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ON DEVELOPING OF A THERMOELASTIC CONTINUUM DAMAGE MODEL FOR DIELECTRIC COMPOSITE MATERIALS

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Abstract

This paper deals with developing a continuum damage mechanics model belonging to constitutive equations which represent linear electro-thermo-elastic behavior of a composite material, where the material was reinforced with arbitrarily distributed single fiber family and which have micro-cracks. The composite medium is assumed to be dielectric, incompressible, homogeneous, and dependent on temperature gradient. The matrix material made of elastic material involving an artificial anisotropy because of fibers reinforcing by arbitrary distributions and the existence of micro-cracks, has been assumed as an isotropic medium. It is accepted that the fiber family is inextensible. Using the basic laws, of continuum damage mechanics and continuum electrodynamics and the equations belonging to kinematic of fiber, the constitutive functionals have been obtained. It has been detected as a result of the thermodynamic constraints that stress potential function depends on two symmetric tensors and two vectors, and the heat flux vector function depends on two symmetric tensors and three vectors. To determine arguments of the constitutive functionals, findings relating to the theory of invariants have been used as a method because of that isotropy constraint is imposed on the matrix material. Finally, the constitutive equations of symmetric stress, polarization field, asymmetric stress, heat flux vector and strain-energy density release rate have been written in material coordinates.

Key Words: Electro-thermoelastic behavior, Continuum damage mechanics, Constitutive equations, Composite materials, Invariants.

DİELEKTRİK KOMPOZİT MALZEMELER İÇİN BİR TERMOELASTİK SÜREKLİ ORTAM HASAR MODELİNİN GELİŞTİRİLMESİ ÜZERİNE

Özet

Bu makale, keyfi dağılımlı tek fiber ailesi ile takviyeli ve mikro çatlaklara sahip bir kompozit malzemenin lineer elektro-termo elastik davranışını temsil eden kurucu denklemlere ait bir sürekli ortam hasar mekaniği modeli geliştirmeyi ele almaktadır. Kompozit ortamın dielektrik, sıkıştırılamaz, homojen olduğu ve sıcaklık gradyanına bağlı olduğu varsayılmaktadır. Keyfi dağılımlı fiber takviyesi ve mikro çatlakların varlığı nedeniyle yapay bir anizotropi içeren elastik malzemeden yapılmış matris malzemesi izotropik bir ortam olarak kabul edilmiştir. Fiber ailesinin uzatılmaz olduğu kabul edilmektedir. Sürekli ortam hasar mekaniğinin ve sürekli ortam elektrodinamiğinin temel kanunları ve süreklilik fiber kinematiğine ait denklemleri kullanılarak bünye fonksiyonelleri elde edilmiştir. Termodinamik kısıtlamaların sonucu olarak, gerilme potansiyeli fonksiyonunun iki simetrik tensör ve iki vektöre bağlı olduğu ve ısı akısı vektör fonksiyonunun ise iki simetrik tensör ve üç vektöre bağlı olduğu belirlenmiştir. Bünye fonksiyonellerinin argümanlarını belirlemek için, invaryantlar teorisine ilişkin bulgular, matris malzemesine uygulanan izotropi kısıtlaması nedeniyle bir yöntem olarak kullanılmıştır. Sonunda, simetrik gerilmenin, polarizasyon alanının, asimetrik gerilmenin, ısı akısı vektörünün ve gerinme-enerjisi yoğunluğunun değişim hızının bünye denklemleri maddesel koordinat sisteminde yazılmıştır.

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1. Introduction

Composite materials are now widely used in the aircraft and motor sport industries, due to their high strength to weight ratio. A common design problem in the aerospace industry is the development of a damage tolerance and damage resistance design to all forms of impacts, from low velocity, such as may occur from tool drops to high velocity impacts from runway debris or in the ballistic regime, micrometeorites. Unlike ductile metals, which can absorb large amounts of energy via plasticity without significant loss of strength, brittle composites absorb energy by elastic deformation and irreversible damage. Hence in practice, many composite structures are overengineered to compensate for their low damage tolerance. The full potential of composite materials remains unused [1].

Damage progression in composite materials and structures is, in general, very complicated and involves multiple failure modes, such as fiber breakage, fiber pullout, delamination between plies, matrix cracking, fiber–matrix debonding, etc [2]. Modeling the damage process accurately poses a very difficult problem because these mechanisms clearly operate at various length scales [3]. However, if more rational design and damage tolerance approaches are to be developed for composite structures, it becomes necessary to develop engineering tools that will enable analysts to model damage and its propagation [2].

Temperature changes frequently represent a significant factor of failure of composite structures subjected to severe environmental loads. The thermal stresses accompanying the uniform, unsteady heating cause thermal fatigue and considerable plastic strains leading to complete or progressive destruction of the composite structures. Further, the repeated action of thermal stresses in some fiberreinforced composite structures leads to debonding of layers, longitudinal cracks, and a thermal buckling in composite thin-walled members [4].

A dielectric is an insulating material that exhibits polarization in the presence of an electric field. Electromechanical behaviors of interest include piezoelectric, pyroelectric, and ferroelectric effects. Piezoelectricity, in a general sense, refers to the coupling between electric field or polarization and stress or deformation. In continuum theories, piezoelectricity of first order is attributed to the particular choice of free energy functional for the body that may depend, for example, on the product of the strain and the polarization [5]. Transversely isotropic piezoelectric materials are widely used in transducers and smart structures. The study of thermoelastic problems has always been an important branch in solid mechanics [6, 7]. In particular, the thermoelastic fracture problems subjected to various types of thermal boundary conditions have been discussed extensively in the literature [8, 9, 10, 11, 12, 13].

Our early studies have provided a basis for the conduction of this study. In our previous study, viscoelastic composites of a single fiber family have been studied assuming that the material has a discontinuity surface [14]. In our study, it has been assumed that a viscoelastic composite material with two different inextensible fiber families does not have a discontinuity surface [15]. Again, in our studies, the exposure of a viscoelastic composite material to the effect of electrical and magnetic fields in addition to its reinforcement by a single fiber family has been researched in the form of separate studies [16, 17]. In our study [18], we have examined the electromechanical behavior of a dielectric viscoelastic composite material with two fiber families. Since temperature was assumed to be constant in all of the these studies, temperature change has not been taken into consideration. Furthermore, in our study [19], we have investigated theoretically electro-thermomechanical behavior of a thermoelastic dielectric body subject to external loading in his study. In study titled [20], the linear thermoelastic behavior of a composite material reinforced by two independent and inextensible fiber families has been analyzed theoretically by Usal. In this study [20], the composite material is assumed to be anisotropic, compressible, dependent on temperature gradient, and showing linear elastic behavior. In our study [21], we have developed constitutive equations for the thermoelastic analysis of composites consisting of an isotropic matrix reinforced by independent and inextensible two families of fibers having an arbitrary distribution. In our study [21], it is assumed that an element from two different continuous fiber families is placed on each point of the composite material. In our study [21], the mechanical interaction and temperature change have been assumed to be linear. In our study [22], a relevant mathematical model is developed in the context of continuum damage mechanics. This mathematical model represents mechanic behavior of an elastic media which have micro-voids and which is subjected to a mechanical loading [22]. Our paper is concerned with developing the continuum damage mechanics model for elastic behavior of composites having micro-cracks consisting of an isotropic matrix reinforced by independent and inextensible two families of arbitrarily fibers [23]. Furthermore, in study titled [24], Usal has developed a continuum damage model for the linear viscoelastic behavior of composites with micro-cracks consisting of an isotropic matrix reinforced by two arbitrarily independent and inextensible fiber families.

In this paper, a systematic approach from balance laws and thermodynamics to constitutive theory is presented for an attempt to make a synthesis in a unified and systematic fashion for the linear electro-thermoelastic behavior of a thermoelastic-dielectric composite having micro-cracks reinforced by arbitrary and inextensible single family of fiber. In this paper, mechanical interactions and effect of damage have been assumed to be linear while electrical interactions have been assumed to be non-linear. Furthermore, since the matrix material has to remain insensitive to directional changes along fiber, even number vector components of fiber vector have been included in the operations. It is assumed that mechanical interactions will be considered linear, from this point of the constitutive relationship is limited to a linear response. In addition to, in the present paper, constitutive equations have been obtained that determine the electro-thermomechanical behavior of a thermoelastic-dielectric composite having micro-cracks reinforced by arbitrary and inextensible single family of fiber. Such constitutive equations relate to polarization, stress, strain-energy density release rate and heat distribution. Since temperature is no longer constant in this study, the temperature gradient has been incorporated into calculations as an independent constitutive variable.

On the other hand, inextensibility of the fiber and incompressibility of the composite are acceptable as broad recognition in practice in terms of formulation. Thus, the composite is assumed to be incompressible and fiber family-inextensible. Due to some technological requirements, it is aspired that specific construction elements have rather elastic properties, provided that they have high durability in certain directions. Fiber-reinforced composite materials are produced sticking fibers in a polymeric matrix which is elastic but with low strength. These fibers are manufactured from high strength grafit or bor. They can be easily bent due to the very small size of their cross-section and it can be assumed that these fibers show a continuous distribution in a medium. Assuming inextensibility of the fibers is a reasonable approach since the rigidity of the fibers is very high according to the rigidity of the matrix [25].

