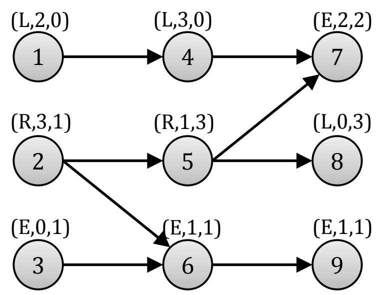

Figure 1: The precedence relationship diagram (adapted from Kim, Kim [14]).

The balancing solution of tasks is presented in Figure 2. As seen from the figure, two mated stations are utilised ($NM = 2$), consisted of a total of three workstations ($NS = 3$) under 5-unit cycle time constraint ($C = 5$). As tasks 2 and 3 must be completed before initialising task 6, one unit idle time occurs in workstation-1. This is called sequence-dependent idle time and sometimes unavoidable due to problem specific constraints.

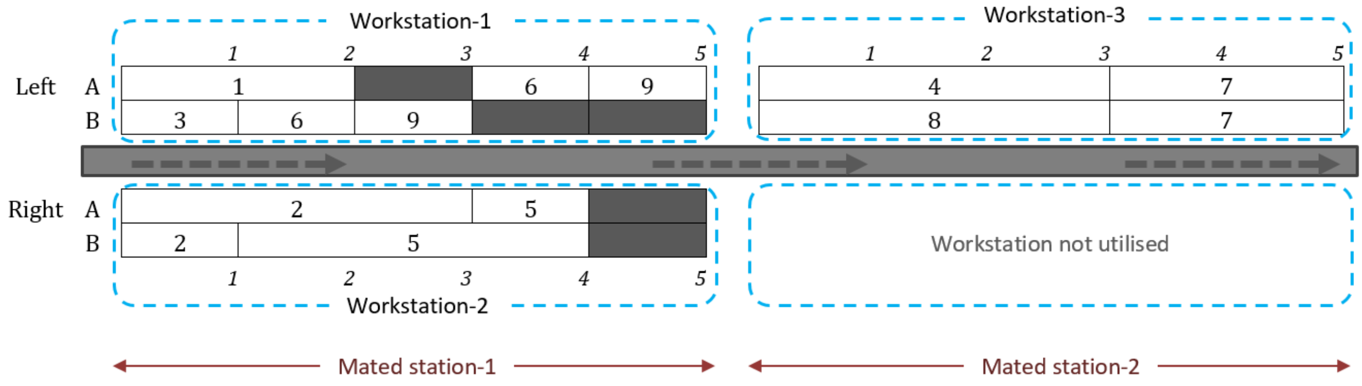

Figure 2: Balancing configuration of tasks (Ozcan and Toklu [22]).

Incompatible tasks are those tasks which cannot be performed concurrently in the same mated station (workstations located on left and right sides and facing each other). Let us assume an assembly plant producing small electrical automobiles and there are two tasks which need to be completed by operators inside the body. If the space of the body is not enough to have operators done their works, these two tasks constitute an ITS. If there would be an incompatible task set such as $ITS = \{5,9\}$, the solution given in Figure 2 would not be feasible as it is not possible to perform tasks 5 and 9 for model B at the same time in the same mated station. However, the solution to be feasible if ITS would include tasks 2 and 9 ($ITS = \{2,9\}$). Note that there may be more than one ITS in the same line and each may contain different and more than two tasks.

The following section presents the proposed solution method for solving MTALBP considering ITS constraint.

3 Nondominated sorting ant colony optimization algorithm (NSACO)

Ant colony optimisation algorithm, developed by Dorigo, Maniezzo [30], is a well-known and powerful nature inspired technique applied widely to solving sophisticated engineering problems, especially combinatorial optimisation problems [28]. Being referred to as an NP-hard class of combinatorial optimisation problem, large-sized assembly line balancing problems require highly powerful solution techniques [25],[31], especially for large-scale instances. Therefore, an ant colony optimisation algorithm, called NSACO, is employed in this study for solving MTALBP considering ITS constraints.

The algorithm makes use of pareto front [32] approach in eliminating solutions obtained by ants in the colonies released. The objectives used by NSACO are cycle time (C), the combination of the number of mated stations and workstations utilised (ST) and smoothness index (SI). Dominated solutions are determined based on these three factors. Thus, the best ST and SI values are kept for each C value tested. This provides the manager of an assembly line the opportunity of choosing the best line configuration based on their cycle time, which is determined by total demand and the planning horizon. In two-sided lines, the length of the line is also an important criterion different from the one-sided lines. Therefore, NM is also considered in this research as an objective and ST is calculated giving more importance to NM ($ST = 100 \times NM + NS$).

If two or more solutions have the same C and ST values, then the solution which has the lower SI value is favoured. This is because the smoother the workload is distributed across the workstations, the more stable the line is. SI value of a solution is calculated as follows:

$$SI = \sqrt{\sum_{m=1}^{M} d_m \sum_{k=1}^{NS} (W_m^{max} - W_{km})^2}, \qquad (1)$$

where W_m^{max} is the maximum workstation workload for model m, W_{km} is the workload of workstation k, and d_m is the proportional demand of model m, which is calculated as follows: $d_m = D_m / \sum_{m=1}^{M} D_m$.

