



An Investigation of the Questions Prepared by Pre-Service Middle School Mathematics Teachers on Fraction Concept¹

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ABSTRACT

Knowledge of assessment is considered as a dimension for professional knowledge of teachers (Magnusson, Krajcik, & Borko, 1999) since assessment and measurement knowledge of teachers is vital for quality issues in education. For this reason, the questions prepared by pre-service elementary mathematics teachers on fractions were investigated in the current study in order to observe pre-service elementary mathematics teachers' knowledge of assessment in mathematics education based on fraction concept. By investigating the questions prepared by pre-service teachers, the quality of questions and the competency of teachers for preparing appropriate questions based on particular objectives were evaluated. 34 pre-service elementary mathematics teachers (3rd grade) were participated to the study. In the study, case study design was employed and the data were analyzed by the use of qualitative data analyses techniques. The tasks were investigated under three categories of cognitive domain framework (Mullis & Martin, 2013), namely knowing, applying, and reasoning. Results showed that pre-service teachers prepared traditional tasks in general. That is, the questions included routine mathematical operations and procedures. Therefore, the developed questions were mostly classified into the knowing and applying categories. In general, routine mathematical tasks were preferred by pre-service teachers rather than non-routine mathematical ones which require reasoning level abilities.

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1. Introduction

Mathematics teachers have an important role for future generations. Therefore, teacher competencies are frequently studied in the literature. Teacher knowledge is one of the most crucial dimensions of teacher competencies (Elbaz, 1983). Consequently, knowledge of mathematics teachers is one of the frequently studied topics in mathematics education. Teaching mathematics is not just knowing school mathematics in front of the students but mastering on the fundamentals of school mathematics, changing the form of mathematics in such a way that students can learn better, interpreting students' questions and mathematical ideas, and explaining mathematical statements in different ways to the students (Hill, Sleep, Jewis, & Ball, 2007). Therefore, a deep knowledge of mathematics is required but not adequate for teaching mathematics effectively.

Mathematics teachers should have pedagogical content knowledge in addition to subject matter knowledge which distinguishes the content specialist from a pedagogue" (Ball, Thames & Phelps, 2008;

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Shulman, 1987). Teacher knowledge is generally investigated under two main topics, namely subject matter knowledge and pedagogical content knowledge (Ball et al., 2008). Subject matter knowledge for teaching mathematics is investigated under three main topics, namely common content knowledge, specialized mathematics knowledge, and horizontal content knowledge (Ball et al., 2008). Pedagogical content knowledge is separated into three categories, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum (Ball et al., 2008). Some researchers investigated measurement and assessment knowledge also under pedagogical content knowledge (Magnusson, Krajcik & Borko, 1999). For this reason, the effective use of measurement and assessment is frequently discussed in the literature (Dwyer, 1998). Research findings showed that attention of researchers has focused on problem solving skills of teachers in general; however, the studies working on problems prepared by the teachers are rare (Crespo, 2003). Ticha and Hospenova (2012) are also stated that writing mathematical problems get teachers be aware of which dimension of knowledge they need for teaching mathematics and get them meet the needs for their inadequate subject matter knowledge. Moreover, Toluk-Uçar (2009) also stated that mathematics knowledge and perception of pre-service teachers towards mathematics were affected positively by the help of writing problems. For this reason, writing mathematical questions has so much attention since it is considered that it provides deeper understanding on mathematical concepts. As a result, the research questions that guided this study were as follows:

- “How are the structure of mathematical problems which prepared by teachers on fraction concept based on the cognitive domain?”
- What are the difficulties and strengths of pre-service middle school mathematics teachers based on their mathematical questions on fraction concept in relation with particular learning objectives?

2. Methodology

The study was a case study within the scope of a non-experimental research design and the data was analyzed by the use of qualitative data analysis. Case study provides answers for “how” and “why” questions which cannot be found easily in an experimental study (Yin, 2013). Moreover, by the help of case study, researchers can get detailed information by investigating particular facts in their natural setting and also develop theories about the research topic (Yin, 2013). Case study design was preferred for the study in order to investigate the questions which prepared by a particular pre-service elementary mathematics teacher group in more detail.