2. Mechanical Representation of Damage

In some researches, in order to be able to define the damage variable, a representative volume element (RVE) has been considered that has a k number of micro cracks. While the open or active part of any k^{th} micro crack has been shown by $A^{(k)}$, its closed or passive surface has been shown by $A^{*(k)}$. Active or passive surfaces of a crack can switch positions among each other depending on stress, temperature and humidity percentage. Despite that, Weitsman states that these open and closed surfaces can be selected as independent variables characterizing the state of a material at a certain time range [26, 27].

Stress and strain at the macro level are average values over the RVE volume. Infinitesimal deformations can also be considered among these macro values. To fully consider the behaviors of RVE it is necessary to deal with a K number of crack parameters representing $\mathbf{A}^{(k)}$ and $\mathbf{A}^{*(k)}$, (no sum on *k*, *k*=1,...,*K*) surfaces. Because the real shape of these surfaces is unknown on the meso scale, assuming them to be equivalent plane surfaces, Weitsman represented them by vectors $\mathbf{A}^{(k)} = A^{(k)}\mathbf{n}^{(k)}$ and $\mathbf{A}^{*(k)} = A^{*(k)}\mathbf{n}^{(k)}$. Here, $\mathbf{n}^{(k)}$ stands for a unit normal vector of a micro crack surface [27]. Let us consider two micro cracks inside a material with different convexities around a material point. Depending on the load applied on the material the cracks having different convexities can demonstrate different types of behavior depending on their crack surfaces. Different infinitesimal crack surfaces with very big curvature radii can be accepted topologically and mechanically equivalent. In this case topologic representation of the crack surface can be expressed independent of the direction of that surface. Mathematically that representation can be shown by using the symmetric tensor, which is a dyadic product of two vectors. Thus, any micro crack can be defined using symmetric dyads as follows.

$$\mathbf{H}^{(k)} = \mathbf{A}^{(k)} \otimes \mathbf{A}^{(k)} \quad \text{and} \quad \mathbf{H}^{*(k)} = \mathbf{A}^{*(k)} \otimes \mathbf{A}^{*(k)} \quad , \quad H_{ij}^{(k)} = A_i^{(k)} A_j^{(k)} \tag{1}$$

Because detailed information about the value and location of surfaces $\mathbf{A}^{(k)}$ and $\mathbf{A}^{*(k)}$ can only be found statistically on the micro scale, on the meso scale where Continuum Mechanics is used, we can show the combined effects of the tensor expressions stated in (1) by the sum of the dyadic products given below. This operation represents homogenization when moving from the micro to the meso scale.

$$\mathbf{H} = \sum_{k=1}^{K} \mathbf{A}^{(k)} \otimes \mathbf{A}^{(k)} \quad \text{and} \quad \mathbf{H}^{*} = \sum_{k=1}^{K} \mathbf{A}^{*(k)} \otimes \mathbf{A}^{*(k)} \tag{2}$$

Thus, the effect of damage on the meso scale can be expressed with two interior conditional variables, the variables bearing second degree symmetric tensor characteristics, as stipulated by their definitions. Dealing with infinitesimal deformations does not mean that tensors representing damage bear separate infinitesimal characteristics [26]. Therefore, while power series is being used for representative strains, is may not be useable for the damage tensors. As the constitutive variable, in this study we are going to deal with only one damage tensor taking into consideration only the effect of open micro surfaces. In this study, due to the existence of fiber distributions and micro voids in the material is it assumed that the material has gained directed medium characteristics, i.e. that an anisotropic structure has appeared due to the damage and the fibers. We assume that initially the material was isotropic and that the anisotropy is only caused by the dispersion of micro voids and fibers. For a medium like that the role of material description vectors will be played by the vector $\mathbf{A}(\mathbf{X},t)$ representing the mean values in RVE and the vector $\dot{\mathbf{A}}(\mathbf{X},t)$ representing the change in time of the preceding vector. We believe that, by dividing these vectors by the area of any characteristic surface pertaining to RVE, we render them dimensionless.

On the other hand, because the material will not be able to detect the positive and negative sides of micro void surfaces, we had previously specified that the dependence on vectors $\mathbf{A}(\mathbf{X},t)$ and $\dot{\mathbf{A}}(\mathbf{X},t)$ can be expressed by a product of tensors.

$$\mathbf{H} \equiv \mathbf{A}\otimes\mathbf{A}, \qquad \dot{\mathbf{H}} \equiv \dot{\mathbf{A}}\otimes\mathbf{A} + \mathbf{A}\otimes\dot{\mathbf{A}} \tag{3}$$

We can specify it as follows in the index form:

$$H_{KL} \equiv A_K A_L, \qquad \dot{H}_{KL} \equiv \dot{A}_K A_L + A_K \dot{A}_L \tag{4}$$

3. Kinematic of Fiber Deformation

Before deformation and after deformation, the fiber family is represented by continuous unit vector $\mathbf{B}(\mathbf{X})$, and $\mathbf{b}(\mathbf{x})$, respectively. The fiber deforms along with the material, i.e. fiber does not have a relative motion with respect to the material in which they are embedded. Relationships given below are true for an B-fiber family [28,29].

$$b_k = \lambda_b^{-1} x_{k,K} B_K, \qquad \lambda_b \equiv \left(\frac{dl}{dL}\right)_b, \qquad \lambda_b^2 = C_{KL} B_K B_L \tag{5}$$

Where dL and dl are respectively arc length of fiber before and after deformation; B_K fiber unit vector component before deformation, b_k fiber unit vector component after deformation, $x_{k,K} = \dfrac{\partial x_k}{\partial X_K}$ deformation gradient, λ_b rate of extension of fiber family, $C_{KL} = x_{k,K} x_{k,L}$ Green deformation tensor.

4. Electro-Thermo-Mechanical Equilibrium Equations and Thermodynamic Conditions

Local electro-thermo-mechanical equilibrium equations can be summarized as follows [30, 31]:

Gauss law;

$$\nabla\cdot\mathbf{D} = 0 \tag{6}$$

$$\mathbf{D} = \varepsilon_0\,\mathbf{E} + \mathbf{P} \tag{7}$$

Faraday law;

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla \phi \tag{8}$$

Conservation of mass;

$$\dot{\rho} + \rho \mathrm{v}_{k,k} = 0 \tag{9}$$

Conservation of Mass in material representation;

$$\rho(\mathbf{x}, t) = \frac{\rho_0(\mathbf{X})}{J(\mathbf{x}, t)} \tag{10}$$

Balance of Linear Momentum;

$$\rho \dot{\mathrm{v}}_p = \rho \mathrm{f}_p + \bar{t}_{rp,r} - P_{r,r} E_p \tag{11}$$

Balance of Moment of Momentum;

$$\varepsilon_{krp} \bar{t}_{rp} = 0, \bar{t}_{rp} \equiv t_{rp} + P_r E_p = \bar{t}_{pr} \tag{12}$$

Conservation of Energy;

$$\rho \dot{\varepsilon} = t_{kl} \mathrm{v}_{l,k} - q_{k,k} + \rho h + \rho \mathbf{E} \cdot \dot{\Pi} \tag{13}$$

Clausius-Duhem Inequality;

$$\rho \theta \dot{\eta} - q_{k,k} + \frac{1}{\theta} q_k \theta_{,k} - \rho \mathrm{h} \geq 0 \tag{14}$$

In these equations, the physical meanings of various symbols are: $\mathbf{D}$ electric displacement vector, $\mathbf{E}$ electric field vector, $\mathbf{P}$ polarization field vector, ε_0 electric permitivity of vacuum, ϕ electrostatic potential, $\nabla = \partial_{\mathbf{k}} \mathbf{i}_k$ gradient operator, ρ_0 mass density before deformation, ρ mass density after deformation, v_k components of velocity vector, *t* time, *J* Jacobian, $\dot{\mathrm{v}}_\mathrm{p}$ components of acceleration vector, f_p mechanical body force per unit mass, t_{kl} asymmetric stress tensor, $\bar{t}_{kl} = t_{kl} + P_k E_l$ a symmetric stress tensor, ε_{krp} permutation tensor, q_k heat flux vector, h heat source per unit mass, ε internal enerji density per unit mass, η entropy density per unit mass, θ absolute temperature of $\mathbf{X}$ at any time, $\rho\gamma$ entropy production per unit mass.

