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THE HIGHER DIMENSIONAL CONFORMAL GEOMETRODYNAMICS AND THE RENORMALIZATION GROUP OF THE SPACETIME METRIC TRANSFORMATIONS

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ABSTRACT

It is shown that the higher dimensional Arnowitt-Deser-Misner (ADM) formalism has an appropriate structure for representing of the ten dimensional spacetime conformal geometrodynamics of the effective string field theory and for the explanation of the observed symmetries of fundamental interactions of physics in this theoretical framework. A class of generalized York transformations is found which give rise to the renormalization of the zeroth order gravitational sector of the effective string field theory.

Key words: *The higher dimensional ADM formalism, the generalized York transformations, the renormalization of the gravitational interactions.*

YÜKSEK BOYUTLU KONFORMAL GEOMETRODİNAMİK VE UZAYZAMAN METRİK DÖNÜŞÜMLERİNİN RENORMALİZASYON GRUBU

ÖZET

Yüksek boyutlu Arnowitt-Deser-Misner (ADM) formalizminin etkin sicim alan kuramının on boyutlu uzayzaman konformal geometrodinamiğinin sunulması için ve bu kuramsal çerçevede fiziğin temel etkileşmelerinin gözlenen simetrilerinin açıklanması için uygun bir yapısı olduğu gösterilir. Etkin sicim alan kuramının sıfırıncı mertebe kütleçekimi sektörünün renormalizasyonunu vermeyi sağlayan bir sınıf genelleşmiş York dönüşümlerinin bir sınıfı bulunur.

Anahtar Kelimeler: *Yüksek boyutlu ADM formalizmi, genelleşmiş York dönüşümleri, kütleçekimi etkileşmelerinin renormalizasyonu.*

1. INTRODUCTION

In according to a popular view, the superstring theory is the strongest candidate for unifying of the fundamental interactions of physics. The effective string field theory is a version of the string theory at the lower energy. The consistency of the popular effective string field theory which describes the very early universe needs a ten dimensional spacetime [3]. The zeroth order gravitational sector of the action of this theory in the Einstein frame is given by an Einstein-Hilbert term. The spacetime topology is determined by the ground state of the effective string field. The ground state manifold has the $M_4 \times C_6$ form, where M_4 denotes the actual 4-dimensional spacetime and C_6 denotes the 6-dimensional compact internal space. But, our universe is four dimensional and how the reason of the excess dimensions i.e. the internal dimensions, can be explained. An explanation is that the lengths of the internal dimensions are at the order of 10^{-33} cm (the Planck length) , for this reason, the internal dimensions are not observable. On the other hand, the lengths of our universe dimensions are at the order of 10^{28} cm. What is the source of this enormous difference? It is understood that there is a need for the consistent renormalization scheme for the gravity sector. For the consistent explanation of the mentioned enormous difference between the actual and the internal universe dimensions, the actual and the internal space sections of higher dimensional universe must be evolved disjointly. This process is realized by the renormalization scheme of this paper, i.e. the presentation of the generalized York transformations group of the spacetime metrics. Unfortunately, even for highly isotropic and homogeneous metrics (for example, Robertson-Walker metrics) for both of the actual and internal space, the field equations derived from the Hilbert-Palatini variation of the zeroth order action do not exhibit this behavior [7], [8]. Moreover, it is well known that the ADM formalism reduces the field equations to first order differential equations and hence this formalism is also used for the Hamiltonian and the conformal structure analysis of the theory [1]. For example, recently, the conformal canonical structure of the 3+1 dimensional spacetime ADM formalism has been investigated in detail [11]. Furthermore, it will be easily seen that the ADM formalism make more clear the symmetries of the action, especially in the higher dimensions [4]. For these reasons, in order to satisfy the renormalization of the gravitational sector of the underlying effective string field theory, a class of generalized York transformations is presented.

2. THE HIGHER DIMENSIONAL ADM FORMALISM

The higher dimensional ADM formalism is a procedure for reducing the $I = \int R\,(-G)^{1/2}\, d^{d+1} x$ Einstein-Hilbert action, to canonical form, where R is the scalar curvature, G is the determinant of the spacetime metric tensor $G_{\mathcal{AB}}$ [9]. In this work, the italic capital Latin letters in the indices such as *(A,B,C,…)* refers to the spacetime coordinates and the capital Latin letters in the indices such as (A,B,C,…) refers to the d-space coordinates. The index o denotes the time coordinate. The d+1 dimensional spacetime is splitted into the time and space constituents by means of the $n_{\mathcal{A}}$ vector which satisfies $n_{\mathcal{A}}\, n^{\mathcal{A}} = -1$. The covariant and the contravariant components of $n_{\mathcal{A}}$ are given by $\{n_o = -N,\ n_A = 0,\ n^o = 1/N,\ n^A = -N^A / N\}$, where N is the Lapse function and N^A is the shift vector [2], [7].

