RESPONSE OF A TWO-LAYER GROUND TO AN INFINITE CABLE CARRYING ALTERNATING CURRENT WITH REFERENCE TO TURAM ANOMALIES

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ABSTRACT. — A formula is developed for the vertical magnetic field due to an alternating current passing through a long horizontal cable placed on a two-layer ground. The variations of the phase and amplitude ratio of the vertical field, along profiles perpendicular to the cable line are investigated. It is found that if the upper layer is conductive within the limits encountered in practice, as compared to the lower layer, the phase may vary appreciably whereas the amplitude ratio changes as in the case of vacuum.

It is known that with some types of electromagnetic prospecting, as in the case of the Turam method, some quantities such as phase and amplitude ratio of the magnetic field due to an energizing alternating current passing through a horizontal cable grounded at both ends are employed. It is usually assumed that the ground on which the cable is situated is uniform with electrical parameters not differing from those of vacuum and the normal variation of the measured quantities are calculated accordingly. If, on the other hand, the area to be prospected consists of different layers with the searched discontinuity in, then the normal variation of the phase and amplitude ratio will be different from those in the case of uniform medium or vacuum. The order of magnitude of these variations should therefore be known with a view to interpreting the measured quantities.

It is our experience that, with most prospecting areas in Turkey, the ground consists essentially of two layers from the point of view of electromagnetic parameters. The first one is a thin conductive layer of soil or young sediments with resistivities varying between 1 and 30 ohm-m, and the second layer with much higher resistivities and larger thicknesses.

We will, therefore, derive the formula for the vertical magnetic field of an infinite straight wire carrying a current (I exp. iwt), placed at a height h parallel to the ground consisting of 2 layers. To this end, we will first obtain the vertical magnetic field of a dipole (I exp. iwt, ds), then integrate the result along the wire, obtaining the field due to the current in the cable.

Let us suppose that the centre of the dipole is situated at $z=h; y=o: x=o$ parallel to the x axis.

Subscripts 0, 1 and 2 are used for constants of the air, the upper and the lower layers respectivelly, while d denotes the depth of the upper layer. The field is symmetrical with . respect to the $x—z$ plane, but not with respect to the $x—y$

plane. For these reasons, it is necessary, in order to satisfy boundary conditions, to assume a Hertzian wave function with 2 components Π_x and Π_z .

The resultant wave functions in the three media are taken as

$$\Pi_o = \Pi_{ox},\ \Pi_{oz};\quad \Pi_1 = \Pi_{1x},\ \Pi_{1z};\quad \Pi_2 = \Pi_{2x},\ \Pi_{2z}$$

Boundary conditions at $z = o$ and at $z = -d$ consist in the continuity of the tangential *(x, y)* components of E and H, as follows : (See E.M. Theory, Stratton, 1941)

at $z = o$

$$\gamma_o^2 \Pi_{oz} = \gamma_2^1 \Pi_{1z} \quad \ldots\ldots \quad (1)$$

$$\gamma_o^2 \frac{\partial \Pi_{ox}}{\partial z} = \gamma_1^2 \frac{\partial \Pi_{1x}}{\partial z} \quad \ldots\ldots \quad (2)$$

$$\frac{\partial \Pi_{ox}}{\partial x} + \frac{\partial \Pi_{oz}}{\partial z} = \frac{\partial \Pi_{1x}}{\partial x} + \frac{\partial \Pi_{1z}}{\partial z} \quad \ldots\ldots \quad (3)$$

$$\gamma_o^2 \Pi_{ox} = \gamma_1^2 \Pi_{1x} \quad \ldots\ldots \quad (4)$$

at $z = -d$

$$\gamma_1^2 \Pi_{1z} = \gamma_2^2 \Pi_{2z} \quad \ldots\ldots \quad (1')$$

$$\gamma_1^2 \frac{\partial \Pi_{1x}}{\partial z} = \gamma_2^2 \frac{\partial \Pi_{2x}}{\partial z} \quad \ldots\ldots \quad (2')$$

$$\frac{\partial \Pi_{1x}}{\partial x} + \frac{\partial \Pi_{1z}}{\partial z} = \frac{\partial \Pi_{2x}}{\partial x} + \frac{\partial \Pi_{2z}}{\partial z} \quad \ldots\ldots \quad (3')$$

$$\gamma_1^2 \Pi_{1x} = \gamma_2^2 \Pi_{2x} \quad \ldots\ldots \quad (4')$$

where

$$\gamma^2 = i \omega \mu (\sigma + i \omega k)$$

From the two pairs of equations 2, 2′ and 4, 4′, the x-components may be determined separately and then used in finding the z-components.