Local energy equation (13) is then suitably combined with the entropy inequality (14) and, using a Legendre transformation such as $\psi \equiv \varepsilon - \theta\eta - \mathbf{E} \cdot \mathbf{\Pi}$ for free energy, entropy inequality is obtained as follows in the material form:

$$-(\dot{\Sigma}+\rho_0\eta\dot{\theta})+\frac{1}{2}\overline{T}_{KL}\dot{C}_{KL}-\frac{1}{\theta}\theta_{,K}Q_K-\Pi_K\dot{E}_K\geq 0 \tag{15}$$

Relationships between material and spatial forms of values appearing in this inequality have been presented as follows [25]:

$$\Sigma \equiv \rho_0\ \psi \tag{16}$$

$$\dot{C}_{KL}=2\mathrm{d}_{kl}x_{k,K}x_{l,L}\Rightarrow d_{kl}=\frac{1}{2}\dot{C}_{KL}X_{K,k}X_{L,l} \tag{17}$$

$$\overline{T}_{KL}\equiv JX_{K,k}X_{L,l}\bar{t}_{kl}\Rightarrow\bar{t}_{kl}=J^{-1}x_{k,K}x_{l,L}\overline{T}_{KL} \tag{18}$$

$$Q_K\equiv J\,X_{K,k}\,q_k\Rightarrow q_k=J^{-1}\,x_{k,K}\,Q_K \tag{19}$$

$$\Pi_K\equiv\frac{\rho_0}{\rho}X_{K,k}\,P_k\Rightarrow P_k=J^{-1}x_{k,K}\Pi_K \tag{20}$$

$$E_K\equiv x_{k,K}\,E_k\Rightarrow E_k=X_{K,k}\,E_K \tag{21}$$

$$G_K\equiv\theta_{,K}=x_{k,K}\theta_{,k}\Rightarrow g_k\equiv\theta_{,k}=X_{K,k}\,\theta_{,K} \tag{22}$$

Where Σ thermodynamical stress potential, ψ generalized free energy density, C_{KL} Green deformation tensor, d_{kl} deformation rate tensor, $x_{k,K}$ deformation gradient, $X_{K,k}$ deformation gradient of inverse motion, $\overline{T}_{KL}$ symmetric stress tensor in material coordinates, Q_K heat vector in material coordinates, G_K temperature gradient, Π_K polarization vector per unit mass in material coordinates.

5. Constitutive Model

To be able to use the inequality (15), which is a general expression of entropy production, we need to know which independent variables the thermodynamic potential Σ depends on and how. Accordingly, selecting the arguments of Σ would formally mean selecting a material. According to the material selected, arguments of Σ and variables it depends on have been found using constitutive axioms. Suggesting that the stress potential Σ at a point in time t of a material point **X** in the material under consideration depends on the motion and the temperature background of all material points constituting the object and on the electrostatic field, formulation of constitutive equations for a dielectric-thermoelastic material under consideration can be summarized as follows.

$$\Sigma=\Sigma(C_{KL},H_{KL},E_K,B_K,\theta) \tag{23}$$

$$\Pi_K = -\frac{\partial \Sigma}{\partial E_K} \quad 24)$$

$$\bar{T}_{KL} = 2\frac{\partial \Sigma}{\partial C_{KL}} \quad (25)$$

$$\bar{Y}_{MN} \equiv -\frac{\partial \Sigma}{\partial H_{MN}}, \qquad Y_{KL} \equiv -\bar{Y}_{KL}, \qquad Y_{KL} \equiv \frac{\partial \Sigma}{\partial H_{KL}} \quad (26)$$

$$Q_K = Q_K\,(C_{KL}, H_{KL}, E_K, B_K, G_K, \theta) \quad (27)$$

$$G_K\, Q_K\ (C_{KL}, H_{KL}, E_K, B_K, G_K, \theta) \geq 0 \quad (28)$$

$$Q_K\,(C_{KL}, H_{KL}, E_K, B_K, 0, \theta) = 0 \quad (29)$$

$$T_{KL} = \bar{T}_{KL} - \Pi_K\, E_M\, C^{-1}_{ML} \quad (30)$$

In the expression (26), Y_{KL} is called as the strain-energy density release [32]. The definition $Y_{KL} \equiv -\bar{Y}_{KL}$ is used in order to deal with a positive value . From the equations provided in the expressions (25), (24) and (26), it is understood that symmetric stress, the polarization and the strain-energy density release rate are derived from the free energy function Σ and that the heat flux vector appears in a isotropic vectorial form with definite arguments independent of the stress potential. Thus, a need arises to determine the explicit forms of Σ and Q_K that appear as constitutive functions with known arguments.

However, firstly we should consider the constraints imposed by the material symmetry axiom onto the material under consideration. Because the symmetry group of the material under consideration is the fully orthogonal group, property $[S_{KL}]^{-1} = [S_{KL}]^T$, $\det \mathbf{S} = \pm 1$ is true for the symmetry operation $[S_{KL}]$. Therefore, each material point conversion matches an orientation of the material medium. Such conversion should, for every $[S_{KL}]$ be in the following form

$$X'_K = S_{KL} X_L\ ,\ X_L = S^T_{LK} X'_K\ ,\ [S_{KL}]^{-1} = [S_{KL}]^T \quad (31)$$

and leave constitutive functionals form invariant. Mathematically this means the validity of the following conversions.

$$\Sigma\,(S_{KP}\, S_{LR}\, C_{PR}, S_{KP}\, S_{LR}\, H_{PR}, S_{KL}\, E_L, S_{KL}\, B_L, \theta) = \Sigma\,(C_{KL}, H_{KL}, E_K, B_K, \theta) \quad (32)$$

$$Q_J\,(S_{KP} S_{LR} C_{PR}, S_{KP} S_{LR} H_{PR}, S_{KP} E_P, S_{KP} B_P, S_{KP} G_P, \theta) = S_{JN} Q_N\,(C_{KL}, H_{KL}, E_K, B_K, G_K, \theta) \quad (33)$$

On the other hand, both incompressibility of the composite and inextensibility of the fibers is broadly accepted in practice for formulation purposes. The following conditions should be satisfied once the composite is assumed to be incompressible and the fiber familY inextensible, respectively [25].

$$J = 1 \qquad \text{or} \qquad \det \mathbf{C} = III = 1 \tag{34}$$

$$\lambda_b^2 = C_{KL}\, B_K\, B_L = 1 \tag{35}$$

Thus, the constitutive equation for the stress is obtained as follows in material coordinates.

$$\overline{T}_{KL} = -p\, C_{KL}^{-1} + \Gamma_b\, B_K\, B_L + 2\frac{\partial \Sigma}{\partial C_{KL}} \tag{36}$$

In this expression, p and Γ_b are Lagrange coefficients and are defined by field equations and boundary conditions. $C_{KL}^{-1} \equiv X_{K,l}\, X_{L,l}$ Piola deformation tensor.

6. Determination of Symmetric Stress, Polarization Field, Asymmtric Stress and Strain-Energy Density Release Rate Constitutive Equations

Since the matrix has been assumed to be isotropic, the relation (32) is expressed as below.

$$\Sigma\,(C_{KL}, H_{KL}, E_K, B_K, \theta) = \Sigma\,(M_{KP}\, M_{LR} C_{PR},\, M_{KP}\, M_{LR} H_{PR},\, M_{KL}\, E_L,\, M_{KL}\, B_L,\, \theta) \tag{37}$$

Here, the orthogonal matrix indicating the symmetry group $\{M_{KL}\}$ shall be expressed for $[M_{KL}] \in O(3)$ and $[M_{KL}]^{-1} = [M_{KL}]^{T} \Rightarrow \det \mathbf{M} = \pm 1$ is true for the $[M_{KL}]$.

On the other hand, since Σ has been assumed to be the analytical function of its arguments, such arguments, which are expected to remain invariant under orthogonal transformations belonging to the symmetry group, should depend on a finite number of invariants. Using the methods of the invariants theory [33], 21 invariants of two symmetric matrices (C_{KL}, H_{KL}) and two polar vectors $(E_K,\, B_K)$ independent of one another have been expressed as follows:

$$\begin{aligned}
&I_1 \equiv C_{KK}, \quad I_2 \equiv C_{KL} C_{LK}, \quad I_3 \equiv C_{KL} C_{LM} C_{MK}, \quad I_4 = H_{KK}, \quad I_5 \equiv E_K E_K, \quad I_6 \equiv B_K B_K, \\
&I_7 \equiv E_K\, B_K, \quad I_8 \equiv E_K\, C_{KL} E_L, \quad I_9 \equiv E_K\, C_{KL}\, C_{LM}\, E_M, \quad I_{10} \equiv B_K\, C_{KL}\, B_L = \lambda_b^2, \\
&I_{11} \equiv B_K\, C_{KL}\, C_{LM}\, B_M, \quad I_{12} \equiv E_K\, C_{KL}\, B_L, \quad I_{13} \equiv E_K\, C_{KL}\, C_{LM}\, B_M, \quad I_{14} \equiv E_K\, H_{KL}\, E_L, \\
&I_{15} \equiv E_K\, C_{KL}\, H_{LM}\, E_M, \quad I_{16} \equiv B_K\, H_{KL}\, B_L, \quad I_{17} \equiv B_K\, C_{KL}\, H_{LM}\, B_M, \quad I_{18} \equiv E_K\, H_{KL}\, B_L, \\
&I_{19} \equiv E_K\, C_{KL}\, H_{LM}\, B_M, \quad I_{20} \equiv C_{KL} H_{LK}, \quad I_{21} \equiv C_{KL} C_{LM} H_{MK}
\end{aligned} \tag{38}$$