The general flow of NSACO is given in Figure 3. As seen, the algorithm starts with initialising all parameters and importing problem data. C is set to C_{low}, which is a user determined parameter, and global best solution indicators (WLE^*, ST^*, and SI^*) are set to default values ($WLE^* \leftarrow 0$, $ST^* \leftarrow B$ and $SI^* \leftarrow B$, where B is a very big positive number). A new colony of ants is released and each ant in the colony builds a balancing solution, using the procedure which will be described in Figure 4. ST value is calculated and pheromone is deposited between task-workstation on the basis of the following rule:

$$\tau_{ik} \leftarrow (1-\rho)\tau_{ik} + \Delta\tau_{ik}, \qquad (2)$$

Where, $\Delta\tau_{ik} = Q/ST$; ρ and Q are evaporation rate and a user determined parameter, respectively. Thus, the solution having the less ST value is favoured depositing more pheromone on the edges of the shorter path.

The colony best solution is updated ($ST_{col} \leftarrow ST$) if the ST value of an individual is lower than the ST value of the colony best solution ($ST < ST_{col}$). All ants build solutions in this way. All colonies are released one-by-one and the colony best solution is added to pareto front followed by the calculation of its WLE and SI values. If WLE of a solution is higher than the global best WLE ($WLE > WLE^*$), the global best solution, C_{best}, WLE^*, ST^* and SI^* are updated ($C_{best} \leftarrow C$, $WLE^* \leftarrow WLE$, $ST^* \leftarrow ST_{col}$, $SI^* \leftarrow SI$). C_{best} is the cycle time value for which the best solution is found. If $WLE = WLE^*$ and $SI < SI^*$, the global best solution, C_{best}, ST^* and SI^* are updated ($C_{best} \leftarrow C$, $ST^* \leftarrow ST_{col}$ and $SI^* \leftarrow SI$). In this case, there is no need to update WLE^* as there is no change.

WLE of a solution is calculated dividing the sum of the task times multiplied by proportional demands to the multiplication of cycle time and the number of workstations as follows:

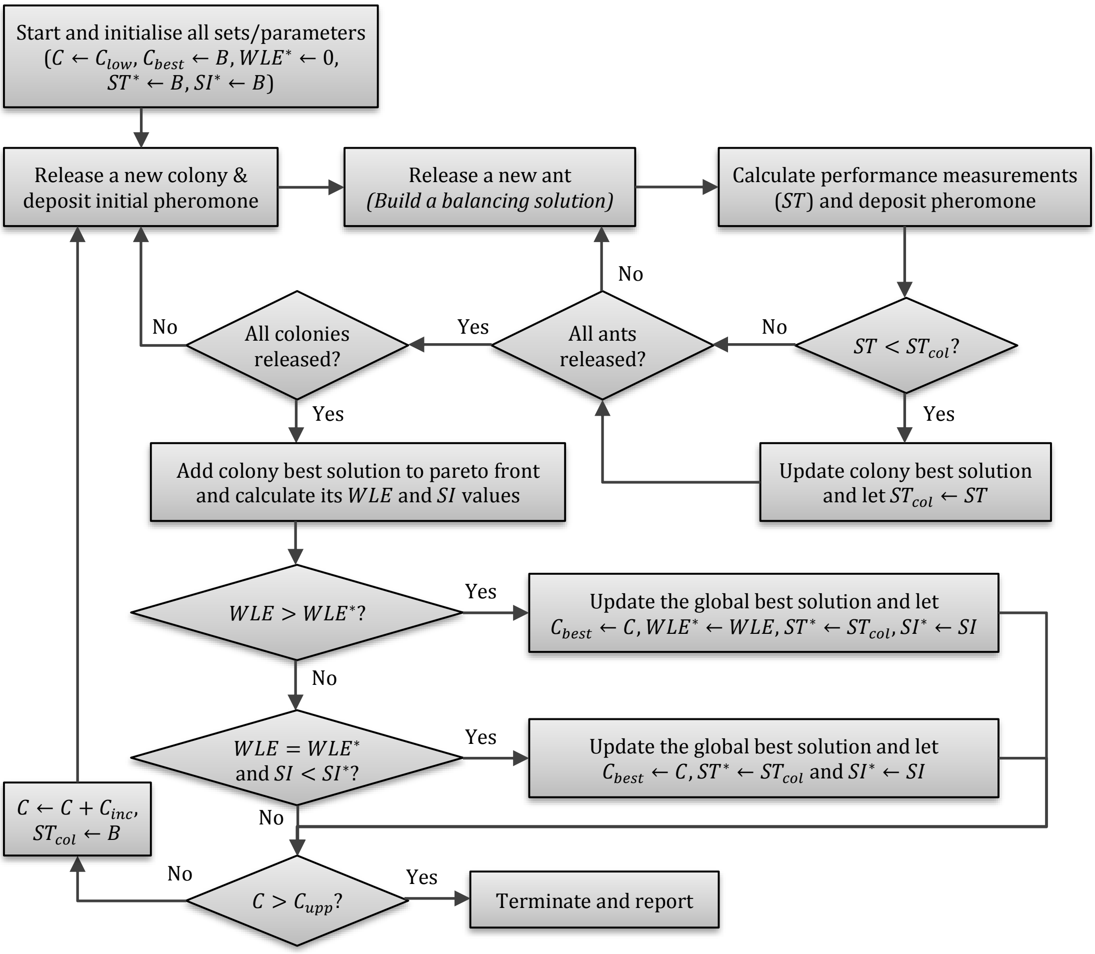

Figure 3: The general outline of NSACO.

