

New soliton solutions in dual-core optical fibers

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Abstract: In this paper, we study a certain class of equations that model the propagating in dual-core fibers. Linear stability analysis is applied to discuss the existence of some types of travelling wave solutions and to compute the wave speed. New doubly periodic solutions are obtained, and new bright and dark soliton solutions are found.

Keywords: Dual-core fiber, dark-bright soliton, elliptic functions, Schrödinger equation.

1 Introduction

Recently, many authors have been working on the methods in constructing the exact solutions of nonlinear partial differential equations (NPDEs). In various applied branches of science, the researchers give special interest in developing the methods to obtain the exact solutions of NPDEs, specifically in terms of traveling solitary wave, soliton forms and Jacobi elliptic functions.

In nonlinear optics, lots of work have been devoted to the study of nonlinear Schrödinger equations (NLS). That is because of enormous number of potential applications [1,2]. A great deal of study focus on a generalized NLS with dual-power which is a generalization of the parabolic law nonlinearity, see [3,4,5,6]. Biswas and co-workers have obtained and studied the soliton solution of NLS type equations, with different nonlinearities [7,8,9,10]. In a recent work, there has been a considerable interest in adopting various methods to construct solutions of Schrödinger equation and systems. There has been a focus on extracting exact solutions in terms of Optical solitons; dark and bright solitons [11,12,13,14,15,16,17,18,19,20,21].

In this article we study the solitary wave in the dual-core fiber. The existence of solitary waves in dual core fiber was discussed in [22] and [23]. The equations that model the wave envelopes, ψ_1 and ψ_2 , which propagate through the dual-core fiber are [24,25]

$$i\left(\frac{\partial \psi_1}{\partial x} + \alpha_1 \frac{\partial \psi_2}{\partial t}\right) + \alpha_2 \frac{\partial^2 \psi_1}{\partial t^2} + \alpha_3 |\psi_1|^2 \psi_1 + \alpha_4 \psi_2 = 0, \quad (1)$$

$$i\left(\frac{\partial \psi_2}{\partial x} + \alpha_1 \frac{\partial \psi_1}{\partial t}\right) + \alpha_2 \frac{\partial^2 \psi_2}{\partial t^2} + \alpha_3 |\psi_2|^2 \psi_2 + \alpha_4 \psi_1 = 0, \quad (2)$$

where the $\alpha_1, \alpha_2, \alpha_3$ and α_4 are constants (for more details about the physical significance, see for example [1] [2]).

In [24], the exact solutions using traveling wave technique are obtained. In [26], soliton solutions are obtained by using

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G'/G -expansion scheme. In this paper different and new exact solutions are obtained. Dark and bright solitons are obtained with constraint. Also, linear stability analysis is used to obtain the travelling wave speed for non oscillatory soliton form solutions.

This article is organized as follows. Section 2 is devoted discussing the existence of travelling wave solutions using linear stability analysis by linearizing the proposed system near the zero steady state. In Section 3, we use the travelling wave method to construct new exact solutions of the system (1)-(2). Our work is concluded in Section 4.

2 Existence of Travelling waves

In this section, linear stability analysis is used to obtain the travelling wave speed for non oscillatory soliton form solutions. Here, in the traveling wave coordinates $(z, t) = (x - Ct, t)$, where C is the wave speed, we perturb the system (1) and (2) near its homogenous steady state, $\psi_1 = \psi_2 = 0$. This procedure enables us to study the behavior of the front solution whether it is monotonic or oscillatory according to the eigenvalues of the obtained characteristic equation. This linear stability analysis scheme is equivalent to the method of phase plane analysis. In the travelling wave coordinates $(z, t) = (x - Ct, t)$ the partial derivatives are

$$\begin{aligned}\frac{\partial}{\partial x} &= \frac{\partial}{\partial z}, \\ \frac{\partial}{\partial t} &= \frac{\partial}{\partial t} - C \frac{\partial}{\partial z}, \\ \frac{\partial^2}{\partial t^2} &= \frac{\partial^2}{\partial t^2} - 2C \frac{\partial^2}{\partial t \partial z} + C^2 \frac{\partial^2}{\partial z^2}.\end{aligned}\quad (3)$$