Instead of the first three invariants of the Green deformation tensor **C**, we can use the principal invariants below.

$$I = I_1, \qquad II = \frac{1}{2}(I_1^2 - I_2), \qquad III = \frac{1}{6}(I_1^3 - 3\, I_1 I_2 + 2\, I_3) = \det \mathbf{C} \tag{39}$$

Given the incompressibility of the composite, inextensibility of the fiber family and the fact that $\mathbf{B}$ is unit vector, the invariants III and I_6 in expressions (38) and (39) are equal to 1 thus eliminating the dependence of Σ on these invariants. As a result, the invariants on which Σ depends are expressed as follows.

$$\Sigma = \Sigma(I,\ II\ ,I_4\ ,I_5\ I_7\ ,I_8,\ I_9\ ,\ I_m\ ,\ \theta),\quad (m = 11,...,21) \tag{40}$$

Taking the derivative of expression (40) according to C_{PR}, H_{PR} and E_R and substituting it into equations (36), (26) and (24), the following expressions are obtained.

$$\bar{T}_{PR} = -\,p\,C_{PR}^{-1} + \Gamma_b B_P B_R + 2\left(\frac{\partial\Sigma}{\partial I}\frac{\partial I}{\partial C_{PR}} + \frac{\partial\Sigma}{\partial II}\frac{\partial II}{\partial C_{PR}} + \frac{\partial\Sigma}{\partial I_i}\frac{\partial I_i}{\partial C_{PR}}\right), \quad (i = 8,9,11,12,13,15,17,19,20,21) \tag{41}$$

$$Y_{PR} = \frac{\partial\Sigma}{\partial I_m}\frac{\partial I_m}{\partial H_{PR}} \qquad (m = 4,14,15,...,21) \tag{42}$$

$$\Pi_R = -\left(\frac{\partial\Sigma}{\partial I_k}\frac{\partial I_k}{\partial E_R}\right),\ (k = 5,7,8,9,12,13,14,15,18,19) \tag{43}$$

It is understood that, as always, repeated indices will undergo summation. If derivatives of invariants appearing in these equations according to C_{PR}, H_{PR} and E_R are taken from expressions (38) and (39) and substituted afterwards, constitutive equation of the symmetric stress in non-linear form is obtained as follows.

$$\begin{aligned}\bar{T}_{PR} = &-\,p\,C_{PR}^{-1} + \Gamma_b B_P B_R + 2\left\{\left(\frac{\partial\Sigma}{\partial I} + \frac{\partial\Sigma}{\partial II}C_{LL}\right)\delta_{PR} - \frac{\partial\Sigma}{\partial II}C_{PR} + \frac{\partial\Sigma}{\partial I_8}E_P E_R + \frac{\partial\Sigma}{\partial I_9}(E_P E_K C_{KR} +\right.\\ &+ C_{PK}E_K E_R) + \frac{\partial\Sigma}{\partial I_{11}}(B_P B_L C_{LR} + C_{PL}B_L B_R) + \frac{\partial\Sigma}{\partial I_{12}}E_P B_R + \frac{\partial\Sigma}{\partial I_{13}}(E_P B_L C_{LR} + C_{PL} E_L B_R) +\\ &\frac{\partial\Sigma}{\partial I_{15}}(E_P E_L H_{LR} + H_{PL} E_L E_R) + \frac{\partial\Sigma}{\partial I_{17}}(B_P B_L H_{LR} + H_{PL} B_L B_R) + \frac{\partial\Sigma}{\partial I_{19}}(E_P B_L H_{LR} + H_{PL} E_L B_R) +\\ &\frac{\partial\Sigma}{\partial I_{20}}H_{PR} + \frac{\partial\Sigma}{\partial I_{21}}(C_{PL}H_{LR} + H_{PL}C_{LR})\end{aligned} \tag{44}$$

And polarization field in non-linear form is obtained as follows.

$$\begin{aligned}\Pi_R = &-\left(2\frac{\partial\Sigma}{\partial I_5}E_R + \frac{\partial\Sigma}{\partial I_7}B_R + 2\frac{\partial\Sigma}{\partial I_8}C_{RL}E_L + 2\frac{\partial\Sigma}{\partial I_9}C_{RL}C_{LK}E_K + \frac{\partial\Sigma}{\partial I_{12}}C_{RL}B_L + \frac{\partial\Sigma}{\partial I_{13}}C_{RL}C_{LK}B_K +\right.\\ &\left.2\frac{\partial\Sigma}{\partial I_{14}}H_{RL}E_L + 2\frac{\partial\Sigma}{\partial I_{15}}C_{RL}H_{LK}E_K + \frac{\partial\Sigma}{\partial I_{18}}H_{RL}B_L + \frac{\partial\Sigma}{\partial I_{19}}C_{RL}H_{LK}B_K\right)\end{aligned} \tag{45}$$

And the strain-energy density release rate in non-linear form is obtained as follows.

$$Y_{PR} = \frac{\partial \Sigma}{\partial I_4}\delta_{PR} + \frac{\partial \Sigma}{\partial I_{14}} E_P E_R + \frac{\partial \Sigma}{\partial I_{15}} C_{PK} E_K E_R + \frac{\partial \Sigma}{\partial I_{16}} B_P B_R + \frac{\partial \Sigma}{\partial I_{17}} C_{PL} B_L B_R + \frac{\partial \Sigma}{\partial I_{18}} E_P B_R + \frac{\partial \Sigma}{\partial I_{19}} C_{PL} E_L B_R + \frac{\partial \Sigma}{\partial I_{20}} C_{PR} + \frac{\partial \Sigma}{\partial I_{21}} C_{PL} C_{LR} \quad (46)$$

More concrete form of the constitutive equations given by (44), (45) and (46) can be obtained provided that Lagrange coefficients $-p$ and Γ_b and the derivatives of Σ based on its invariants are known. It has been already stated that $-p$ and Γ_b can be obtained from field equations and boundary conditions. To obtain the derivatives of Σ according to its invariants it should be estimated how Σ depends on the invariants it is shown to depend on in expression (40). In this study, the matrix material has been considered as an isotropic medium. According to that Σ is an analytical function of those invariants, assuming that this function is analytic, the stress potential is expanded in the power series around natural condition. To obtain a quadratic theory, the terms in this series expanding should be kept to second order, therefore the stress potential can be represented by a polynomial [25, 34]. However, the grade and number of terms of the polynomial representing Σ depends on the size of its deformation invariant and their shares of interaction in the case, shortly on their nonlinearity grades [35-37].

In this study, mechanical interactions and effect of damage have been assumed to be linear while electro-mechanical interactions have been assumed to be non-linear. Furthermore, considering that the material remains insensitive to directional changes along fiber, double components of fiber vector have been included in the operation. Because mechanical interactions and effect of damage are assumed to be linear, the symmetric stress, the polarization field and the strain-energy density release rate should remain linear according to the deformation tensor and the damage tensor. Therefore function Σ could be represented by a second degree polynomial according to the invariants it depends on. On the other hand, because internal energy is defined as a positive definite form, for a polynomial to be positively defined and for the order of invariants not to affect Σ, the polynomial must have symmetric coefficients, i.e. be in a quadratic form. Accordingly, if polynomial approximation is selected, the following expression can be recorded for the stress potential Σ in terms of the existing invariants.

$$\Sigma = \sum_{i.j} a_{ij} I_i I_j \,, (i, j = 1, 2, 4, 7, 9, 11, \ldots, 21), \qquad a_{ij} = a_{ji} \quad (47)$$

In the expansion (47) I_1 and I_2 have been substituted by principal invariants I and II, respectively. The derivatives of Σ based on its invariants in the equations (44), (45) and (46) are obtained from the expression (47) as follows.

$$\frac{\partial \Sigma}{\partial I} = 2(a_{1,1} I + a_{1,2} II + a_{1,k} I_k), \quad \frac{\partial \Sigma}{\partial II} = 2(a_{1,2} I + a_{2,2} II + a_{2,k} I_k),$$

$$\frac{\partial \Sigma}{\partial I_m} = 2(a_{m,1} I + a_{m,2} II + a_{m,k} I_k), \ (m=4,5,7,8,9,11,\ldots,21), (k=4,5,7,8,9,11,\ldots,21) \quad (48)$$

At this stage, derivatives of the stress potential have been taken without paying attention to whose functions the invariants are. Expressions (38) and (39) have shown on what the invariants in the expression (48) depend. Due to the existence of the relationship $C_{KL} = \delta_{KL} + 2\mathrm{E}_{KL}$ between the Green deformation tensor with the strain tensor, and assuming mechanic interactions are linear ($\mathrm{E}_{KL} \cong \tilde{\mathrm{E}}_{KL} = \frac{1}{2}(U_{K,L} + U_{L,K})$), those invariants that depend on the Green deformation tensor (C_{KL}) can be expressed in terms of strain tensor (E_{KL}), which is a more useful parameter.