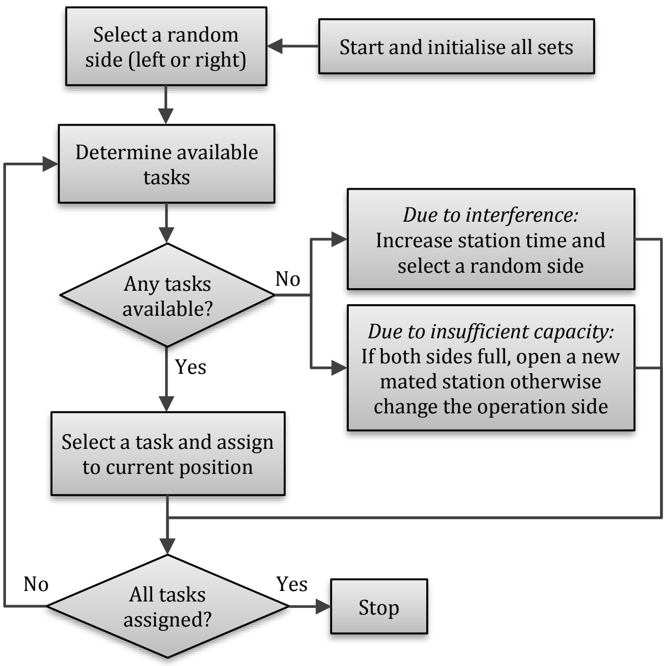

Figure 4: The procedure of building a balancing solution.

$$WLE = \frac{\sum_{m=1}^{M}\sum_{i=1}^{I} d_m t_{im}}{C \times NS}. \quad (3)$$

Note that WLE is calculated only for *colony best* solutions to avoid unnecessary computation. Thus, the global best solution is determined among the pareto front solutions. C is increased by C_{inc} ($C \leftarrow C + C_{inc}$), ST_{col} is set to its default value ($ST_{col} \leftarrow B$) and new colonies of ants are released for building new solutions considering the new C value. The pheromones and best solutions are updated in the same way as above and this cycle continues until the upper bound for cycle time (C_{upp}) is achieved. The algorithm is terminated and the best solution is reported when the stopping criterion is met ($C > C_{upp}$).

Figure 4 outlines the procedure of building a balancing solution adapted from Simaria and Vilarinho [21]. As seen, a task is selected based on a selection criterion and assigned to the current position (workstation). The selection of a task (i) to a workstation (k) is determined by the probability of $p_{ik} = \left([\tau_{ik}]^{\alpha}[\eta_i]^{\beta}\right)/\left(\sum_{y \epsilon Z_i}[\tau_{yk}]^{\alpha}[\eta_y]^{\beta}\right)$, where α and β are weighting parameters which determine the influence of pheromone and heuristic information in the task selection process, respectively [5]. Z_i and τ_{ik} are the list of tasks available and the pheromone amount existing between task i and workstation k, respectively. The term η_i denotes the heuristic information for greedy search and ranked positional weight method [33] is employed for this aim.

The station time of workstation k for model m (st_{km}) is increased by the operation time of task i ($st_{km} \leftarrow st_{\bar{k}m} + t_{im}$) and all tasks are assigned to workstations one-by-one. When there is no task available, one of the two options is selected based on the reason. If the preceding tasks assigned to the companion station prevent availability, the station time of the current station is increased for all models, $st_{km} \leftarrow st_{\bar{k}m}$ (where $\bar{k}$ denotes the companion of workstation k). The procedure ends when each task is assigned to a workstation.

4 A numerical example

A numerical example is provided in this section to exemplify the methodology proposed. Let us assume an MTALBP consisting of 16 tasks. Table 1 presents the input data for the numerical example, taken from Ozcan and Toklu [22].

Table 1: Input data for the numerical example.

Task	Side	Time for A	Time for B	Immediate Predecessor(s)
1	E	6	0	-
2	E	5	2	-
3	L	2	0	1
4	E	0	9	1
5	R	8	0	2
6	L	4	8	3
7	E	7	7	4,5
8	E	4	3	6,7
9	R	0	5	7
10	R	4	1	7
11	E	6	3	8
12	L	0	5	9
13	E	6	9	9,10
14	E	4	5	11
15	E	3	8	11,12
16	E	4	7	13

The algorithm is coded in Java and run on Intel Xeon CPU E3-1270 3.5 GHz PC with 16 GB of RAM using the parameters, $\alpha = 0.5$, $\beta = 0.3$, $\rho = 0.1$, $Q = 50$, initial pheromone ($initPher$) = 30, maximum number of colonies ($maxCol$) = 20 and colony size ($colSize$) = 10, determined based on preliminary tests. $C_{inc} = 1$ and both models have the same proportional demand ($d_A = d_B$). Lower and upper bound for cycle time are assumed to be $C_{low} = 14$ and $C_{upp} = 24$. The range between C_{low} and C_{upp} is kept large to have a more inclusive example in terms of the visualisation of the results. The incompatible task set is $ITS = \{15,16\}$, which means these two tasks cannot be handled concurrently in the same mated station.

Table 2 reports the best solution obtained for each C value through the iterations in which C is increased by $C_{inc} = 1$ starting from $C = 14$. As seen from the table, the solution found in the first step for $C = 14$ requires four mated stations and 7 workstations ($NM = 4$ and $NS = 7$) while the lower bound is $NM = 3$ and $NS = 6$. The $WLE\%$ and SI values of this solution are calculated as 68.88 and 9.38, respectively. In step 2, C is increased to 15 and a solution is obtained requiring one lower workstation, which increases the $WLE\%$ to 75.00. The best solution having the highest $WLE\%$ is obtained for $C = 21$ (in step 8) for which the optimal number of mated stations and workstations are investigated by NSACO as $NM = 2$ and $NS = 4$. The pareto front chart of the solutions obtained is presented in Figure 5. The blue points denote the nondominated solutions while the best solution among them is obtained at $C = 21$ and identified in green.