Now assume that the perturbations

$$\psi_1 = \phi_1(z, t) \quad \text{and} \quad \psi_2 = \phi_2(z, t), \quad (4)$$

then system (1) and (2) is linearized near its steady state in the coordinates $(z, t) = (x - Ct, t)$ and appears as

$$i\left(\frac{\partial \phi_1}{\partial z} + \alpha_1\left(\frac{\partial \phi_2}{\partial t} - C \frac{\partial \phi_2}{\partial z}\right) + \alpha_2\left(\frac{\partial^2 \phi_1}{\partial t^2} - 2C \frac{\partial^2 \phi_1}{\partial t \partial z} + C^2 \frac{\partial^2 \phi_1}{\partial z^2}\right) + \alpha_4 \phi_2\right) = 0, \quad (5)$$

$$i\left(\frac{\partial \phi_2}{\partial z} + \alpha_1\left(\frac{\partial \phi_1}{\partial t} - C \frac{\partial \phi_1}{\partial z}\right) + \alpha_2\left(\frac{\partial^2 \phi_2}{\partial t^2} - 2C \frac{\partial^2 \phi_2}{\partial t \partial z} + C^2 \frac{\partial^2 \phi_2}{\partial z^2}\right) + \alpha_4 \phi_1\right) = 0. \quad (6)$$

Now assume that the modulating traveling wave perturbation is in the Fourier form

$$\phi_1 = \phi_2 = \varepsilon e^{-i\lambda t} e^{\rho z}, \quad (7)$$

where $\varepsilon \ll 1$, λ is real (modulating frequency) and ρ is complex. This perturbation is periodic in time with periodic time $2\pi/\lambda$. Substitute the modulating Fourier ansatz (7) into either equations (5) or (6) to obtain the characteristic equation

$$\alpha_2 C^2 \rho^2 + i(1 - \alpha_1 C + 2\alpha_2 \lambda C) \rho + \alpha_4 + \alpha_1 \lambda - \alpha_2 \lambda^2 = 0. \quad (8)$$

When the eigenvalue ρ is purely real, the traveling wave solution reaches its zero steady state monotonically, and this happens if

$$\alpha_2 C^2 \rho^2 + \alpha_4 + \alpha_1 \lambda - \alpha_2 \lambda^2 = 0 \quad \text{and} \quad 1 - (\alpha_1 - 2\alpha_2 \lambda) C = 0. \quad (9)$$

From the above conditions displayed in (9), we argue that the system displayed in equations (1) and (2) supports a soliton-like solutions with a speed of propagation, C_p , given by

$$C_p = \frac{1}{(\alpha_1 - 2\alpha_2\lambda)}, \tag{10}$$

which can be considered as the transition speed from monotonic to oscillating fronts. These results motivates us to search for some of these types of solutions, and the next section is devoted to extracting new solutions.

3 The exact solution

In this section, we investigate the system displayed in equations (1) and (2) to construct new exact traveling wave solutions including bright and dark solitons and doubly periodic solutions in terms of Jacobi elliptic functions. We assume that the system admits the traveling wave solutions, $\psi_1(\xi, \theta)$ and $\psi_2(\xi)$, in the form [27,28]

$$\psi_1 = U(\xi)e^{i(lx-vt)}, \quad \psi_2 = V(\xi)e^{i(lx-vt)}, \quad \xi = kx - wt, \tag{11}$$

where k, w, l and v are real constants. Now, substitute (11) into equations (1) and (2) to obtain

$$\alpha_2 w^2 U'' + i(k + 2\alpha_2 vw)U' - i\alpha_1 wV' + \alpha_3 U^3 - (l + \alpha_2 v^2)U + (\alpha_4 + \alpha_1 v)V = 0, \tag{12}$$

$$\alpha_2 w^2 V'' + i(k + 2\alpha_2 vw)V' - i\alpha_1 wU' + \alpha_3 V^3 - (l + \alpha_2 v^2)V + (\alpha_4 + \alpha_1 v)U = 0, \tag{13}$$

where the prime corresponds to the differentiation with respect to ξ . We get the two equations (result from setting the imaginary parts in (12) and (13) to zero)

$$(k + 2\alpha_2 vw)U' - \alpha_1 wV' = 0, \quad (k + 2\alpha_2 vw)V' - \alpha_1 wU' = 0. \tag{14}$$

Integrate the system displayed in (14), then set the integration constants to zero (without loss of generality), to obtain

$$(k + 2\alpha_2 vw)U - \alpha_1 wV = 0, \quad \alpha_1 wU - (k + 2\alpha_2 vw)V = 0. \tag{15}$$

The system in (15) possesses a nontrivial envelope solution ($U \neq 0$ and $V \neq 0$)

$$V = U, \tag{16}$$

in a condition

$$k + 2\alpha_2 vw = \alpha_1 w. \tag{17}$$

From this condition the speed of the wave, $C = w/k$ can be written as

$$C = \frac{w}{k} = \frac{1}{\alpha_1 - 2v\alpha_2}, \tag{18}$$

which exactly the same computed speed C_p (with the time modulating frequency $v = \lambda$), deduced in the previous section and displayed in (10).