The higher dimensional spacetime metric has the form of $ds^2 = -dt^2 + G_{AB}\, dx^A\, dx^B$. From the field redefineable freedom of the theory, the substitutions of dt with $N\,dt$ and dx^A with $dx^A + N^A dt$ can be applied. As the result, the final metric has the form of
$ds^2 = -N^2 dt^2 + G_{AB}\,(dx^A + N^A dt)\,(dx^B + N^B dt)$. Hence the covariant and contravariant components of the G_{AB} metric tensor are $\{ G_{oo} = -N^2 + N^A N_A,\ G_{oA} = N_A,\ G_{AB} = g_{AB},\ G^{oo} = -1/N^2,\ G^{oA} = N^A/N^2,\ G^{AB} = g^{AB} - N^A N^B / N^2\}$ where g_{AB} is the d-space metric tensor and g^{AB} is the inverse of g_{AB}. The G_{oA} component represents spin 1 gauge fields. Therefore, the $n_A\, n^A = -1$ condition is one of the necessary conditions for the renormalization of the spin 1 gauge fields. The G_{AB} components represent spin 2 gravitons as the mediating particles of gravitational interactions. Thus, it is desirable to find the group of transformations of the spacetime metrics satisfies to obtain the metrics which has the unit determinant. The unit determinant condition of the metrics is one of the necessary conditions for the renormalization of the gravitational sector of the theory. It will be shown in this paper that these transformations are the generalized York transformations. The extrinsic curvature tensor or the second fundamental form of the d-space geometry K_{AB} is defined by $K_{AB} = [N_{A|B} + N_{B|A} - (\partial g_{AB} / \partial t)] / 2N$, where | denotes the d-space covariant derivative [5],[6],[7].

On the other hand, the extrinsic curvature tensor K_{AB} can also be expressed as $K_{AB} = [\mathcal{L}_N g + - (\partial g_{AB} / \partial t)] / 2N$, where $\mathcal{L}_N g = N_{A|B} + N_{B|A}$ is the Lie derivative of the g_{AB} d-space metric tensor with respect to the N_A vector field. When the N_A vectors are thought as the generators of the isometry group of the d-space, the $\mathcal{L}_N g$ Lie derivative vanishes. This means that the isometry group symmetries of the d-space give rise to the observed symmetries of the fundamental interactions of physics. For instance, from the antisymmetric $N_{A|B}$ quantities, the totally antisymmetric tensor $H_{ABC} = N_{A|B|C} + N_{B|C|A} + N_{C|A|B}$ can be constrıcted which has 84 degrees of freedom of the relevant string theory. (d! / (3! (d-3)! = 84 for d=9). Hence, the internal symmetries of the fundamental interactions of physics in our universe are the spacetime symmetries in higher dimensional universe [10].

In according to the ADM formalism, the zeroth order Einstein-Hilbert action is $I = \int [\pi^{AB} (\partial g_{AB} / \partial t) - N\, C^o - N_A\, C^A]\, d^{d+1} x$, where a total divergence term has been discarded. The Hilbert-Palatini variation is taken with respect to the π^{AB}, g_{AB}, N and N_A quantities, separately, where, $\pi^{AB} = g^{1/2} (g^{AB}\, \mathrm{Tr}\, K - K^{AB})$, $\mathrm{Tr}\, K = g_{AB} K^{AB}$, $C^o = g^{-1/2} [\, \mathrm{Tr}\, \pi^2 - (\mathrm{Tr}\, \pi)^2 /(d-1)] - g^{1/2}\ {}^{d}R$, $\mathrm{Tr}\, \pi^2 = \pi_{AB}\, \pi^{AB}$, $\mathrm{Tr}\, \pi = \pi_A{}^A = g_{AB}\, \pi^{AB}$, $C^A = -2\, \pi^{AB}{}_{|B}$, g is the determinant of the d-space metric g_{AB}, and ${}^{d}R$ is the d-space curvature scalar which is constructed by d-space metric tensor g_{AB} [7].