Table 2: The best solutions obtained for different cycle time values.

Step	C	LB	$NM[NS]$	$WLE\%$	SI
1	14	3[6]	4[7]	68.88	9.38
2	15	3[5]	4[6]	75.00	8.74
3	16	3[5]	4[6]	70.31	8.34
4	17	3[5]	4[6]	66.17	8.34
5	18	2[4]	3[5]	75.00	12.00
6	19	2[4]	3[5]	71.05	9.89
7	20	2[4]	3[5]	67.50	8.94
8	21	2[4]	2[4]	**80.35**	9.56
9	22	2[4]	2[4]	76.70	8.33
10	23	2[4]	2[4]	73.37	7.04
11	24	2[3]	2[4]	70.31	7.31

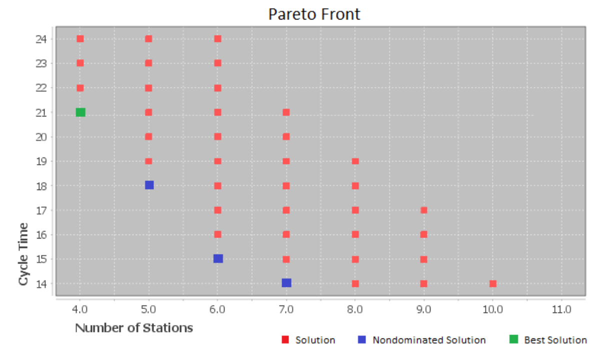

Figure 5: The pareto front diagram of the solutions obtained.

The detailed balancing configuration of tasks for this solution is depicted in Figure 6. As seen from the figure, task 16 starts in workstation-4 after task 15, which is performed on the other side of the line (see workstation-3) for both models (A and B). If no ITS constraint was subject to consideration during the balancing solution, a slightly different balancing configuration having a smoother workload distribution could be obtained with $WLE\% = 80.35$ and $SI = 9.35 (< 9.56)$.

5 Computational tests

This section reports the results of the computational tests conducted through solving test problems derived/adapted from the literature using the proposed solution approach, NSACO. The tests have been performed under two conditions:

- Incompatible task set constraints are not considered (called con-I),
- Incompatible task set constraints are considered (called con-II).

In con-I, the aim is to measure the performance of NSACO, by solving the test problems whose results have been published in the literature. This is because no comparable result was published in the literature when incompatible task set constraints have been considered.

5.1 Results achieved for con-I

There are no results published regarding the MTALBP considering incompatible task set constraints in the literature.

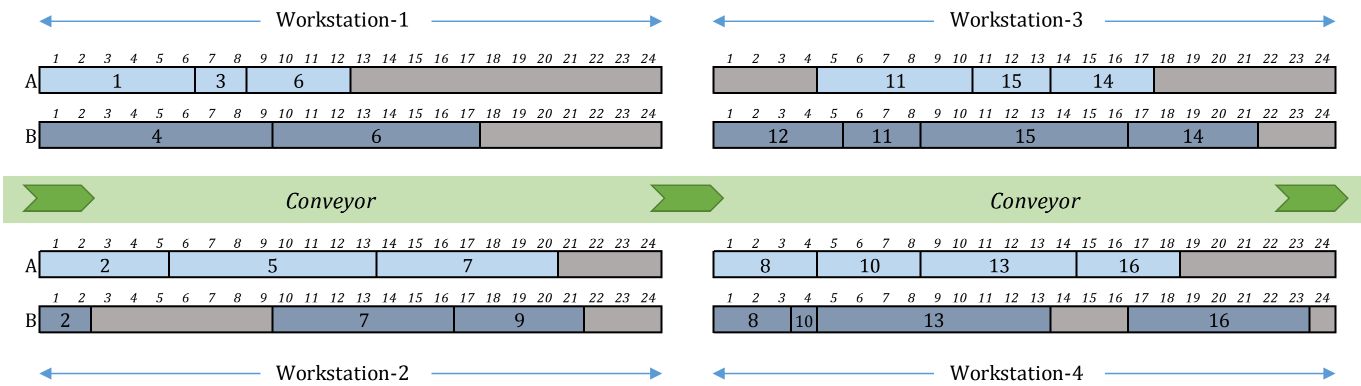

Figure 6: The best balancing solution under the ITS constraint.

Therefore, benchmarks derived and solved by Ozcan and Toklu [22] have been solved using NSACO to measure its performance. The algorithm has been coded in JAVA and run on Intel Xeon CPU E3-1270 3.5 GHz PC with 16 GB of RAM. The parameters are selected based on preliminary tests and similar studies in the literature. Some parameters are the same for all test problems ($\alpha = 0.5$, $\beta = 0.3$, $\rho = 0.1$, $Q = 50$ and $initPher = 30$. However, $maxCol$ and $colSize$ are increased in parallel to the increasing problem complexity caused by the growing problem size. Therefore, $maxCol$ and $colSize$ are set to 20 and 10 for test problems P9, P12 and P16; 30 and 15 for test problems P20, P24 and P36; and 40 and 20 for test problems P65, P148 and P205, respectively. C_{inc} is assumed to be '1' for test problems P9, P12, P16, P20, P24 and P36; and '5' for test problems P65, P148 and P205. The proportional demands of the models are assumed to be the same for all test problems ($d_A = d_B = \cdots = d_M$).