Now referring to equation (12) and equate the real part of the left hand side to zero to get

$$\alpha_2 w^2 U'' + \alpha_3 U^3 - (l + \alpha_2 v^2)U + (\alpha_4 + \alpha_1 v)V = 0, \tag{19}$$

and replacing V by U , from (16), results in

$$\alpha_2 w^2 U'' + \alpha_3 U^3 + [(\alpha_4 + \alpha_1 v) - (l + \alpha_2 v^2)]U = 0. \quad (20)$$

Multiply the obtained equation (20) by U' and integrate once to get a first order nonlinear ordinary differential equation in the form

$$U'^2 = \frac{2c_0}{\alpha_2 w^2} - \left(\frac{\alpha_4 + \alpha_1 v}{\alpha_2 w^2} - \frac{l + \alpha_2 v^2}{\alpha_2 w^2} \right) U^2 - \frac{\alpha_3}{2\alpha_2 w^2} U^4, \quad (21)$$

where c_0 is a constant of integration. Fortunately, equation (21) has many solutions in terms of Jacobi elliptic functions (see [29]). These new extracted solutions are listed in the following two subsections.

3.1 Doubly-periodic solutions

Equation (21) has doubly-periodic solutions in the following forms

- (1) Jacobi elliptic sine function, $sn\xi$,

$$U_1(\xi) = sn(\xi, M), \quad (22)$$

where M is the modulus of $sn\xi$ and (with $\alpha_2 \alpha_3 < 0$)

$$c_0 = \frac{\alpha_2 w^2}{2}, \quad M = \pm \frac{1}{w} \sqrt{\frac{-\alpha_3}{2\alpha_2}} \quad \text{and} \quad \alpha_3 = -2[(\alpha_4 + \alpha_1 v) - (l + \alpha_2 v^2) - \alpha_2 w^2]. \quad (23)$$

- (2) Jacobi elliptic cosine function $cn\xi$

$$U_2(\xi) = cn(\xi, M), \quad (24)$$

where M is the modulus of $cn\xi$ and (with $\alpha_2 \alpha_3 > 0$)

$$c_0 = \frac{1}{4}(2\alpha_2 w^2 - \alpha_3), \quad M = \pm \frac{1}{w} \sqrt{\frac{\alpha_3}{2\alpha_2}} \quad \text{and} \quad \alpha_3 = \alpha_2 w^2 + l + \alpha_2 v^2 - \alpha_4 - \alpha_1 v. \quad (25)$$

- (3) A third type elliptic function, $dn\xi$,

$$U_3(\xi) = dn(\xi, M), \quad (26)$$

where M is the modulus of $dn\xi$ and

$$c_0 = \frac{1}{2}\alpha_2 (M^2 - 1) w^2, \quad \alpha_3 = 2\alpha_2 w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v - 2\alpha_2 w^2. \quad (27)$$

- (4) Elliptic function $ns\xi$,

$$U_4(\xi) = ns(\xi, M) = \frac{1}{sn(\xi, M)}, \quad (28)$$

where M is the modulus of $ns\xi$ and

$$c_0 = \frac{1}{2}\alpha_2 M^2 w^2, \quad \alpha_3 = -2\alpha_2 w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v + \alpha_2 w^2. \quad (29)$$

- (5) Elliptic function $nc\xi$,

$$U_5(\xi) = nc(\xi, M) = \frac{1}{cn(\xi, M)}, \quad (30)$$

where M is the modulus of $nc\xi$ and

$$c_0 = -\frac{1}{2}\alpha_2 M^2 w^2, \quad \alpha_3 = 2\alpha_2 (M^2 - 1) w^2 \quad \text{and} \quad \alpha_4 = l - 2\alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v + \alpha_2 w^2. \quad (31)$$

(6) Elliptic function $nd\xi$,

$$U_6(\xi) = nd(\xi, M) = \frac{1}{dn(\xi, M)}, \quad (32)$$

where M is the modulus of $nd\xi$ and

$$c_0 = -\frac{1}{2}\alpha_2 w^2, \quad \alpha_3 = -2\alpha_2 (M^2 - 1) w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v - 2\alpha_2 w^2. \quad (33)$$