The K_{AB} is expressed in terms of the conjugate momentum π_{AB} that $K_{AB} = - g^{-1/2} (\pi_{AB} - g_{AB} \mathrm{Tr}\, \pi / (d-1))$, which is one of the necessary conditions for an extremum of I [7].

3. THE GENERALIZED YORK TRANSFORMATIONS

The generalized York transformations are defined in the $M_4 \times C_6$ manifold which has the dimension $N = d + 1 = 3 + 6 + 1 = 10$ as follows (The tranformed quantities are denoted by an overtilde) :
$\tilde{g}_{ab} = Ø\, h_{ab}$, $\tilde{g}_{\alpha\beta} = Ø\, h_{\alpha\beta}$, $h_{ab} = (g_3)^{-(N-2)\,/\,(N-3)\,(N-4)}\,(g_6)^{-1\,/\,(N-3)\,(N-4)}\, g_{ab}$,
$h_{\alpha\beta} = (g_3\, g_6)^{-(N-6)\,/\,(N-3)\,(N-4)}\, g_{\alpha\beta}$, $Ø = [g_3\,(g_6)^{-(N-5)\,/\,(N-8)}]^{(N-8)\,/\,(N-3)(N-4)(N-7)}$ where Ø is the conformal factor, g_{ab} is the 3-space metric tensor of the M_4 , $g_{\alpha\beta}$ is the 6-space metric tensor of the C_6 , g_3 is the determinant of the g_{ab} and g_6 is the determinant of the $g_{\alpha\beta}$. Obviously, the determinant of the transformed space metric $\tilde{g}_{AB}$ is unit.

In this section, the Gaussian normal coordinates has been used for all calculations. The metric in the Gaussian normal coordinates has the form $ds^2 = -d n^2 + g_{ab}\, dx^a\, dx^b + g_{\alpha\beta}\, dy^\alpha\, dy^\beta$ with the $K_{AB} = (-1/2)\, (\partial g_{AB} / \partial n)$, $d n = N\, d t$,where the x^a coordinates refer to the 3-actual space and y^α coordinates refer to the 6-internal space [7].

The generalized York transformations eliminate the $\mathrm{Tr}\, \pi_3$. $\mathrm{Tr}\, \pi_6$ terms from the Einstein-Hilbert ADM action I, where $\mathrm{Tr}\, \pi_3 = g^{ab}\, \pi_{ab}$, $\mathrm{Tr}\, \pi_6 = g^{\alpha\beta}\, \pi_{\alpha\beta}$. ($\tilde{K}_{AB} = (-1/2)\, (\partial h_{AB} / \partial n)$, $\tilde{K}^{AB}\, \tilde{K}_{AB} - \mathrm{Tr}\, \tilde{K}^2 = K_{ab}\, K^{ab} - (5 / 21)\, \mathrm{Tr}\, K_3{}^2 + K_{\alpha\beta}\, K^{\alpha\beta} - (11 / 42)\, \mathrm{Tr}\, K_6{}^2$, $\mathrm{Tr}\, K_3 = g_{ab}\, K^{ab}$,$\mathrm{Tr}\, K_6 = g_{\alpha\beta}\, K^{\alpha\beta}$ hence, there are no the $\mathrm{Tr}\, K_3$. $\mathrm{Tr}\, K_6$ type terms and $\tilde{g}_{AB}$ depends on h_{AB} conformally). Applying the generalized York transformations, the trace of the transformed conjugate momentum is vanished, namely, $\mathrm{Tr}\, \tilde{\pi} = \tilde{g}^{AB}\ \tilde{\pi}_{AB} = 0$, and hence the generalized York transformations group of the spacetime metrics satisfies to obtain the metrics which has the unit determinant.

As the final result, the disjointly evolution process of the actual and the internal space sections of higher dimensional universe, i.e. the renormalization of the gravity sector, is realized by the renormalization scheme of this paper, i.e. the presentation of the generalized York transformations group of the spacetime metrics.

These transformations has been expressed in such a manner that the substitutions of g_{ab} with $\psi\, g_{ab}$, and of $g_{\alpha\beta}$ with $\Phi\, g_{\alpha\beta}$ do not change the results, where ψ and Φ are arbitrary conformal factors.

4. CONCLUSION

The higher dimensional conformal geometrodynamics has been reviewed. The $M_4 \times C_6$ decomposition of the formalism has been explained. In order to renormalize the gravitational sector of the effective string field theory, as an alternative way, a class of generalized York transformations has been presented. The presentation of the new class of the generalized York transformations opens new horizons to recent research.

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