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ON PARA-SASAKIAN MANIFOLDS

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ABSTRACT

The object of the present paper is to study Para-Sasakian manifolds satisfying certain conditions on the curvature tensor.

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Key words: *Sasakian manifolds, Para-Sasakian manifolds, Weyl-pseudosymmetric manifolds.*

PARA-SASAKIAN MANİFOLDLAR ÜZERİNE

ÖZET

Bu çalışmanın amacı eğrilik tensörü üzerinde belirli şartları sağlayan Para-Sasakian manifoldları incelemektir.

Anahtar Kelimeler: *Sasakian manifoldlar, Para-Sasakian manifoldlar, Weyl-pseudosimetrik manifoldlar.*

1.Introduction

In ([1]), T. Adati and K. Matsumoto defined para-Sasakian and special para-Sasakian manifolds which are considered as special cases of an almost paracontact manifold introduced by I. Sato and K. Matsumoto ([10]). In the same paper, the authors studied conformally symmetric para-Sasakian manifolds and they proved that an n-dimensional (n>3) conformally symmetric para-Sasakian manifold is conformally flat and special para-Sasakian (n>3). In ([5]), U. C. De and N. Guha showed that an n-dimensional Weyl-semisymmetric para-Sasakian manifold is conformally flat.

Let (M,g) be an n-dimensional differentiable manifold of class C^{∞}. We denote by ∇ the Levi-Civita connection. We define endomorphisms $R(X,Y)$ and $X \wedge Y$ by

$$R(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z, \tag{1}$$

$$(X \wedge Y)Z = g(Y,Z)X - g(X,Z)Y, \tag{2}$$

respectively, where $X, Y, Z \in \chi(M)$ and $\chi(M)$ is being the Lie algebra of vector fields on M. The Riemannian-Christoffel tensor $`R$ is defined by $`R(X,Y,Z,W)=g(R(X,Y)Z,W), W \in \chi(M)$.

By the definition of the Weyl conformal curvature tensor C of n-dimensional (n>3) differentiable manifold (M,g) is given by

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\begin{bmatrix} g(Y,Z)QX - g(X,Z)QY \\ +S(Y,Z)X - S(X,Z)Y \end{bmatrix} \tag{3}$$

$$+\frac{\tau}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y],$$

where Q denotes Ricci operator, i.e. $S(X,Y)=g(QX,Y)$ and τ is the scalar curvature of M ([11]). The Weyl conformal curvature tensor C is defined by $C(X,Y,Z,W)=g(C(X,Y)Z,W)$. If $C=0$, then M is called *conformally flat*.

For a *(0,k)*-tensor field T, $k\geq 1$, on (M,g) we define $R\cdot T$, $C\cdot T$, and $Q(g,T)$ by

$$(R(X,Y)\cdot T)(X_1,...,X_k) = -T(R(X,Y)X_1,X_2,...,X_k) - T(X_1,R(X,Y)X_2,...,X_k)$$

$$-...-T(X_1,X_2,...,R(X,Y)X_k), \tag{4}$$

$$(C(X,Y)\cdot T)(X_1,...,X_k) = -T(C(X,Y)X_1,X_2,...,X_k) - T(X_1,C(X,Y)X_2,...,X_k)$$

$$-...-T(X_1,X_2,...,C(X,Y)X_k), \tag{5}$$

$$Q(g,T)(X_1,X_2,...,X_k) = -T((X\wedge Y)X_1,X_2,...,X_k) - T(X_1,(X\wedge Y)X_2,...,X_k)$$

$$-...-T(X_1,X_2,...,(X\wedge Y)X_k), \tag{6}$$

respectively ([7]).

If the tensor $R\cdot C$ (respectively $C\cdot R$) and $Q(g,C)$ are linearly dependent then M is called Weyl-pseudosymmetric. This is equivalent to

$$R\cdot C = L_C Q(g,C) \tag{7}$$

(respectively $C\cdot R = L_C Q(g,C)$), which holds on the set $U_C = \{x\in M : C\neq 0 \text{ at } x\}$ where L_C is some function on U_C. If $R\cdot C = 0$ then M is called Weyl-semisymmetric (see ([6]), ([7]), ([8])). If $\nabla C = 0$ then M is called conformally symmetric (see [4]). It is obvious that a conformally symmetric manifold is Weyl-semisymmetric.