Terms after the third term on the right side of the equation (44) and all terms of the right side of the equation (45) and (46) have been calculated using the partial derivatives given in the expression (48) and invariants that depend on the strain tensor ($\tilde{\mathrm{E}}_{KL}$). Due to the assumptions made in this study, of the first grade components of the strain tensor ($\tilde{\mathrm{E}}_{KL}$) and the damage tensor H_{KL} and of the external multiplication components of vector B_K, those whose number is even have been taken into consideration. Thus, in the beginning, the elastic stress is expressed for the condition without stress and without load (with the term $\alpha_1 \delta_{PR}$ assumed to be zero) by taking common coefficients into common parenthesis.

$$\begin{aligned}
\bar{T}_{PR} = & -pC^{-1}_{PR} + \Gamma_b B_P B_R + \alpha_2 \tilde{\mathrm{E}}_{KK}\delta_{PR} + \alpha_3 H_{KK}\delta_{PR} + \alpha_4 E_K E_K \delta_{PR} + \alpha_5 E_K \tilde{\mathrm{E}}_{KL} E_L \delta_{PR} + \\
& \alpha_6 E_K H_{KN} E_N \delta_{PR} + \alpha_7 B_K H_{KN} B_N \delta_{PR} + \alpha_8 E_K E_K \tilde{\mathrm{E}}_{LL}\delta_{PR} + \alpha_9 \tilde{\mathrm{E}}_{PR} + \alpha_{10} E_K E_K \tilde{\mathrm{E}}_{PR} + \alpha_{11} E_P E_R + \\
& \alpha_{12}\tilde{\mathrm{E}}_{KK} E_P E_R + \alpha_{13} H_{KK} E_P E_R + \alpha_{14} B_K H_{KN} B_N E_P E_R + \alpha_{15}(E_P E_Q \tilde{\mathrm{E}}_{QR} + \tilde{\mathrm{E}}_{PQ} E_Q E_R) + \\
& \alpha_{16}\tilde{\mathrm{E}}_{KK} B_P B_R + \\
& \alpha_{17} H_{KK} B_P B_R + \alpha_{18} E_K E_K B_P B_R + \alpha_{19} E_K \tilde{\mathrm{E}}_{KL} E_L B_P B_R + \alpha_{20} E_K H_{KN} E_N B_P B_R + \alpha_{21}[B_P B_R + \\
& (B_P B_L \tilde{\mathrm{E}}_{LR} + \tilde{\mathrm{E}}_{PL} B_L B_R)] + \alpha_{22} E_K E_K (B_P B_L \tilde{\mathrm{E}}_{LR} + \tilde{\mathrm{E}}_{PL} B_L B_R) + \alpha_{23} E_K B_K E_P B_R + \\
& \alpha_{24} E_K \tilde{\mathrm{E}}_{KN} B_N E_P B_R + \alpha_{25} E_K H_{KN} B_N E_P B_R + \alpha_{26} E_K B_K (E_P B_L \tilde{\mathrm{E}}_{LR} + \tilde{\mathrm{E}}_{PL} E_L B_R) + \\
& \alpha_{27}(E_P E_L H_{LR} + H_{PL} E_L E_R) + \alpha_{28}(B_P B_L H_{LR} + H_{PL} B_L B_R) + \\
& \alpha_{29} E_K B_K (E_P B_L H_{LR} + H_{PL} E_L B_R) + \alpha_{30} H_{PR} + \alpha_{31} E_K E_K H_{PR} \qquad (49)
\end{aligned}$$

The polarization field has been obtained as follows.

$$\begin{aligned}
\Pi_R \equiv -\{ & \beta_1 E_R + \beta_2 \tilde{\mathrm{E}}_{KK} E_R + \beta_3 E_K B_K B_R + \beta_4 (E_K H_{KN} B_N B_R + E_K B_K H_{RL} B_L) + \\
& \beta_5 \tilde{\mathrm{E}}_{RL} E_L + \beta_6 E_K B_K \tilde{\mathrm{E}}_{RL} B_L + \beta_7 E_K \tilde{\mathrm{E}}_{KL} B_L B_R + \beta_8 H_{RL} E_L \} \qquad (50)
\end{aligned}$$

The strain-energy density release rate is expressed as follows by taking common coefficients into parenthesis in the beginning, without micro-cracks (the term $\gamma_1 \delta_{PR}$ is taken here as zero) has been obtained as follows.

$$Y_{PR} = \gamma_2 \tilde{\mathrm{E}}_{KK}\delta_{PR} + \gamma_3 H_{KK}\delta_{PR} + \gamma_4 E_K E_K \delta_{PR} + \gamma_5 E_K \tilde{\mathrm{E}}_{KL} E_L \delta_{PR} + \gamma_6 E_K H_{KL} E_L \delta_{PR} + \gamma_7 B_K H_{KL} B_L \delta_{PR} +$$

$$\gamma_8 E_P E_R + \gamma_9 \tilde{\mathrm{E}}_{KK} E_P E_R + \gamma_{10} H_{KK} E_P E_R + \gamma_{11} B_K H_{KL} B_L E_P E_R + \gamma_{12} \tilde{\mathrm{E}}_{PL} E_L E_R + \gamma_{13} B_P B_R +$$
$$\gamma_{14} \tilde{\mathrm{E}}_{KK} B_P B_R + \gamma_{15} H_{KK} B_P B_R + \gamma_{16} E_K E_K B_P B_R + \gamma_{17} E_K \tilde{\mathrm{E}}_{KL} E_L B_P B_R + \gamma_{18} E_K H_{KL} E_L B_P B_R +$$
$$\gamma_{19} \tilde{\mathrm{E}}_{PL} B_L B_R + \gamma_{20} E_K E_K \tilde{\mathrm{E}}_{PL} B_L B_R + \gamma_{21} E_K B_K E_L B_R + \gamma_{22} E_K \tilde{\mathrm{E}}_{KL} B_L E_P B_R + \gamma_{23} E_K H_{KL} B_L E_P B_R +$$
$$\gamma_{24} E_K B_K \tilde{\mathrm{E}}_{PL} E_L B_R + \gamma_{25} \tilde{\mathrm{E}}_{PR} + \gamma_{26} E_K E_K \tilde{\mathrm{E}}_{PR} \tag{51}$$

The coefficients { $\alpha_i\,(i=1,2,3,...,31)$, $\beta_j\ (j=1,2,3,...,8)$ and $\gamma_m\ (m=1,2,3,...,26)$ } in the equations (49), (50), and (51) have been and depend on the medium temperature θ and a_{ij} .

In a composite material that consist of an isotropic matrix reinforced by one arbitrary and inextensible fiber family, the medium is assumed to be linear, dielectric, isotropic, incompressible, has micro-cracks and dependent on temperature gradient. The equation (49) is the linear constitutive equation of symmetric stress. First and second terms of the equation (49) are hydrostatic pressure and contribution of fiber tension to the symmetric stress respectively; third and tenth terms combined are the contribution of the elastic deformation; fourth and thirty second terms combined are the contribution of the damage tensor; fifth and twelfth terms are the second grade electrostatic contribution; sixth, ninth, eleventh, thirteenth and sixteenth terms are the stress produced by the interaction of the non-linear electric field with the deformation field; seventh, fourteenth, twenty ninth and thirty third terms are the stress produced by the interaction of the non-linear electric field with the damage tensor; eighth, eighteenth and thirtieth terms are the stress arising of the interaction between the fiber distribution $\mathbf{B}$ and the damage tensor; fifteenth, twenty first, twenty seventh and thirty first terms are the contribution arising of the triple interaction between the non-linear electrostatic field, the damage tensor and the fiber field $\mathbf{B}$; seventeenth and twenty third terms are the stress arising of the interaction between the fiber distribution $\mathbf{B}$ and the elastic deformation; nineteenth and twenty fifth terms are the contribution of the non-linear electrostatic field and the fiber field $\mathbf{B}$ to the stress; twentieth, twenty fourth, twenty sixth and twenty eighth terms are the contribution arising of the triple interaction between the non-linear electrostatic field, elastic field and the fiber field $\mathbf{B}$; twenty second term is the contribution of the fiber distribution $\mathbf{B}$ to the stress.

The equation (50) is the linear constitutive equation of the polarization field. First term of the equation (50) shows the well-known electrical sensitivity. The second and fifth terms show the interaction of the linear electric field with the deformation field. The third term shows the interaction of the linear electric field with the fiber field $\mathbf{B}$. The fourth term is the contribution arising of the triple interaction between the linear electrostatic field, the damage tensor and the fiber field $\mathbf{B}$; sixth and seventh terms are the contribution arising of the triple interaction between the linear electrostatic field, the damage tensor and the fiber field $\mathbf{B}$; eighth term is the polarization produced by the interaction of the linear electric field with the damage tensor.