2.1. Participants

The study is conducted with 34 pre-service elementary mathematics teachers (3th grade) in elementary mathematics education program of a governmental university in Western Black Sea Region in Turkey. The participants were selected to the study by the use of typical case sampling method of purposive sampling since we considered the participants as typical 3th grade pre-service elementary mathematics teachers. Since the study was conducted in the spring semester of 2015-2016 academic year, the participants had completed the measurement and assessment course and methods of teaching I and II courses in elementary mathematics education program. In measurement and assessment course, pre-service teachers were taught how to prepare, conduct, and assess measurement tools. In methods of teaching I and II courses, pre-service teachers learned how to teach mathematical concepts, got required methods, techniques, and materials for teaching particular mathematical concepts, and investigated the curriculum of middle school mathematics. Therefore, they were familiar with the mathematical objectives and different methods for teaching mathematics.

2.2. Data collection tools

In the study, pre-service teachers prepared mathematical questions related to particular objectives on fraction concept in middle school mathematics curriculum (MoNE, 2013) and they were given two hours for the development of questions (See Table 1 for the objectives).

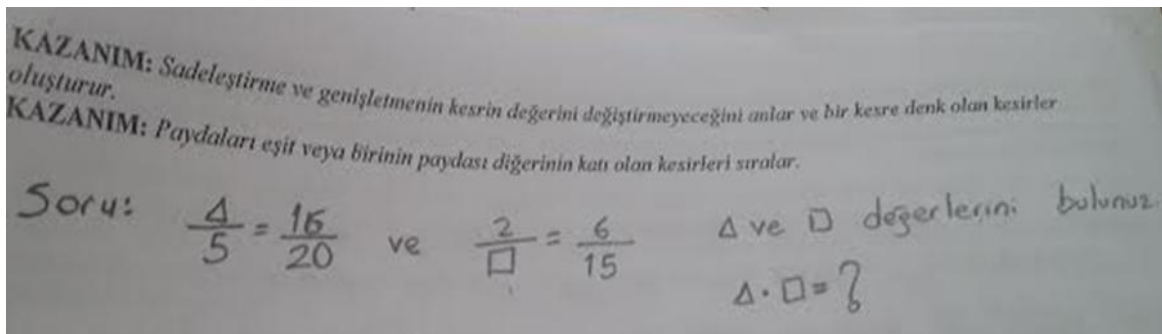
Table 1. Objectives related to fractions in middle school mathematics curriculum (MoNE, 2013)

Topic	Objectives
Simple fraction	Aligns the unit fractions. Represents the unit fractions on numerical axis.
Simple fraction Mixed fraction Improper fraction	Understands that mixed fraction is the summation of a natural number and a proper fraction and transforms a mixed fraction to an improper fraction and an improper fraction to mixed fraction.
Improper fraction	Makes comparison between a natural number and an improper fraction (Uses appropriate fraction models.).
Equivalent fraction	Understands that simplification and enlargement do not make any change on the value of a fraction and create fractions equivalent to a fraction. Align the fractions whose denominators are equivalent or whose denominator is a multiple of another.
Unit fraction Simple fraction	Calculates the multiplication of a simple fraction and a quantity and the whole of a quantity whose multiplication with a simple fraction is given by using unit fractions.
Alignment in fractions	Compares and aligns the fractions and represents them on numerical axis. [Hint: Students are encouraged to use the appropriate strategies while aligning the fractions. The strategies: the closeness to the whole, becoming smaller or greater than the half, the closeness to the half, comparison of unit fractions, making the denominators equal.]

The groups of objectives for the development of the questions were prepared by the collection of objectives related to proper fraction, improper fraction, mixed fraction, and equivalent fraction in middle school mathematics curriculum. Since some objectives were similar to each other based on knowledge and ability requirements, we made some of them groups of two objectives and required just one question for each of the group. Moreover, pre-service teachers did not be restricted based on the types of the questions (i.e. open ended, multiple choice, or true-false).