Table 3 presents data for the test problems and reports the results of the computational tests when incompatible task group constraints are not considered. The precedence relationships and operation directions of P9, P12 and P24 are taken from Kim, Kim [14]. Those data for P16, P65 and P205 are taken from Lee, Kim [15] and for P148 are taken from Bartholdi [12]. The task processing times of P9, P12, P16, P24, P65 and P148 are retrieved from Ozcan and Toklu [22]. P205 was not solved by Ozcan and Toklu [22], therefore task times for this problem are taken from Kucukkoc [34]. Two other test problems P20 and P36, have also been considered in the current study in addition to those in Ozcan and Toklu [22]. All data required for P20 and P36 are gathered from Kucukkoc and Zhang [26].

Each test problem has been solved using NSACO considering a lower and an upper bound for the cycle time (C_{low} and C_{upp}, respectively). These values have been given as an input to the algorithm and the solution with the best $WLE\%$ value is reported as the best solution. C_{best} denotes the cycle time value for which the best solution is investigated. $NM[NS]_3$ column reports the number of mated-stations (NM) and the number of workstations (NS) required for the best solution. LB_C is the lower bound of $NM[NS]_3$ under the condition that the cycle time is C_{best}. LB_C is calculated using the formulae provided by Ozcan and Toklu [22]. Thus, it is possible to make a direct comparison between LB_C and $NM[NS]_3$ to measure the performance of the algorithm. When making such a comparison, it should be noted here that the solution is optimal if $NM[NS]_3$ is equal to LB_C. The reason lying behind this idea is that it is not theoretically possible to have a solution less than LB_C number of NM and NS under the condition that cycle time is C_{best}.

It should be noted here that Ozcan and Toklu [22] have not handled the MTALBP with the aim of optimising cycle time as well as the number of workstations. For each test problem, Ozcan and Toklu [22] reported the number of mated-stations and the number of stations (given different cycle times in each case) found by mixed integer programming (MIP) model and simulated annealing (SA) algorithm. Therefore, the problem addressed by Ozcan and Toklu [22] is referred to as a type-I problem while the current work deals with type-E MTALBP for which cycle time is incremented by C_{inc} within an interval between C_{low} and C_{upp}. Thus, it is not possible to make a direct comparison to Ozcan and Toklu [22]. However, the results reported by Ozcan and Toklu [22] have also been presented in Table 3 to have an idea on the overall performance of the proposed NSACO algorithm. One can compare the WLE value of the best solution obtained by NSACO with the best WLE value reported by Ozcan and Toklu [22]. Note that NSACO can find the best solution for a cycle time value not tested by Ozcan and Toklu [22], which is basically an advantage of the proposed approach. This is because it is not needed to test different cycle time values manually as NSACO increments it by C_{inc}, systematically. It is also worthy to declare that $NM[NS]_1$ values are the same with $NM[NS]_2$ for those cases solvable by MIP model reported by Ozcan and Toklu [2]. This shows how competitive the SA proposed by Ozcan and Toklu [2] is.

As seen from the table, NSACO finds solutions having the same WLE values (87.50, 80.35 and 89.28) with MIP and SA for P12, P16 and P24, respectively. For P65, the WLE value found by NSACO (84.76) is the same with MIP and SA when cycle time is considered as 490. However, MIP and SA found the best WLE for the cycle time value of 326, which is skipped by NSACO. This is because NSACO starts with $C_{low} = 180$, increments by $C_{inc} = 5$ in each iteration, and tries $C = 325$ and $C = 330$, but not $C = 326$. As for P9 and P148, NSACO investigates better WLE values (89.28 and 80.90, respectively) for cycle times not tested by Ozcan and Toklu [22]. While this does not mean that NSACO outperforms SA, it clearly shows the powerful solution building capacity of NSACO and the nondominated sorting solution methodology proposed in this research. When the $NM[NS]_3$ values are compared to LB_C, it is observed that NSACO obtains optimal solutions for P9, P12, P16, P20, P24, P36 and P65. For P148, the solution found by NSACO requires one more workstation than the theoretical lower bound while this does not always mean that the solution is not optimal. NSACO solves P205 with requiring eight more workstations than the lower bound, which still seems reasonable considering similar studies in the literature and the growing search space with increasing number of tasks.

5.2 Results achieved for con-II

In con-II, ITS constraints have been included in the problem sets and the problems were solved using NSACO method on the same computer using the same parameters with con-I. The precedence relationships and operation sides have been kept the same as in con-I. However, some changes have been done in the number of models considered and the processing times of tasks. That information has been taken from Kucukkoc [34] and presented in Appendix for interested readers and researchers.

The proportional demands of the models are assumed to be the same for all test problems ($q_A = q_B = \cdots = q_M$). The results of the computational tests have been reported in Table 4. In addition to those columns given in Table 3, three new columns have been added in Table 4, which are incompatible task set (ITS), maximum task processing time ($max\{t_{im}\}$) and the cycle time increment value (C_{inc}). ITS column shows tasks which cannot be performed at the same time in the same mated-station for the corresponding test problem. The algorithm starts with $C = C_{low}$, finds solutions releasing colonies of ants, increments C by C_{inc} and repeats this until $C = C_{upp}$. The solution with the highest $WLE\%$ value is reported, where C_{best} is the cycle time for which the best solution is found. LB_C is the lower bound for $NM[NS]$ given the cycle time is C_{best}.

Table 3: Computational test results for con-I.

