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A Case Study on the Ways How Engagement with Spatial Visualization Problem Solving Activities Helps Pre-Service Mathematics Teachers in Solving Mental Rotation Problems

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Uzamsal Görselleştirme Problemleri Çözme Etkinliklerinin Öğretmen Adaylarının Zihinsel Döndürme Problemleri Çözmelerine Nasıl Yardımcı Olduğu Üzerine Bir Durum Çalışması

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İÇİNDEKİLER / CONTENTS

İÇİNDEKİLER / CONTENTS

İÇİNDEKİLER / *CONTENTS*

A Case Study on the Ways How Engagement with Spatial Visualization Problem Solving Activities Helps Pre-Service Mathematics Teachers in Solving Mental Rotation Problems

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Abstract: The purpose of the present study is to investigate in what ways engagement with spatial visualization problem solving activities helps pre-service mathematics teachers in solving mental rotation problems. To achieve this purpose, a case study is conducted and the case of the study is the group of five pre-service mathematics teachers studying in a public university in Ankara. Spatial Ability Test (SAT) and task-based interviews are used as data collecting instruments. The results of the study show that participants use different ways in solving mental rotation problems; *mentally rotating* the figure or *comparing* the relative positions or patterns of the parts of the given figure. Additionally, solving spatial problems help participants in *exploring*, *practicing* or *shifting* their ways of solving problems to cope with the difficulties in solving spatial visualization problems requiring mental rotation accurately and practically.

Key Words: Spatial ability, spatial visualization, mental rotation problems, pre-service mathematics teachers, task-based interviews

Uzamsal Görselleştirme Problemleri Çözme Etkinliklerinin Öğretmen Adaylarının Zihinsel Döndürme Problemleri Çözmelerine Nasıl Yardımcı Olduğu Üzerine Bir Durum Çalışması

Özet: Bu çalışmanın amacı uzamsal görselleştirme problemleri çözme etkinliklerinin öğretmen adaylarının zihinsel döndürme problemleri çözmelerine nasıl yardımcı olduğunu araştırmaktır. Bu amaçla bir durum çalışması yapılmıştır ve çalışmada incelenen durum Ankara'da bir devlet üniversitesinde eğitim gören matematik öğretmen adaylarıdır. Veri toplama araçları olarak Uzamsal Yetenek Testi (SAT) ve görev temelli görüşmeler kullanılmıştır. Çalışmanın bulguları göstermiştir ki katılımcılar zihinsel döndürme problemleri çözerken farklı yollar kullanmışlardır; verilen şekli *zihinde döndürme* veya verilen şeklin parçalarının konumlarını veya desenlerini *karşılaştırma*. Buna ek olarak, uzamsal görselleştirme problemleri çözmek katılımcıların çözüm yollarını *keşfetmelerini, değiştirmelerini* veya *pratik yapmalarını* sağlayarak zihinsel döndürme problemlerini doğru ve pratik bir biçimde çözerken karşılaştıkları zorlukları aşmalarına yardımcı olmuştur.

Anahtar Kelimeler: Uzamsal yetenek, uzamsal görselleştirme, zihinsel döndürme problemleri, matematik öğretmen adayları, görev temelli görüşmeler

1. INTRODUCTION

Since Sir Francis Galton first reported on his experimental inquiries in to mental imagery in 1880, spatial ability has been accepted as a necessary ability not only for our daily lives but also for many other areas such as arts, design, engineering, science, geometry and mathematics (Delialioğlu, 1999; Gardner, 1985; Gutierrez, 1996; Kayhan, 2005; Mohler, 2008; Olkun, 2003; Usiskin, 1987). Although there is not a consensus on the terminology *spatial ability* could be defined as the mental skills concerned with understanding, manipulating, reorganizing or interpreting relationships visually (Tartre, 1990). Additionally, Battista (1998) used the term *spatial ability* for the ability to formulate mental images and to manipulate these images in the mind.

According to McGee (1979) the factor analytic studies reveal that spatial ability has distinct factors. Research studies provide consistent support for the existence of at least two spatial abilities; visualization and orientation. Spatial visualization is accepted to involve the ability to mentally rotate, manipulate, and twist two- and three-dimensional stimulus objects (Mc Gee, 1979). According to Tartre (1990) spatial visualization is defined as mentally moving an object and this mental movement can be in two ways; mental rotation or mental transformation. In mental rotation, the entire object is transformed by turning in space (Sorby, 1999). Kolb and Wishow (1981) defined mental rotation as the ability to adopt novel perspectives, to see other side of things. In the present study mental rotation is accepted as mentally turning the entire object in space (Mc Gee, 1979).

Previous research studies have found a positive relation between the spatial visualization ability and the mathematics performance (Usiskin, 1987; Herskowithz, 1989; Hodgson, 1996, Presmeg, 1997). Gutierrez (1985) mentions that he usefulness of visualization and graphical representations in the teaching of mathematics had been recognized by most mathematics educators and teacher of mathematics. Similarly, Presmeg (2000) expresses that mathematics is a subject that has diagrams, tables, spatial arrangements of signifiers such as symbols, and other inscriptions, so spatial visualization is one of the primary abilities that are important in learning and doing mathematics. Turğut and Yılmaz (2012) also found that for pre-service mathematics teachers, academic success was positively related to spatial ability.

The importance of spatial visualization ability in learning and doing mathematics highlights the importance of development of the ability. Various research studies show that there is a positive relationship between spatial training and spatial ability development

(Herskowitz, Parzysz & Van Dormolen, 1996; Kayhan, 2005; Olkun, 2003; Osberg, 1997; Owens&Clements, 1998; Pallascio & Allaire & Mongeau, 1993, Smith et. al., 2009). Olkun (2003) mentions that, although there are somewhat conflicting results in the literature regarding whether spatial ability could be improved, numerous studies have indicated that it could be improved through training if appropriate materials are provided. Furthermore, Bennie and Smith (1999) insist that spatial sense could not be taught, but have to be developed over a period of time. Moreover they state that a number of researchers referred to the importance of learners engaging with concrete spatial activities before being able to form and to manipulate visual images. Another study that focuses on the improvement of spatial visualization was constructed by Battista (1989). In the study, elementary education majors enrolled in an informal geometry course which primarily used hand-on activities and manipulative aids. The researchers concluded that the types of activities used in the course might have a direct influence on the improvement of spatial visualization ability. In other words, students' spatial ability could be developed by different teaching methods. Similarly, Sorby (1999) expressed that several research studies had been conducted to determine what type of pre-collage activities tended to be present in those students who had well developed spatial ability. And they concluded that, activities that required eye-to-hand coordination were those that helped to develop spatial ability. Another study conducted by Cakmak, Işıksal and Koç (2013) showed that origami-based instruction had a positive effect on the students' spatial ability. Moreover, recent research studies also show that computer supported activities and virtual environment is also effective in the development of spatial thinking skill as concrete objects (Güven & Kosa, 2008; Toptaş, Çelik & Karaca, 2012; Yurt & Sünbül, 2012).

Therefore it is important for the mathematics teachers to have high spatial visualization ability and to know how to develop their students' spatial visualization for better mathematics education. The related literature argues that appropriate training is helpful in the development of spatial visualization ability of the individuals. One of the ways of development of the spatial visualization ability is the activities that include solving spatial visualization problems or completing spatial visualization tasks.

In the new elementary mathematics and secondary geometry curricula designed by Turkish Ministry of National Education (MoNE), the importance of spatial ability is also acknowledged and developing spatial ability of students is accepted as one of the main purposes of geometry education (MoNE, 2005, 2006, 2009, 2010, 2012). Additionally, in order

21

A Case Study on the Ways How Engagement with Spatial Visualization Problem Solving...
E. Banu KAYHAN KIRMAÇ - Safure BULUT

to develop spatial ability of students, spatial activities and materials are introduced by Turkish Board of Education (BoE) (BoE, 2005). National Council of Teachers of Mathematics (NCTM) acknowledged the importance of spatial visualization ability by including spatial skills in the US curriculum standards for primary and secondary school geometry education and stated that rather than a separate subject, spatial ability of students should be developed through the all subjects of mathematics (2000). Moreover, the published standards of NCTM in 2003 for middle level mathematics teachers emphasized that, teacher candidates should use spatial visualization to explore and analyze geometric shape, build and manipulate representations of two- and three-dimensional objects and apply transformation.