2.3. Data analysis

The questions developed by pre-service teachers were analyzed by the use of descriptive analysis based on TIMSS 2015 cognitive domain framework (Mullis & Martin, 2013). The framework was comprised of three categories, namely knowing, applying, and reasoning. Firstly, the knowing category includes basic mathematics concepts and their properties, basic mathematical operations, and using mathematical symbols correctly (see Figure 1).

**Figure 1.** A sample question from knowing category

The sample question in Figure 1 is categorized into knowing level since it requires basic mathematical operations, namely finding the values of unknowns given in the ratios and multiplication of them. The objectives given to the pre-service teachers were respectively “understands that simplification and enlargement do not make any change on the value of a fraction and create fractions equivalent to a fraction” and “align the fractions whose denominators are equivalent or whose denominator is a multiple of another” in Figure 1. Secondly, the applying category includes identification of appropriate solutions to routine problems, using those solutions effectively, and representation of mathematical statements. The question prepared by P₁₈ was from applying category since it requires a series of operations for solving a routine problem. A sample question from applying category was as follows:

P₁₈: Ayşe read a book, $\frac{1}{4}$ of it in the first day, $\frac{1}{3}$ of the remaining part in the second day, and $\frac{1}{8}$ of the total part which was read in the first and second days. Therefore, how many pages does Ayşe should read to complete the book now? (*Objective: Calculate the multiplication of a simple fraction and a quantity and the whole of a quantity whose multiplication with a simple fraction is given by using unit fractions.*)

Lastly, the reasoning category includes the facts which require logical reasoning and interpretations for non-routine problems. Moreover, the reasoning category comprises cognitively high level mathematical facts and justification of those facts. One of the questions from reasoning category was as follows:

P₁₈: In a birthday party, Ayşe ate $\frac{1}{4}$ of the pie, Meliha ate $\frac{1}{4}$ of the pie, and Ezgi ate $\frac{1}{4}$ of the pie. Therefore, who ate the pie at most in the birthday party? (*Objective: Align the unit fractions⁶ and represent the unit fractions on numerical axis.*)

In the question prepared by P₁₈, students were required to calculate the quantities stated in the question and make comparison by asking who ate the most among those quantities. Since, it was expected to do logical reasoning in the question written by P₁₈, the question was classified into the reasoning category.

As a result, the questions prepared by pre-service teachers were investigated based on the categories of cognitive domain framework (Mullis & Martin, 2013). The questions were analyzed whether they were mostly related with the conceptual meaning of mathematical operations or just the ability of doing mathematical operations. At the end of the analyses, the results were represented as frequencies and percentages. In addition, deeper analyses were done on the questions in order to investigate the ability of pre-service teachers for preparing questions based on the levels of cognitive domain in more detail.

3. Findings

Within the scope of the study, the questions prepared by the pre-service teachers were analyzed based on the framework of cognitive domain and in which level the questions were most was analyzed.

3.1. Analyses of the questions written by pre-service teachers about unit fraction

The questions written by pre-service teachers for the objective “Aligns the unit fractions and shows them on the numerical axis” were investigated and the findings were given in Table 2.

Table 2. Analysis of the questions written by pre-service teachers about unit fraction

Objective	Knowing		Applying		Reasoning		Invalid	
	f	%	f	%	f	%	f	%
Aligns the unit fractions and shows them on the numerical axis.	1	2.9	5	73.5	2	8	6	17.4

According to Table 2, it was seen that most of the pre-service teachers (%73.5) wrote questions in the applying level. Moreover, some of the pre-service teachers (%17.4) wrote invalid questions for the objective. Besides, it was observed that some pre-service teachers had deficient knowledge about unit fractions. It was realized that they could not distinguish unit fractions and proper fractions. Moreover, most of the pre-service teachers asked the students to show the given unit fractions on the numerical axis in the questions they wrote but they did not emphasize that each unit fraction should be shown with a separate numerical axis. Similarly, some pre-service teachers asked the students to draw a numerical axis and show more than one unit fractions on this numerical axis.