Furthermore we define the tensors $R(\xi,X)\cdot C$ and $C(\xi,X)\cdot R$ on (M,g) by

$$(R(\xi,X)\cdot C)(Y,Z)W = R(\xi,X)C(Y,Z)W - C(R(\xi,X)Y,Z)W$$
$$-C(Y,R(\xi,X)Z)W - C(Y,Z)R(\xi,X)W, \tag{8}$$

$$(C(\xi,X)\cdot R)(Y,Z)W = C(\xi,X)R(Y,Z)W - R(C(\xi,X)Y,Z)W$$
$$-R(Y,C(\xi,X)Z)W - R(Y,Z)C(\xi,X)W. \tag{9}$$

In this study our aim is to obtain the characterization of P-Sasakian manifolds satisfying the conditions $R(\xi,X)\cdot C - C(\xi,X)\cdot R = 0$ and $R(\xi,X)\cdot C - C(\xi,X)\cdot R = L_C Q(g,C)$.

2.Preliminaries

Let M be an n-dimensional contact manifold with contact form η, i.e. $\eta \wedge (d\eta)^n \neq 0$. It is well known that a contact manifold admits a vector field ξ, called the characteristic vector field, such that $\eta(\xi)=1$ and $d\eta(\xi, X)=0$ for every $X\in\chi(M)$. Moreover, M admits a Riemannian metric g and a tensor field ϕ of type (1,1) such that

$$\phi^2 = -I + \eta \otimes \xi , \qquad g(X,\xi)=\eta(X) , \qquad g(X,\phi Y) = d\eta(X,Y) .$$

We then say that (ϕ,ξ,η,g) is a contact metric structure. A contact metric manifold is said to be a Sasakian if

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X ,$$

holds, where ∇ denotes the operator of covariant differentiation with respect of g ([3]). In this case, we have

$$\nabla_X \xi = -\phi X, \qquad R(X,Y)\xi = \eta(Y)X - \eta(X)Y .$$

Now we give a structure similar to Sasakian but not contact.

An n-dimensional differentiable manifold M is said to admit an almost paracontact Riemannian structure (ϕ,ξ,η,g), where ϕ is a (1,1)- tensor field, ξ is a vector field, η is a 1-form and g is a Riemannian metric on M such that

$$\phi\xi = 0 , \quad \eta\circ\phi = 0 , \quad \eta(\xi)=1 , \quad g(X,\xi)=\eta(X) ,$$

$$\phi^2 X = X - \eta(X)\xi , \quad g(\phi X,\phi Y) = g(X,Y) - \eta(X)\eta(Y) ,$$

for all vector fields X and Y on M. The equation $\eta(\xi)=1$ is equivalent to $|\eta| \equiv 1$, and then ξ is just metric dual of η, where g is the Riemannian metric on M. If (ϕ,ξ,η,g) satisfy the following equations

$$d\eta = 0 , \qquad \nabla_X \xi = \phi X,$$

$$(\nabla_X \phi)Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

then M is called a Para-Sasakian manifold or, briefly, a P-Sasakian manifold. Especially , a P-Sasakian manifold M is called a special para-Sasakian manifold or, briefly, a SP-Sasakian manifold if M admits a 1-form η satisfying

$$(\nabla_X \eta)(Y) = -g(X,Y) + \eta(X)\eta(Y).$$

In a P-Sasakian manifold the following relations hold:

$$S(X,\xi) = (1-n)\eta(X) , \tag{10}$$

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{11}$$

$$Q\xi = -(n-1)\xi, \tag{12}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{13}$$

$$R(\xi,X)Y = \eta(X)Y - g(X,Y)\xi, \tag{14}$$

for any vector fields $X, Y, Z \in \chi(M)$,(see ([2]),([9]) and ([10])).

A Para-Sasakian manifold M is said to be η-Einstein if its Ricci tensor S is of the form $S = ag + b\eta \otimes \eta$, where a, b are smooth functions on M ([2]).