In the light of all these previous research studies, this study focuses on the mental rotation abilities of pre-service mathematics teachers. The present study attempts to investigate in what ways engagement with spatial visualization problem solving activities help pre-service mathematics teachers in solving mental rotation problems.

The review of literature shows that individuals use different ways of solutions while solving spatial problems or tasks. As an example, Gluck and Fitting (2003) concluded that reliable individual differences existed in the strategies that people used when solving spatial tasks. Thus, the complexity of spatial information is reduced; the focus is on parts of the object or the environment rather than on the object as a whole (which would include the spatial relations among the parts). Similarly, Schulz (1991) mentions that in mental rotation tasks there are three different ways used by the individuals. One was *Move Oneself* that requires imaginary change of one's own viewpoint. The other one was *Move Object* which requires imaginary manipulation of the stimulus object. And the last one was *Key Feature* which requires the analysis and manipulation of important features of the stimulus object. According to Burin and Prieto (2000) there were two general kinds of solution strategies for spatial visualization tasks. One is an *analytic* or *feature comparison* approach, in which the examinee seeks to verify the identity of key features of the probes to match them with the target stimulus. A variant of this analytic strategy is verbal labeling of the features. The other is a *holistic* or *spatial manipulation* strategy, which involves mental movements of the probes, such as rotation, translation, synthesis, etc.

In the present study, to have a better understanding of the participants' mental rotation problem solving processes, their ways of solving mental rotation problems are considered rather than just considering their answers' being true or false.

2. METHODOLOGY

This study is designed as a case study to investigate in what ways engagement with spatial visualization problem solving activities help pre-service mathematics teachers in solving mental rotation problems.

2.1. Participants of the Study

Participants of the study are five pre-service mathematics teachers those are studying secondary or elementary mathematics education at a university in Ankara in the spring semester of 2007-2008 academic years who voluntarily participate the present study. In the present study, to protect the personal rights of the participants the codes are used to define participants rather than using their names. They are coded as participants from 1 to 5 (P1, P5). The characteristics of the participants; the departments of the participants, the degree that the participants had already taken and they are studying and the average grade of the mathematics courses (AGM) of the participants are given in Table 1.

Table1: The Participants' Characteristics

Participants	Department			AGM (over 4)
	BS	*MS*	*PhD*	
P_1	Math		SSME	2.5
P_2	Math		SSME	3
P_3	EME	SSME		3.2
P_4	EME (Senior)			2.15
P_5	EME (Senior)			2.6

(Math: Mathematics, EME: Elementary Mathematics Education, SSME: Secondary Science and Mathematics Education)

2.2. Spatial Visualization Problem Solving Activities

The study is conducted within the "teaching of geometry concepts" course that lasts 14 weeks. During the course sessions, the researcher attends to the classes and in each session additional spatial visualization problem solving activities are given to the participants. In the selection of the spatial visualization problem solving activities, the main focus is making the participants engage with the spatial problems that required the basic abilities of spatial visualization defined by Mc Gee (1979).

For the selection of appropriate spatial visualization problem solving activities, first of all the literature is reviewed and the activities that have already been found to be helpful in the development of spatial visualization abilities are listed. Then, the activity sheets for the selected activities are designed and the required materials are obtained. The prepared spatial

visualization problem solving activities were applied to the pre-service teachers that attended the "teaching of geometry concepts" course in the autumn semester of the 2008-2009 academic years, before used with the participants of the study. By using the observations and the experiences in this application, some of the activities are revised and some of them are changed with the new ones. By this way the spatial visualization problem solving activities appropriate for the participants' level, the course schedule and the study are selected.

The instruction of the course and the spatial visualization problem solving activities are followed on as follows;

In the first week of the semester, information was given about the objectives and the requirements of the course and about the study that would be held during the semester within the course sessions. Moreover, the participants discussed on the questions; "What is geometry?", "Why/how do you learn geometry?", "Why/how will you teach geometry?"

In the second week, the participants of the course became definite so the SAT was applied as the pre-test. Furthermore, the students made three groups and each group selected one of the subjects given in the schedule as the project subject. Moreover, the instructor leaded a discussion on teaching mathematics in the new context of the new elementary mathematics curriculum development.

For the third week the participants were given journals about Van Hiele Geometric Thinking, meanings of theorem, definition, axiom/postulate, non-metric geometry (Point, Space, Plane, etc.) and Euclidean geometry and non-Euclidean geometry (postulates). In the course session the instructor gave additional information about these subjects. Additionally, the researcher applied the "*Polygons Constructed by Tying Paper Knots*" activity. The purpose of the activity was to use spatial visualization ability to find out the way of tying the knot for the desired polygon. This activity is designed to lead participants to discover and demonstrate such relationships as reflections, transformations, and symmetry.
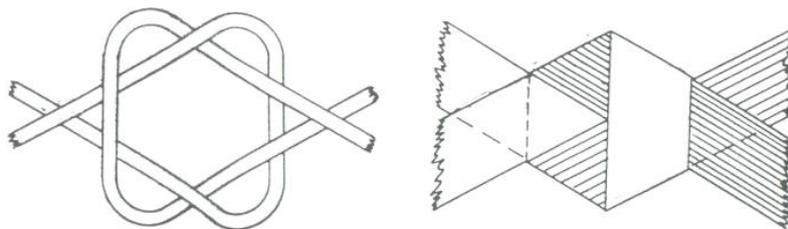
Figure 1: Tying Paper Knots: Obtain a hexagon with two papers

In the fourth week course, the participants engaged with the basic constructions with compass and straight edge. They constructed the basic geometric figures and then discussed on the proofs of their constructions. Moreover, the researcher applied *"Shading Turns"* activities, where the participants were expected to mentally turn the given figures (Mc Gee,1979).
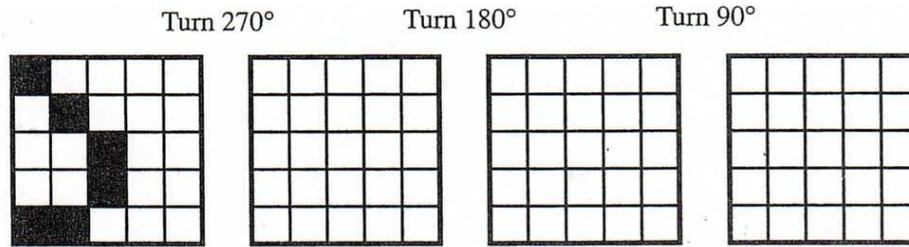
Figure 2: Shading Turns: Shade the grids to show the results of the turns in the clockwise direction with given angles.

In the course of the fifth week, more complex constructions were done and discussed by the participants. Moreover, the researcher gave information about the types of reflections and then "*Reflections*" activities applied. The purpose of the activity was to make use of spatial visualization ability to find out the reflection of the given figure with and without the symmetry mirror.

In the sixth week course spatial activities were applied by the researcher. The activities were; "*Symmetry*" and "*Tangrams*". Symmetry activity required to use a symmetry mirror to obtain the given symmetries. The main challenge was where to place the symmetry mirror. The aim of the activity was again to make the participants use spatial visualization ability to solve the problem.

Figure 3: Symmetry: Place a symmetry mirror to the first figure to obtain the second one.

In the seventh week course, the participants applied the Origami and Spatial Ability activities they prepared to the class. The eighth week course the first group introduced the "Polygons" to the class. Moreover they applied three activities about the Polygons. At the rest of the course, the researcher introduced pentomino to the class and applied the "*Pentomino*" activity. Pentomino is a polygon that is made up of five unit squares and in the activity participants were expected to obtain the given figures by using the given pentomino pieces.
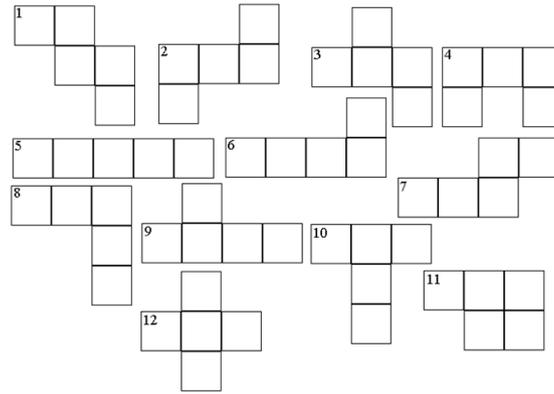
Figure 4: 12 Pieces of Pentominoes

The ninth week course session the participants presented the activities they prepared as the course project and then the researcher introduced the "*Archimedes's Stomachion*" and applied the related activity. Archimed's Stomachion was a kind of tangram puzzle but its ieces were more detailed so the activity was more challenging. The students were expected to place the pieces of Archimedes's Stomachion to obtain the given figures.