Problem	M	Ozcan and Toklu [22]					Current Work						
				MIP	SA		NSACO						
		C	LB	$NM[NS]_1$	$NM[NS]_2$	$WLE\%$	C_{low}	C_{upp}	C_{best}	LB_C	$NM[NS]_3$	$WLE\%$	CPU (x10³ms)
P9	2	4	4	3[4]	3[4]	78.12	4	7	7	1[2]	1[2]	**89.28**	66
		5	3	2[3]	2[3]	83.33							
		6	3	2[3]	2[3]	69.44							
P12	2	5	5	3[5]	3[5]	84.00	4	7	6	2[4]	2[4]	**87.50**	**113**
		6	4	2[4]	2[4]	**87.50**							
		7	3	2[4]	2[4]	75.00							
		8	3	2[3]	2[3]	**87.50**							
P16	2	15	5	4[6]	4[6]	75.00	14	24	21	2[4]	2[4]	**80.35**	**533**
		16	5	4[6]	4[6]	70.31							
		18	4	3[5]	3[5]	75.00							
		19	4	3[5]	3[5]	71.05							
		21	4	2[4]	2[4]	**80.35**							
		22	4	2[4]	2[4]	76.40							
P20	2	-	-	-	-	-	12	24	18	3[5]	3[5]	80.55	598
P24	2	20	7	4[7]	4[7]	**89.28**	16	35	35	2[4]	2[4]	**89.28**	**1612**
		24	6	3[6]	3[6]	86.80							
		25	5	3[6]	3[6]	83.33							
		30	5	3[5]	3[5]	83.33							
		35	4	2[4]	2[4]	**89.28**							
		40	4	2[4]	2[4]	78.12							
P36	2	-	-	-	-	-	16	35	34	3[5]	3[5]	87.35	2805
P65	3	326	8	-	5[9]	**84.93**	310	560	490	3[6]	3[6]	84.76	14574
		381	7	-	4[8]	81.75							
		435	6	-	4[7]	81.83							
		490	6	-	3[6]	84.76							
		544	5	-	3[6]	76.34							
P148	4	204	13	-	9[17]	75.81	180	510	325	5[9]	5[10]	**80.90**	107868
		255	11	-	7[14]	73.64							
		306	9	-	6[12]	71.60							
		357	8	-	5[10]	73.64							
		408	7	-	5[10]	64.44							
		459	6	-	4[8]	71.60							
		510	6	-	4[8]	64.44							
P205	3	-	-	-	-	-	550	1020	755	20[40]	25[48]	72.47	132426

Table 4: Results of the computational tests for con-II.

Problem	M	ITS	$max\{t_{im}\}$	C_{low}	C_{upp}	C_{inc}	C_{best}	LB_C	$NM[NS]_4$	WLE
P9	3	{8,9}	4	4	7	1	6	2[4]	2[4]	72.33
P12	3	{2,3}	3	4	7	1	6	2[4]	3[4]	87.50
P16	3	{12,13}	9	14	24	1	24	3[5]	3[5]	73.55
P20	3	{9,11}	9	12	24	1	15	4[7]	5[7]	76.04
P24	3	{11,17}	9	16	35	1	18	4[8]	4[8]	72.47
P36	3	{2,3}, {30,32}	9	16	35	1	30	3[6]	3[6]	81.04
P65	3	{2,3,13}, {12,43,46}	272	300	550	5	505	6[11]	7[13]	80.40
P148	3	{56,74,93}, {115,124,130}	170	180	485	5	450	11[22]	12[24]	70.81
P205	3	{42,63,70}, {156,160}	452	490	800	5	765	20[39]	27[49]	70.06

From the comparison of LB_C and $NM[NS]_4$ columns, NSACO found optimal solutions in terms of the NS values for test problems P9, P12, P16, P20, P24 and P36. For the remaining ones, i.e. P65, P148 and P205, NSACO finds 2, 2 and 10 more workstations than the lower bound, respectively. The gap gets bigger in P205, which seems reasonable considering the large number of tasks subject to balancing. It is also worthy to express that the optimal solutions tend to have more number of workstations than the lower bound when the problem size increases.

The weighted line efficiency values reported in $WLE\%$ column are different from those reported for con-I. This is caused by the fluctuation in the task processing times between the models. As the processing times are generated randomly, they show considerable variation in comparison to those used in con-I (see Appendix). However, finding optimal solutions as discussed above indicates that NSACO has a promising solution building capacity for MTALBP with and without ITS constraints.

6 Conclusions

This paper addressed to the MTALBP considering ITS constraints and reported the first research results. MTALBP has NP-hard complexity and is hard to optimally solve the large-sized instances using traditional methods. Therefore, a nondominated sorting solution approach is proposed for the solution of the problem. Three important performance criteria have been considered in the solution method, i.e. cycle time, the number of mated stations/workstations, and smoothness index. The proposed method, NSACO, has been described and a numerical example has been solved to exhibit the solution mechanism of the proposed methodology. The best solution obtained for the numerical example is depicted in details and the pareto front diagram is presented highlighting nondominated solutions. The best solution among the nondominated solutions is identified based on the $WLE\%$ value. When there is more than one solution having the same $WLE\%$ value, the solution having the smaller SI value is chosen. Test problems have been derived from the literature and solved under two conditions: ITS constraints were considered and not considered. The results obtained when ITS constraints were not considered have been compared to the results existing in the literature and it was observed that NSACO performs quite well. The results obtained when ITS constraints were considered have been presented and compared to the lower bounds. The comparison indicated that NSACO has quite promising solution capacity.

The problem and the solution method proposed in this research can be extended in several ways. As the MTALBP relates to a real-world industrial engineering problem, the managers of assembly lines used for producing large-sized products can easily adopt the concept proposed in this paper to their problems. Also, researchers studying on line balancing problems can adopt the ITS constraint to other problem types, such as U-shaped lines and parallel lines with two sides. Furthermore, researchers interested in this topic can use the results reported as a benchmark for their research. New solution methods (exact or approximate) can be developed for MTALBP with ITS constraints and their performances can be compared to NSACO solving the test problems, for which the input data have been provided in Appendix.