3. Main results

In the present section our aim is to find the characterization of the P-Sasakian manifolds satisfying the conditions $R(\xi,X)\cdot C - C(\xi,X)\cdot R = 0$ and $R(\xi,X)\cdot C - C(\xi,X)\cdot R = L_C Q(g,C)$.

Theorem 1. Let M be an n-dimensional, n>3, P-Sasakian manifold. If the condition $R(\xi,X)\cdot C - C(\xi,X)\cdot R = 0$ holds on M then the manifold is an η-Einstein manifold.

Proof. Let M^n (n>3) a Para-Sasakian manifold. Then from (8) and (9) we have

$$\begin{aligned}(R(\xi,X)\cdot C)(Y,Z)W &= R(\xi,X)R(Y,Z)W - R(R(\xi,X)Y,Z)W \\ &\quad - R(Y,R(\xi,X)Z)W - R(Y,Z)R(\xi,X)W \\ &\quad - \frac{1}{n-2}\begin{bmatrix} S(R(\xi,X)Y,W)Z + g(R(\xi,X)Y,W)QZ \\ -S(R(\xi,X)Z,W)Y - g(R(\xi,X)Z,W)QY \\ -S(Z,R(\xi,X)W)Y + S(Y,R(\xi,X)W)Z \\ -g(Z,R(\xi,X)W)QY + g(Y,R(\xi,X)W)QZ \end{bmatrix} \\ &\quad + \frac{\tau}{(n-1)(n-2)}\begin{bmatrix} g(R(\xi,X)Y,W)Z - g(R(\xi,X)Z,W)Y \\ +g(Y,R(\xi,X)W)Z - g(Z,R(\xi,X)W)Y \end{bmatrix},\end{aligned} \tag{15}$$

and

$$\begin{aligned}(C(\xi,X)\cdot R)(Y,Z)W &= C(\xi,X)R(Y,Z)W - R(C(\xi,X)Y,Z)W \\ &\quad - R(Y,C(\xi,X)Z)W - R(Y,Z)C(\xi,X)W\end{aligned}$$

$$-\frac{1}{n-2}\begin{bmatrix}\mathrm{S(X,R(Y,Z)W)}\xi\text{-}\mathrm{S(}\xi\mathrm{,R(Y,Z)W)X}\\ \mathrm{+(1\text{-}n)g(X,R(Y,Z)W)}\xi\text{-}\eta\mathrm{(R(Y,Z)W)QX}\\ \mathrm{\text{-}S(X,Y)R(}\xi\mathrm{,Z)W+(1\text{-}n)}\eta\mathrm{(Y)R(X,Z)W}\\ \mathrm{\text{-}(1\text{-}n)g(X,Y)R(}\xi\mathrm{,Z)W+}\eta\mathrm{(Y)R(QX,Z)W}\\ \mathrm{+S(X,Z)R(}\xi\mathrm{,Y)W+(1\text{-}n)}\eta\mathrm{(Z)R(Y,X)W}\\ \mathrm{+(1\text{-}n)g(X,Z)R(}\xi\mathrm{,Y)W+}\eta\mathrm{(Z)R(Y,QX)W}\\ \mathrm{\text{-}S(X,W)R(Y,Z)}\xi\mathrm{+(1\text{-}n)}\eta\mathrm{(W)R(Y,Z)X}\\ \mathrm{\text{-}(1\text{-}n)g(X,W)R(Y,Z)}\xi\mathrm{+}\eta\mathrm{(W)R(Y,Z)QX}\end{bmatrix} \quad (16)$$

$$+\frac{\tau}{(n-1)(n-2)}\begin{bmatrix}\mathrm{g(X,R(Y,Z)W)}\xi\text{-}\mathrm{g(}\xi\mathrm{,R(Y,Z)W)X}\\ \mathrm{\text{-}g(X,Y)R(}\xi\mathrm{,Z)W+}\eta\mathrm{(Y)R(X,Z)W}\\ \mathrm{+g(X,Z)R(}\xi\mathrm{,Y)W+}\eta\mathrm{(Z)R(Y,X)W}\\ \mathrm{\text{-}g(X,W)R(Y,Z)}\xi\mathrm{+}\eta\mathrm{(W)R(Y,Z)X}\end{bmatrix}.$$