The tenth week, the subject was the Circles and the second Group gave information about the characteristics of circles and applied three activities about circles. Additionally, the "Egg Tangram" which was another kind of Tangram puzzle was introduced and activity about it was given. The activity required to use the pieces of Egg Tangram to obtain the given figures. Pentomino, Tangram, Archimedes's Stomachion and Egg Tangram were both designed to make participants to use spatial visualization ability to manipulate or transform the image of spatial patterns into other arrangements (Mc Gee, 1979).

In the eleventh week, the researcher introduced "*Perspective Drawing*" to the class and the participants did some exercises about perspective drawing. Perspective is the geometrical technique in drawing that creates the illusion of three-dimensional space on a two-dimensional plane. The participants were introduced one-point and two-point perspectives and asked to make perspective drawings of given 3-D block figures.

For the twelfth week, the third group introduced the properties of solid figures and applied three activities about the solid figures. Moreover, the researcher gave exercises that require isometric drawings of 3-D objects.

In the thirteenth week, the researcher introduced "*Soma Cubes*" to the class and the Soma Cube activity was applied. Additionally, exercises about orthogonal drawing and

isometric drawing of 3-D block figures obtained by Soma Cubes were held. The aim of the activity was to arrange the pieces of soma cubes to obtain a 3-D object.
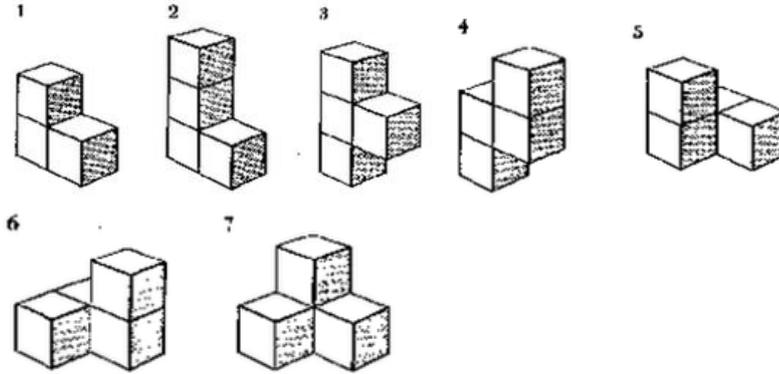
Figure 5: Seven Pieces of Soma Cubes

In the last week of the term, the exercises on isometric drawing, and orthogonal drawing of 3-D block figures were conducted.

2.3. Data Sources

In the present study, 1) Spatial Ability Test (SAT); 2) Structured Task-Based Interviews are used to collect data.

2.3.1. Spatial Ability Test (SAT)

Spatial Ability Test (SAT) was administered to the participants as pre- and post- test to determine their spatial visualization abilities. SAT is a paper-pencil test that is designed to establish the spatial ability and developed by Ekstrom (1976) on the other hand, according to the classification of McGee, solving the problems of SAT requires spatial visualization ability. There are four different tests in SAT;, Paper Folding Test (PFT) and Surface Development Test (SDT), Card Rotation Test (CRT) and Cube Comparison Test (CCT). In the present study CRT and CCT are used as a measure of mental rotation.

Card Rotation Test (CRT): The test consists of true-false items that require mentally rotating of 2-D figures. For each figure, there were 8 turned figures to be compared and for each true item one gets 1 point. The test consists of two parts with 10 questions each so, the total score is 160. Participants were given 3 minutes to complete the each part of the test.

Cube Comparison Test (CCT): The test consists of true-false items that require mental rotation of cubes with different patterns on its faces. Each true answer is scored as 1 and there

are 42 items, therefore the test is scored over 42. The test consists of two parts with 21 items each and each part was applied in 3 minutes.

The Spatial Ability Tests were translated in to Turkish and reliability coefficient of each test was calculated by Delialioğlu (1996). Reliability coefficients, number of questions and the durations for each test is given in Table 2.

Table 2: Reliability Coefficients, Number of Questions, Total Scores and Durations for the Tests of SAT (Delialioğlu, 1996).

Sub-Test	Cronbach Alpha	Number of Questions/Total Scores	Duration (min)
CRT	0.80	160	6
CCT	0.84	42	6

2.3.2. Structured Task-Based Interviews

This study intends to determine in what ways engagement with spatial visualization problem solving activities help pre-service mathematics teachers in solving mental rotation problems. Therefore, it is important to investigate the participants' mental rotation problem solving process in detail. So, participants' verbal and non verbal behavior and interactions are analyzed to make inferences about the spatial visualization problem solving of the participants (Goldin, 2000). So, task-based interview is particularly the appropriate way of collecting data for the research questions of the present study.

The research attention is focused on the participants' mental rotation problem solving process before and after their engagement with spatial visualization problem solving activities therefore task-based interviews were held at two different sessions. The first task-based interview was held at the beginning of the semester, just after applying SAT. The second task-based interview was held at the end of the semester, after all the course sessions had finished.

In the selection of the problems of the task-based interviews, the main criterion is that they require mental rotation abilities. Those are; mentally rotate a pictorially presented stimulus object. The mental rotation problems asked in the task-based interviews can be summarized as follows;

• Card Rotation problems require mentally rotating of a 2-D figure,

• Cube Comparison problems require mentally rotating a 3-D object,

The numbers and categorizations of the spatial visualization problems asked in the task-based interviews were given in Table 3.

Table 3: Categories of Spatial Visualization Problems of Task-Based Interviews

Required Ability		Number of Questions	
		Task-Based Int. I	*Task-Based Int. II*
Mental Rotation	Cube Comparison	2	3
	Card Rotation	2	3

Additionally, at the end of the second task-based interview the participants were asked the following question; "How do you think engagement with spatial visualization activities during the semester help your mental rotation problem solving?" and the answers for this question is used to interpret the results of SAT and task-based interviews.

2.4. Overall Procedure

The study was conducted during the spring semester of the 2007-2008 academic years with five pre-service mathematics teachers attending the course "teaching geometry concepts" at METU as a part of a PhD thesis study.

Before conducting the study, the literature reviewed and the spatial visualization problem solving activities were selected and then the activity sheets were prepared. During the autumn semester of the same academic year, the spatial visualization problem solving activities were applied in the classes of the same course with different attendants. By observing the activity sessions, some of the activities were taken out from the study, some of them were revised and the appropriate activities were selected for the level of the participants, the course schedule and the present study. Therefore, the spatial visualization problem solving activities were prepared before the semester began. The spatial visualization problem solving activities applied in the class were given in detail in part 2.2.

Moreover, Spatial Ability Tests designed by Ekstrom (1976) was selected as a spatial aptitude test. Ekstrom and his colleagues (1976) designed the SAT to measure the spatial ability aptitudes of people and it consist of four sub tests. However, in the present study, the SAT problems those require mental rotation as mental activity, are used. These problems are cube comparison and mental rotation problems.

Before conducting the study, the problems of task-based interviews were also prepared. In preparing the spatial visualization problems of task-based interviews, the researcher focused on the problems of SAT requiring mental rotation. In the first task-based interview the problems of cube comparison test (CCT) and card rotation test (CRT) were used.

However, in the second task-based interview, different problems were selected which require the same mental activities. By this way, the effect of participants' memorizing the problems and the solutions tried to be overcome.

At the beginning of the spring semester of 2007-2008 academic years, the pre-service teachers that attended the course were given information about the study and asked whether they wanted to be in the study voluntarily. Five of them accepted to be a part of the study and became the participants of this research.