The equation (51) is the linear constitutive equation of strain-energy density release rate. First and twenty fourth terms combined are the contribution of the elastic deformation; second term is the contribution of the damage tensor; third and seventh terms are the second grade electrostatic contribution; fourth, eighth, eleventh and twenty fifth terms are the strain-energy density release arising of the interaction between the non-linear electric field and the deformation field; fifth and ninth terms are the strain-energy density release produced by the interaction of the non-linear electric field with the damage tensor; sixth and fourteenth terms are the strain-energy density release stress arising of the interaction between the fiber distribution $\mathbf{B}$ and the damage tensor; tenth, seventeenth and twenty second terms are the contribution arising of the triple interaction between the non-linear electrostatic field, the damage tensor and the fiber field $\mathbf{B}$; twelfth term is the contribution of the fiber distribution $\mathbf{B}$ to the strain-energy density release; thirteenth and eighteenth terms are the strain-energy density release arising of the interaction between the fiber distribution $\mathbf{B}$ and the elastic deformation; fifteenth and twentieth terms are the contribution of the non-linear electrostatic field and the fiber field $\mathbf{B}$ to the strain-energy density release; sixteenth, nineteenth, twenty first and twenty third terms are the contribution arising of the triple interaction between the non-linear electrostatic field, elastic field and the fiber field $\mathbf{B}$ to the strain-energy density release.

Equations of the symmetric stress provided by the expression (49) and of the polarization field provided by the expression (50) are substituted into the equation (30), thus the total stress (the asymmetric stress) has been obtained as follows.

$$
\begin{aligned}
&T_{PR} = -pC^{-1}_{PR} + \Gamma_b B_P B_R + \alpha_2 \tilde{\mathrm{E}}_{KK}\, \delta_{PR} + \alpha_3 H_{KK}\delta_{PR} + \alpha_4 E_K E_K \delta_{PR} + \alpha_5 E_K \tilde{\mathrm{E}}_{KL} E_L \delta_{PR} + \\
&\alpha_6 E_K H_{KN} E_N \delta_{PR} + \alpha_7 B_K H_{KN} B_N \delta_{PR} + \alpha_8 E_K E_K \tilde{\mathrm{E}}_{LL}\delta_{PR} + \alpha_9 \tilde{\mathrm{E}}_{PR} + \alpha_{10} E_K E_K \tilde{\mathrm{E}}_{PR} + \alpha_{11} E_P E_R + \\
&\alpha_{12}\tilde{\mathrm{E}}_{KK} E_P E_R + \alpha_{13} H_{KK} E_P E_R + \alpha_{14} B_K H_{KN} B_N E_P E_R + \alpha_{15}(E_P E_Q \tilde{\mathrm{E}}_{QR} + \tilde{\mathrm{E}}_{PQ} E_Q E_R) + \\
&\alpha_{16}\tilde{\mathrm{E}}_{KK} B_P B_R + \alpha_{17} H_{KK} B_P B_R + \alpha_{18} E_K E_K B_P B_R + \alpha_{19} E_K \tilde{\mathrm{E}}_{KL} E_L B_P B_R + \alpha_{20} E_K H_{KN} E_N B_P B_R + \\
&\alpha_{21}[B_P B_R + (B_P B_L \tilde{\mathrm{E}}_{LR} + \tilde{\mathrm{E}}_{PL} B_L B_R)] + \alpha_{22} E_K E_K (B_P B_L \tilde{\mathrm{E}}_{LR} + \tilde{\mathrm{E}}_{PL} B_L B_R) + \alpha_{23} E_K B_K E_P B_R + \\
&\alpha_{24} E_K \tilde{\mathrm{E}}_{KN} B_N E_P B_R + \alpha_{25} E_K H_{KN} B_N E_P B_R + \alpha_{26} E_K B_K (E_P B_L \tilde{\mathrm{E}}_{LR} + \tilde{\mathrm{E}}_{PL} E_L B_R) + \\
&\alpha_{27}(E_P E_L H_{LR} + H_{PL} E_L E_R) + \alpha_{28}(B_P B_L H_{LR} + H_{PL} B_L B_R) + \alpha_{29} E_K B_K (E_P B_L H_{LR} + H_{PL} E_L B_R) + \\
&\alpha_{30} H_{PR} + \alpha_{31} E_K E_K H_{PR} + \beta_1 E_P E_L C^{-1}_{LR} + \beta_2 \tilde{\mathrm{E}}_{KK} E_P E_L C^{-1}_{LR} + \beta_3 E_K B_K B_P E_L C^{-1}_{LR} + \\
&\beta_4 (E_K H_{KN} B_N B_P + E_K B_K H_{PN} B_N) E_L C^{-1}_{LR} + \beta_5 \tilde{\mathrm{E}}_{PN} E_N E_L C^{-1}_{LR} + \beta_6 E_K B_K \tilde{\mathrm{E}}_{PN} B_N E_L C^{-1}_{LR} + \\
&\beta_7 E_K \tilde{\mathrm{E}}_{KN} B_N B_P E_L C^{-1}_{LR} + \beta_8 H_{PN} E_N E_L C^{-1}_{LR}
\end{aligned}
\tag{52}
$$

Expression (52) is the constitutive equation of the stress that occurs asymmetrically in a polarized, has micro-cracks, arbitrary fiber-reinforced thermoelastic composite material that is undergoing deformation due to the electro-thermo mechanical loading, is in interference with an electrostatic field and is considered as an isotropic medium. According to the expression (52), the last nine terms arising of polarization cause the stress to be asymmetric. Assuming that the electrical interactions are linear, too, terms of this equation arising of polarization and the terms arising of the symmetric stress and containing the second degree electric field will be neglected. As understood from here, asymmetry of the stress on material coordinates occurring inside the material is caused by strong electrical interactions.

7. Determination of Heat Flux Vector Constitutive Equation

It has been determined that the heat flux vector depends on the deformation tensor, the damage tensor, the electric field vector, the fiber field vector and temperature gradient and expressions (27)-(29) have been provided. Additional constraints imposed on the heat flux vector by constitutive functions originate from the material symmetry of the medium. The structure of the heat flux vector should be in compliance with following transformation for each orthogonal matrix $[M_{KL}] \in O(3)$ belonging to the symmetry group of the material.

$$M_{JN}Q_N(C_{KL},H_{KL},E_K,B_K,G_K,\theta) = \\ Q_J(M_{KP}M_{LR}C_{PR},M_{KP}M_{LR}H_{PR},M_{KP}E_P,M_{KP}B_P,M_{KP}G_P,\theta) \quad (53)$$

Where the matrix is isotropic, the relation (53) is valid for each orthogonal matrix of the fully orthogonal group. The heat flux vector is an isotropic function of the symmetric matrices C_{KL} and H_{KL} and polar vectors E_K, B_K and G_K. To obtain an explicit expression of the Q_K vector, a scalar multiplication of the vector Q_K with the vector **V** is required with **V** being an arbitrary vector. Such multiplication is defined as a scalar function $\Re$ as indicated below [33]. This review article [33] summarizes the subject of representation methods for constitutive equations based on material symmetry.

$$\Re(C_{KL},H_{KL},E_K,B_K,G_K,V_K) \equiv V_K\,Q_K(C_{PR},H_{PR},E_P,B_P,G_P) \quad (54)$$

Taking the partial derivative of the expression (54) according to V_K, the following can be recorded.

$$Q_K(C_{PR},H_{PR},E_P,B_P,G_P)=\frac{\partial\Re(C_{KL},H_{KL},E_K,B_K,G_K,V_K)}{\partial V_K}\Bigg| \quad (55)$$

Because the left part of this expression is independent of the vector **V,** the equation (55) should also be valid for **V=0**. Thus, the isotropic vector function Q_K is expressed as follows.

$$Q_K(C_{PR},H_{PR},E_P,B_P,G_P)=\frac{\partial\Re(C_{KL},H_{KL},E_K,B_K,G_K,V_K)}{\partial V_K}\Bigg|_{V_K=0} \quad (56)$$

In this situation, in order to find the vector Q_K from the relation (56), one needs to define the structure of the scalar $\Re$ depending on the arguments $C_{PR},H_{PR},E_P,B_P,G_P,V_P$ and calculate the partial derivative of this function based on the vector **V** for **V=0**. Let us first remove the arbitrary vector **V** from the arguments of the scalar function $\Re$ and define a scalar function with arguments $C_{PR},H_{PR},E_P,B_P,G_P$.

$$F \equiv F(C_{KL},H_{KL},E_K,B_K,G_K) \quad (57)$$

For the function F, which is an isotropic function, to remain invariant under orthogonal coordinate transformations, its arguments must depend on a finite number of invariants. Using the methods in the theory of invariants [33], 36 invariants of the two symmetric tensors C_{KL}

and H_{KL} and the three polar vectors E_K, B_K and G_K independent of one another have been expressed below.