The general solution for the Π function is

$$\Pi = \begin{pmatrix} A_\phi \cos n\phi \\ + \\ B_\phi \sin n\phi \end{pmatrix} \int_o \begin{pmatrix} p_1 (v) e^{-\beta z} \\ + \\ q_1 (v) e^{\beta z} \end{pmatrix} \begin{pmatrix} p_2 (v) J_n (rv) \\ + \\ q_2 (v) Y_n (rv) \end{pmatrix} dv \quad \ldots\ldots \quad (5)$$

For $r = o$, $z \neq o$ the field must remain finite; and the field must be symmetrical with opposite directions with respect to x—z plane. These conditions are satisfied when $Y_n (rv)$ and $\sin n\phi$ terms are omitted from (5), so that the solution for the Π function becomes

$$\Pi = \cos n\phi \int_o^\infty \left[p (v) e^{\beta z} + q (v) e^{-\beta z} \right] J_n (rv)\, dv \quad \ldots\ldots \quad (6)$$

where n is a numeric, $\cos \phi = \frac{x}{r}$, $\beta = (v^2 + \gamma^2)^{1/2}$

For the x — components (6) applies with $n = o$, viz.,

$$\Pi_{ox} = \int_o^\infty \left(p_o e^{\beta_o z} + q_o e^{-\beta_o z} \right) J_o(rv)\, dv \qquad o \leq z < h \quad \text{.......... (7)}$$

$$\Pi_{1x} = \int_o^\infty \left(p_1 e^{\beta_1 z} + q_1 e^{-\beta_1 z} \right) J_o(rv)\, dv \qquad -d \leq z < o \quad \text{.......... (8)}$$

$$\Pi_{2x} = \int_o^\infty p_2 e^{\beta_2 z} \cdot J_o(rv)\, dv \qquad z \leq -d \quad \text{..................... (9)}$$

The arbitrary functions are then determined from 2, 2′ and 4, 4′ :

$$\gamma_o^2 \beta_o (p_o - q_o) = \gamma_1^2 \beta_1 (p_1 - q_1)$$

$$\gamma_o^2 (p_o + q_o) = \gamma_1^2 (p_1 + q_1)$$

$$\gamma_1^2 \beta_1 \left(p_1 e^{-\beta_1 d} + q_1 e^{\beta_1 d} \right) = \gamma_2^2 \beta_2 p_2 e^{-\beta^2 d}$$

$$\gamma_1^2 \left(p_1 e^{-\beta_1 d} + q_1 e^{\beta_1 d} \right) = \gamma_2^2 p_2 e^{-\beta_2 d}$$

the solution of which is,

$$p_1 = p_o^2 \cdot \frac{\gamma_o^2 \cdot 2 \beta_o (\beta_1 + \beta_2)}{\gamma_1^2 \cdot T}$$

$$q_1 = p_o \cdot \frac{\gamma_o^2 \cdot 2 \beta_o (\beta_1 - \beta_2)}{\gamma_1^2 \cdot T} e^{-2 d \beta_1}$$

$$T = (\beta_o + \beta_1)(\beta_1 + \beta_2) + (\beta_o - \beta_1)(\beta_1 - \beta_2) e^{-2d \beta_1}$$

where $\beta_i^2 = v^2 + \gamma_i^2$

The function p_o may now be determined by noting that with $\gamma_1 = \gamma_2 = \gamma_o$, the solution must be the same as for a uniform medium of infinite extent in all directions. In this case $q_1 = o$ and $p_1 = p_o$. To obtain the same field at $z = -h$ as from

$$\Pi = \frac{i \omega \mu I\, ds}{4 \pi \gamma_o^2} \int_o^\infty \frac{v}{\beta_o} \cdot e^{-\beta_o z} \cdot J_o(rv)\, dv$$

where $\beta_o^2 = v^2 + \gamma_o^2$, $\gamma_o = i \omega (\mu_o \cdot k_o)^{1/2}$

which is the Hertzian function of the dipole in the air, p_o must then equal

$$I.\ ds.\ e^{-\beta_o h} \cdot i\,\omega\, v / 4\,\pi\, \gamma_o^2\, \beta_o$$

therefore

$$p_1 = I\ .\ ds.\ \frac{i\,\omega\,\mu}{4\,\pi}\ .\ \frac{2\,v\,(\beta_1 + \beta_2)}{\gamma_1^2\ .\ T}\ .\ e^{-\beta_o h} \quad \text{.....................} \quad (10)$$

$$q_1 = I\ .\ ds\ .\ \frac{i\,\omega\,\mu}{4\,\pi}\ .\ \frac{2\,v\,(\beta_1 - \beta_2)}{\gamma_1^2\ .\ T}\ .\ e^{-2d\,\beta_1}\ .\ e^{-\beta_o h} \quad \text{...............} \quad (11)$$

II_{1x} may now be obtained from (8) and in turn II_{ox} and II_{2x} by use of (4), (4').