At the beginning of the first course session, after they were given information about the study, spatial ability test (SAT) was administered to the participants as pre-test. The tests were administered by the researcher to the participants in the classroom where they took the course. Before the administration of the tests, the directions were explained to the participants. In addition, before administering each part of the SAT the participants were given instructions about how to answer the part.

Two task-based interviews were conducted during the semester to analyze the pre-service teachers' solving spatial visualization problems. The first interview was conducted at the beginning of the semester just after the SAT was applied to the participants. Then the course began and during the course sessions the researcher applied the spatial visualization problem solving activities that has already been prepared parallel to the course schedule. The second task-based interview was conducted at the end of the semester, after all the sessions had finished. Task-based interview responses were collected as video recordings and these recordings were then transcribed by the researcher. Moreover, the products of the participants obtained in the task-based interviews were collected. At the end of the semester SAT was administered to the participants as post-test. Therefore, transcriptions of the interviews, SAT results of the participants and their drawings of the task-based interviews were the data sources collected during the study.

2.5. Data Analysis

In case studies, it is important to collect multiple data such as conducting interview, observation and examining documents (Yin, 1984) to increase the validity and reliability of the study. In the present study, the test scores, the interviews, observation and the document analysis are used as data sources. The pre- and post-SAT scores of the participants are analyzed to define participants' spatial ability for mental rotation problems. To determine detailed information the number of unanswered and the incorrect responses of the

participants are also analyzed. Additionally, the video recordings of the task-based interviews are transcribed; the conversation and the observations are denoted in the transcriptions. The task-based interviews are analyzed according to determine the spatial visualization abilities of the participants in detail.

Responses of the participants to the interviews are analyzed through content analysis. Content analysis is a method that arranges the concepts in a logical way upon conceptualization of the data collected and explains the relationship between these concepts (Yıldırım & Şimşek, 2006). First of all, an initial category and code list based on the literature are formed for content analysis. Furthermore, new category and code additions are performed when required during review of the answers. New arrangement is made in order to finalize categories and codes once all answers were reviewed. As a result of the content analysis, concepts are presented in a descriptive way. Direct quotations from the interviews to increase the internal reliability of the study (Silverman, 1993) are included while reporting the results of the research. After all the codes are finished a doctorate student from the department of secondary science and mathematics checked the codes of two of the participants. Then the necessary changes in the codes are made and a full agreement with the second rater is reached. Additionally, all the participants checks the interview codes held with themselves and asked whether the codes and the inferences are matching with what they wanted to express, and according to their responses, the codes are reviewed.

3. RESULTS AND CONCLUSIONS

In this section, findings for the following research question will be presented;

"In what ways do spatial visualization problem solving activities help pre-service mathematics teachers in solving spatial visualization problems requiring mental rotation?"

To answer this research question first of all pre- and post- SAT scores are analyzed for each of the participant. The results are given in Table 4.

Table 4: Number of True, Incorrect and Empty Responses of Participants in Pre and Post CCT and CRT

Tests		Number of Responses	Participants				
			P_1	P_2	P_3	P_4	P_5
Pre	CCT (42)	True	10	31	20	12	18
		False	6	8	5	10	4
		Empty	26	3	17	20	19
	CRT (160)	True	58	123	108	84	109

31

A Case Study on the Ways How Engagement with Spatial Visualization Problem Solving...
E. Banu KAYHAN KIRMAÇ - Safure BULUT

		False	11	8	4	12	7
		Empty	91	29	47	64	44
		True	20	38	40	30	41
	CCT (42)	False	5	4	2	7	3
Post		Empty	17	0	0	5	0
		True	121	150	144	142	145
	CRT (160)	False	5	5	4	5	5
		Empty	34	5	8	13	10

To have a better interpretation the transcriptions of structured task-based interviews of the participants and the SAT results of each participant are analyzed together and the results could be explained as follows;

When the SAT scores of P_1 is analyzed in Table 4, it is seen that, in the pre-test she has nearly half of the problems unanswered and these are mostly at the end of the tests. So, she couldn't manage to answer the questions in given periods. Moreover, her incorrect responses are nearly 25% of the tests.

In the first task-based interview, for the card rotation problem given in Figure 6, P_1 explains her solution as;

P_1: I tried to rotate the figure in my mind.

Figure 6: 1st Card Rotation Problem of the 1st Task-Based Interview

Similarly, for the cube comparison problems she tries to rotate the cubes mentally. On the other hand, in the second task-based interview, for the card rotation problem given in Figure 7 she mentions;

P_1: I control the positions and the distances of the parts. The black square should always be on the right and 2 units away from the white one.
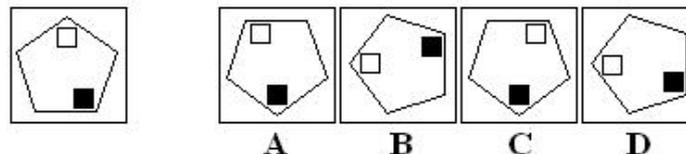
Figure 7: 1st Card rotation Problem of the 2nd Task-Based Interview

The post-test results of P_1 show that number of her true responses increases where the number of her unanswered and incorrect responses decreases. When she is asked in what ways solving spatial problem activities help her in solving spatial visualization problems she mentions that engagement with spatial visualization problem solving activities help her imagine the manipulations or rotations better. She says;

P_1: Especially "shading turns" and "reflection" activities helped me mentally turn the figure… Moreover, I become more practical in comparing the relative parts or patterns of the objects.

When the task-based interview transcriptions of P_1 revised, it was concluded that, she shifts her strategy in mental rotation problems and for the complex objects she starts to *compare the relative positions* of the parts of the object.

SAT scores of P_2 shows that she has the best results among all the participants. Although she has high scores in the pre-test, she has higher scores in the post-test. She becomes faster and gives more accurate answers. In the task-based interview she mentions her solution for the card rotation problem in Figure 8 as;

P_2: I take the asymmetric parts of the figure and fixed their positions with respect to each other.

Figure 8: 2nd Card Rotation Problem of the 1st Task-Based Interview

Similarly, for the cube comparison test in Figure 9 she says, "If the pattern is symmetric like O then I control the position of that face. And then I take two asymmetric parts and fix their positions." In the second task-based interview she uses similar ways to solve the mental rotation problems. As an example she explains her solution for the card rotation problem in Figure 12 as;

P_2: I take one reference point but it is not enough I will use one more. Now, I will control the positions of them with respect to each other.
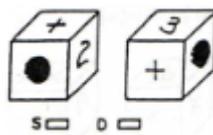
Figure 9: 2nd Cube Comparison Problems of the 1st Task-Based Interview

Similarly, for the cube comparison problem in Figure 10 she says;

P$_2$: I control whether the patterns on the faces follow each other.
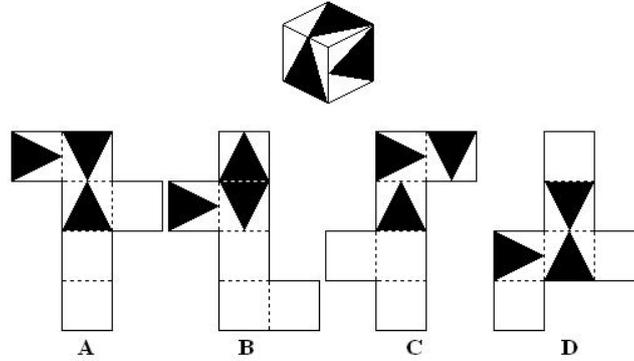
Figure 10: 1st Cube Comparison Problem of the 2nd Task-Based Interview

When P$_2$ is asked about the help of spatial visualization problem solving activities, she mentions that the "Tangram" and "Arschimed's Stomachion" activities are open ended and weak for developing her spatial ability. But "Pentomino" and "Soma Cubes" are helpful in practicing the rearrangement of the parts of a figure. P$_2$ expresses she has no difficulty in rotation problems and she could easily compare the relative positions of the parts or the patterns of the figures. So, the spatial visualization problem solving activities don't lead any change in her solving such problems. Moreover she expresses;

P$_2$: I found a practical way for each kind of problem and then used it in all other problems of the same kind.