The sample questions given by pre-service teachers P₁₅ and P₁₇ to the objective “Aligns the unit fractions.” and “Represents the unit fractions on numerical axis.” were identified as invalid. P₁₅ used only one unit fraction in the question; therefore, it might be inferred that P₁₅ did not have enough knowledge about unit fraction.

P₁₅: Align $\frac{1}{4}, \frac{6}{3}, -7, -\frac{2}{3}$ and $\frac{4}{3}$ and show them on the numerical axis.

Similarly, P17 identified such points on the same numerical axis as if there were unit fractions although they were not unit fractions and asked students to write numerical form of those fractions based on their positions on the numerical axis and align them in a descending order.

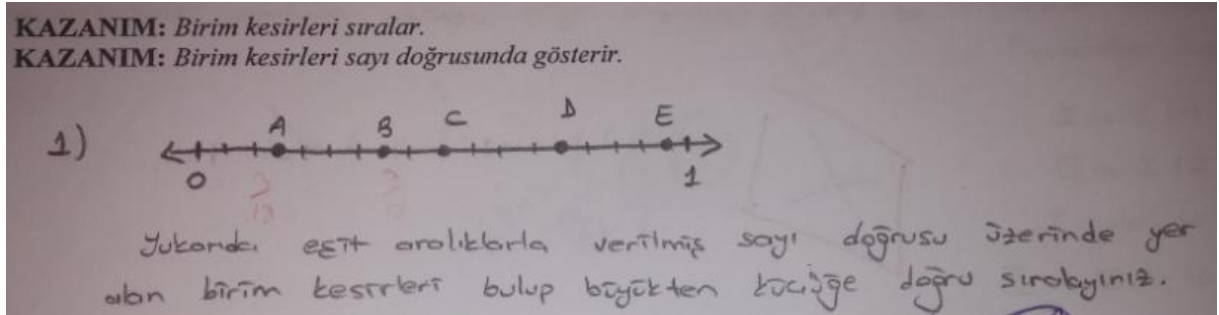


Figure 2. Sample question given by P17

In the question given by P34 for the same objectives, the unit fractions were given to the students within a problem situation and the students were expected to make logical reasoning and solve the problem. Therefore, the question was coded in the interpretation level.

P₃₄: Ayşe and Ahmet went to a pastry shop and both of them ordered cakes in the same size. Ayşe divided her cake into 3 equal slices and ate one slice. Ahmet divided his cake into 4 equal slices and ate one slice. Which one ate more cake? Show them on the numerical axis and compare them.

3.2. Analyses of the questions written by pre-service teachers about proper fractions and mixed fractions

The analyses of the questions written by pre-service teachers for the objective "Understands that a mixed fraction is the sum of a natural number and a unit fraction and converts a mixed fraction into an improper fraction and an improper fraction into a mixed fraction" are given in Table 3.

Table 3. Analysis of the questions written by the pre-service teachers about proper fractions and mixed fractions

Objective	Knowing		Applying		Reasoning		Invalid	
	f	%	f	%	f	%	f	%
Understands that a mixed fraction is the sum of a natural number and converts a mixed fraction into an improper fraction and an improper fraction into a mixed fraction.	26	76.4	3	8.8	0	0	5	14.7

As Table 3 revealed, it was seen that pre-service teachers wrote questions appropriate for the knowing level most (76.4 %). However, the problems written by 5 pre-service teachers were invalid. One of the questions categorized into knowing level was given by P9 was represented in Figure 3. The question was an example of a knowing level question for this objective. Since the question requires translation of a visual expression to numerical symbols and making routine mathematical operations, it was categorized into the knowing level.