(7) Elliptic function $sc\xi$,

$$U_7(\xi) = sc(\xi, M) = \frac{sn(\xi, M)}{cn(\xi, M)}, \quad (34)$$

where M is the modulus of $sc\xi$ and

$$c_0 = \frac{\alpha_2 w^2}{2}, \quad \alpha_3 = 2\alpha_2 (M^2 - 1) w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v - 2\alpha_2 w^2. \quad (35)$$

(8) Elliptic function $sd\xi$,

$$U_8(\xi) = sd(\xi, M) = \frac{sn(\xi, M)}{dn(\xi, M)}, \quad (36)$$

where M is the modulus of $sd\xi$ and

$$c_0 = \frac{\alpha_2 w^2}{2}, \quad \alpha_3 = -2\alpha_2 M^2 (M^2 - 1) w^2 \quad \text{and} \quad \alpha_4 = l - 2\alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v + \alpha_2 w^2. \quad (37)$$

(9) Elliptic function $cs\xi$,

$$U_9(\xi) = cs(\xi, M) = \frac{cn(\xi, M)}{sn(\xi, M)}, \quad (38)$$

where M is the modulus of $cs\xi$ and

$$c_0 = -\frac{1}{2}\alpha_2 (M^2 - 1) w^2, \quad \alpha_3 = -2\alpha_2 w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v - 2\alpha_2 w^2. \quad (39)$$

(10) Elliptic function $cd\xi$,

$$U_{10}(\xi) = cd(\xi, M) = \frac{cn(\xi, M)}{dn(\xi, M)}, \quad (40)$$

where M is the modulus of $cd\xi$ and

$$c_0 = \frac{\alpha_2 w^2}{2}, \quad \alpha_3 = -2\alpha_2 M^2 w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v + \alpha_2 w^2. \quad (41)$$

(11) Elliptic function $ds\xi$,

$$U_{11}(\xi) = ds(\xi, M) = \frac{dn(\xi, M)}{sn(\xi, M)}, \quad (42)$$

where M is the modulus of $ds\xi$ and

$$c_0 = \frac{1}{2}\alpha_2 M^2 (M^2 - 1) w^2, \quad \alpha_3 = -2\alpha_2 (2M^2 - 1) w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 v^2 - \alpha_1 v - \alpha_2 w^2. \quad (43)$$

(12) Elliptic function $dc\xi$,

$$U_{12}(\xi) = dc(\xi, M) = \frac{dn(\xi, M)}{cn(\xi, M)}, \quad (44)$$

where M is the modulus of $dc\xi$ and

$$c_0 = \frac{1}{2}\alpha_2 M^2 w^2, \quad \alpha_3 = -2\alpha_2 w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 M^2 w^2 + \alpha_2 v^2 - \alpha_1 v + \alpha_2 w^2. \quad (45)$$

3.2 Soliton solutions

The Jacobi elliptic functions degenerate into hyperbolic functions, as the modulus tends to unity. That is as $M \rightarrow 1$

$$sn(\xi, M) \rightarrow \tanh(\xi), \quad cn(\xi, M) \rightarrow \operatorname{sech}(\xi), \quad \text{and} \quad dn(\xi, M) \rightarrow \operatorname{sech}(\xi). \quad (46)$$

We use this property to obtain the following bright and dark soliton solutions. From equations (22) and (23) and when $M = 1$, we get a solution in the form

$$U_{13}(\xi) = \tanh(\xi), \quad (47)$$

provided that

$$c_0 = \frac{\alpha_2 w^2}{2}, \quad \alpha_3 = -2\alpha_2 w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 v^2 + 2\alpha_2 w^2 - \alpha_1 v. \quad (48)$$

Also, when we substitute $M = 1$ into the obtained solution displayed in (22) and (23), we obtain the solution

$$U_{14}(\xi) = \operatorname{sech}(\xi), \quad (49)$$

along with the conditions

$$c_0 = 0, \quad \alpha_3 = 2\alpha_2 w^2 \quad \text{and} \quad \alpha_4 = l + \alpha_2 v^2 - \alpha_2 w^2 - \alpha_1 v. \quad (50)$$

4 conclusion

In summary, we studied a class of vector Schrödinger equations that models the wave propagation in dual-core fibers. A linear stability analysis mechanism is used to examine the existence of monotonic front solutions and to the speed of this type of waves. New exact solutions in the form of Jacobi elliptic functions are obtained using the traveling waves method with the help of known solutions to a specific class of nonlinear ordinary differential equation of first order. Special cases of the obtained solution are derived using the limiting property of the elliptic functions that results in new dark and bright soliton solutions of the studied system.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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