So, while solving mental rotation problems in task-based interviews she explores an analytic way such as *comparing the pattern* or *comparing the relative positions* of the parts of the given figure for each kind of problems and then used it and she has no tendency to shift her solution ways to mentally rotating.

When SAT results of P$_3$ are analyzed in Table 4, they show that for the pre-test she leaves nearly half of the questions unanswered and these questions are at the end of the tests. So, we can conclude that in pre-test she has speed problem. On the other hand, her responses are mostly correct. That is her solutions are correct but not fast enough. The post-test results of P$_3$ show that the number of her unanswered responses decreases and her scores become one of the highest in the group. When the task-based transcriptions of P$_3$ analyzed, it is seen that she use similar ways with P$_2$. For the card rotation problem in Figure 6 she explains her solution as;

P₃: Each has an asymmetric part. For instance, in this problem, the thin and long part should always be on left. I took distinct points and then control their positions with respect to each other.

Similarly for the card rotation problem in the second task-based interview in Figure 7;

P₃: I choose distinct points and control their positions with respect to each other. In this problem, on the left there should be two white squares.

For the cube comparison problem in Figure 11 she mentions that; "If the patterns on the faces of the cubes are asymmetric than I will control their positions. For instance, in this second problem the short part of T and the square should be on the same hand and on the left of the long part of T there should be triangles."
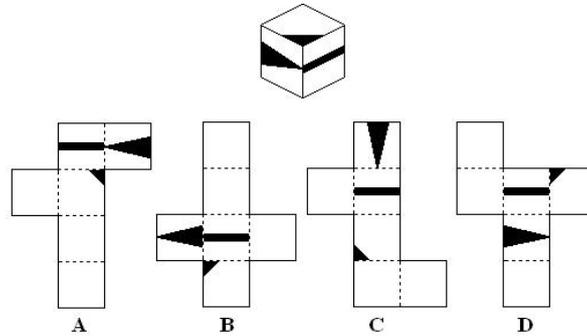
Figure 11: 3rd Cube Comparison Problem of the 2nd Task-Based Interview

P₃ tends to use the way of *comparing the positions* of the parts of the figure in solving mental rotation problems. She expresses that during the spatial visualization problem solving activities she becomes more practical in analytic methods and she explains;

P₃: In mental rotation problems I don't have difficulty but I become more practical....When I couldn't imagine the rotation I compare its parts.

To sum up, P₃ shifts her solution ways to analytical thinking rather than visualizing and moreover becomes more practical in using such ways.

The SAT scores of P₄ show that has half of the questions unanswered in the pre-test and she also has 20% of her responses incorrect (see Table 4). However, in the post-test she has nearly all of the questions answered and number of her incorrect answers decreases. In the first task-based interview she explains her solution for the card rotation problem given in Figure 8 as follows;

P₄: I tried to imagine the rotation. For instance this is not the same; the long part changed its place.
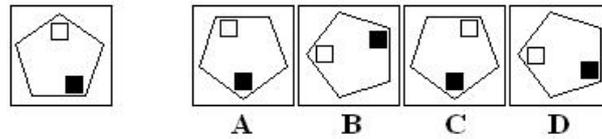
Figure 12: 1st Card rotation Problem of the 2nd Task-Based Interview

Similarly, for the card rotation problem of the second task-based interview given in Figure 12 she mentions "I establish distinct parts and they should be on the same direction with the same distance." Her explanation for the cube comparison problem for the second task-based interview given in Figure 13 is on the same way;

P₄: First, I tried to understand the rotation and I looked at the faces with the same pattern. For example in this one I should firstly turn right than upwards. Then I tried to imagine how would be the other faces.
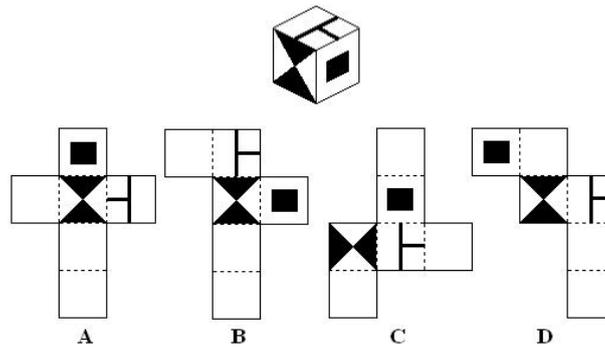
Figure 13: 2nd Cube Comparison Problem of the 2nd Task-Based Interview

When the help of the spatial visualization problem solving activities were asked to P₄ she mentions;

P₄: In mental rotation tasks I could imagine the rotations for easy objects anyway. However, for the complex objects I practice to compare the positions of the parts of the object.

On the other hand, she thinks, all the activities are not so helpful, for example;

P₄: But, "Reflections" activity for example had no help for me. I had already had that skill.

So, P₄ mostly try to imagine the required rotation of the given figure and engagement with spatial visualization solving activities practices her existing solution ways and helps her become more competent.

The pre-test results of P₅ shows that she has one of the lowest scores as could be seen in Table 4. Even though, P₅ doesn't have too many wrong answers, she has nearly the half of

the questions of the tests unanswered and these are the last questions of the tests as the other participants. In the post-test on the other hand, she nearly answered all of the questions and the number of incorrect answers are still low. Therefore, her problem solving speed increases. For the card rotation problems in the first task-based interview she says;

P_5: I mentally do such problems. I tried to imagine the rotation.

On the other hand, in the second task-based interview for the card rotation problem given in Figure 14 she mentions;

P_5: In this problem the distance between the squares should remain the same.
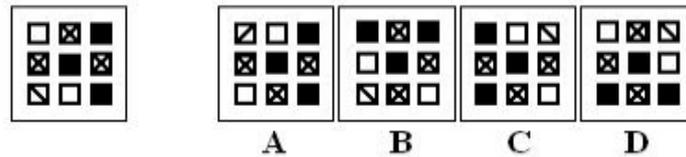
Figure 14: 3rd Card Rotation Problem of the 2nd Task Based Interview

For the cube comparison test in the first task-based interview she explains;

P_5: I looked at both of the cubes and try to find the turns by looking at the patterns, then rotate the first figure. If the pattern is symmetric as , then I control the asymmetric ones as 2.

For the cube comparison test in the second task-based interview given in Figure 13;

P_5: I control the positions of the distinct figures with each other. For example for this one the tip of the arrow should be followed by the line.

P_5 had similar responses to the question about the help of spatial visualization problem solving activities. For the mental rotation problems she mentioned;

P_5: I become faster in rotation tasks, I could compare the asymmetric parts more practically.

Thus, P_5 tries to mentally rotate the given figures at the beginning. On the other hand while solving spatial visualization problems during the course sessions she explores *comparing patterns* and *comparing relative positions* of the parts of the given figures and shifts her way of solution. She uses techniques faster and more accurately than *mentally rotating* the figures.

The SAT results and task-based interview responses of the participants are analyzed to understand in what ways the spatial visualization problem solving activities helps participants in solving mental rotation problems. To have a better understanding, rather than investigating the increase in the true responses the ways of solutions and the changes in their ways of

solutions are also analyzed. The results show that participants use two main different ways; *mentally rotating* the given figure or *comparing* the relative positions or patterns of the parts of the given figures which can be accepted as analytic. When the responses are analyzed in terms of in what ways the spatial visualization problem solving activities helps participants in solving mental rotation problems; it is seen that there are three main aspects of help; *exploring, practicing* and *shifting* the way of solution. The ways that spatial visualization problem solving activities help the participants in solving the spatial visualization problems are given in Table 5.

Table 5: The Ways that Spatial Visualization Problem Solving Activities Help the Participants in Solving the Spatial Visualization Problems

Participants	Spatial Visualization Problems	
	Card Rotation	Cube Comparison
P$_1$	*Practice imagining the rotation* *Shift comparing the relative positions*	*Shift* *Comparing pattern*
P$_2$		
P$_3$	*Practice comparing relative positions*	*Shift comparing pattern*
P$_4$	*Shift comparing relative positions*	*Shift comparing pattern*
P$_5$	*Practice comparing the relative positions*	*Practice Comparing pattern*

Analyzing the results of task-based interviews the following could be concluded:

• If the participants could solve the mental rotation problems by using a way of solution then they don't change that.