Multiplying equations (15) and (16) with ξ and using the condition $R(\xi,X)\cdot C-C(\xi,X)\cdot R=0$, we can write

$$-\frac{1}{n-2}\begin{bmatrix}\mathrm{S(R(}\xi\mathrm{,X)Y,W)}\eta\mathrm{(Z)+(1\text{-}n)g(R(}\xi\mathrm{,X)Y,W)}\eta\mathrm{(Z)}\\ \mathrm{\text{-}S(R(}\xi\mathrm{,X)Z,W)}\eta\mathrm{(Y)\text{-}(1\text{-}n)g(R(}\xi\mathrm{,X)Z,W)}\eta\mathrm{(Y)}\\ \mathrm{\text{-}S(Z,R(}\xi\mathrm{,X)W)}\eta\mathrm{(Y)+S(Y,R(}\xi\mathrm{,X)W)}\eta\mathrm{(Z)}\\ \mathrm{\text{-}(1\text{-}n)g(Z,R(}\xi\mathrm{,X)W)}\eta\mathrm{(Y)+(1\text{-}n)g(Y,R(}\xi\mathrm{,X)W)}\eta\mathrm{(Z)}\\ \mathrm{\text{-}S(X,R(Y,Z)W)\text{-}(1\text{-}n)g(X,R(Y,Z)W)}\\ \mathrm{+S(X,Y)g(Z,W)\text{-}(1\text{-}n)g(X,Y)g(Z,W)}\\ \mathrm{+S(X,Z)g(Y,W)+(1\text{-}n)g(X,Z)g(Y,W)}\end{bmatrix} \quad (17)$$

$$+\frac{\tau}{(n-1)(n-2)}\begin{bmatrix}\mathrm{g(R(}\xi\mathrm{,X)Y,W)}\eta\mathrm{(Z)\text{-}g(R(}\xi\mathrm{,X)Z,W)}\eta\mathrm{(Y)}\\ \mathrm{+g(Y,R(}\xi\mathrm{,X)W)}\eta\mathrm{(Z)\text{-}g(Z,R(}\xi\mathrm{,X)W)}\eta\mathrm{(Y)}\\ \mathrm{\text{-}g(X,R(Y,Z)W)\text{-}g(X,Y)g(Z,W)+g(X,Z)g(Y,W)}\end{bmatrix}=0.$$

Putting $Y=W=\xi$ in (17) and then using (10) and (11), we get

$$-\frac{1}{n-2}\left[\mathrm{5(1\text{-}n)g(X,Z)\text{-}8(1\text{-}n)}\eta(X)\eta(Y)+3S(X,Z)\right]$$

$$+\frac{4\tau}{(n-1)(n-2)}\left[\mathrm{g(X,Z)}\text{-}\eta(X)\eta(Z)\right]=0. \quad (18)$$

From equation (18), we obtain

$$S(X,Z)=\left(\frac{4\tau}{3(n-1)}-\frac{5}{3}(1-n)\right)g(X,Z)$$

$$+\left(\frac{8}{3}(1-n)-\frac{4\tau}{3(n-1)}\right)\eta(X)\eta(Z). \quad (19)$$

Thus *M* is an η-Einstein manifold.

Theorem 2. Let *M* be an *n*-dimensional, *n*>3, P-Sasakian manifold. If the condition $R(\xi,X)\cdot C-C(\xi,X)\cdot R=L_C Q(g,C)$ is satisfied on *M*, then *M* is either conformally flat, in which case *M* is a SP-Sasakian manifold, or $L_C=-1$ holds on *M*.

Proof. Assume that *M*, (*n*>3), is satisfying the condition $R(\xi,X)\cdot C-C(\xi,X)\cdot R=L_C Q(g,C)$. So we have

$$\begin{aligned}
&R(\xi,X)C(Y,Z)W-C(R(\xi,X)Y,Z)W-C(Y,R(\xi,X)Z)W\\
&-C(Y,Z)R(\xi,X)W-C(\xi,X)R(Y,Z)W+R(C(\xi,X)Y,Z)W\\
&+R(Y,C(\xi,X)Z)W+R(Y,Z)C(\xi,X)W\\
&=L_C\begin{bmatrix}(\xi\wedge X)C(Y,Z)W-C((\xi\wedge X)Y,Z)W\\ -C(Y,(\xi\wedge X)Z)W-C(Y,Z)(\xi\wedge X)W\end{bmatrix}.
\end{aligned}\tag{20}$$