$$I_1 \equiv C_{KK}\,,\; I_2 \equiv C_{KL}C_{LK}\,,\; I_3 \equiv C_{KL}C_{LM}C_{MK}\,,\; I_4 \equiv H_{KK}\,,\; I_5 \equiv E_K E_K\,,\; I_6 \equiv B_K B_K\,,\; I_7 \equiv G_K G_K\,,$$
$$I_8 \equiv E_K B_K\,,\; I_9 \equiv E_K G_K\,,\; I_{10} \equiv B_K G_K\,,\; I_{11} \equiv E_K C_{KL} E_L\,,\; I_{12} \equiv E_K C_{KL} C_{LM} E_M\,,$$
$$I_{13} \equiv B_K C_{KL} B_L = \lambda_b^2\,,\; I_{14} \equiv B_K C_{KL} C_{LM} B_M\,,\; I_{15} \equiv G_K C_{KL} G_L\,,\; I_{16} \equiv G_K C_{KL} C_{LM} G_M\,,$$
$$I_{17} \equiv E_K C_{KL} B_L\,,\; I_{18} \equiv E_K C_{KL} C_{LM} B_M\,,\; I_{19} \equiv E_K C_{KL} G_L\,,\; I_{20} \equiv E_K C_{KL} C_{LM} G_M\,,$$
$$I_{21} \equiv B_K C_{KL} G_L\,,\; I_{22} \equiv E_K C_{KL} C_{LM} G_M\,,\; I_{23} \equiv E_K H_{KL} E_L\,,\; I_{24} \equiv E_K C_{KL} H_{LM} E_M\,,$$
$$I_{25} \equiv B_K H_{KL} B_L\,,\; I_{26} \equiv B_K C_{KL} H_{LM} B_M\,,\; I_{27} \equiv G_K H_{KL} G_L\,,\; I_{28} \equiv G_K C_{KL} H_{LM} G_M\,,$$
$$I_{29} \equiv E_K H_{KL} B_L\,,\; I_{30} \equiv E_K C_{KL} H_{LM} B_M\,,\; I_{31} \equiv E_K H_{KL} G_L\,,\; I_{32} \equiv E_K C_{KL} H_{LM} G_M\,,$$
$$I_{33} \equiv B_K H_{KL} G_L\,,\; I_{34} \equiv B_K C_{KL} H_{LM} G_M\,,\; I_{35} \equiv C_{KL} H_{LK}\,,\; I_{36} \equiv C_{KL} C_{LM} H_{MK} \qquad (58)$$

However, the arguments of the scalar isotropic function $\Re$, the function which the main function to be found, are $C_{PR}, H_{PR}, E_P, B_P, G_P, V_P$. A linear function of the vector $\mathbf{V}$, the scalar $\Re$ is also dependent on the following invariants in addition to the invariants in (58).

$$K_1 \equiv V_K E_K\,,\; K_2 \equiv V_K B_K\,,\; K_3 \equiv V_K G_K\,,\; K_4 \equiv V_K C_{KL} E_L\,,\; K_5 \equiv V_K C_{KL} B_L\,,\; K_6 \equiv V_K C_{KL} G_L\,,$$
$$K_7 \equiv V_K C_{KM} C_{ML} E_L\,,\; K_8 \equiv V_K C_{KM} C_{ML} B_L\,,\; K_9 \equiv V_K C_{KM} C_{ML} G_L\,,\; K_{10} \equiv V_K H_{KL} E_L\,,$$
$$K_{11} \equiv V_K H_{KL} B_L\,,\; K_{12} \equiv V_K H_{KL} G_L\,,\; K_{13} \equiv V_K C_{KM} H_{ML} E_L\,,$$
$$K_{14} \equiv V_K C_{KM} H_{ML} B_L\,,\; K_{15} \equiv V_K C_{KM} H_{ML} G_L \qquad (59)$$

Thus, the function $\Re$ can be written down as follows.

$$\Re(C_{KL}, H_{KL}, E_K, B_K, G_K, V_K) = \sum_{\alpha=1}^{15} \lambda_\alpha K_\alpha \qquad (60)$$

Coefficients λ_α in the equation (60) are each a scalar function of the invariants given in the equation (58). Furthermore, the heat flux vector has been obtained as follows using the relation (56) given the assumptions made on the mechanical interaction in this study and considering the first-grade terms of the tensor $\mathbf{C}$.

$$Q_R = \lambda_1 E_R + \lambda_2 B_R + \lambda_3 G_R + \lambda_4 C_{PR} E_P + \lambda_5 C_{PR} B_P + \lambda_6 C_{PR} G_P + \lambda_7 H_{PR} E_P +$$
$$\lambda_8 H_{PR} B_P + \lambda_9 H_{PR} G_P \qquad (61)$$

Because mechanical interactions and effect of damage have been assumed to be linear, coefficients in the equation (61) are each a scalar function of invariants that do not contain square or higher grade terms of tensor $\mathbf{C}$ and terms in the form $(\mathbf{CH})$ in the equation (58). Besides, taking into account that values of the invariant I_{13} is equal to 1 due to the inextensibility of fiber family and value of the invariant I_6 is equal to 1 because it is unit vector pertaining to the distribution of fiber $\mathbf{B}$ before deformation, invariant on which the said coefficients depend have been recorded as follows in terms of $\tilde{E}_{KL}$.

$$J_1=3+2\tilde{\mathrm{E}}_{KK}\,,\;\; J_2=H_{KK}\,,\;\; J_3=E_K E_K\,,\;\; J_4=G_K G_K\,,\;\; J_5=E_K B_K\,,\;\; J_6=E_K G_K\,,\;\; J_7=B_K G_K\,,$$
$$J_8=E_K E_K+2E_K\tilde{\mathrm{E}}_{KL}E_L,\quad J_9=G_K G_K+2G_K\tilde{\mathrm{E}}_{KL}G_L,\quad J_{10}=E_K B_K+2E_K\tilde{\mathrm{E}}_{KL}B_L,$$
$$J_{11}=E_K G_K+2E_K\tilde{\mathrm{E}}_{KL}G_L,\quad J_{12}=B_K G_K+2B_K\tilde{\mathrm{E}}_{KL}G_L,\quad J_{13}=E_K H_{KL}E_L,\quad J_{14}=B_K H_{KL}B_L,$$
$$J_{15}=G_K H_{KL}G_L,\quad J_{16}=E_K H_{KL}B_L,\quad J_{17}=E_K H_{KL}G_L,\quad J_{18}=B_K H_{KL}G_L \tag{62}$$

Coefficients in the equation (61) can be defined as follows as a scalar function of the invariants in (62).

$$\lambda_\alpha=\beta_0+\sum_{i=1}^{18}\beta_i J_i,\qquad 1\le\alpha\le 9,\qquad (\alpha=1\Rightarrow\beta=a,\;\alpha=2\Rightarrow\beta=b,\;\alpha=3\Rightarrow\beta=c,$$
$$\alpha=4\Rightarrow\beta=d,$$
$$\alpha=5\Rightarrow\beta=f,\;\alpha=6\Rightarrow\beta=k,\;\alpha=7\Rightarrow\beta=l,\;\alpha=8\Rightarrow\beta=m,\;\alpha=9\Rightarrow\beta=s) \tag{63}$$

Due to the existence of the relationship $C_{KL}=\delta_{KL}+2\mathrm{E}_{KL}$ between the Green deformation tensor with the strain tensor, and assuming mechanical interactions are linear { $\mathrm{E}_{KL}\cong\tilde{\mathrm{E}}_{KL}=\frac{1}{2}(U_{K,L}+U_{L,K})$ }, those invariants that depend on the Green deformation tensor (C_{KL}) can be expressed in terms of strain tensor $(\mathrm{E}_{KL}\cong\tilde{\mathrm{E}}_{KL})$ which is a more useful parameter. Using the expressions (63) in the equation (61), considering mechanical interactions, effect of damage and temperature changes have been assumed to be linear, electrical interactions have been assumed to be non-linear, the following expression has been obtained.

$$\begin{aligned}Q_R=&\,\Omega_1E_R+\Omega_2\tilde{\mathrm{E}}_{KK}E_R+\Omega_3H_{KK}E_R+\Omega_4E_KG_KE_R+\Omega_5E_K\tilde{\mathrm{E}}_{KL}G_LE_R+\Omega_6B_KH_{KL}B_LE_R+\\&\Omega_7E_KH_{KL}G_LE_R+\Omega_8E_KB_KB_R+\Omega_9B_KG_KB_R+\Omega_{10}E_K\tilde{\mathrm{E}}_{KL}B_LB_R+\Omega_{11}B_K\tilde{\mathrm{E}}_{KL}G_LB_R+\\&\Omega_{12}E_KH_{KL}B_LB_R+\Omega_{13}B_KH_{KL}G_LB_R+\Omega_{14}G_R+\Omega_{15}\tilde{\mathrm{E}}_{KK}G_R+\Omega_{16}H_{KK}G_R+\Omega_{17}E_KE_KG_R+\\&\Omega_{18}E_K\tilde{\mathrm{E}}_{KL}E_LG_R+\Omega_{19}E_KH_{KL}E_LG_R+\Omega_{20}B_KH_{KL}B_LG_R+\Omega_{21}\tilde{\mathrm{E}}_{PR}E_P+\Omega_{22}E_KG_K\tilde{\mathrm{E}}_{PR}E_P+\\&\Omega_{23}E_KB_K\tilde{\mathrm{E}}_{PR}B_P+\Omega_{24}B_KG_K\tilde{\mathrm{E}}_{PR}B_P+\Omega_{25}\tilde{\mathrm{E}}_{PR}G_P+\Omega_{26}E_KE_K\tilde{\mathrm{E}}_{PR}G_P+\Omega_{27}H_{PR}E_P+\\&\Omega_{28}E_KG_KH_{PR}E_P+\Omega_{29}E_KB_KH_{PR}B_P+\Omega_{30}B_KG_KH_{PR}B_P+\Omega_{31}H_{PR}G_P+\Omega_{32}E_KE_KH_{PR}G_P\end{aligned} \tag{64}$$