The boundary conditions for the z-components may be satisfied by taking $n = 1$ in (6) so that,

$$II_{oz} = \cos\phi \int_o^\infty l_o\, e^{-\beta_o z}\ .\ J_1\,(rv)\,dv \quad \text{............................} \quad (12)$$

$$II_{1z} = \cos\phi \int_o^\infty \left(l_1\, e^{\beta_1 z} + w_1\, e^{-\beta_1 z} \right) J_1\,(rv)\,dv \quad \text{..................} \quad (13)$$

$$II_{2z} = \cos\phi \int_o^\infty l_2\, e^{\beta_2 z}\ .\ J_1\,(rv)\,dv \quad \text{...............................} \quad (14)$$

From the boundary conditions (1), (1') and (3), (3'), together with the expressions for the x-components as already determined, the following solutions are obtained :

$$l_1 = \frac{(\gamma_o^2 - \gamma_1^2)\ \ T_1'\ \ (p_1 + q_1) - (\gamma_1^2 - \gamma_2^2)\ T_o''\left(p_1\ e^{-2d\,\beta_1} + q_1\right)}{T_o'\ \ T_1' + T_o''\ T_1''\ e^{-2\,d\,\beta_1}}\ v$$

$$w_1 = \frac{(\gamma_o^2 - \gamma_1^2)\ \ T_1''\ (p_1 + q_1)\ \ e^{-2\,d\,\beta_1} + (\gamma_1^2 - \gamma_2^2)\ T_o'\ \ (p_1\, e^{-2\,d\,\beta_1} + q)}{T_o'\ \ T_1' + T_o''\ \ T_1''\ e^{-2\,d\,\beta_1}}\ \ v$$

where

$$T_o' = \beta_o\,\gamma_1^2 + \beta_1\,\gamma_o^2 \qquad T_o'' = \beta_o\,\gamma_1^2 - \beta_1\,\gamma_o^2$$

$$T_1' = \beta_1\,\gamma_2^2 + \beta_2\,\gamma_1^2 \qquad T_1'' = \beta_1\,\gamma_2^2 - \beta_2\,\gamma_1^2$$

Now, since $H_z = -\frac{\gamma^2}{i\omega\mu} \cdot \frac{\partial \Pi_z}{\partial y}$

at $z=o$, the vertical magnetic force is from (8),

$$Hz = -\frac{\gamma^2}{i\omega\mu} \cdot \frac{\partial}{\partial y} \int_o^{\infty} (p_1 + q_1)\, J_o\,(rv)\, dv \qquad (15)$$

From (10) and (11),

$$p_1 = p_1' \cdot ds = p_1' \cdot dx$$
$$q_1 = q_1' \cdot ds = q_1' \cdot dx$$

where p_1' and q_1' are the remaining cofactors in (10) and (11).

The total vertical magnetic force, can be obtained by integrating (15) along the x direction :

$$Hz\,(\text{total}) = -\frac{\gamma^2}{i\omega\mu} \cdot \int_{-\infty}^{+\infty} \left[\frac{\partial}{\partial y} \cdot \int_o^{\infty} (p_1' + q_1')\, J_o\,(rv)\, dv \right] dx$$

since $\int_o^{\infty} J_o\,(rv) \cdot dx = \frac{\cos yv}{v}$

$$Hz\,(\text{total}) = -\frac{2\gamma^2}{i\omega\mu} \cdot \frac{\partial}{\partial y} \int_o^{\infty} (p_1' + q_1') \frac{\cos yv}{v}\, dv \qquad (16)$$

which can be evaluated numerically.

Case 1. Uniform ground

In the special case of uniform ground

$$\gamma_1 = \gamma_2 = \gamma$$
$$\beta_1 = \beta_2 = \beta$$

After making necessary alteration in formula (16), we can obtain,

$$Hz = -\frac{I}{\pi} \cdot \frac{\partial}{\partial y} \int_o^{\infty} \frac{\cos yv}{\beta_o + \beta}\, dv$$

$$Hz = -\frac{I}{\pi} \cdot \frac{\partial}{\partial y} \left\{ \frac{1}{(\gamma^2 - \gamma_o^2)\, y^2} \left[\gamma_o y\, K_1\,(\gamma_o y) - \gamma y\, K_1\,(\gamma y) \right] \right\}$$

where $K_1\,(\gamma_o y)$ is the second kind modified Bessel function of first order with complex argument.