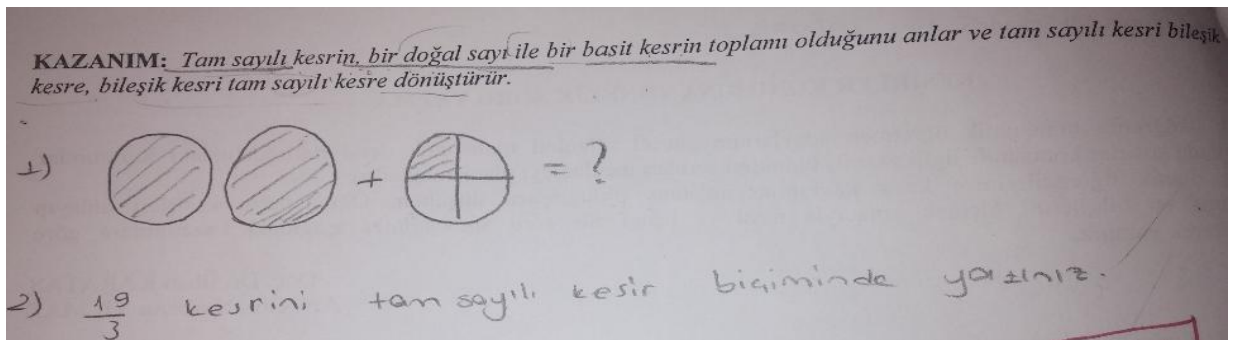


Figure 3. Sample question given by P9

Another question given by P₁₀ was categorized into the applying level for the same objective since it included the given objective by the use of a problem situation.

P₁₀: Ali's mother told him that there was a half loaf of bread at home but it would not be enough. Ali bought 2 loaves of bread from the grocer's and they had breakfast together. Since there was no bread left after the breakfast, how many loaves of bread did Ali and his mother eat? Convert the fraction you find into a mixed fraction and write it.

3.3. Analyses of the questions written by pre-service teachers about improper fractions

The findings obtained from the analyses of the questions written by pre-service teachers for the objective "Compares a natural number with an improper fraction" are given in Table 4.

Table 4. Analysis of the questions written by the pre-service teachers about improper fractions

Objective	Knowing		Applying		Reasoning		Invalid	
	f	%	f	%	f	%	f	%
Compares a natural number to an improper fraction	6	17.6	20	58.8	0	0	8	23.5

According to Table 4, most of the pre-service teachers (58.8 %) gave questions in applying level. However, the questions written by 8 pre-service teachers were evaluated as invalid questions. One of the questions given for this objective categorized into the applying level was developed by P₃₀. Since modeling is required for the solution of this question, it was categorized into the applying level.

T₃₀: Compare 2 and $\frac{3}{2}$. Show which one occupies more place by modeling the fractions.

One of the questions identified as invalid was given by P7. A proper fraction was given in the question instead of an improper fraction. Moreover, this question did not measure the skill required for the objective.

T₇: Ayşe ate a whole cake. Ali ate $\frac{3}{5}$ of the cake. Therefore, how much cake did Ali eat less than Ayşe? Show it by modeling.

3.4. Analyses of the questions written by pre-service teachers about equivalent fractions

The results obtained by the analyses of the questions for the equivalent fractions are given in Table 5. As demonstrated in Table 5, most of the questions formed by the pre-service teachers (70.6 %) were in the knowing level.

Table 5. Analysis of the questions written by the pre-service teachers about equivalent fractions

Objective	Knowing		Applying		Reasoning		Invalid	
	f	%	f	%	f	%	f	%
Understand that simplification and enlargement do not make any change on the value of a fraction and create fractions equivalent to a fraction. Align the fractions whose denominators are equivalent or whose denominator is a multiple of another.	24	70.6	4	11.7	0	0	6	17.7

The percentage of the questions identified as invalid was 17.7 %. One of the questions in the knowing level was given by P₆. Since the question sample given by P₆ was only related with one of the objectives, it was coded as deficient (See Figure 1).

P₆: Find the values of the \diamond and \square in the expressions of $\frac{\diamond}{5} = \frac{16}{20}$ and $\frac{2}{\square} = \frac{6}{15}$.

Since the question given by P₇ was not suitable for the given objective, it was identified as invalid.

T₇: Ali's father put sticks around the garden and he painted half of these sticks. Later, Ali's father, who thought that the sticks were not enough, put 20 sticks more. How many sticks does he have to paint?

3.5. Analyses of the questions written by pre-service teachers about unit fractions and proper fractions

The findings for the objective "Calculate the multiplication of a simple fraction and a quantity and the whole of a quantity whose multiplication with a simple fraction is given by using unit fractions." are given in Table 6.