Coefficients $\Omega_k\,(k=1,2,3,...,32)$ have been depend on the medium temperature θ and λ_α. Because the tensor C_{KL} can be expressed in the terms of the tensors $\mathrm{E}_{KL}\cong\tilde{\mathrm{E}}_{KL}$ the expression (29) imposes a constraint on the coefficients in the equation (64). Accordingly, the following expression can be recorded.

$$\begin{aligned}0=&\,\Omega_1E_R+\Omega_2\tilde{\mathrm{E}}_{KK}E_R+\Omega_3H_{KK}E_R+\Omega_6B_KH_{KL}B_LE_R+\Omega_8E_KB_KB_R+\Omega_{10}E_K\tilde{\mathrm{E}}_{KL}B_LB_R+\\&\Omega_{12}E_KH_{KL}B_LB_R+\Omega_{21}\tilde{\mathrm{E}}_{PR}E_P+\Omega_{23}E_KB_K\tilde{\mathrm{E}}_{PR}B_P+\Omega_{27}H_{PR}E_P+\Omega_{29}E_KB_KH_{PR}B_P\end{aligned} \tag{65}$$

Because, according to the expression (65), the above-mentioned are arbitrary, the necessary and sufficient condition for validity of this equation is the following coefficients being zero.

$$\Omega_1 = \Omega_2 = \Omega_3 = \Omega_6 = \Omega_8 = \Omega_{10} = \Omega_{12} = \Omega_{21} = \Omega_{23} = \Omega_{27} = \Omega_{29} = 0 \quad (66)$$

In this situation, expression providing the material form of the heat flux vector is obtained as follows.

$$\begin{aligned} Q_R =& \Omega_{14} G_R + \Omega_{15} \tilde{E}_{KK} G_R + \Omega_{16} H_{KK} G_R + \Omega_{17} E_K E_K G_R + \Omega_{18} E_K \tilde{E}_{KL} E_L G_R + \Omega_{19} E_K H_{KL} E_L G_R + \\ & \Omega_{20} B_K H_{KL} B_L G_R + \Omega_4 E_K G_K E_R + \Omega_5 E_K \tilde{E}_{KL} G_L E_R + \Omega_7 E_K H_{KL} G_L E_R + \Omega_9 B_K G_K B_R + \\ & \Omega_{11} B_K \tilde{E}_{KL} G_L B_R + \Omega_{13} B_K H_{KL} G_L B_R + \Omega_{22} E_K G_K \tilde{E}_{PR} E_P + \Omega_{24} B_K G_K \tilde{E}_{PR} B_P + \Omega_{25} \tilde{E}_{PR} G_P + \\ & \Omega_{26} E_K E_K \tilde{E}_{PR} G_P + \Omega_{28} E_K G_K H_{PR} E_P + \Omega_{30} B_K G_K H_{PR} B_P + \Omega_{31} H_{PR} G_P + \Omega_{32} E_K E_K H_{PR} G_P \end{aligned} \quad (67)$$

Expression (67) is constitutive equation of the heat flux vector on material coordinates in terms of its components in a composite material that consist of an isotropic matrix reinforced by one arbitrary and inextensible fiber family, the medium is assumed to be linear, dielectric, isotropic, incompressible, has micro-cracks and dependent on temperature gradient, the mechanical interactions and the temperature changes–linear and the electrical interactions–non-linear. As understood from this equation, interactions of the temperature gradient on its own and of the deformation field, the damage tensor, fiber distribution field and square of electric field separately and collectively contribute to the formation of the heat flux vector.

8. Conclusions

In the present paper we present a continuum damage model for the linear electro-thermo-elastic behavior of the temperature dependent dielectric composite materials, which have micro-cracks and consisting of an isotropic matrix reinforced by inextensible single fiber family. The developed model is based on continuum damage mechanics (CDM), continuum electrodynamics and equations belonging to kinematic and deformation geometry of fiber. Damage is incorporated by means of two second-rank, symmetric tensors that represented the total areas of open (active) and closed (passive) micro-voids contained within a representative volume element (RVE). It is further assumed that an element from single continuous fiber family is placed on each point of the composite material. The matrix material has been assumed to be an isotropic medium, however, due to the distribution of fiber and the existence of micro-cracks, it has gained the property of a directed object, thus gaining the appearance of an anisotropic structure. The composite material is assumed showing linear thermoelastic behavior. Both incompressibility of the medium and inextensibility of the fiber is broadly recognized in terms of formulation. Thus, fiber family is assumed to be inextensible and composite medium is assumed to be incompressible. In this context, the composite expresses itself behaviorally in terms of the symmetric stress, the electrical polarization, the heat flux and the strain-energy density release rate. Since the matrix has been assumed to be isotropic, findings of the theory of invariants have been suitably used to concretely determine arguments of both the stress potential and the heat flux vector functions. To obtain a more concrete expression of non-linear constitutive equations of the symmetric stress, the polarization field and the strain-energy density release rate given by expressions (44), (45) and (46), derivatives of Σ must be known according to the arguments it depends on. Thus, stress potential Σ has been represented by a second degree polynomial and its derivatives according to its invariants have been calculated. During these operations mechanical interactions and effect of damage have been assumed to be linear while electrical interactions have been assumed to be nonlinear. Furthermore, since the matrix material has to remain insensitive to directional changes along fiber, even-numbered exterior products of vector field representing fiber distribution have been considered. The linear constitutive equations of the symmetric stress, the polarization field and the strain- energy density release rate have been found by expressions (49), (50) and (51). From these equations, it can be seen that the deformation field, the damage tensor, electric field, distribution of fiber and interactions of them both contribute to the creation of the symmetric stress, the polarization field and the strain-energy density release rate. Using the symmetric stress and polarization field, the asymmetric stress has been determined by the expression (52) in material coordinates.

Since the heat flux is a vector-valued isotropic function, the equation (61) has obtained by using the invariants of the arguments depends on of heat flux. Coefficients in this equation have been expressed in terms of the invariants on which they depend and each term has been calculated. When these calculations are made, the mechanical interaction, temperature change and the effect of damage have been assumed to be linear while electrical interactions have been assumed to be nonlinear and even number vector components of fiber vectors have been included in the operations, since the composite remains indifferent to change of direction along the fiber. The linear constitutive equation of the heat flux vector has been expressed by the equation (67). From this equation, it is observed that the temperature change contributes to the creation of the heat flux vector alone, with the deformation field, with the damage tensor, with the fiber distribution field, with square terms of the electric field and with interaction among them.

This paper is concerned with developing the continuum damage mechanics model for the linear electro-thermoelastic behavior of composites having micro-cracks consisting of an isotropic matrix reinforced by inextensible single family of arbitrarily fiber. The symmetric stress, the polarization field and strain-energy density release rate are expressed in terms of the thermodynamic stress potential, a function of the left Cauchy-Green tensor, the damage tensor, the electric field vector, the fiber distribution and the temperature. The heat flux vector depends on these quantities and the temperature gradient. The material symmetry group is assumed to be the full isotropy group. Standard methods in invariant theory are used to construct representations for the constitutive equations for symmetric stress, polarization field, strain energy density release rate and heat flux vector. The symmetric stress, polarization field, strain-energy density release rate and heat flux vector are treated in separate sections. The appropriate invariants used as arguments for thermodynamic stress potential function and heat flux vector function are introduced and the symmetric stress, polarization field, strain-energy density release rate and heat flux vector constitutive equations are worked out. This is followed by specializations for incompressibility, inextensibility of the fiber, the special case when thermodynamic stress potential is a quadratic polynomial in the invariants and a linearization based on small strains.

After this paper, practical problems will be solved by forming $\mathbf{B}(\mathbf{X})$ vector field for various fiber distribution whose parametric equations in the material medium are in the form of $\mathbf{X} = \mathbf{X}(s)$ and necessary interpretations will be made in a more concrete way. Also, in a future work we will study the development of numerical methods for this model. In this work, developing a theoretical investigation for formulation constitutive modeling based on continuum damage mechanics has been purposed for the thermoelastic behavior of a dielectric composite material reinforced by single inextensible fiber family.

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