When the displacement currents are neglected in the air, $\gamma_o = 0$, then,

$$Hz = -\frac{I}{\pi} \cdot \frac{\partial}{\partial y}\left\{\frac{1}{\gamma^2 y^2}\left[1 - \gamma y\, K_1(\gamma y)\right]\right\}$$

For $\gamma y < 0.25$, this expression can be expanded to an approximation as follows :

$$Hz \simeq -\frac{I}{\pi} \cdot \frac{\partial}{\partial y}\left[\frac{1}{2}\log\frac{2}{e\gamma y} + \frac{1}{1} - \text{Euler's Const.}\right]$$

$$Hz \simeq -\frac{I}{2\pi} \cdot \frac{1}{y}$$

From the last formula, it can be seen that, for practical purposes, the phase angle does not vary with distance from the cable. The field ratios are also the same as in the case of vacuum.

Case 2. Two-layer ground with a thin upper layer of high conductivity

If we suppose that the earth consists of a thin upper layer of low resistivity underlain by a layer of infinite resistivity, then,

$$\gamma_o = 0 \qquad \gamma_2 = 0$$

$$\beta_o = v \qquad \beta_2 = v$$

Since $exp \cdot -2\beta_1 d \simeq 1 - 2\beta_1 d$

$$Hz = -\frac{I}{\pi} \cdot \frac{\partial}{\partial y}\int_0^{\infty} \frac{\cos vy}{2v + \gamma_1^2 d} \cdot dv$$

$$Hz = -\frac{I}{\pi} \cdot \frac{\partial}{\partial y}\left[e^{\eta y}\, Ei\,(\eta y) + e^{-\eta y}\, Ei\,(-\eta y)\right]$$

where $\eta = \dfrac{\omega \mu \sigma_1 d}{2}$

and Ei is the exponential integral function.

When $\eta y < 0.25$

$$Hz \simeq -\frac{I}{2\pi} \cdot \frac{\partial}{\partial y}\left[-i\frac{\pi}{2}e^{-\eta y} + \log\frac{1}{\eta y} - \text{Euler's Const.}\right]$$

$$Hz \simeq -\frac{I}{2\pi}\left[i\frac{\pi}{2}\,\eta\, e^{-\eta y} - \frac{1}{y}\right]$$

The phase ϕ of the vertical field Hz and the field ratios are respectively,

$$\phi = -Arctg\frac{\pi}{2} \cdot \eta y \cdot e^{-\eta y} \qquad (17)$$

$$\frac{Hz_1}{Hz_2} = \frac{\left[\left(\frac{\pi}{2} \cdot \eta\, e^{-\eta y_1}\right)^2 + \frac{1}{y_1^2}\right]^{1/2}}{\left[\left(\frac{\pi}{2} \cdot \eta\, e^{-\eta y_2}\right)^2 + \frac{1}{y_2^2}\right]^{1/2}} \qquad (18)$$

Phase. — To find out about the magnitudes of the phase change with distance from the cable, phase angles were calculated assuming that the first layer has a resistivity of 5 ohm-m, a thickness of 20 m and magnetic permeability of 1 c.g.s. Using a frequency of 660 C.P.S., following values were obtained :

y	ηy	ϕ
36 m	0.03	— 2.33 degrees
60 m	0.05	— 4.17 »
120 m	0.10	— 7.58 »
180 m	0.15	— 11.19 »
240 m	0.20	— 14.35 »

As shown in the above table, the phase angle increases regularly with distance and its presence can be detected from Turam measurements; but with a conductive layer of varying thickness, this variation can be quite irregular with phase amplitudes comparable to those caused by the searched conductive body. The phase effect of the hidden body lying in or under a conductive layer can therefore be masked to a large extent.

Amplitude ratio- — The amplitude ratio in formula (18) can be simplified to an approximation by expanding the exponential term, discarding the higher than second order terms and using the binomial theorem discarding again the higher than second order terms. After carrying out these steps, the following approximation was obtained :

$$\frac{Hz_1}{Hz_2} = \frac{\dfrac{1}{y_1}}{\dfrac{1}{y_2}} \cdot \frac{1 + \dfrac{\pi}{4}\,\eta y_1}{1 + \dfrac{\pi}{4}\,\eta y_2}$$

The second cofactor on the right side of this equation, represents the percentage change from the case of uniform ground. Its effect is negligible for most of the surface layer conductivities encountered in practice.

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BIBLIOGRAPHY

JAHNKE, E. and EMDE, R. (1944) : Tables of Functions. Dover.

STRATTON, J.A. (1941,) : Electromagnetic Theory. Mc. Graw-Hill.