7 References

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Appendix A

This section presents the task times of the test problems taken from Kucukkoc [34] and used in Section 5.2.

	P9			P12			P16			P20			P24			P36		
Task	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
1	2	4	2	3	2	2	6	7	6	4	7	3	3	3	0	9	5	4
2	3	3	1	3	3	2	5	2	0	3	5	9	7	0	2	5	3	9
3	2	2	1	0	2	1	2	5	9	0	2	3	7	1	1	7	7	9
4	3	0	0	2	3	2	9	2	8	4	1	3	5	0	0	2	3	0
5	4	2	3	2	1	2	8	9	5	1	2	2	4	6	1	6	4	5
6	3	2	0	0	1	1	4	8	0	4	8	1	3	5	1	4	8	9
7	0	3	3	2	2	2	7	8	9	3	4	9	4	8	5	4	1	4
8	2	1	1	2	3	3	4	6	3	5	4	5	3	0	7	1	4	1
9	1	2	2	1	2	1	5	0	8	7	7	6	6	4	4	0	6	2
10				3	2	1	4	4	7	8	3	0	4	2	9	8	5	3
11				2	0	1	6	5	7	2	6	3	4	8	3	0	8	0
12				1	1	2	5	6	6	1	6	5	3	1	1	6	5	0
13							6	4	9	2	1	9	3	5	3	7	6	7
14							4	2	7	5	8	9	9	4	3	1	1	1
15							3	6	9	3	5	3	5	1	4	5	9	1
16							4	8	8	2	1	4	9	1	2	6	5	7
17										5	2	5	2	7	3	0	0	0
18										0	8	6	7	4	4	7	3	1
19										2	0	1	9	2	1	8	3	5
20										4	0	9	9	1	1	2	2	3
21													8	9	7	8	0	8
22													8	7	9	1	3	8
23													9	9	5	1	2	0
24													9	3	5	7	1	8
25																5	1	3
26																5	0	6
27																9	0	0
28																1	3	5
29																2	6	8
30																5	0	5
31																5	1	4
32																0	5	1
33																7	8	1
34																7	4	7
35																8	7	3
36																8	1	2

Appendix B

P65											
Task	A	B	C	Task	A	B	C	Task	A	B	C
1	49	57	133	23	104	120	88	45	97	119	57
2	49	68	71	24	84	52	89	46	37	102	52
3	71	62	135	25	113	95	208	47	25	46	144
4	26	145	110	26	72	9	5	48	89	118	50
5	42	104	43	27	62	7	104	49	27	17	114
6	30	47	101	28	272	47	84	50	50	79	5
7	167	83	133	29	89	66	84	51	46	56	17
8	91	95	126	30	49	127	63	52	46	140	94
9	52	53	114	31	11	133	144	53	55	17	50
10	153	265	200	32	45	91	70	54	118	86	63
11	68	81	33	33	54	73	135	55	47	28	0
12	52	64	29	34	106	128	268	56	164	39	65
13	135	28	211	35	132	145	118	57	113	149	174
14	54	87	67	36	52	124	17	58	69	23	18
15	57	0	18	37	157	149	73	59	30	40	115
16	151	86	58	38	109	141	90	60	25	48	84
17	39	93	36	39	32	31	145	61	106	15	25
18	194	53	27	40	32	144	182	62	23	59	1
19	35	69	115	41	52	118	27	63	118	122	121
20	119	86	15	42	193	256	103	64	155	44	108
21	34	10	89	43	34	80	43	65	65	34	44
22	38	86	48	44	34	21	29				

Appendix C

P148

Task	A	B	C	Task	A	B	C	Task	A	B	C	Task	A	B	C
1	16	24	69	38	80	84	84	75	101	165	129	112	162	128	87
2	30	13	125	39	7	109	31	76	5	33	70	113	11	113	28
3	7	25	109	40	41	119	67	77	28	76	64	114	19	19	96
4	47	113	69	41	47	21	3	78	8	24	126	115	14	70	121
5	29	16	63	42	16	25	95	79	111	43	149	116	31	80	36
6	8	53	6	43	32	80	37	80	7	70	11	117	32	55	10
7	39	76	123	44	66	85	52	81	26	103	24	118	26	32	57
8	37	30	36	45	80	46	21	82	10	45	92	119	55	55	69
9	32	60	88	46	7	33	121	83	21	68	114	120	31	100	3
10	29	7	5	47	41	94	4	84	26	74	87	121	32	85	35
11	17	99	54	48	13	8	123	85	20	8	7	122	26	59	71
12	11	115	30	49	47	121	45	86	21	92	102	123	19	48	51
13	32	120	35	50	33	0	28	87	47	6	4	124	14	45	32
14	15	117	72	51	34	27	30	88	23	53	71	125	19	44	119
15	53	28	64	52	11	51	81	89	13	59	87	126	48	51	120
16	53	81	116	53	118	59	99	90	19	32	53	127	55	79	33
17	8	43	2	54	25	49	41	91	115	0	25	128	8	109	43
18	24	4	95	55	7	3	52	92	35	0	66	129	11	123	106
19	24	18	58	56	28	73	14	93	26	40	95	130	27	90	1
20	8	78	31	57	12	96	43	94	46	88	46	131	18	46	48
21	7	61	68	58	52	49	110	95	20	60	72	132	36	3	119
22	8	48	48	59	14	41	74	96	31	4	119	133	23	74	109
23	14	115	96	60	3	20	78	97	19	52	22	134	20	80	85
24	13	52	81	61	3	86	25	98	34	54	28	135	46	29	63
25	10	123	124	62	8	32	129	99	51	29	91	136	64	78	91
26	25	2	40	63	16	48	46	100	39	63	4	137	22	126	31
27	11	24	59	64	33	37	71	101	30	15	81	138	15	28	6
28	25	45	43	65	8	63	89	102	26	30	127	139	34	122	48
29	11	47	80	66	18	121	57	103	13	57	47	140	22	54	42
30	29	26	4	67	10	31	63	104	45	107	52	141	151	90	30
31	25	50	33	68	14	47	95	105	58	129	119	142	148	24	106
32	10	34	71	69	28	15	60	106	28	70	78	143	64	32	55
33	14	29	92	70	11	45	97	107	8	67	80	144	170	37	9
34	41	11	49	71	18	3	107	108	43	68	40	145	137	4	89
35	42	117	54	72	25	104	55	109	40	39	88	146	64	24	28
36	47	43	83	73	40	30	60	110	34	69	111	147	78	24	26
37	7	91	34	74	40	97	1	111	23	103	75	148	78	86	111