Using (6) and multypling equation (20) with ξ, we have

$$\begin{aligned}
&\eta(R(\xi,X)C(Y,Z)W)-\eta(C(R(\xi,X)Y,Z)W)-\eta(C(Y,R(\xi,X)Z)W)\\
&-\eta(C(Y,Z)R(\xi,X)W)-\eta(C(\xi,X)R(Y,Z)W)+\eta(R(C(\xi,X)Y,Z)W)\\
&+\eta(R(Y,C(\xi,X)Z)W)+\eta(R(Y,Z)C(\xi,X)W)\\
&=L_C\begin{bmatrix}g(X,C(Y,Z)W)-\eta(C(Y,Z)W)\eta(X)\\ -g(X,Y)\eta(C(\xi,Z)W)+\eta(Y)\eta(C(X,Z)W)\\ -g(X,Z)\eta(C(Y,\xi)W)+\eta(Z)\eta(C(Y,X)W)\\ +\eta(W)\eta(C(Y,Z)X)\end{bmatrix}.
\end{aligned}\tag{21}$$

Using equations (10) and (11) in (21), we have

$$\begin{aligned}
&\eta(C(Y,Z)W)\eta(X)-g(X,C(Y,Z)W)-\eta(C(R(\xi,X)Y,Z)W)\\
&-\eta(C(Y,R(\xi,X)Z)W)-\eta(C(Y,Z)R(\xi,X)W)-\eta(C(\xi,X)R(Y,Z)W)\\
&+g(C(\xi,X)Y,W)\eta(Z)-g(Z,W)\eta(C(\xi,X)Y)+g(Y,W)\eta(C(\xi,X)Z)\\
&-g(C(\xi,X)Z,W)\eta(Y)+g(Y,C(\xi,X)W)\eta(Z)-g(Z,C(\xi,X)W)\eta(Y)\\
&=L_C\begin{bmatrix}g(X,C(Y,Z)W)-\eta(C(Y,Z)W)\eta(X)\\ -g(X,Y)\eta(C(\xi,Z)W)+\eta(Y)\eta(C(X,Z)W)\\ -g(X,Z)\eta(C(Y,\xi)W)+\eta(Z)\eta(C(Y,X)W)\\ +\eta(W)\eta(C(Y,Z)X)\end{bmatrix}.
\end{aligned}\tag{22}$$

Interchanging *X* and *Y* in (22), we obtain

$$\begin{aligned}&\eta(\mathrm{C(X,Z)W})\eta(\mathrm{Y})\text{-}\mathrm{g(Y,C(X,Z)W)}\text{-}\eta(\mathrm{C(R(}\xi,\mathrm{Y)X,Z)}\\&\text{-}\eta(\mathrm{C(X,R(}\xi,\mathrm{Y)Z)W})\text{-}\eta(\mathrm{C(X,Z)R(}\xi,\mathrm{Y)W})\text{-}\eta(\mathrm{C(}\xi,\mathrm{Y)R(X,Z)W})\\&+\mathrm{g(C(}\xi,\mathrm{Y)X,W)}\eta(\mathrm{Z})\text{-}\mathrm{g(Z,W)}\eta(\mathrm{C(}\xi,\mathrm{Y)X})+\mathrm{g(X,W)}\eta(\mathrm{C(}\xi,\mathrm{Y)Z})\\&\text{-}\mathrm{g(C(}\xi,\mathrm{Y)Z,W)}\eta(\mathrm{X})+\mathrm{g(X,C(}\xi,\mathrm{Y)W)}\eta(\mathrm{Z})\text{-}\mathrm{g(Z,C(}\xi,\mathrm{Y)W)}\eta(\mathrm{X})\\&=L_C\begin{bmatrix}\mathrm{g(Y,C(X,Z)W)}\text{-}\eta(\mathrm{C(X,Z)W})\eta(\mathrm{Y})\\\text{-}\mathrm{g(Y,X)}\eta(\mathrm{C(}\xi,\mathrm{Z)W})+\eta(\mathrm{X})\eta(\mathrm{C(Y,Z)W})\\\text{-}\mathrm{g(Y,Z)}\eta(\mathrm{C(X,}\xi\mathrm{)W})+\eta(\mathrm{Z})\eta(\mathrm{C(X,Y)W})\\+\eta(\mathrm{W})\eta(\mathrm{C(X,Z)Y})\end{bmatrix}.\end{aligned} \tag{23}$$