Table 6. Analysis of the questions written by the pre-service teachers about unit fractions and proper fractions

Objective	Knowing		Applying		Reasoning		Invalid	
	f	%	f	%	f	%	f	%
Calculate the multiplication of a simple fraction and a quantity and the whole of a quantity whose multiplication with a simple fraction is given by using unit fractions.	19	55.9	9	26.5	0	0	6	7.6

As presented in Table 6, while most of the pre-service teachers (55.9 %) prepare questions in the knowing level, 26.5 % of them prepared applying level questions. Analyses of the questions for this objective showed that some of the pre-service teachers used improper or mixed fractions instead of proper fractions. One of the questions from knowing level was given by P₃₄. Since the question related to the objective required only doing basic mathematical operations on fractions, it was classified into knowing level.

P₃₄: How much is the whole of the number whose $\frac{1}{2}$ is 24?

Since the question given by T₁₉ was a problem which required routine mathematical operations, it was evaluated in the applying level.

T₁₉: Ayşe bought 3 kilograms of apples from the market. She took $\frac{3}{8}$ of the apples to her friends at school. How many apples did Ayşe take to school? (There are 8 apples in 1 kg)

3.6. Analyses of the questions written by pre-service teachers about aligning the fractions

The results of the analyses of the questions for the objective "Compare and align the fractions and represent them on numerical axis." are given in Table 7. The participants were given an explanation in addition to the objective which was related with the use of appropriate strategies such as making relation with the whole, becoming smaller or bigger than the half, making relation with the half, comparison of unit fractions, and making the denominators of fractions equal (highlighting equivalent fractions).

Table 7. Analyses of the questions written by the pre-service teachers about aligning the fractions

Objective	Knowing		Applying		Reasoning		Invalid	
	f	%	f	%	f	%	f	%
Compares and aligns the fractions and shows them on the numerical axis.	1	2.9	9	85.3	1	2.9	3	8.8

As presented in Table 7, most of the pre-service teachers (85.3 %) were in applying level. Most of the questions were in applying level because of the skills they contain. For example, the question given by P₁₀ required showing the fractions given in the models on the numerical axis, it was evaluated in applying level (See Figure 4).

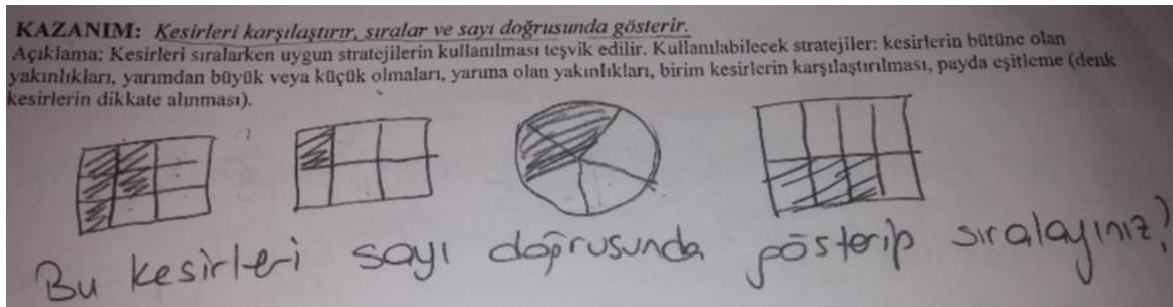


Figure 4. Question given by P₁₀

The question given by pre-service teacher P₆ was evaluated in reasoning level. In this question, there was a problem situation and the students were asked to make a logical reasoning.

T₆: Ali, Mehmet and Hasan decide to start a journey. The place they will travel is Muğla and the way is 100 km long. Ali goes $\frac{3}{10}$ of the way and turns back. Mehmet goes $\frac{2}{5}$ of the way and he goes out of gas on the way and he cannot go there. Hasan goes $\frac{1}{3}$ of the way and decides to have a break at a station. Who have the longest and who have the shortest journey?