Appendix D

P205

Task	A	B	C	Task	A	B	C	Task	A	B	C	Task	A	B	C
1*	*39*	151	204	53	85	185	53	105	232	84	6	157	83	113	189
2	42	104	75	54	43	49	34	106	122	396	410	158	35	84	35
3	261	52	126	55	97	116	59	107	151	222	120	159	58	61	40
4	261	447	394	56	37	206	113	108	31	76	105	160	42	108	70
5	157	75	10	57	13	36	143	109	97	16	42	161	68	165	100
6	90	7	139	58	35	73	299	110	308	426	429	162	68	139	74
7	54	167	145	59	217	102	219	111	116	196	151	163	68	44	42
8	67	296	168	60	72	199	62	112	312	291	22	164	103	71	90
9	30	77	48	61	85	373	154	113	34	136	195	165	103	35	108
10	106	124	200	62	25	137	44	114	128	15	83	166	103	94	46
11	32	34	89	63	37	210	89	115	54	89	3	167	103	87	58
12	62	79	19	64	37	38	139	116	175	180	76	168	103	134	44
13	54	176	188	65	103	41	253	117	55	111	221	169	68	19	37
14	67	8	192	66	140	424	425	118	306	368	15	170	103	138	23
15	30	116	172	67	49	59	152	119	59	198	122	171	68	28	5
16	106	69	297	68	35	143	139	120	59	204	46	172	103	342	431
17	32	225	15	69	51	51	163	121	66	130	234	173	103	93	258
18	62	300	37	70	88	221	123	122	66	118	99	174	68	7	42
19	56	156	234	71	53	341	288	123	23	95	127	175	103	230	201
20	67	35	224	72	144	301	102	124	244	250	50	176	103	185	139
21	86	71	4	73	337	83	394	125	54	83	292	177	10	69	64
22	37	12	59	74	107	184	156	126	294	10	266	178	187	96	145
23	41	64	16	75	371	100	444	127	84	136	110	179	134	82	163
24	72	105	3	76	97	276	11	128	61	113	127	180	89	156	147
25	86	61	40	77	166	104	6	129	57	237	177	181	58	113	148
26	16	54	355	78	92	87	50	130	38	89	221	182	49	120	15
27	51	167	107	79	92	160	95	131*	*57*	2	4	183	134	254	203
28	66	29	116	80	106	210	238	132*	*129*	122	211	184	53	129	21

Appendix D (cont.)

P205															
29	41	285	181	81	49	257	252	133*	*276*	115	238	185	334	50	380
30	72	30	78	82	92	219	285	134	445	383	309	186	24	84	56
31	51	36	16	83	371	256	49	135	68	81	244	187	76	85	56
32	16	49	5	84	87	57	248	136	53	89	78	188	76	57	36
33	15	13	65	85	162	183	435	137	49	138	128	189	192	201	98
34	15	84	63	86	96	208	154	138	92	71	127	190	98	85	67
35	85	34	241	87	79	267	22	139	236	339	103	191	258	187	241
36	59	384	116	88	96	175	134	140	116	216	259	192	165	251	189
37	23	26	110	89	42	262	309	141	265	314	133	193	38	3	62
38	13	115	179	90	88	128	404	142	149	270	357	194	115	262	89
39	19	114	36	91	90	46	39	143	74	118	176	195	83	68	39
40	108	203	7	92	97	196	172	144	332	306	384	196	56	21	17
41	214	166	64	93	270	199	29	145	324	264	22	197	29	62	39
42	80	46	180	94	452	363	91	146	104	253	119	198	303	433	225
43	37	40	96	95	48	319	297	147	51	65	238	199	18	0	43
44	84	28	106	96	338	423	188	148	58	131	98	200	29	56	37
45	18	72	29	97	34	140	196	149	67	49	148	201	154	126	248
46	12	51	7	98	65	39	178	150	49	39	84	202	90	74	18
47	29	86	252	99	50	120	136	151	107	364	272	203	93	95	38
48	37	184	179	100	112	25	122	152	38	17	31	204	94	118	36
49	13	154	219	101	48	197	137	153	27	98	33	205	165	203	186
50	70	91	111	102	117	287	42	154	68	41	78				
51	217	304	61	103	50	170	179	155	207	386	168				
52	72	167	107	104	68	123	370	156	202	227	177				