• If the participants use an analytic way such as *comparing* the relative positions or patterns of the parts of the given figure then rather than shifting it to way of *mentally rotating* they practice their strategy while solving the spatial problems.

• If the participants have a difficulty in imagining the rotation then they explore analytic ways and shift their strategy to ways such as *comparing* the relative positions or patterns of the parts of the figures.

Participants are given spatial visualization problem solving activities during the semester and, they solve spatial visualization problems requiring mental rotation in task-based interviews. Therefore, they engage in solving spatial visualization problems. By analyzing the task-based interviews with pre- and post- SAT results, we try to find in what ways spatial visualization problem solving activities help participants in solving mental rotation problems. (See Table 5) In the spatial visualization problems requiring mental rotation, the participants who try to imagine the required mental process at the beginning mostly have difficulties, especially for the complex problems. Then while solving spatial problems during the class sessions or task-based interviews, they mention that they explore more practical ways and they shift their ways of solutions. The participants, who have already used ways such as *comparing* the parts or patterns of the given figures rather than imagining the rotation, on the other hand, practice their solution ways during the activities, and become more competent in using those ways. Therefore, we could conclude that solving spatial problems help in *exploring*, *practicing* or *shifting* ways of solving problems to cope with the difficulties in solving spatial visualization problems requiring mental rotation accurately and practically.

4. DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

4.1. Discussion

The main purpose of the present study is to investigate in what ways engagement with spatial visualization problem solving activities helps pre-service mathematics teachers in solving mental rotation problems. To achieve this purpose, the participants' ways of solving mental rotation problems are tried to be identified deeply. By this way the pre- and post- SAT scores and the task-based interview responses of the participants are tried to be interpreted in terms of the ways that engagement with spatial visualization problem solving activities helps participants in solving mental rotation problems.

The present study is conducted during the spring semester of 2007-2008 academic years with five adults studying elementary or secondary mathematics education at a university in Ankara who accept participating in this study voluntarily. To obtain related data, pre- and post-Spatial Ability Test (SAT) (Ekstrom, 1979) and two task-based interviews are used.

Analyzing pre- and post- test results of cube comparison (CCT) and card rotation (CRT) tests of Spatial Ability Test (SAT) of each participant in Table 4, it is seen that the number of correct answers for each participant increases where the number of incorrect responses

decreases. That is, the participants solve mental rotation problems more correctly than before. Additionally, when the responses of the participants are examined in detail, it is found out that the empty responses are mostly at the end of the tests; therefore we may say that the main reason for the empty responses is lack of time. So, it could be concluded that decrease of empty responses may be a result of increase in the speed of problem solving. Therefore, we could say that engagement with spatial visualization activities helps participants to solve mental rotation problems faster and more correctly. The analysis of the task-based interviews also supports this view.

To have a better understanding of the collected data the participants' ways of solving mental rotation problems are considered also. The review of related literature shows that, individuals use different ways of solutions while solving spatial problems. One of the ways of solutions is, maintaining and using information about spatial relations among elements in the mental representation (Glück & Fitting, 2003). Another group of ways involves representing and manipulating spatial information by reducing it to a systematic step by step approach that requires minimal visualization (Hsi et al., 1997). The results of the study also show that participants use similar ways of solutions while solving mental rotation problems. One of the ways is trying to imagine the required rotation such as defined by Glück and Fitting (2003). On the other hand the other way requires comparing the patterns or relative positions of the parts of the given figure such as the ways defined by His et al. (1997).

Kyllonen et al. (1984) studied the strategies of subjects solving paper-folding task and they concluded that subjects differed in the strategies they normally used and often shifted strategies between and also within different spatial tests. As items within a test became more difficult, the tendency to use analytic, non-spatial strategies appeared to increase. The analysis of the first task-based interview also shows that, while solving mental rotation problems, the participants who try to imagine the required mental process at the beginning mostly have difficulties, especially for the complex problems and these participants have problems in solving SAT problems practically and accurately. Then while solving spatial problems during the class sessions or task-based interviews, they mention that they explore more practical ways and they shift their ways of solutions. And analyzing the second task based interviews and the post-SAT results shows that they explore *comparing* the parts or patterns of the given figures and they shifted their ways of solutions to these ones. Additionally, post-SAT results and second task-based interview responses show that, the participants, who have already used

ways such as *comparing* the parts or patterns of the given figures rather than imagining the rotation, on the other hand, practice their solution ways during the activities, and become more competent in using those ways. Therefore, we could conclude that solving spatial problems help in *exploring*, *practicing* or *shifting* ways of solving problems to cope with the difficulties in solving spatial visualization problems requiring mental rotation accurately and practically. The findings of the study are parallel to the findings of Baron (1978). According to Baron (1978) failure to use the appropriate strategy could be overcome through training and proficiency limitations could be overcome by practice on the task which the strategy in question is used. Baron (1978) insisted that strategies can be modified by educational manipulations and concluded that if one knows what strategies to use, one would be in a better position to discover which capacities were most important.

4.2. Implications and Recommendations

The main interests of the present study is the ways that engagement with spatial visualization problem solving activities help pre-service mathematics teachers in solving mental rotation problems. The results of the study show that; while solving mental rotation problems, the participants use two main different ways of solutions; *imagining the rotation* and *comparing the patterns or relative positions* of the parts of the given figures. Additionally, we could conclude that solving spatial problems help in *exploring*, *practicing* or *shifting* ways of solving problems to cope with the difficulties in solving spatial visualization problems requiring mental rotation accurately and practically.

The analysis of the results of the present study shows that in most of the cases participants reduced the required mental rotation in to systematic step by step approach with minimal visualization rather than using visualization. Moreover, in the cases participants try to imagine the rotation, they have difficulties in finding the accurate solution in the determined time period. These results bring out the need of development of mental rotation ability. Even the adults studying mathematics education are lack of visualization. Therefore, Turkish policy makers should give some thought to the educational poorness about spatial visualization ability of students.

It is concluded in the present study that, solving spatial problems help in *exploring*, *practicing* or *shifting* ways of solving mental rotation problems. In other words, spatial visualization problem solving activities help the participants to solve mental rotation problems more accurately and practically. Therefore, mathematics educators should design courses

containing such activities. Additionally, while mathematics teachers are preparing and applying spatial visualization activities, they should be aware of that there are different ways of solving mental rotation problems to be developed for more competencies. Therefore, for mathematics teachers seminars should be planned about how to use different ways of solving mental rotation problems in the development of spatial visualization ability. The findings of the present study will be used effectively in such seminars. Addition to the above recommendations, the curriculum developers also can use the findings of the present research study. In preparing the geometry curriculum for different levels, spatial activities that are developing students' use of different ways of solutions should be used.

Based on the analysis of data, some recommendation for further research studies can be reported. First of all, this study is conducted with five adults studying mathematics education. By using their expressions, spatial abilities and difficulties in spatial visualization problem solving are tried to be defined in depth. However, this study is limited to selected five adults. Therefore, this study can be replicated with a larger group, by this way, different ways of solutions for mental rotation problems could be identified. Additionally, different categories of help in ways of solving problems to cope with the difficulties in solving mental rotation problems could be found out. Finally, the present study focuses on mental rotation problems, a similar study can be conducted with mental manipulation problems.

REFERENCES

Baron, J. (1978). Intelligence and general strategies. In G. Underwood (Ed.), Strategies in information pro- cessing, pp. 403-450). London: Academic Press.

Battista, M. T., Wheatley, G. H., & Talsma, G. (1989). Spatial Visualization, Formal Reasoning, and Geometric Problem-Solving Strategies of Pre-service Elementary Teachers, *Focus on Learning Problems in Mathematics*, *11*(4), 17-30.

Bennie, K., & Smit, S. (1999). *"Spatial Sense": Translating Curriculum Innovation in to Classroom Practice, Proceedings of the Fifth Annual Congress of the Association for Mathematics Education of South Africa*: 1, 22-29. Port Elizabeth: Port Elizabeth Technikon.