4. Discussion and Results

The purpose of this study was to analyze the questions written by pre-service elementary mathematics teachers on fraction concept in consideration of the cognitive domain framework. Based on the results, pre-service teachers produced questions in the knowing and applying levels mostly. They included routine mathematical procedures, operations and problems frequently on the fraction concept. Questions which are not routine and requiring logical reasoning and interpretation were rarely developed pre-service teachers. An, Kulm and Wu (2010) analyzed the students' learning under two domains: knowledge memorized and not linked to each other and knowledge interiorized and linked to each other. Therefore, it might be said that the questions written by pre-service teachers included mostly the ability of doing mathematical operations since they were rarely asking the conceptual meaning of mathematical concepts and operations.

Moreover, it was observed that pre-service teachers had some misconceptions. For example, it was observed that some of the pre-service teachers did not have an adequate knowledge about the concept of unit fraction. It was found that pre-service teachers produced questions including proper fractions or improper fractions although they were asked to write questions including just unit fractions. This result coincides with the study of Ball (1991) which states that pre-service teachers have a lack of knowledge about the concept of fractions. Moreover, the questions written by pre-service teachers had erroneous parts in consideration of the representation of unit fractions on the numerical axis in contrast to the representation of other fractions. Therefore, we may infer that pre-service teachers might have misconceptions at this point.

Bright, Behr, Post and Wachsmuth (1988) expressed that the numerical axis model was different than the region model and set model since all numbers aligned consecutively on the numerical axis and each part of the wholes can be shown at the same time on numerical axis. Although there were gaps between the wholes in other models, there were no gaps between the wholes on numerical axis. Moreover, although the use of symbols is not important in other models, it is important to use symbols in numerical axis since they have a numerical meaning. Therefore, the reason of the erroneous representation of pre-service teachers related to the representation of unit fractions on the numerical axis might be related with their unawareness about the fact that more than one unit fractions with different denominator values cannot be shown on the same numerical axis simultaneously.

Pesen (2008) investigated how students represented the fractions on the numerical axis and observed that students had difficulties in perceiving the expression a/b as a unique number and they considered

that the numbers a and b were different numbers in the study conducted with the primary school students. Moreover, the researcher stated that students had difficulties about dividing a whole number into pieces and substituting a proper fraction between the points 0 and 1 on the numerical axis. These misconceptions observed by Pesen (2008) were not observed in the questions given by participants of the current study. The errors of pre-service teachers were mostly related with the substitution of unit fractions on the numerical axis. For this reason, it might be inferred that similar misconceptions may be observed on the students in the future since the knowledge of the students might be affected by their teachers. For this reason, the deficiencies about the representation of the unit fractions on the numerical axis might be eliminated by emphasizing the points mentioned by Bright et al. (1988) above. To sum up, it is shown that some of the pre-service teachers were inadequate in writing questions for the given objectives. Also, some of the questions were not appropriate for the given objectives. To illustrate, a limited number of the questions given for the objective "Compares a natural number with an improper fraction" were appropriate for that objective. Lastly, it might be concluded that pre-service teachers generally prefer to develop questions at knowing and applying levels.

5. Conclusion

Results showed that pre-service teachers prepared questions especially for the use of routine mathematical operations. Although there was not a restriction related to the structure of the questions based on cognitive domain framework, the questions requiring logical reasoning and interpretations were rarely observed. Moreover, it was seen that pre-service teachers had misconceptions on the concept of fractions, types of fractions, and representation of fractions by the use of models. Furthermore, it was concluded that pre-service teachers generally had a traditional approach towards understanding an objective. Also, some of them were inadequate in preparing questions appropriate for the given objective. In summary, the findings of the current study may provide useful information and clues for the content of methods of teaching mathematics courses given for pre-service elementary mathematics teachers on fraction concept. Graeber, Tirosh and Glover (1989) remarked that teachers had a crucial role for the identification and revision of misconceptions of students. As the questions developed by pre-service elementary mathematics teachers revealed, they have some misconceptions on fraction concept. Therefore, further diagnostic and remedial studies on the knowledge of pre-service and in-service teachers on fraction concept might be helpful to overcome this problem.

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