Substracting (23) from (22), we get

$$\begin{aligned}&\eta(\mathrm{C(Y,Z)W})\eta(\mathrm{X})\text{-}\eta(\mathrm{C(X,Z)W})\eta(\mathrm{Y})\text{-}\mathrm{g(X,C(Y,Z)W)}+\mathrm{g(Y,C(X,Z)W)}\\&+\eta(\mathrm{C(R(X,Y)}\xi,\mathrm{Z)W})\text{-}\eta(\mathrm{C(Y,R(}\xi,\mathrm{X)Z)W})+\eta(\mathrm{C(X,R(}\xi,\mathrm{Y)Z)W})\\&\text{-}\eta(\mathrm{C(Y,Z)R(}\xi,\mathrm{X)W})+\eta(\mathrm{C(X,Z)R(}\xi,\mathrm{Y)W})\text{-}\eta(\mathrm{C(}\xi,\mathrm{X)R(Y,Z)W})\\&+\eta(\mathrm{C(}\xi,\mathrm{Y)R(X,Z)W})+\mathrm{g(Y,W)}\eta(\mathrm{C(}\xi,\mathrm{X)Z})\text{-}\mathrm{g(X,W)}\eta(\mathrm{C(}\xi,\mathrm{Y)Z})\\&=L_C\begin{bmatrix}\mathrm{g(X,C(Y,Z)W)}\text{-}\mathrm{g(Y,C(X,Z)W)}+2\eta(\mathrm{C(X,Z)W})\eta(\mathrm{Y})\\\text{-}2\eta(\mathrm{C(Y,Z)W})\eta(\mathrm{X})\text{-}2\eta(\mathrm{Z})\eta(\mathrm{C(X,Y)W})+\mathrm{g(X,Z)}\eta(\mathrm{C(}\xi,\mathrm{Y)W})\\\text{-}\mathrm{g(Y,Z)}\eta(\mathrm{C(}\xi,\mathrm{Y)W})+\eta(\mathrm{W})\eta(\mathrm{C(Y,X)Z})\end{bmatrix}.\end{aligned} \tag{24}$$

Putting $Z=\xi$ in (24) we get

$$=[1+L_C]\begin{bmatrix}3\eta(\mathrm{C(X,}\xi\mathrm{)W})\eta(\mathrm{Y})\text{-}3\eta(\mathrm{C(Y,}\xi\mathrm{)W})\eta(\mathrm{X})+\mathrm{g(X,C(Y,}\xi\mathrm{)W)}\\\text{-}\mathrm{g(Y,C(X,}\xi\mathrm{)W)}\text{-}2\eta(\mathrm{C(X,Y)W})\end{bmatrix}=0. \tag{25}$$

So a contraction of (25) with respect to *X* gives us

$$[1+L_C][\eta(\mathrm{C(}\xi,\mathrm{Y)W})]=0. \tag{26}$$

If $L_C=0$ then *M* is Weyl-semisymmetric and so equation (26) is reduced to

$$\eta(\mathrm{C(}\xi,\mathrm{Y)W})=0, \tag{27}$$

which gives

$$\mathrm{S(Y,W)}=\left(\frac{\tau}{(n-1)}+1\right)g(Y,W)-\left(\frac{\tau}{(n-1)}+n\right)\eta(Y)\eta(W). \tag{28}$$

Therefore *M* is an η-Einstein manifold. So using (27) and (28) the equation (24) takes the form

$$\mathrm{C(Y,Z,W,X)}=0,$$

which means that *M* is conformally flat. So by ([2]), *M* is a SP-Sasakian manifold.

If $L_C\neq 0$ and $\eta(\mathrm{C(}\xi,\mathrm{Y)W})=0$, then $1+L_C=0$, which gives $L_C=-1$. This completes the proof of the our Theorem.

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