Burin, D. I., Delgado, A. R. & Prieto, G. (2000) Solution strategies and gender differences in spatial   visualization tasks. *Psicológica,* 21, 275-286.

Çakmak, S., Işıksal, M., Koç, Y. (2013). Investigating Effect of Origami-Based Instruction on Elementary Students' Spatial Skills and Perceptions. *Journal of Educational Researc, Sep 2013.*

Delialioğlu, Ö. (1996). *Contribution of students' logical thinking ability, mathematical skills and spatial ability on achievement in secondary school physics*, Unpublished Master Thesis, Middle East Technical University, Ankara, Turkey.

Ekstrom, R. B., et al. (1976). *Kit of Factor-Referenced Cognitive Tests*. ETS, Princeton, NJ. Ekstrom, R. B.,French, J. W., & Harman, H. H. (1979). Cognitive factors: Their identification and replication. *Multivariate Behavioral Research Monographs*, *79*(2).

Glück, J., & Fitting, S., (2003). Spatial Strategy Selection: Interesting Incremental Information. *International Journal of Testing, 3*(3), 293-308.

Güven, B.,& Kosa, T. (2008). The Effect of Dynamic Geometry Software on Student Mathematics Teachers' Spatial Visualization Skills. *The Turkish Online Journal of Educational Technology, Vol 7(4),100-107.*

Hershkowitz, R. (1989). Visualization in Geometry- Two Sides of the Coin, *Focus on Learning Problems in Mathematics*, *11*(1), 61-76.

Hershkowitz, R., Parzysz, B., & Dormolen J. V. (1996). Space and Shape, *International Handbook of Mathematics Education,* 161-204.

Hsi, S., Linn, M.C. & Bell, J.E. (1997). The Role of Spatial Reasoning in Engineering and the Design of Spatial Instruction. *Journal of EngineeringEducation,* 151-158.

Hodgson, T. (1996). Students' Ability to Visualize Set Expressions: An Initial Investigation. *Educational Studies in Mathematics, 30*, 159- 178.

Kayhan, E. B. (2005). *Investigation of high school students' spatial ability*. Unpublished master's thesis, Middle East Technical University, Ankara, Turkey.

Kolb, B., & Whishaw, I. Q. (1981). Neonatal frontal lesions in the rat: Sparing of learned but not species-typical behavior in the presence of reduced brain weight and cortical thickness. *Journal of Comparative and Physiological Psychology, 95*(6), 863-879.

Kyllonen, P. C.; Lohman, D. F. & Woltz, D. J. (1984). Componental modeling of alternative strategies for perfoming spatial tasks. Journal of Educational Psychology, 76 , 1325-1345.

McGee, M.G. (1979) Human Spatial Abilities: Psychometric Studies and Environmental, Genetic, Hormonal, and Neurological Influences. *Psychological Bulletin*, *86*(5), 889-918.

Milli Eğitim Bakanlığı, (2005). *Talim Terbiye Kurulu Başkanşığı, İlköğretim Matematik Dersi Öğretim Programı ve Kılavuzu 6-8. Sınıflar.* Milli Eğitim Basımevi-Ankara.

Milli Eğitim Bakanlığı, (2006). *Talim Terbiye Kurulu Başkanşığı, İlköğretim Matematik Dersi Öğretim Programı ve Kılavuzu 6-8. Sınıflar.* Milli Eğitim Basımevi-Ankara.

Milli Eğitim Bakanlığı, (2009). *Talim Terbiye Kurulu Başkanşığı, Ortaöğretim Geometri Dersi Öğretim Programı ve Kılavuzu 9-10. Sınıflar.* Milli Eğitim Basımevi-Ankara.

Milli Eğitim Bakanlığı, (2010). *Talim Terbiye Kurulu Başkanşığı, Ortaöğretim Geometri Dersi Öğretim Programı ve Kılavuzu 11. Sınıflar.* Milli Eğitim Basımevi-Ankara.

Milli Eğitim Bakanlığı, (2011). *Talim Terbiye Kurulu Başkanşığı, Ortaöğretim Geometri Dersi Öğretim Programı ve Kılavuzu 12. Sınıflar.* Milli Eğitim Basımevi-Ankara.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council.

National Council of Teachers of Mathematics (2001). *Navigating through Geometry in Grades 9-12.* Reston Va.:20191-9988.

National Council of Teachers of Mathematics (2003). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council.

Olkun, S. (2003) Making Connections: Improving Spatial Abilities with Engineering Drawing Activities, *International Journal of MathematicsTeaching and Learning*, (Apr,17). [Online]: http://www.ex.ac.uk/cimt/ijmtl /ijabout.htm.

Osberg, K. M. (1997). *Spatial Cognition in the Virtual Environment, Technical* R-97-18. Seattle: HIT Lab.

Owens, K. D., & Clements, M.A. (1998). Representations in Spatial Problem Solving in the Classroom, *Journal of Mathematical Behavior, 17* (2), 197-218.

Pallascio, R., Allaire, R., & Mongeau, P. (1993). The Development of Spatial Competencies through Alternating Analytic and Synthetic Activities, For the Learning of Mathematics, 13(3), 175-190.

Presmeg, N. (1997). Research on Visualization in Learning and Teaching Mathematics.

Sorby, S.A. (1999) Computer aided design instruction. *1999 ASEE Annual Conference Proceedings Session 1438.* [Online] In M. Alias, T. R. Black, & D. E.Gray, (2002). Attitudes Towards Sketching and Drawing and the Relationship with Spatial Visualization Ability in Engineering Students, *International Education Journal, 3*(3), 1-12.

Schultz, K. (1991) *The contribution of solution strategy to spatial performance.* Canadian Journal of Psychology/Revue canadienne de psychologie, Vol 45(4), Dec 1991, 474-491.

Smith, G.G., Gerretson, H., Olkun ,S., Yuan., Y., Dogbey, J., & Erdem, A. (2009). Stills, not full motion, for interactive spatial training: American, Turkish and Taiwanese female pre-service teachers learn spatial visualization. *Computers& Education, Vol 52(1), Jan 2009, 201-209.*

Sorby, S. A. (1999). Developing 3-D Spatial Visualization Skills, *Engineering Design Graphics Journal,63*(2), 21-32.

Tartre, L. A. (1990). Spatial Orientation Skill and Mathematical Problem Solving, *Journal for Research in Mathematics Education,21*(3), 216- 229.

Toptaş, V., Çelik, S., & Karaca, E. T. (2012). Improving 8th Grades Spatial Thinking Abilities Through a 3D Modeling Program. *The Turkish Online Journal of Educational Technology – April 2012, volume 11(2), 128-134.*

Turğut, M. & Yılmaz, S. (2012). Relationships Among Preservice Primary Mathematics Teachers' Gender, Academic Success and spatial ability. *International Journal of Instruction, July 2012,Vol.5(2), 5-20*

Usiskin Z. (1987) Resolving the Continuing Dilemmas in School Geometry. In R. Hershkowitz, Visualization in Geometry: Two Sides of the Coin, *Focus in Learning Problems in Mathematics*, *11*(1), 61- 76.

Yin, R. K. (1994). *Case Study Research; Design and Methods,* Sage Publications, California, 1994.

Yıldırım, A.& Şimşek, H. (2006). Sosyal Bilimlerde Nitel Araştırma Yöntemleri. Ankara: Seçkin.

GENİŞ ÖZET

Uzamsal yetenek günlük hayatımızda olduğu kadar sanat, mimarlık, mühendislik, tasarım, fen bilimleri, matematik ve geometri gibi birçok alan için önemli bir yetenek olarak kabul edilmektedir. Uzamsal yeteneği oluşturan alt yeteneklerden biri olarak kabul edilen uzamsal görselleştirme yeteneği, nesnenin parçalarının hareketinin ardından durumlarının görselleştirilmesi, bir nesnenin katlanması ve açılması, uzayda nesnelerin ilişkisel olarak konumundaki değişikliğin zihinde canlandırılabilmesi, uzamsal bir örüntünün başka bir şekilde düzenlenmesi ya da manipüle edilebilmesi ve üçüncü boyutta hareketin zihinde canlandırılması ve zihinde nesnelerin manipüle edilebilmesi yeteneği olarak tanımlanmıştır. Uzamsal görselleştirme yeteneğinin matematik öğrenmede ve kullanmada en önemli yeteneklerden biri olarak kabul edilmesi, matematik öğretiminde bu yeteneğin geliştirilmesinin önemini ortaya koymuştur. Yapılan çalışmalar uzamsal görselleştirme yeteneğinin eğitimle ve doğru materyaller ve etkinlikler kullanılarak geliştirilebildiğini göstermiştir. Bu nedenle matematik öğretmenlerinin ve matematik öğretmen adaylarının uzamsal görselleştirme yeteneğinin geliştirilmesi ve kendi öğrencilerinin uzamsal görselleştirme yeteneklerini geliştirme yöntemlerini bilmeleri gerekmektedir.

Bu çalışmada uzamsal görselleştirme problemleri çözme etkinliklerinin öğretmen adaylarının zihinsel döndürme problemleri çözmelerine nasıl yardımcı olduğu araştırılmaktadır. Araştırma bir durum çalışması olarak tasarlanmıştır ve çalışmanın katılımcıları 2007-2008 yılı yaz döneminde Ankara'da bir devlet üniversitesinde eğitim gören ve çalışmaya gönüllü olarak katılan beş orta öğretim ve ilköğretim matematik eğitimi bölümü öğrencisidir. Çalışma, 14 hafta süren "Geometri Kavramlarını Öğretme" dersi içerisinde yürütülmüştür. Dersler süresince katılımcılara ders konularına ek olarak araştırmacı tarafından uzamsal görselleştirme problemleri çözme etkinlikleri verilmiş ve çözmeleri sağlanmıştır.

Çalışmada kullanılan veri toplama araçları Uzamsal Yetenek Testi (SAT) ve görev temelli görüşmelerdir. Uzamsal yetenek testlerinden Kart Döndürme (CRT) ve Küp Karşılaştırma (CCT) testleri zihinsel döndürme yeteneğini ölçmek için ön test ve son test olarak dönem başında ve sonunda katılımcılara uygulanmıştır. Test sonuçları incelenirken, daha detaylı sonuç elde etmek için doğru cevapların sayısının yanı sıra yanlış verilen cevapların ve boş bırakılan soruların sayıları da göz önünde bulundurulmuştur. Çalışma süresince katılımcılarla biri dönem başında diğeri de derslerin bitiminden sonra olmak üzere ikişer kez görev temelli görüşme yapılmıştır. Görüşmelerde katılımcılara kart döndürme ve küp karşılaştırma problemleri verilerek çözmeleri ve çözümlerini anlatmaları istenmiştir. Görüşme bulguları incelenirken katılımcıların verilen zihinsel döndürme problemlerini çözmede kullandıkları yöntemler ele alınmıştır. Çalışmanın verileri incelenirken uzamsal yetenek testi sonuçları ve görev temelli görüşmeler birlikte ele alınmıştır.

Uzamsal yetenek testi sonuçları incelendiğinde, ön test olarak uygulanan uzamsal yetenek testi sonuçları göstermiştir ki katılımcıların doğru cevaplarının sayıları genellikle çok düşüktür. Verilen cevaplar incelendiğinde yanlış cevap sayısından çok, cevaplanmayan soru sayılarının oldukça fazla

olduğu görülmüştür. Cevaplanamayan soruların testlerin son kısmında olması, katılımcıların testleri verilen sürelerde bitiremediklerini göstermektedir. Katılımcılar ön testte zihinsel döndürme sorularını çözerken seri ve doğru bir şekilde çözmekte zorlandıkları söylenebilir. Son test olarak uygulanan uzamsal yetenek testi sonuçları incelendiği zaman ise doğru cevap sayısında artış olurken cevaplanmayan soru sayısının oldukça düşük olduğu gözlemlenmiştir. Cevaplanamayan sorular yine testlerin sonundadır ama testlerin hemen her sorusunun incelenebildiği görülmüştür. Buradan da katılımcıların son testte daha hızlı ve daha doğru şekilde soruları cevapladıkları sonucu çıkarılabilir.

Görev temelli görüşmeler incelenirken daha ayrıntılı sonuçlar elde etmek için katılımcıların zihinsel döndürme problemlerini çözme yöntemleri göz önüne alınmıştır. İlk görüşmede zihinsel döndürme problemleri çözerken katılımcılardan bazılarının dönmeyi hayal etmeye çalıştıkları ve bu yöntemi kullanırken oldukça zorlandıkları gözlemlenmiştir. Bir başka deyişle, kullandıkları yöntemle doğru çözümleri bulamadıkları ya da çok uzun sürelerde çözüme ulaştıkları görülmüştür. Bu yöntemi kullanan katılımcıların uzamsal yetenek testlerinden de düşük sonuçlar elde ettiği görülmüştür. Katılımcıların bazılarının ise daha analitik yollarla sonuca gitmeye çalıştıkları gözlemlenmiştir. Bu yollar, döndürülecek şeklin parçalarının ya da üzerindeki desenlerin birbirlerine göre konumlarını ve uzaklıklarını karşılaştırmaktır. Bu yöntemleri kullanan katılımcıların uzamsal yetenek testi sonuçları daha yüksektir. İkinci görüşmeler incelendiğinde, öncelikle ilk görüşmelerde zihinsel döndürme problemleri çözerken dönmeyi zihinde canlandırma yöntemini seçen ve bunda zorlanan katılımcıların dersler sırasında yeni yöntemler *keşfettiklerini* belirttikleri görülmüştür. Katılımcılar şeklin tamamının dönmesini hayal etmeye çalışmak yerine parçalarının birbirlerine göre konumlarını ve uzaklıklarını kontrol etmeyi keşfettiklerini belirtmişlerdir. Böylece, katılımcılar kullandıkları yöntemleri *değiştirerek* analitik yöntemleri kullanmaya başlamışlardır. Bununla birlikte, ilk görüşmede de döndürülecek şeklin parçalarının ya da üzerindeki desenlerin birbirlerine göre konumlarını ve uzaklıklarını karşılaştırma yöntemini kullanmış olan katılımcılar ise dersler sırasında bu yöntemlerini *patrikleştirdiklerini* belirtmişlerdir. Dersler süresince uzamsal görselleştirme problemleri çözme etkinliklerinde analitik yöntemleri kullanmanın, bu yöntemleri daha hızlı bir şekilde kullanarak doğru sonucu yardımcı olduğu görülmüştür. Katılımcıların son test olarak uygulanan uzamsal yetenek testinde doğru sayısının artarak cevaplanamayan soru sayısının çok az olması bu bulguları desteklemektedir.

Çalışmanın sonuçları göstermiştir ki; uzamsal görselleştirme problemleri çözme etkinlikleri öğretmen adaylarının zihinsel döndürme problemleri çözerken kullandıkları yöntemleri *değiştirme, keşfetme* ve *pratikleştirme* konusunda yardımcı olmuşlardır. Bunun sonucunda da katılımcılar zihinsel döndürme problemlerini daha hızlı ve doğru şekilde cevaplandırmaya başlamışlardır.

Çalışmanın bulgularından yola çıkarak öncelikle matematik öğretmen adaylarının bile zihinsel döndürme problemleri çözümünde gerekli döndürmeyi zihninde canlandırmada zorlandıklarını, dolayısıyla Türkiye'de eğitim programları hazırlanırken uzamsal görselleştirme yeteneğinin

geliştirilmesinin üzerinde durulması gerektiği söylenebilir. Çalışmada da görüldüğü gibi zihinsel döndürme problemleri çözmenin tek yolu gerekli döndürmeyi zihinde canlandırmak değildir. Matematik öğretmenleri uzamsal yeteneği geliştirme etkinlikleri hazırlarken ve öğrencilerin uzamsal yeteneklerini ölçmek için araçlar hazırlarken bu durumu göz önünde bulundurmalıdır. Öğrenciler uzamsal yetenek problemleri çözerken farklı yöntemler kullanmaya yönlendirilebilir. Matematik öğretmenlerine bu konuda hizmet içi eğitimler ve seminerler düzenlenebilir. Ayrıca matematik öğretmeni yetiştiren eğitimcilerin de bu konuya önem vermeleri gereklidir.